Heterotic Line Bundle Models

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Pascos 2013, Taipei, Nov 2013

based on: arXiv:1311.1941, arXiv:1307.4787, arXiv:1106.4804, arXiv:1202.1757,
with Lara Anderson, Evgeny Buchbinder, Andrei Constantin, James Gray and Eran Palti

Overview

- Introduction: Heterotic line bundle models
- Arena: Specific Calabi-Yau manifolds and line bundles
- An exhaustive scan over favourable Cicys
- An example
- Continuation to non-Abelian bundles
- Conclusion and outlook

Introduction: Heterotic line bundle models

Data to define a heterotic line bundle model we need:

- A Calabi-Yau 3-fold X
- A line bundle sum $V = L_1 \oplus \cdots \oplus L_5$ on X,
  $c_1(V) = 0$, so structure group is
  $S(U(1)^5) \subset SU(5) \subset E_8$
- vanishing slopes $\mu(L_a) \equiv c_1(L_a) \wedge J^2 \stackrel{!}{=} 0$
- Anomaly: $c_2(TX) - c_2(V) - c_2(\tilde{V}) = [C]$
  in practice: $c_2(V) \leq c_2(TX)$

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standard-like model (hopefully) with gauge group $G_{\rm SM} \times S(U(1)^5)$

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$\langle S^\alpha \rangle = 0$: line bundle model, $\langle S^\alpha \rangle \neq 0$: non-Abelian bundle

U(1) symmetries constrain 4d theory, e.g. superpotential:

$$W = \mu(S) H \bar{H} + Y^{(d)}_{pq}(S) H \overline{\mathbf{5}}^p \mathbf{10}^q + Y^{(u)}_{pq}(S) \mathbf{10}^p \mathbf{10}^q + \cdots$$

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Two ways to explore non-Abelian bundles:

- VEVs $\langle S^\alpha \rangle \neq 0$, spontaneously breaks U(1)s
- Construct non-Abelian bundles which ``split'' to line bundle sum

Arena: complete intersection CY manifolds (CICYs)

CICYs defined as common zero locus $X = \{p_i = 0\} \subset \mathcal{A}$ of homogeneous polynomials p_i in ambient space $\mathcal{A} = \bigotimes_{r=1}^{m} \mathbb{P}^{n_r}$.

for example: quintic $X \sim [\mathbb{P}^4|5]$ or bi-cubic $X \sim \left[\begin{array}{c|c} \mathbb{P}^2 & 3 \\ \mathbb{P}^2 & 3 \end{array} \right]$

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Classification of freely-acting discrete symmetries

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Line bundles on CY manifolds

Line bundles, L, are classified by their first Chern class:

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No a priori bounds on k_a^i , so for $-k_{\rm max} \leq k_a^i \leq k_{\rm max}$ we have

$\sim (2k_{\rm max}+1)^{4h^{1,1}(X)}$ line bundle sums V

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*standard model: SM gauge group times (anomalous) U(1)s, exact MSSM matter spectrum, one or more pairs of Higgs doublets, no exotics charged under standard model group.

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Feasible because some constraints can be checked while line bundle sum is built up, e.g if

$$h^1(X, L) > 3|\Gamma|$$

we do not need to consider line bundle L (too many families).

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Scan over all (k_a^i) with $|k_a^i| \leq k_{\max}$ and find number of viable models as a function of $k_{\max}$:

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Scan over all (k^i_a) with $|k^i_a| \leq k_{\max}$ and find number of viable models as a function of $k_{\max}$:

Table 6: *Number of models as a function of k_{max} on CICYs with $h^{1,1}(X) = 6$. Total number of models: 41036*

X, $\|\Gamma\|$	$k_{\rm m} = 1$	$k_{\rm m} = 2$	$k_{\rm m} = 3$	$k_{\rm m} = 4$	$k_{\rm m} = 5$	$k_{\rm m} = 6$	$k_{\rm m} = 7$	$k_{\rm m} = 8$	$k_{\rm m} = 9$	$k_{\rm m} = 10$, 11, 12, 13
3413, 3	0	2278	2897	2906	2906	2906				
4190, 2	11	766	1175	1243	1246	1247	1249	1249	1249	
5273, 2	29	4895	7149	7738	7799	7810	7810	7810		
5302, 2	0	4314	5978	6360	6369	6369	6369			
5302, 4	0	11705	16988	17687	17793	17838	17868	17868	17868	
5425, 2	0	2381	3083	3305	3337	3337	3337			
5958, 2	0	148	224	240	253	253	253			
6655, 5	0	92	178	189	194	194	198	201	202	203
6738, 2	1	2733	4116	4346	4386	4393	4399	4399	4399	

Number of consistent SU(5) GUT models with correct indices:

$h^{1,1}(X)$	1	2	3	4	5	6	total
#models	0	0	6	552	21731	41036	63325

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Have started a similar programme on CY manifolds defined in toric 4-folds (Kreuzer-Skarke list) -> Chuang Sun's talk

An example

CY data:

- **Cicy 7862, Symmetry 3**

$$X = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$$

$\eta(X) = -128 \quad h^{1,1}(X) = 4 \quad h^{2,1}(X) = 68 \quad c_2(TX) = \{24, 24, 24, 24\}$

$\kappa = 12\, t_1\, t_2\, t_3 + 12\, t_1\, t_2\, t_4 + 12\, t_1\, t_3\, t_4 + 12\, t_2\, t_3\, t_4$

symmetry: 3 order: 4

Abelian: True block diagonal: True factors: {2, 2}

Action on coordinates: $\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \right\}$

Action on polynomials: $\{(\ 1\ ), (\ 1\ )\}$

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$\mathbb{Z}_2 \times \mathbb{Z}_2$ generators

bundle data:

Basic properties

standard model? True   massless U(1): 1   number of 5 $\overline{5}$ pairs: 3 $c_2(V) = \{24, 8, 20, 12\}$

$$V:\ (k^i{}_a) = \begin{pmatrix} -1 & -1 & 0 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -2 \end{pmatrix}$$

Cohomology of V:

L_2	=	{−1, −3, 2, 2}	$h[L_2]$	=	{0, 8, 0, 0}	$h[L_2,R]$	=	{{0, 0, 0, 0}, {2, 2, 2, 2}, {0, 0, 0, 0}, {0, 0, 0, 0}}
L_5	=	{1, 1, 0, −2}	$h[L_5]$	=	{0, 4, 0, 0}	$h[L_5,R]$	=	{{0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_2\times L_4$	=	{0, −2, 1, 1}	$h[L_2\times L_4]$	=	{0, 4, 0, 0}	$h[L_2\times L_4,R]$	=	{{0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}}
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$L_4\times L_5$	=	{2, 2, −1, −3}	$h[L_4\times L_5]$	=	{0, 8, 0, 0}	$h[L_4\times L_5,R]$	=	{{0, 0, 0, 0}, {2, 2, 2, 2}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_1\times L_2^*$	=	{0, 3, −2, −1}	$h[L_1\times L_2^*]$	=	{0, 0, 12, 0}	$h[L_1\times L_2^*,R]$	=	{{0, 0, 0, 0}, {0, 0, 0, 0}, {3, 3, 3, 3}, {0, 0, 0, 0}}
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$L_2\times L_3^*$	=	{−1, −4, 3, 2}	$h[L_2\times L_3^*]$	=	{0, 20, 0, 0}	$h[L_2\times L_3^*,R]$	=	{{0, 0, 0, 0}, {5, 5, 5, 5}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_2\times L_4^*$	=	{−2, −4, 3, 3}	$h[L_2\times L_4^*]$	=	{0, 12, 0, 0}	$h[L_2\times L_4^*,R]$	=	{{0, 0, 0, 0}, {3, 3, 3, 3}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_3\times L_5^*$	=	{−1, 0, −1, 2}	$h[L_3\times L_5^*]$	=	{0, 0, 4, 0}	$h[L_3\times L_5^*,R]$	=	{{0, 0, 0, 0}, {0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}}

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Transfer format: {{6, 1, 1, 4, 6, 5, 9, 9, 8, 10, 1, 7, 17}, {6, 6, −1, −1, −1, −1}}

$\mathrm{rk}(Y^{(u)}) = \{2, 2\}$ $\mathrm{rk}(Y^{(d)})) = \{0, 0\}$ dim. 4 operators absent: {True, True}   dim. 5 operators absent: {True, True}

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integer matrix defining line bundle sum

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$L_2\times L_4$	=	{0, −2, 1, 1}	$h[L_2\times L_4]$	=	{0, 4, 0, 0}	$h[L_2\times L_4,R]$	=	{{0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}}
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$L_2 \times L_4$	= {0, −2, 1, 1}	$h[L_2 \times L_4]$	=	{0, 4, 0, 0}	$h[L_2 \times L_4,R]$	=	{{0, 0, 0, 0}, {1, 1, 1, 1}, {0, 0, 0, 0}, {0, 0, 0, 0}}
$L_2 \times L_5$	= {0, −2, 2, 0}	$h[L_2 \times L_5]$	=	{0, 3, 3, 0}	$h[L_2 \times L_5,R]$	=	{{0, 0, 0, 0}, {0, 1, 1, 1}, {0, 1, 1, 1}, {0, 0, 0, 0}}
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$\mathrm{rk}(Y^{(u)})$ = {2, 2} $\mathrm{rk}(Y^{(d)})$) = {0, 0} dim. 4 operators absent: {True, True} dim. 5 operators absent: {True, True}

spectrum: $\mathbf{10}_2,\ \mathbf{10}_2,\ \mathbf{10}_5,\ \overline{\mathbf{5}}_{2,4},\ \overline{\mathbf{5}}_{4,5},\ \overline{\mathbf{5}}_{4,5},\ H_{2,5},\ \bar{H}_{2,5}$

$3\,\mathbf{1}_{2,1},\ 3\,\mathbf{1}_{5,1},\ 5\mathbf{1}_{2,3},\ 3\,\mathbf{1}_{2,4},\ \mathbf{1}_{5,3}$

allowed operators:

■ Operators

basic superpotential terms:

$\overline{H}10^p 10^q$: $Y^{(u)} = \begin{pmatrix} (0) & (0) & (1) \\ (0) & (0) & (1) \\ (1) & (1) & (0) \end{pmatrix}$

$H\overline{5}^p 10^q$: $Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$

$H\overline{H}$: $\mu = \{1\}$

$W_{sing} = \{0\}$

R−parity violating terms in superpotential:

$\overline{H}L^p$: $\rho = \begin{pmatrix} 0 \\ S_{2,4} \\ S_{2,4} \end{pmatrix}$

$10^p\overline{5}^q\overline{5}^r$: λ = {{{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}}

Dimension 5 operators in superpotential:

$\overline{5}^p 10^q 10^r 10^s$: λ' = {{{{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}},
{{{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}, {{{0}, {0}, {0}}, {{0}, {0}, {0}}, {{0}, {0}, {0}}}}, {{{{0}, {0}, {0}}, {{0},

D−terms:

FI−terms: $k^i{}_a \kappa_i = \begin{pmatrix} 4 t_1 t_2 + 4 t_1 t_3 - 4 t_2 t_4 - 4 t_3 t_4 \\ 16 t_1 t_2 - 4 t_1 t_3 + 4 t_2 t_3 - 4 t_1 t_4 + 4 t_2 t_4 - 16 t_3 t_4 \\ -4 t_1 t_2 + 4 t_1 t_3 - 4 t_2 t_4 + 4 t_3 t_4 \\ -8 t_1 t_2 + 8 t_3 t_4 \\ -8 t_1 t_2 - 4 t_1 t_3 - 4 t_2 t_3 + 4 t_1 t_4 + 4 t_2 t_4 + 8 t_3 t_4 \end{pmatrix}$

singlet D−terms: $q_{\alpha a} S^\alpha \overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1} S^\dagger{}_{2,1} - S_{5,1} S^\dagger{}_{5,1} \\ S_{2,1} S^\dagger{}_{2,1} + S_{2,3} S^\dagger{}_{2,3} + S_{2,4} S^\dagger{}_{2,4} \\ -S_{2,3} S^\dagger{}_{2,3} - S_{5,3} S^\dagger{}_{5,3} \\ -S_{2,4} S^\dagger{}_{2,4} \\ S_{5.1} S^\dagger{}_{5.1} + S_{5.3} S^\dagger{}_{5.3} \end{pmatrix}$

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- **Operators**

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$$\overline{H}10^p10^q:\ Y^{(u)} = \begin{pmatrix} (0) & (0) & (1) \\ (0) & (0) & (1) \\ (1) & (1) & (0) \end{pmatrix}$$

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Dimension 5 operators in superpotential:

$\overline{5}^p10^q10^r10^s:\ \lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$
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$$\text{singlet D−terms: } q_{\alpha a}S^\alpha\overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1}S^\dagger{}_{2,1} - S_{5,1}S^\dagger{}_{5,1} \\ S_{2,1}S^\dagger{}_{2,1} + S_{2,3}S^\dagger{}_{2,3} + S_{2,4}S^\dagger{}_{2,4} \\ -S_{2,3}S^\dagger{}_{2,3} - S_{5,3}S^\dagger{}_{5,3} \\ -S_{2,4}S^\dagger{}_{2,4} \\ S_{5,1}S^\dagger{}_{5,1} + S_{5,3}S^\dagger{}_{5,3} \end{pmatrix}$$

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Dimension 5 operators in superpotential:

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Dimension 5 operators in superpotential:

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Continuation to non-Abelian bundles

Superpot. for example: $W = \lambda_i \overline{H}_{2,5}(Q_2^{(i)} u_5 + Q_5 u_2^{(i)}) + \rho_{\alpha i} \mathbf{1}_{2,4}^{(\alpha)} L_{4,5}^{(i)} \overline{H}_{2,5}$

Singlets for example: $3\,\mathbf{1}_{2,1}$, $3\,\mathbf{1}_{5,1}$, $5\mathbf{1}_{2,3}$, $3\,\mathbf{1}_{2,4}$, $\mathbf{1}_{5,3}$

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Suggests: massless Higgs doublet pair throughout moduli space as long as $\langle \mathbf{1}_{2,4}^{\alpha} \rangle = 0$.

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What happens to bundle when singlets are switched on?

- All singlet VEVs non-zero: $V = \bigoplus_{a=1}^{5} L_a \to \tilde{V}$

$$S(U(1)^5) \to SU(5)$$

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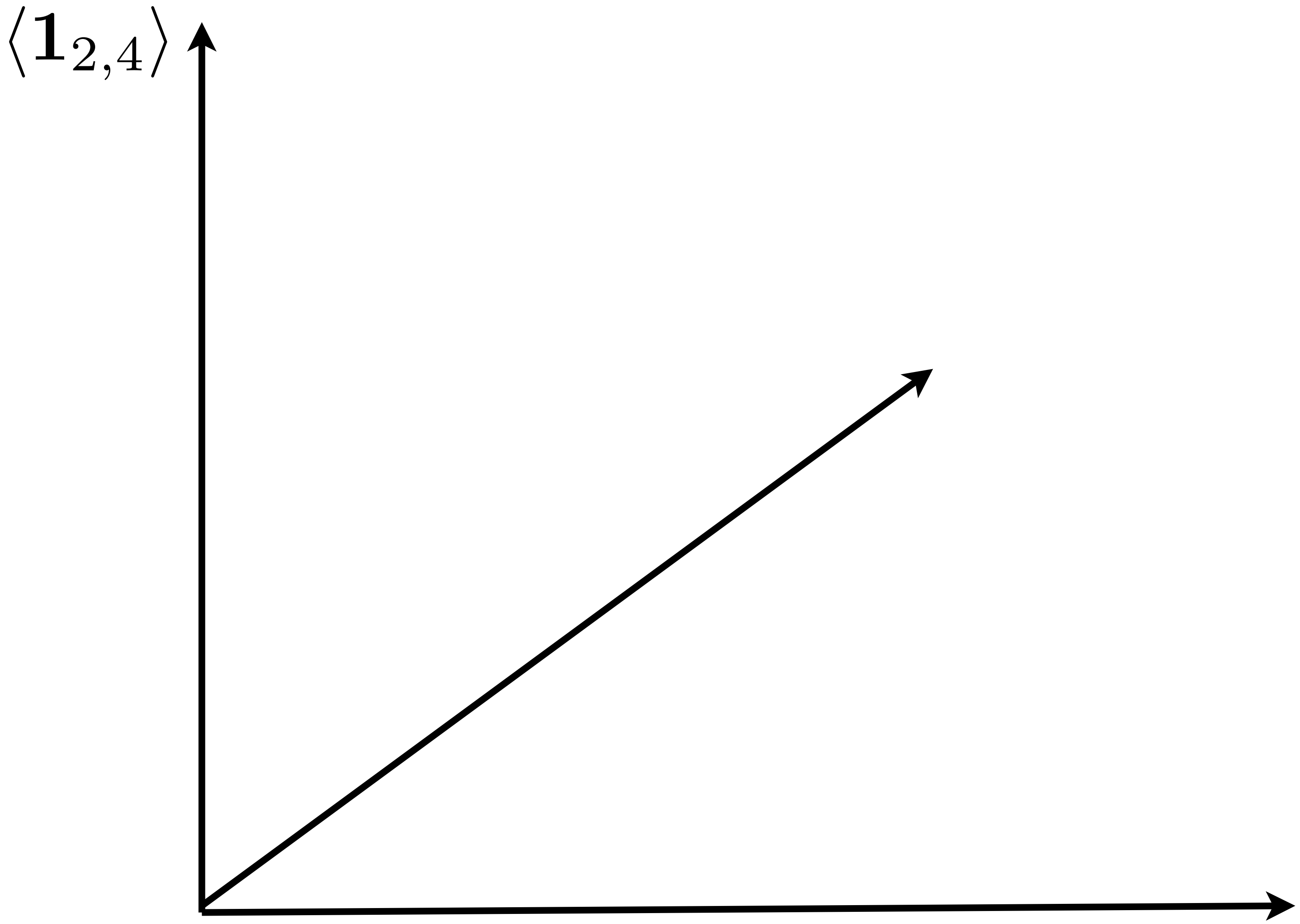
Schematic structure of moduli space:



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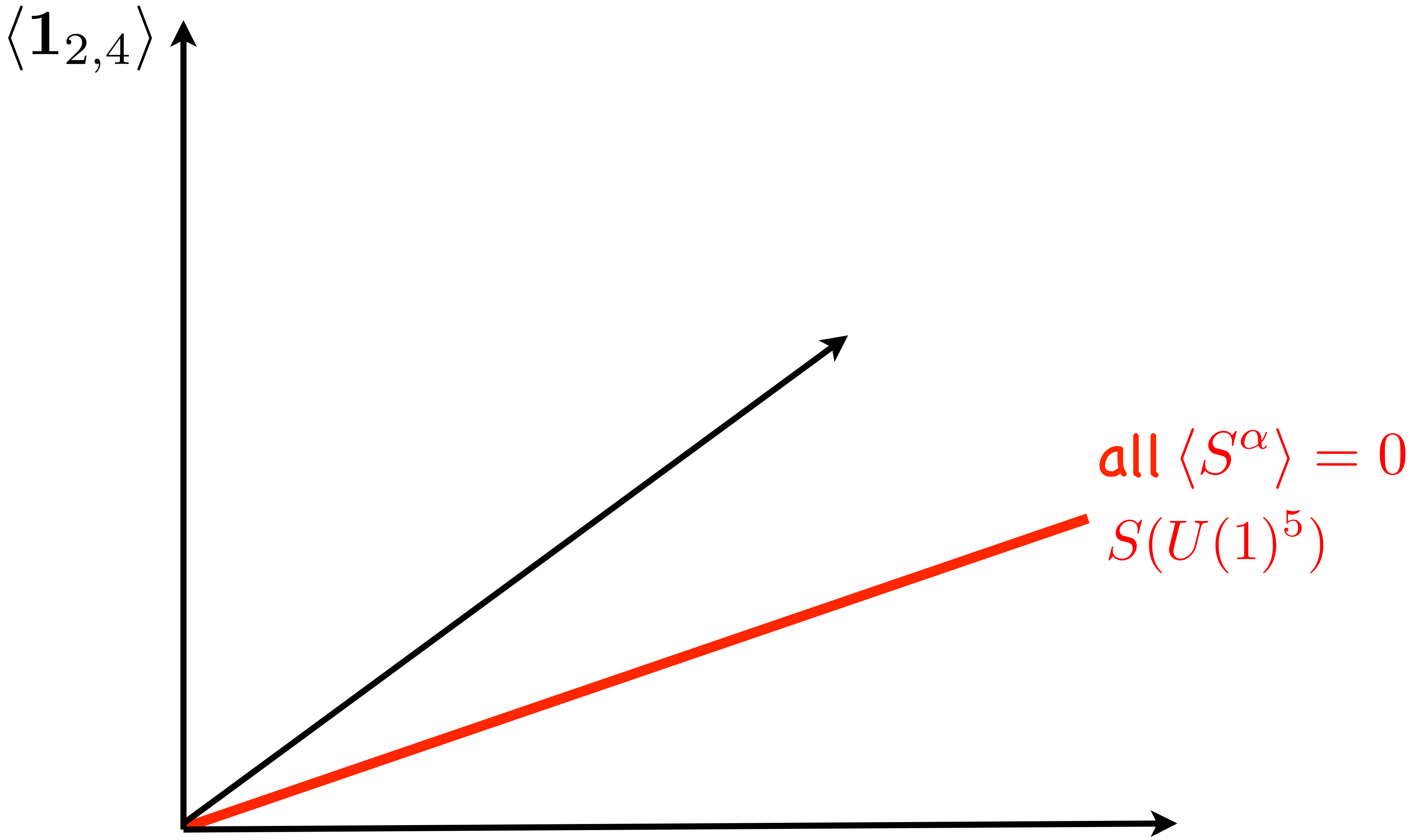

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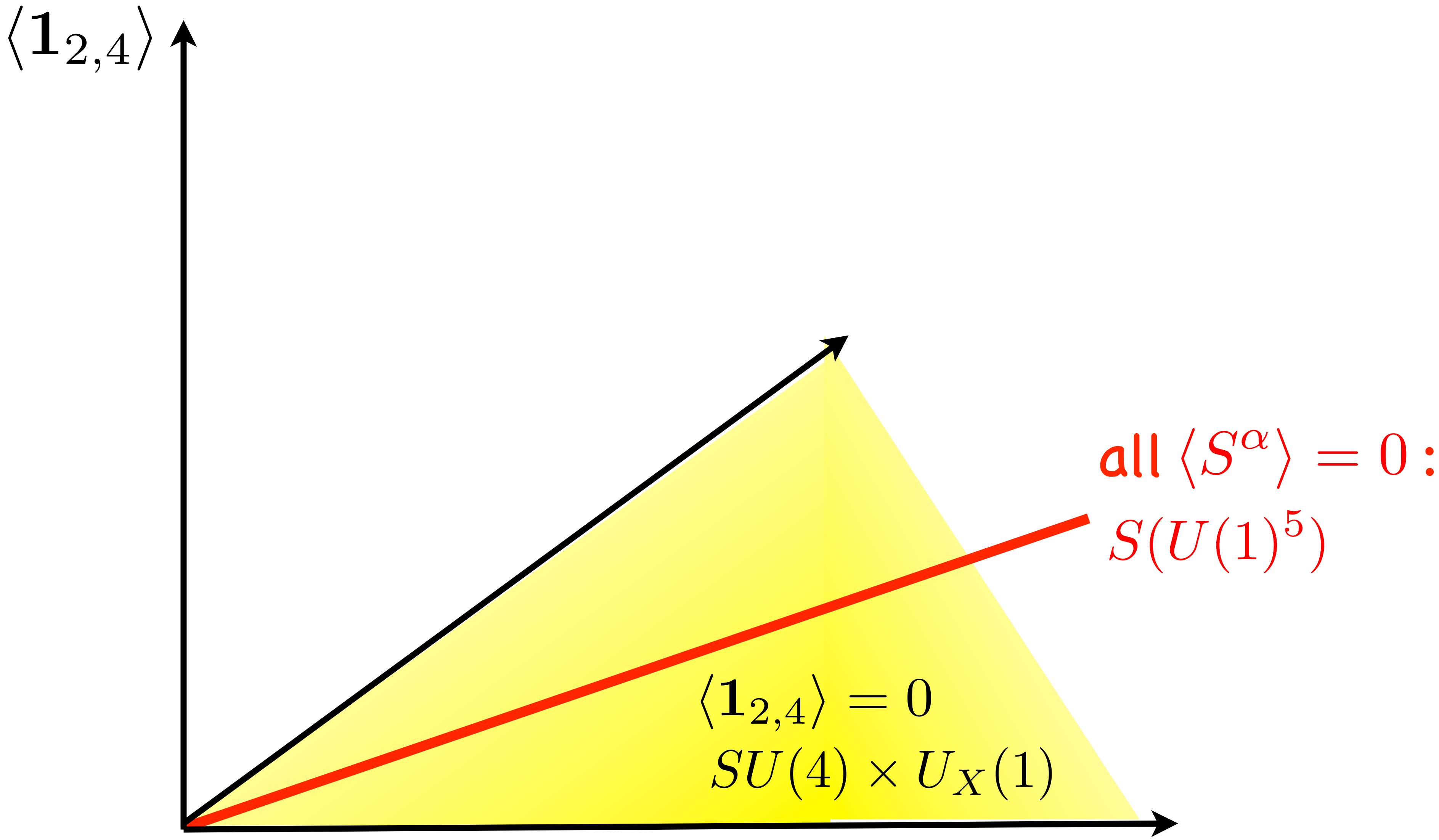

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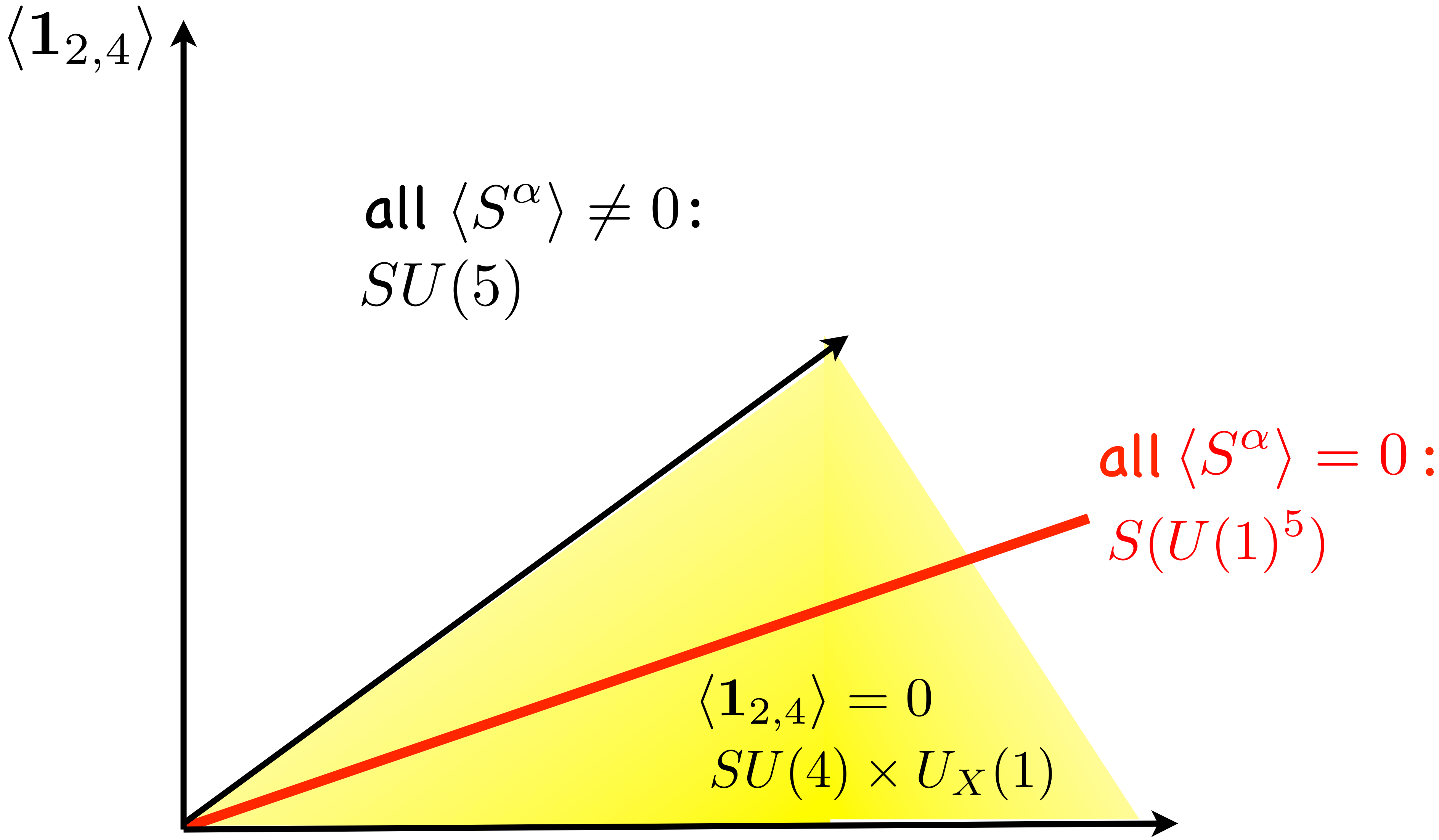
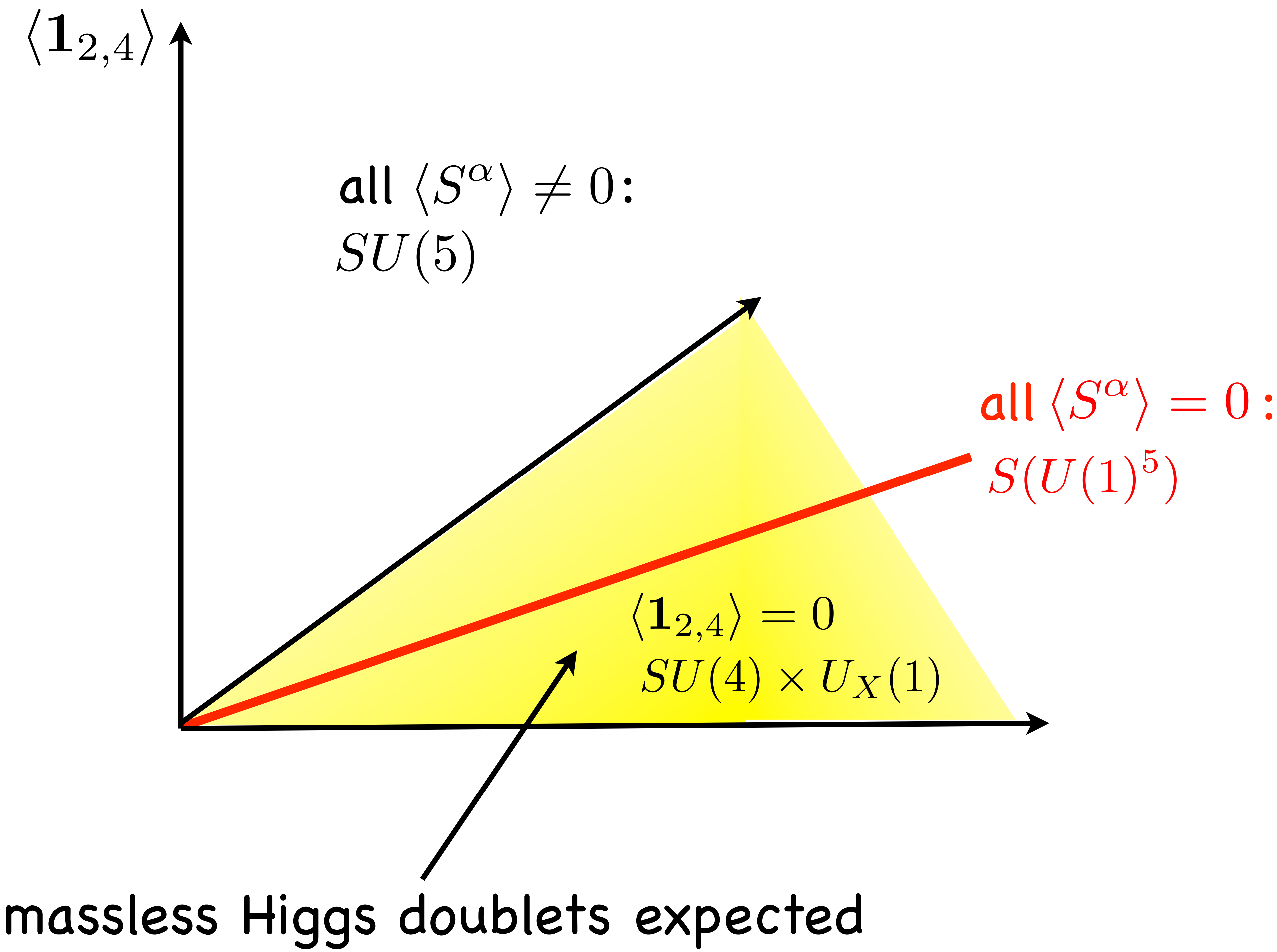


Schematic structure of moduli space:



Check fate of Higgs by constructing non-Abelian bundle

Recall: $L_1 = \mathcal{O}_X(-1,0,0,1) \quad , \quad L_2 = \mathcal{O}_X(-1,-3,2,2) \quad , \quad L_3 = \mathcal{O}_X(0,1,-1,0)$

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1) Extension bundles

For $V_1 = L_2 \oplus L_5,\ V_2 = L_1 \oplus L_3 \oplus L_4$ define extension

$$0 \longrightarrow V_1 \longrightarrow \tilde{V} \longrightarrow V_2 \longrightarrow 0$$

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Compute $\#\mathbf{5} = h^2(X, \wedge^2 \tilde{V}) = \begin{cases} 3 & \text{for} \quad \langle \mathbf{1}_{2,4} \rangle = 0 \\ 0 & \text{for} \quad \langle \mathbf{1}_{2,4} \rangle \neq 0 \end{cases}$

2) Monads

$$0 \longrightarrow \widetilde{V} \longrightarrow B \xrightarrow{f} C \longrightarrow 0$$

$$B \sim \begin{bmatrix} -1 & -1 & -1 & 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & -1 & -1 & 0 & 0 \\ 1 & 2 & 2 & 0 & -1 & 0 & 0 \end{bmatrix} \qquad C \sim \begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 0 \\ 2 & 2 \end{bmatrix}$$

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$$\begin{array}{ccccc} L_1 & L_2 & L_3 & L_4 & L_5 \end{array}$$

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$$f \sim \begin{pmatrix} f_{(0,1,2,1)} & f_{(0,2,0,0)} & f'_{(0,2,0,0)} & 0 & 0 & 0 & 0 \\ f_{(2,1,0,1)} & 0 & 0 & f_{(1,0,1,2)} & f_{(0,0,1,3)} & f_{(0,0,0,2)} & f'_{(0,0,0,2)} \end{pmatrix}$$

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$L_1 \quad L_2 \quad L_3 \; L_4 \quad L_5$

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bundle splits if zero

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We can show for $\langle \mathbf{1}_{2,4} \rangle = 0$, $\tilde{V} = U \oplus L_4$:

- bundle $\tilde{V}$ is supersymmetric
- $\#\mathbf{5} = h^2(X, \wedge^2 \tilde{V}) = 3$

Features of the $SU(4) \times U_X(1)$ model:

- $U_X(1) \longrightarrow U_{B-L}(1)$

- μ-term forbidden

- dangerous dim. 4 terms forbidden by $U_{B-L}(1)$

- $\overline{\mathbf{5}}\,\mathbf{10}\,\mathbf{10}\,\mathbf{10}$ operators still absent, due to existence of line bundle locus

Conclusions and outlook

- We can "mass-produce" heterotic CY standard models from line bundles. 2000 models have been found from 200 GUT models.
- We have found all viable line bundle GUT models on favourable Cicys with freely-acting symmetries: 35000 models
- These models will lead to a large number of standard models which will form the starting point for a detailed phenom. analysis.
- Higgs can be kept light away from the line bundle locus.
- We can study the non-Abelian continuation of line bundle models: ``Unexpected" absences of operators due to line bundle locus.
- What is the total number of string standard models?

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