

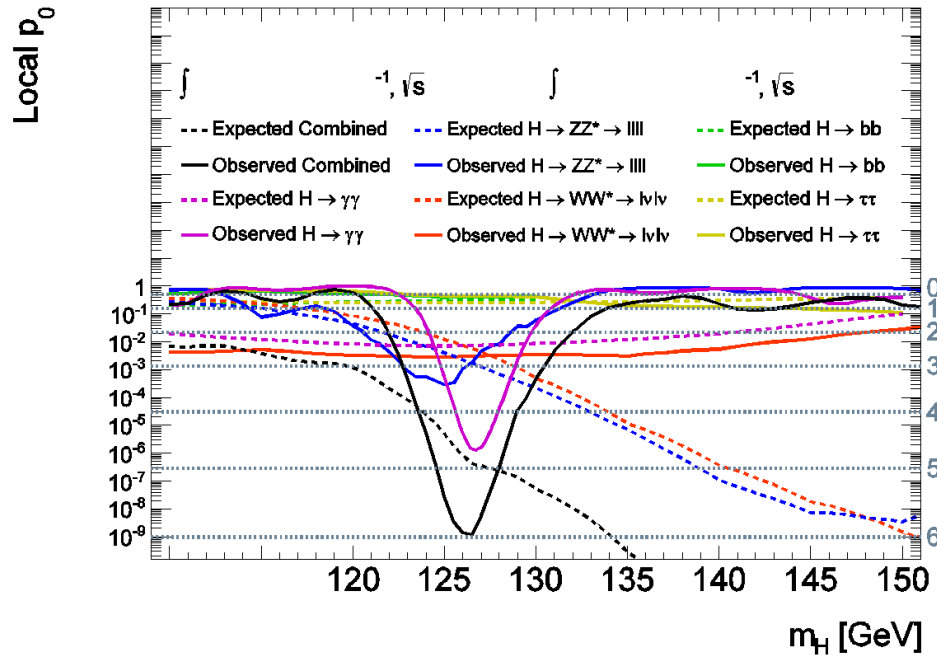
The Higgs boson mass in a natural MSSM with nonuniversal gaugino masses

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with
H. Abe and J. Kawamura

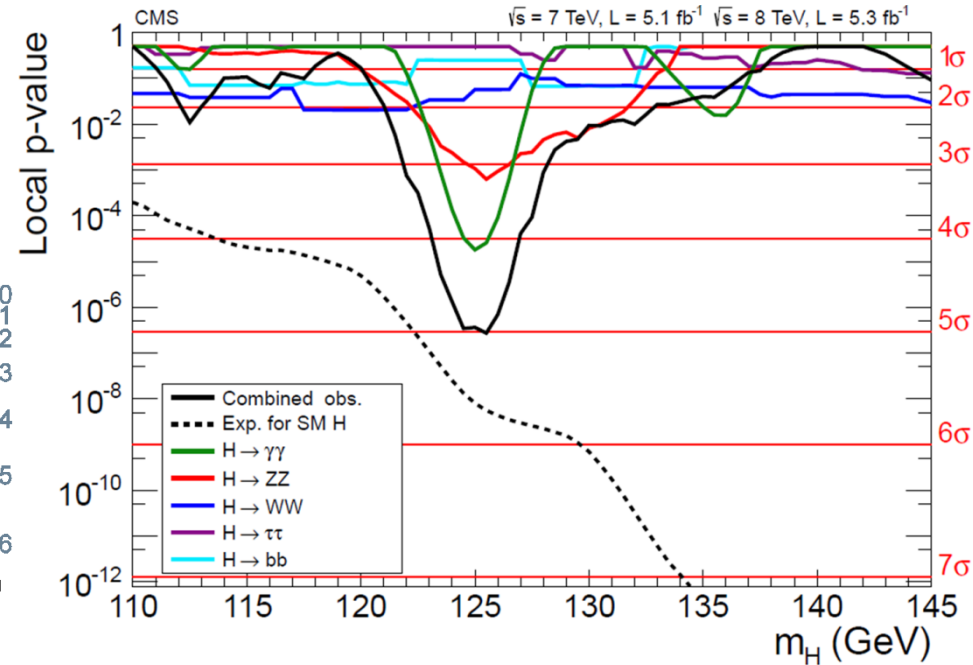
● A Standard Model-like Higgs particle has been discovered.

ATLAS



$$m_{Higgs} = 126.0 \pm 0.4 \pm 0.4 [\text{GeV}] (\text{ATLAS})$$

CMS



$$m_{Higgs} = 125.3 \pm 0.4 \pm 0.5 [\text{GeV}] (\text{CMS})$$

Fine-tuning problem in the Standard Model

$$m_{Higgs}^2 = m_{bare}^2 - \Lambda_{CUT}^2 + \dots$$

If the cut-off scale is of order GUT scale,

$$(125 \text{ [GeV]})^2 \simeq (10^{16} \text{ [GeV]})^2 - (10^{16} \text{ [GeV]})^2$$

In the MSSM,

$$m_{Higgs}^2 = m_{bare}^2 + \underbrace{(\Lambda_{CUT}^2 - \Lambda_{CUT}^2)}_0 + m_{soft}^2 \log \left(\frac{\Lambda_{CUT}}{m_{soft}} \right) \dots$$

One of motivations for SUSY ; Solution of the fine-tuning problem.

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SUSY little hierarchy problem in the MSSM

(Fine-tuning problem for EWSB)

$$\frac{1}{2}m_Z^2 = -|\mu|^2 - m_{H_u}^2 + \dots \quad (\tan\beta \gg 1)$$

m_Z ··· Z boson mass μ ··· higgsino mass

m_{H_u} ··· SUSY breaking soft term of H_u

$$\tan\beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}$$

We need $\mu \sim m_{H_u} \sim O(m_Z)$ to avoid the tuning problem.

**We examined the possibility to
realize ~ 125 GeV Higgs
and
avoid the SUSY little hierarchy problem.**

In the MSSM, lightest CP-even Higgs boson mass is evaluated as[1]

$m_h^2 \simeq m_Z^2 \cos^2(2\beta)$ at the tree level

$$m_h^2 \simeq m_Z^2 \cos^2(2\beta) \left(1 - \frac{3}{8\pi^2} \frac{\overline{m}_t^2}{v^2} t \right) + \frac{3}{4\pi^2} \frac{\overline{m}_t^4}{v^2} \left[\frac{1}{2} X_t + t + \frac{1}{16\pi^2} \left(\frac{3\overline{m}_t^2}{2v^2} - 32\pi\alpha_3 \right) (X_t t + t^2) \right]$$

There are two possibilities to realize ~ 125 GeV Higgs.

① High-scale SUSY breaking($m_{\tilde{t}} > 10$ [TeV])

$$t \equiv \ln(m_{\tilde{t}}^2/\overline{m}_t^2)$$

$\overline{m}_t \dots$ top mass

$m_{Q_3}^2 \dots$ left handed stop mass

$m_{U_3}^2 \dots$ right handed stop mass

-----> SUSY little hierarchy problem

$$\delta m_{H_u}^2 \sim -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln \frac{\Lambda_{CUT}}{m_{\tilde{t}}}$$

$$\begin{cases} \tilde{A}_t \equiv A_t(m_Z) - \mu(m_Z) \cot \beta \\ m_{\tilde{t}}^2 \equiv \sqrt{m_{U_3}^2(m_Z) m_{Q_3}^2(m_Z)} \end{cases}$$

② Large stop mixing

$$r_a = \frac{|\tilde{A}_t|}{m_{\tilde{t}}} \simeq \sqrt{6}$$



Higgs mass

is maximally enhanced.

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$$\frac{1}{2}m_Z^2 = -|\mu|^2 - m_{H_u}^2 + \dots \quad \delta m_{H_u}^2 \sim -\frac{3y_t^2}{4\pi^2} m_{\tilde{t}}^2 \ln \frac{\Lambda_{CUT}}{m_{\tilde{t}}}$$

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② Large stop mixing

$$r_a = \frac{|\tilde{A}_t|}{m_{\tilde{t}}} \simeq \sqrt{6} \implies X_t(r_a) \text{ is large.} \implies \text{Higgs mass is maximally enhanced.}$$

② Large stop mixing

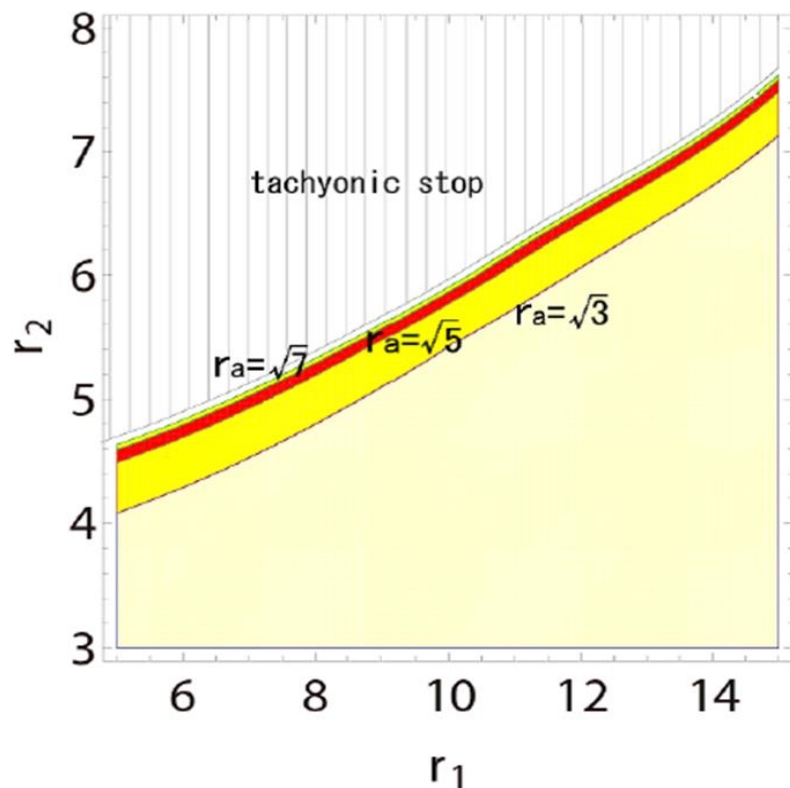
How do we get the large stop mixing ?

$$r_a = \frac{|\tilde{A}_t|}{m_{\tilde{t}}} \simeq \sqrt{6}$$

After running a one-loop RG eq. from the **GUT scale** to the EW scale,

$$\begin{aligned} \tilde{A}_t & A_t(m_Z) \simeq -0.04M_1(m_{GUT}) - 0.21M_2(m_{GUT}) - 1.90M_3(m_{GUT}) + \dots \\ m_{\tilde{t}} & \left[\begin{aligned} m_{Q_3}^2(m_Z) & \simeq -0.02M_1^2(m_{GUT}) + 0.38M_2^2(m_{GUT}) + 5.63M_3^2(m_{GUT}) + \dots \\ m_{U_3}^2(m_Z) & \simeq 0.07M_1^2(m_{GUT}) - 0.21M_2^2(m_{GUT}) + 4.61M_3^2(m_{GUT}) + \dots \end{aligned} \right. \end{aligned}$$

$M_1 \cdot \cdot$ Bino mass
 $M_2 \cdot \cdot$ Wino mass
 $M_3 \cdot \cdot$ Gluino mass



Ratios of the gaugino masses

$$r_1 \equiv \frac{M_1(m_{GUT})}{M_3(m_{GUT})}, \quad r_2 \equiv \frac{M_2(m_{GUT})}{M_3(m_{GUT})}$$

If we take $r_2 \simeq \sqrt{\frac{4.6}{0.2}} \simeq 4.8$

$$\Rightarrow m_{U_3}^2(m_Z) < m_{Q_3}^2(m_Z), A_t^2(m_Z)$$

$$\Rightarrow r_a > 1$$

Stop mixing is enhanced due to the RG effects.



Higgs mass is enhanced.

② Large stop mixing

Solution of the SUSY little hierarchy problem.

$$\frac{1}{2}m_Z^2 = -|\mu|^2 - m_{H_u}^2 + \cdots$$

After running a one-loop RG eq. from the GUT scale to the EW scale,

M_1 · · Bino mass

M_2 · · Wino mass

M_3 · · Gluino mass

$$m_{H_u}^2(m_Z) \simeq 0.17M_2^2(m_{GUT}) - 3.09M_3^2(m_{GUT}) + \cdots$$



$$r_2 \equiv \frac{M_2}{M_3} \simeq \sqrt{\frac{3.09}{0.17}} \simeq 4.3$$

$\Rightarrow \delta m_{H_u}^2(m_Z)$ is suppressed.

SUSY little hierarchy problem is relaxed due to the RG effects.

Gaugino mass ratio is almost same as the large stop mixing.

Higgs mass and tuning

There are the parameter spaces which realize the 125 GeV Higgs and avoid the tuning problem.

Tuning parameter

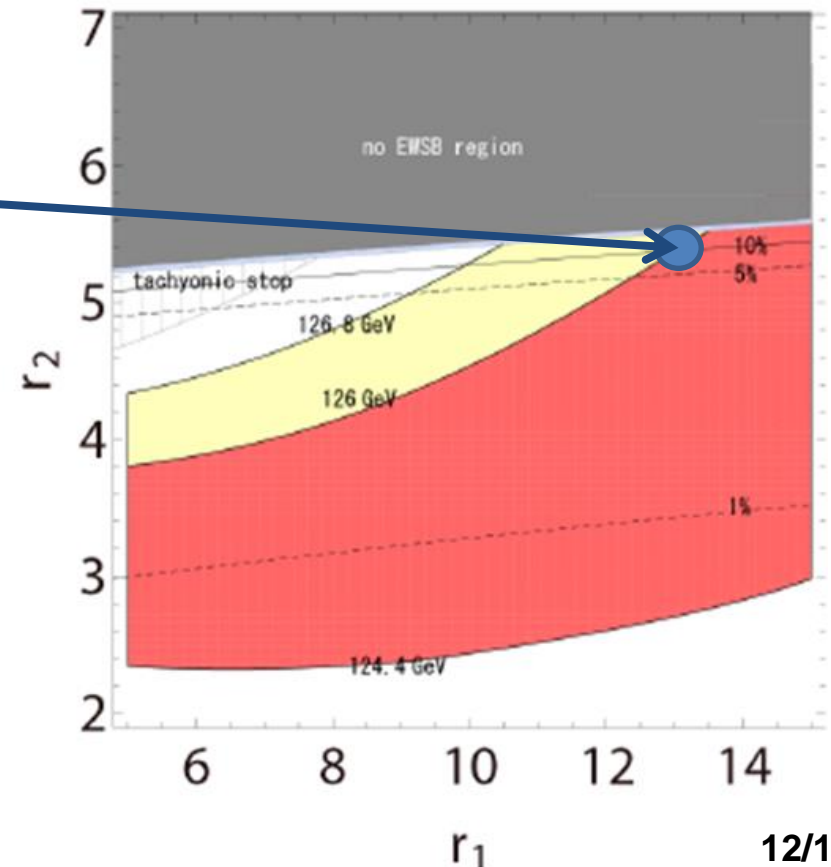
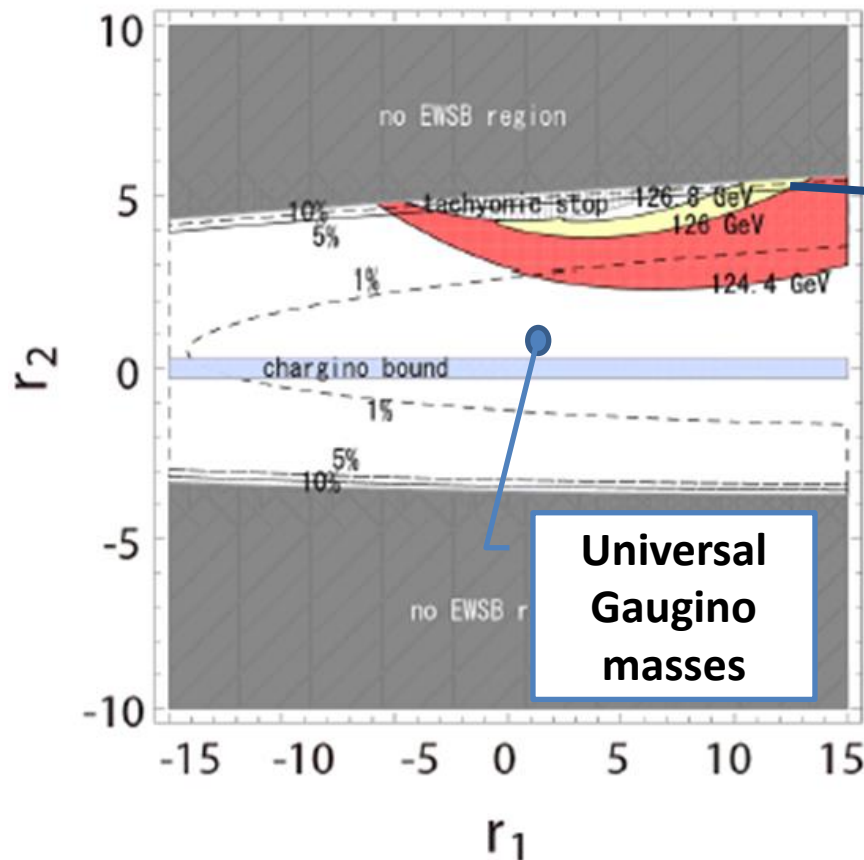
$$\Delta_\mu = \frac{|\mu|}{2m_Z^2} \frac{\partial m_Z^2}{\partial |\mu|}, \quad \frac{100}{|\Delta_\mu|} (\%)$$

Input parameter at the GUT scale

$$M_3 = 385[\text{GeV}], \tan\beta = 15,$$

$$(3\text{rd. generation}, H_u, H_d) = 200[\text{GeV}], A_t = -400[\text{GeV}]$$

$$(1\text{st. and 2 nd. generation}) = 1500[\text{GeV}]$$



Sparticle spectrums

Reference point : $(r_1, r_2) = (13, 5.4)$

m_h [GeV]	m_H [GeV]	m_A [GeV]	m_{H^\pm} [GeV]
126.0	1639	1639	1641
$100 \times \Delta_\mu^{-1} $ (%)	$M_1(m_Z)$ [GeV]	$M_2(m_Z)$ [GeV]	$M_3(m_Z)$ [GeV]
11.48	2063	1709	1104

sparticle	mass [GeV]	sparticle	mass [GeV]
\tilde{u}_1	2209	\tilde{e}_1	2302
\tilde{u}_2	2324	\tilde{e}_2	2456
\tilde{c}_1	2204	$\tilde{\mu}_1$	2301
\tilde{c}_2	2320	$\tilde{\mu}_2$	2439
\tilde{t}_1	752.1	$\tilde{\tau}_1$	1735
\tilde{t}_2	1457	$\tilde{\tau}_2$	1936
\tilde{d}_1	1903	$\tilde{\chi}_1^0$	2064
\tilde{d}_2	2326	$\tilde{\chi}_2^0$	1713
\tilde{s}_1	1874	$\tilde{\chi}_3^0$	187.4
\tilde{s}_2	2322	$\tilde{\chi}_4^0$	192.2
\tilde{b}_1	1071	$\tilde{\chi}_1^\pm$	189.4
\tilde{b}_2	1436	$\tilde{\chi}_2^\pm$	1713

In Summary, LSP is Higgsino.

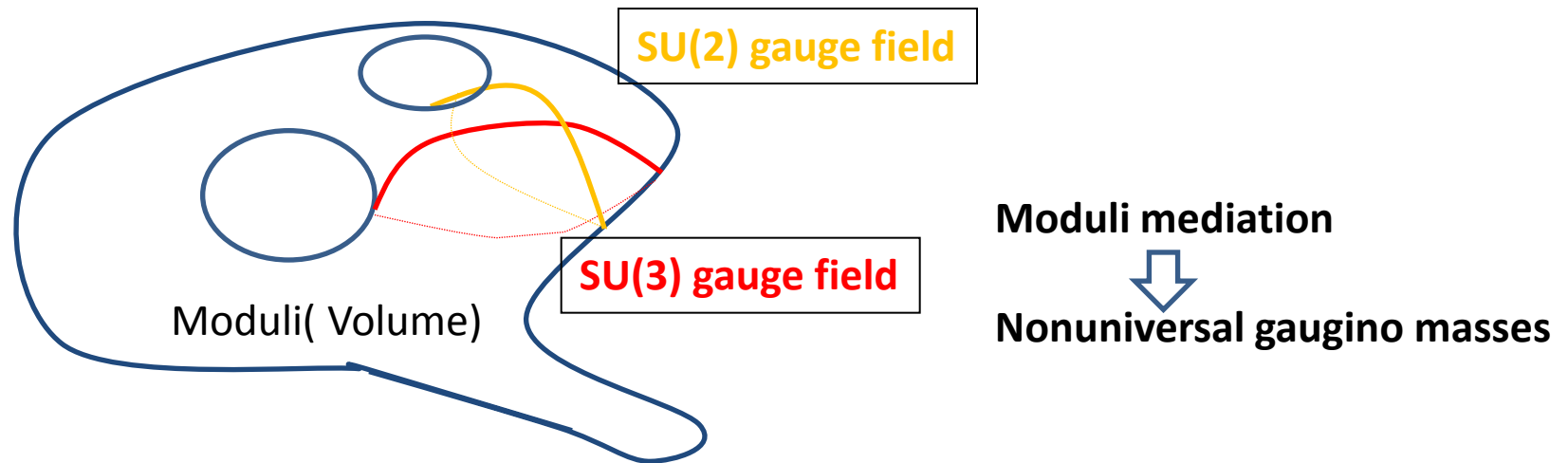
Wino is heavier than gluino.

Stop is relatively light.

SUSY breaking mechanism

There are two interesting candidates to realize the nonuniversal gaugino masses at the GUT scale.

i) Moduli mixing(in the higher-dimensional theory)



ii) Mirage mediation

Moduli mediation + Anomaly mediation



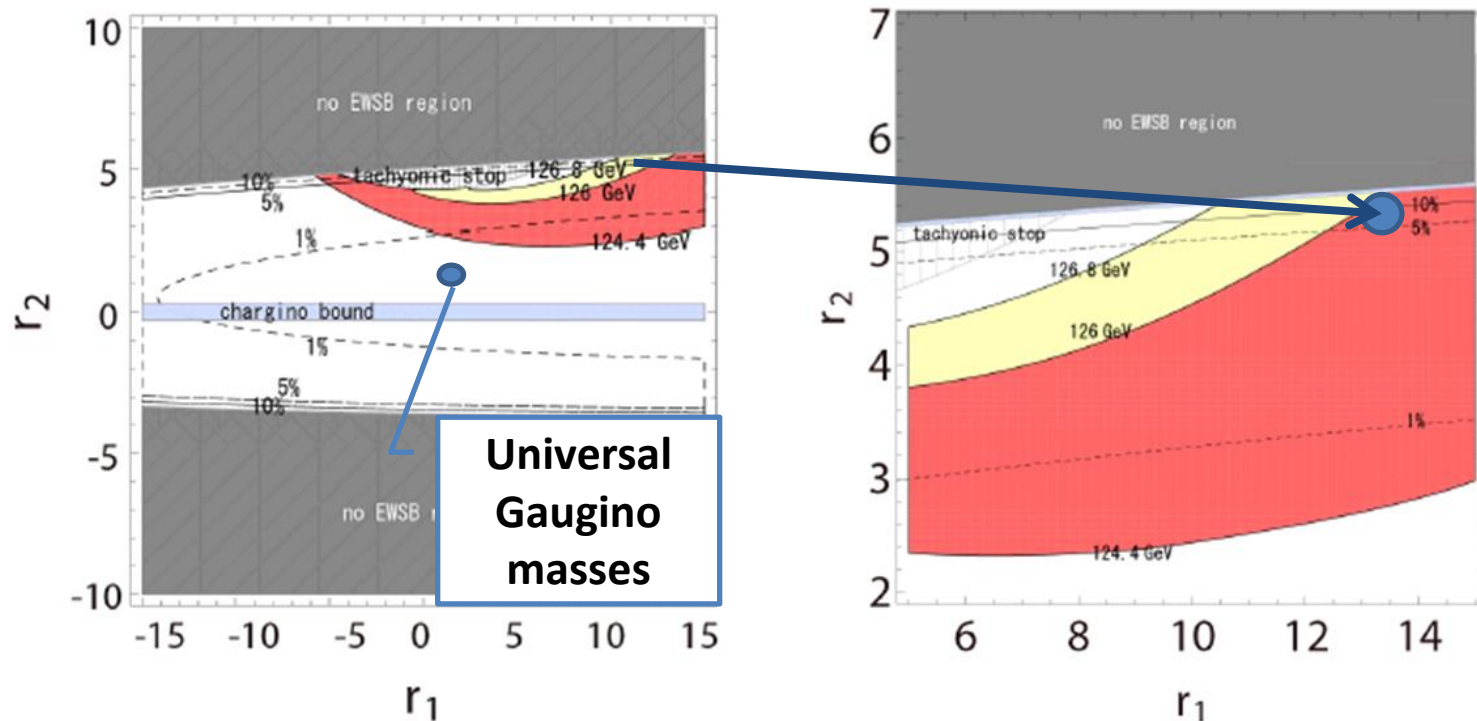
Nonuniversal gaugino masses

Summary

Nonuniversal gaugino masses at the GUT scale



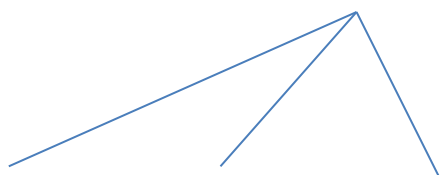
We can realize ~ 125 GeV Higgs and avoid the SUSY little hierarchy problem at the same time.



Appendix

RG eq.

Casimir Invariant


$$\begin{aligned}\frac{dm_Q^2}{dt} &\simeq -\frac{1}{4\pi^2} \left(\frac{8}{3}g_3^2|M_3|^2 + \frac{3}{2}g_2^2|M_2|^2 + \frac{1}{30}g_1^2|M_1|^2 - \frac{1}{20}g_1^2S \right) \hat{\mathbf{1}} \\ &\quad + \frac{1}{8\pi^2} \left(\frac{1}{2}y^u(y^u)^\dagger m_Q^2 + \frac{1}{2}m_Q^2 y^u(y^u)^\dagger + y^u m_U^2 (y^u)^\dagger + (m_{H_u}^2) y^u(y^u)^\dagger + A^u(A^u)^\dagger \right) \\ \frac{dm_U^2}{dt} &= -\frac{1}{4\pi^2} \left(\frac{8}{3}g_3^2|M_3|^2 + \frac{8}{15}g_1^2|M_1|^2 + \frac{1}{5}g_1^2S \right) \hat{\mathbf{1}} \\ &\quad + \frac{1}{4\pi^2} \left(\frac{1}{2}(y^u)^\dagger y^u m_U^2 + \frac{1}{2}m_U^2 (y^u)^\dagger y^u + (y^u)^\dagger m_Q^2 y^u + (m_{H_u}^2)(y^u)^\dagger y^u + (A^u)^\dagger A^u \right), \\ \frac{dm_{H_u}^2}{dt} &= -\frac{1}{4\pi^2} \left(\frac{3}{2}g_2^2|M_2|^2 + \frac{3}{10}g_1^2|M_1|^2 - \frac{3}{20}g_1^2S \right) \\ &\quad + \frac{3}{8\pi^2} \text{tr} \left\{ y^u m_Q^2 (y^u)^\dagger + y^u m_U^2 (y^u)^\dagger + m_{H_u}^2 y^u (y^u)^\dagger + A^u (A^u)^\dagger \right\}, \\ S &= m_{H_u}^2 - m_{H_d}^2 + \text{tr} \left(m_Q^2 - m_L^2 - 2m_U^2 + m_D^2 + m_E^2 \right),\end{aligned}$$

Casimir Invariant, gauge couplings and the value of Yukawa coupling determine the ratio of the gaugino masses.

$$\begin{aligned}
m_{Q_3}^2(m_Z) \simeq & -0.02M_1^2 + 0.38M_2^2 - 0.02M_1M_3 - 0.07M_2M_3 + 5.63M_3^2 \\
& +(0.02M_2 + 0.09M_3 - 0.02A_t)A_t \\
& -0.14m_{H_u}^2 + 0.86m_{Q_3}^2 - 0.14m_{U_3}^2
\end{aligned}$$

$$\begin{aligned}
m_{U_3}^2(m_Z) \simeq & 0.07M_1^2 - 0.01M_1M_2 - 0.21M_2^2 - 0.03M_1M_3 - 0.14M_2M_3 + 4.61M_3^2 \\
& +(0.01M_1 + 0.04M_2 + 0.18M_3 - 0.05A_t)A_t \\
& -0.27m_{H_u}^2 - 0.27m_{Q_3}^2 + 0.73m_{U_3}^2
\end{aligned}$$

$$A_t(m_Z) \simeq -0.04M_1 - 0.21M_2 - 1.90M_3 + 0.18A_t$$

$$\begin{aligned}
m_{H_u}^2(m_Z) \simeq & -0.01M_1M_2 + 0.17M_2^2 - 0.05M_1M_3 - 0.20M_2M_3 - 3.09M_3^2 \\
& +(0.02M_1 + 0.06M_2 + 0.27M_3 - 0.07A_t)A_t \\
& +0.59m_{H_u}^2 - 0.41m_{Q_3}^2 - 0.41m_{U_3}^2.
\end{aligned}$$

CCB breaking minima

CCB breaking minima exist unless the following condition is satisfied.

$$|A_t|^2 \leq 3 \left(m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + |\mu|^2 \right)$$

In the case of $r_a \simeq \sqrt{6}$ is dangerous with satisfying $m_{Q_3} = m_{U_3}$

In our parameter region, $|A_t(m_Z)| \sim m_{Q_3}(m_Z)$

and $0 < m_{U_3}(m_Z) < m_{Q_3}(m_Z)$

This inequality is guaranteed.