An Non-abelian Self-duality Equation in 6d and Multiple M5-branes in String Theory

Chong-Sun Chu
National Tsing-Hua University and Durham University

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based on

1. A Theory of Non-Abelian Tensor Gauge Field with Non-Abelian Gauge Symmetry *GxG*, NPB, arXiv:1108.5131.
2. Non-abelian Action for Multiple Five-Branes with Self-Dual Tensors, with Sheng-Lan Ko, JHEP, arXiv:1203.4224.
3. Non-Abelian Self-Dual String Solutions, with Sheng-Lan Ko, Pichet Vanichchapongjaroen, JHEP, arXiv:1207.1095.
4. Non-abelian Self-Dual String and M2-M5 Branes Intersection in Supergravity with Pichet Vanichchapongjaroen, JHEP, arXiv:1304.4322.
5. Instanton String and M-Wave in Multiple M5-Branes System, with Hiroshi Isono, EJPC, arXiv:1305.6808.
6. Non-Abelian Self-Dual Strings in Six Dimensions from Four Dimensional 1/2-BPS Monopoles, arXiv:1310.7710.

Consider a Lie-algebra valued one-form gauge field (connection) $A = A_\mu^a T^a dx^\mu$ with the field strength (curvature)

$$F = dA + A^2.$$

The self-dual Yang-Mills equation

$$F = *F$$

is an interesting and important equation.

- Physically, these are instantons which describe the nonperturbative vacuum structure of Yang-Mills gauge theory.
- It has also leaded to many important results in mathematics: geometry of 4-manifolds and instantons (Donaldson), ADHM construction, twistor, ...

This talk is concerned about a generalization of the self-duality YM equation in 4-dimensions to a 2-form gauge field $B = \frac{1}{2}B^a_{\mu\nu}T^a dx^\mu dx^\nu$:

$$H = *H.$$

Obviously the equation lives in 6-dimensional spacetimes (Lorentzian).

My main goal is to explain:

1. How to write down such an equation? in particular what is the definition of H?

$$H = dB + (?)$$

2. How did this equation come up in physics? What does it describe in physics (string theory)?

Outline

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Mysteries of M5-branes

What we know:

- The low energy worldvolume dynamics is given by a 6d (2,0) SCFT with $SO(5)$ R-symmetry.

  (Strominger, Witten)

  The (2,0) tensor multiples contains 5 scalars and a selfdual antisymmetric 3-form field strength + fermions.

  (Gibbons, Townsend; Strominger; Kaplan, Michelson)

What we don't know:

- What is the form of the gauge symmetry for multiple M5-branes ?
- Interacting self-dual dynamics on M5-branes worldvolume?

Enhanced gauge symmetry of multiple M5-branes (?)

- For multiple D-branes, symmetry is enhanced from $U(1)$ to $U(N)$:

$$\delta A^a_\mu = \partial_\mu \Lambda^a + [A_\mu, \Lambda]^a, \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + [A_\mu, A_\nu]^a.$$

- For multiple M5-branes, it is not known how to non-Abelianize 2-form (or higher form) gauge fields:

$$\delta B^a_{\mu\nu} = \partial_\mu \Lambda^a_\nu - \partial_\nu \Lambda^a_\mu + (?), \quad H^a_{\mu\nu\lambda} = \partial_\mu B^a_{\nu\lambda} + \partial_\nu B^a_{\lambda\mu} + \partial_\lambda B^a_{\mu\nu} + (?).$$

  to have nontrivial self interaction.
  Besides, what is a?

- Moreover, exists no-go theorems: there is no nontrivial deformation of the Abelian 2-form gauge theory if *locality* of the action and the transformation laws are assumed.

(Henneaux; Bekaert; Sevrin; Nepomechie)

- These no-go theorems, however, suggest an important direction of given up locality.
- The need of nonlocality for M5-branes should not be surprising: ABJM and BLG theory for multiple M2-branes are also non-local if one eliminates the auxillary Chern-Simons gauge field.

Below I will explain a proposal for the equation of motion for the low energy worldvolume theory of multiple M5-branes by similarly introducing a set of auxillary fields.

(Chu, Ko 2012)

Outline

A proposal

- We generalize Perry-Schwarz's formulation for a single free M5-brane.
  1. A direction, say x_5, is treated differently: denote the 5d and 6d coord. by x^μ and $x^M = (x^\mu, x^5)$.
  2. the tensor gauge field potential is represented by a 5×5 antisymmetric tensor field $B_{\mu\nu}$. It can be thought of as a (tensor) gauge fixed formulation in which

$$B_{\mu 5} = 0.$$

  (c.f. Yang-Mills theory in axial gauge $A_3 = 0$).

The self-duality equation $H = *H$ reads,

$$\begin{aligned} H_{5\mu\nu} &= (*H)_{5\mu\nu} \\ &= \frac{1}{6}\epsilon_{\mu\nu\rho\lambda\sigma}H^{\rho\lambda\sigma} := \tilde{H}^{\mu\nu} \end{aligned}$$

In the gauge $B_{\mu 5} = 0$, we have

$$\partial_5 B_{\mu\nu} = \tilde{H}_{\mu\nu}.$$

The equation is not manifestly 6d Lorentz covariant.

- We propose a definition of the nonabelian H by promoting the partial derivative to a covariant derivative:

$$
\begin{aligned}
H_{MNL} &= D_M B_{NL} + D_N B_{LM} + D_L B_{MN}, \\
D_\mu &= \partial_\mu + A_\mu, \quad D_5 = \partial_5
\end{aligned}
$$

  for some 1-form gauge field A_m with $A_5 = 0$.
- The self-duality equation $H = *H$ reads in the gauge $B_{\mu 5} = 0$:

$$\boxed{\partial_5 B_{\mu\nu} = \tilde{H}_{\mu\nu}}.$$

- M5-brane supermultiplet structure leaves no room for a new set of propagating degrees of freedom like the A_μ's. Therefore they must be auxillary and be given in terms of the other fields. Our proposal is:

$$\boxed{F_{\mu\nu} = c \int dx_5 \ \tilde{H}_{\mu\nu}}.$$

The equation is invariant under:

1. Yang-Mills gauge symmetry

$$\delta A_\mu = \partial_\mu \Lambda + [A_\mu, \Lambda], \quad \delta B_{\mu\nu} = [B_{\mu\nu}, \Lambda].$$

2. Tensor gauge symmetry:

$$\delta_T A_\mu = 0, \quad \delta_T B_{\mu\nu} = D_{[\mu}\Lambda_{\nu]},$$

for arbitrary $\Lambda_\mu(x^M)$ such that $[F_{[\mu\nu}, \Lambda_{\lambda]}] = 0$.

Note:

- Our equation may be compared with the SD YM equation in the axial gauge $A_3 = 0$

$$\partial_3 A_\alpha = \tilde{F}_\alpha := \frac{1}{2}\epsilon_{\alpha\beta\gamma}F_{\beta\gamma}, \quad \alpha, \beta, \gamma = 0, 1, 2.$$

  Only a portion of the Lorentz symmetry is manifest. Still need to discover the 6d covariant version.
- c is a constant that is initially arbitrary but actually fixed by properties of its solutions (charge quantization). The whole proposal would be wrong otherwise as there is no free tunable parameter in M-theory.

Outline

Self-dual string on M5-brane

- M2-branes can end on M5-brane. The endpoint gives strings living on the M5-brane.
  These self-dual strings appear as solitons of the M5-branes theory.
- In a series of papers, we constructed non-abelian self-dual string solutions to our self-duality equation and obtained full agreement with the description of the M2-M5 intersection branes system in terms of supergravity.

(Chu, Ko, Vanichchapongjaroen 2012, 2013)

- For the abelian case, Perry-Schwarz has obtained a self-dual string solution:

$$B_{ij} = -\frac{1}{2}\frac{\beta\epsilon_{ijk}x_k}{r^3}\left(\frac{x^5 r}{\rho^2} + \tan^{-1}(x^5/r)\right), \qquad B_{04} = -\frac{\beta}{2\rho^2},$$

$i, j = 1, 2, 3.$

- Although the auxillary field does not appear in the PS construction, it is amazing that

$$F_{ij} = -\frac{c\beta\pi}{2}\frac{\epsilon_{ijk}x_k}{r^3}, \qquad F_{04} = 0$$

i.e. a Dirac monopole in the (x, y, z) subspace if $c\beta = -2/\pi$!

It turns out the use of an non-abelian monopole in place of the Dirac monopole is precisely what is needed to construct the non-abelian self-dual string solution.

Non-abelian Wu-Yang monopole

Wu-Yang

- Consider $SU(2)$ gauge group

$$[T^a, T^b] = i\epsilon^{abc} T^c, \quad a, b, c = 1, 2, 3.$$

- The non-abelian Wu-Yang monopole is given by

$$A_i^a = -\epsilon_{aik} \frac{x_k}{r^2}, \qquad F_{ij}^a = \epsilon_{ijm} \frac{x_m x_a}{r^4}, \qquad i, j = 1, 2, 3.$$

- Note that the field strength for the Wu-Yang solution is related to the field strength of the Dirac monopole by a simple relation:

$$F_{ij}^a = F_{ij}^{(\mathrm{Dirac})} \frac{x^a}{r}.$$

Non-abelian self-dual string solution

- Inspired by the relation of Dirac monopole to the Wu-Yang solution, try the ansatz

$$H^a_{\mu\nu\lambda} = H^{(\mathrm{PS})}_{\mu\nu\lambda}\frac{x^a}{r}$$

Here $r = \sqrt{x^2 + y^2 + z^2}$ and $H^{(\mathrm{PS})}_{\mu\nu\lambda}$ is the field strength for the linearized Perry-Schwarz solution aligning in the x^4 direction. Self-duality is automatically satisfied!

- B can be obtained by integrating $H_{\mu\nu 5} = \partial_5 B_{\mu\nu}$ and we obtain

$$B^a_{\mu\nu} = B^{(\mathrm{PS})}_{\mu\nu}\frac{x^a}{r},$$

- It is amusing that the auxillary field configuration is given by

$$F^a_{ij} = -\frac{c\beta\pi}{2}\frac{\epsilon_{ijm}x_m x_a}{r^4}, \qquad F^a_{tw} = 0.$$

This is the Wu-Yang monopole if we take $c\beta = -\frac{2}{\pi}$.

- The BPS equation of Howe-Lambert-West:

$$H_{ijk} = \epsilon_{ijk}\partial_5\phi, \qquad H_{ij5} = -\epsilon_{ijk}D_k\phi$$

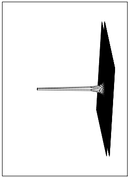
can be solved with

$$\phi^a = -(u + \frac{\beta}{2\rho^2})\frac{x^a}{r},$$

- The transverse distance $|\phi|$ defined by $|\phi|^2 = \phi^a\phi^a$ gives

$$|\phi| = |u + \frac{\beta}{2\rho^2}|.$$

- This describes a system of M5-branes with a spike at $\rho = 0$ and level off to u as $\rho \to \infty$. Hence the physical interpretation of our self-dual string is that two M5-branes are separating by a distance u and with an M2-brane ending on them.



- Asymptotic $U(1)$ B-field is $\mathcal{B}_{\mu\nu} \equiv \hat{\phi}^a_\infty B^a_{\mu\nu} = \pm B^{(\mathrm{PS})}_{\mu\nu}$ and we obtain

$$P = Q = -\frac{4\pi}{|c|}.$$

Charge quantization

$$e^{i(PQ' + QP')} = 1$$

implies

$$PQ' + QP' = 2\pi Z$$

This fixes

$$c = \pm 4\sqrt{\pi}$$

- One may generalize the above to a system of N_5 coincident M5-branes with a spike with N_2 self-dual strings.
- In particular, since for $U(N_5)$ theory with adjoint fields, there is a nontrivial center Z_N in the gauge group. Charge quantization condition is modified to

$$PQ' + QP' = 2\pi \frac{Z}{N_5}$$

(Corrigan, Olive 1976)

- Making use of this, we find the spike

$$|\phi| = u + \frac{N_2}{N_5}\frac{1}{\rho^2}.$$

The N_2, N_5 dependence agree precisely with the supergravity solution for intersecting M2-M5 branes.

(Niarchos, Siampos 2013)

Instanton String

- The previous solution was based on a configuration of the auxillary gauge field being given by the monopole. We can call our self-dual string solution monopole string. And we have shown that it describes precisely the M2-M5 intersections.
- Another well known configuration in Non-abelian gauge theory is the instanton.
  construct instanton string? what does it describe in M-theory?

- Turns out such a solution is not difficult to construct.

(Chu, Isono 2013)

- Consider an ansatz

$$B_{ab} = F_{ab}\, f(x^0, x^5), \quad B_{a0} = 0, \qquad a = 1, 2, 3, 4.$$

then our self-duality eqn reads

$$F_{ab}\partial_5 f = \frac{1}{2}\epsilon_{abcd} F_{cd}\partial_0 f.$$

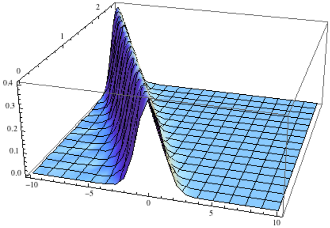
This can be solved with

$$\begin{aligned} F_{ab} \text{ being SD and} \quad & f = f(x^0 + x^5), \\ \text{or, } F_{ab} \text{ being ASD and} \quad & f = f(x^0 - x^5), \end{aligned}$$

where (A)SD stands for (anti)self-dual.

- This solution describes a wave supported by an instanton.

A detailed studies of the properties of the solution reveals that the solution corresponds to M-wave (MW) on the worldvolume of multiple M5-branes.



Outline

- Supersymmetry on the system of multiple M5-branes tell us that the worldvolume theory is govern by a non-abelian self-duality equation. We have constructed such an equation of motion for the multiple M5-branes

$$H = *H$$

and show that it contains soltonic solutions whose properties agree with known brane systems in M-theory:

Auxillary A_μ	M-theory system
Wu-Yang monopole	M2-branes ending on M5-branes
Instanons	M-wave propagating on M5's
⋮	⋮

This provides some dynamical support to our proposed theory.

Further questions

- Supersymmetry: (2,0)? (1,0)?
  Scalar potential and BPS equation?
- Covariant PST extension of our model?
- Classical integrability?
  In some sense, our non-abelian self-duality eqn generalizes the self-dual Yang-Mills instanton equation

$$F = *F$$

. The instanton equation is exactly solvable.

(ADHM; Penrose; Ward; Atiyah; ...)

Q: Could our non-abelian self-duality eqn for H be integrable also?