Calabi-Yau, D-Branes, and Orientifolds

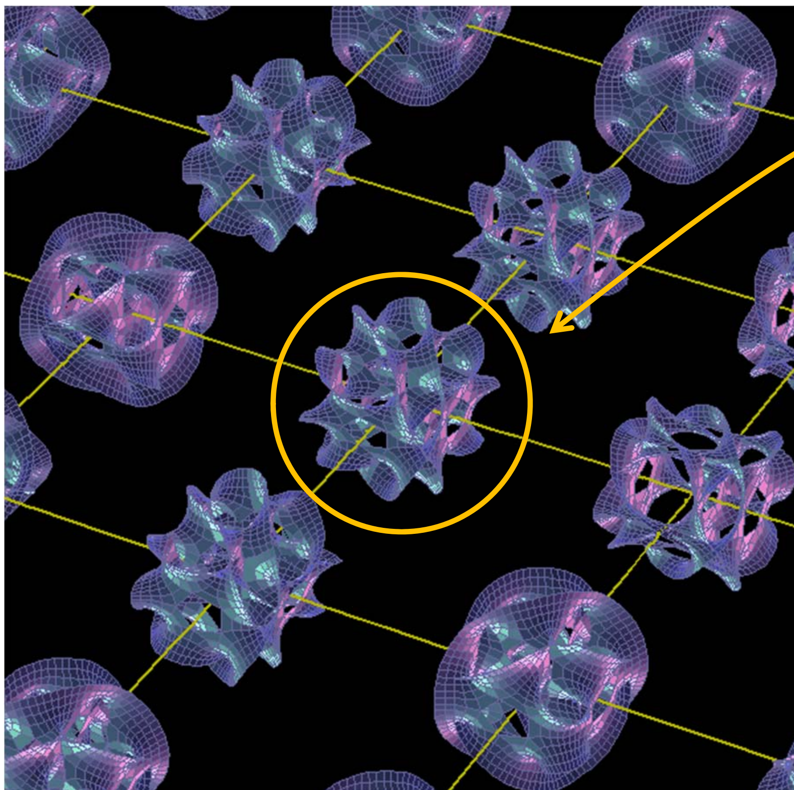
worldsheet partition functions and spacetime topology

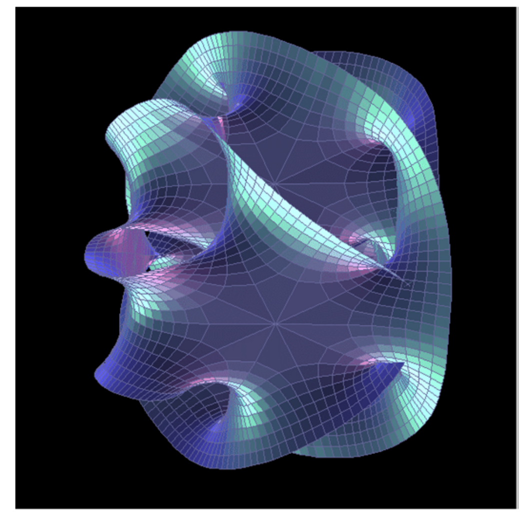
PILJIN YI
Korea Institute for Advanced Study

PASCOS 2013, TAIPEI

string theory on Calabi-Yau → 4D physics

$$L_{4D} \sim \int d\theta^2 d\bar{\theta}^2 K(\rho, \bar{\rho}; \cdots) + \cdots$$

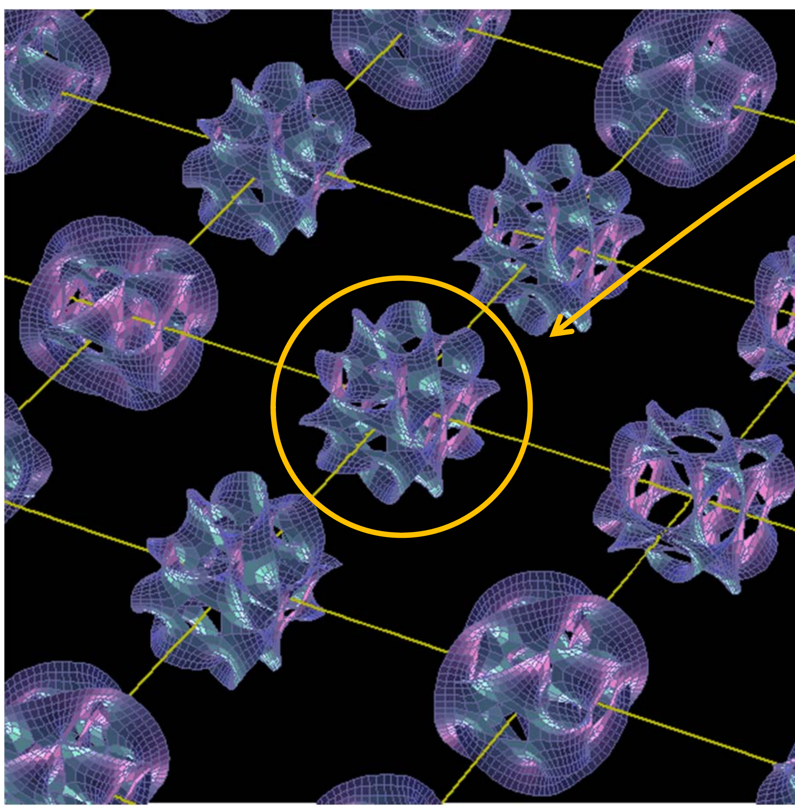


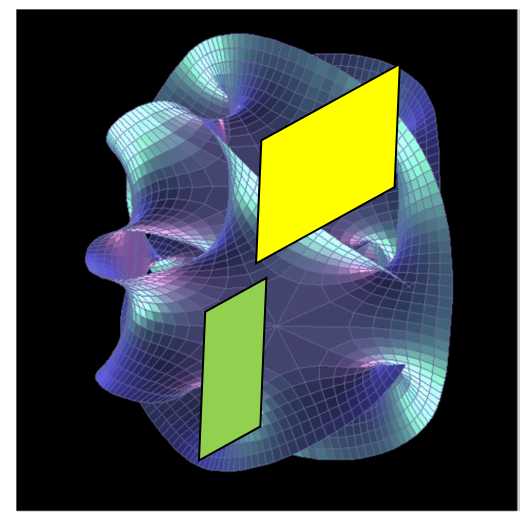


$$K(\rho, \bar{\rho}; \cdots) \sim -\log(\mathrm{Vol}_{CY3})$$

string theory on Calabi-Yau + D-branes/O-planes → 4D physics

$$L_{4D} \sim \int dx^{p-3} e^{-\Phi} \mathrm{tr} \mathcal{F}^2 + \cdots$$





can we compute L_{4D} exactly with stringy corrections when CY and D/O are given ?

GLSM on S^2 and exact moduli space metric

anomaly inflow and characteristic classes

partition functions on RP^2 and on hemisphere

the Gamma class & quantum volume

Gauged Linear Sigma Models

UV field content → N=1 D=4 gauge theory dimensionally reduced to N=(2,2) D=2

gauge fields $(A_{\mu=0,1}, \sigma = A_2 + iA_3, \lambda_\alpha, D)^a$

chiral matter $(X, \psi_\alpha, F)^I$

Edward Witten 1993

Gauged Linear Sigma Models

gauge fields $(A_\mu, \lambda_\alpha, \sigma, D)^a$ with theta angles θ^i

chiral matter $(X, \psi_\alpha, F)^I$ & FI constants ξ^i for U(1)'s

$2\pi\alpha'$ ← gauge coupling^2

↓

infrared (2,2) superconformal field theory
= part of superstring worldsheet theory

Edward Witten 1993

Gauged Linear Sigma Models

gauge fields $(A_\mu, \lambda_\alpha, \sigma, D)^a$ with theta angles θ^i

chiral matter $(X, \psi_\alpha, F)^I$ & FI constants ξ^i for U(1)'s

$\theta^i + i2\pi\xi^i$ → complexified Kaehler parameters of Calabi-Yau

Landau-Ginzburg CFT (Calabi-Yau) NLSM

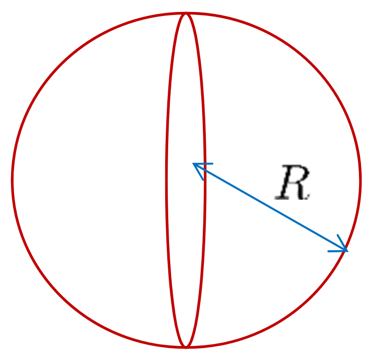
$-\xi \gg 1$ $\xi \gg 1$

infrared (2,2) superconformal field theory
= part of superstring worldsheet theory

Gauged Linear Sigma Models on S^2

gauge fields $(A_\mu, \lambda_\alpha, \sigma, D)^a$ with theta angles θ^i

chiral matter $(X, \psi_\alpha, F)^I$ & FI constants ξ^i for U(1)'s



$$Z_{S^2}(\theta^i, \xi^i) = \int [dA \cdots] e^{-S_{S^2}^{GLSM}}$$

D=2 N=(2,2) GLSM and Exact Partition Functions on S^2 :
Nima Doroud, Jaume Gomis, Bruno Le Floch, Sungjay Lee1210.6022 [hep-th]
Francesco Benini, Stefano 1206.2356 [hep-th]

how? → localization procedure

the partition function
is invariant as the gauge
coupling is deformed
to a very large value

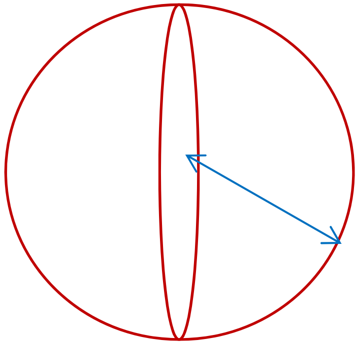
$(A_\mu, \lambda_\alpha, \sigma, D)^a$ with theta angles θ^i

$(X, \psi_\alpha, F)^I$ & FI constants ζ^i for U(1)'s

$$\sigma = i\eta$$

$$D = \eta/R$$

$$F^i = dA^i = (m^i/2) vol_{S^2}$$



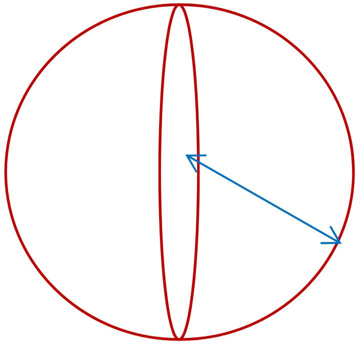
$$R^2 \sim 2\pi\alpha' \Rightarrow 1$$

$$Z_{S^2}(\theta^i, \xi^i) \sim \sum_m \int d\eta \, e^{-2\pi\xi\cdot\eta + i\theta\cdot m} \frac{\prod_{fermions} \mathrm{Det}_{(m,\eta)}}{\prod_{bosons} \mathrm{Det}^{1/2}_{(m,\eta)}}$$

what does it compute ?

$(A_\mu, \lambda_\alpha, \sigma, D)^a$ with theta angles θ^i

$(X, \psi_\alpha, F)^I$ & FI constants ξ^i for U(1)'s



$R^2 \sim 2\pi\alpha' \Rightarrow 1$

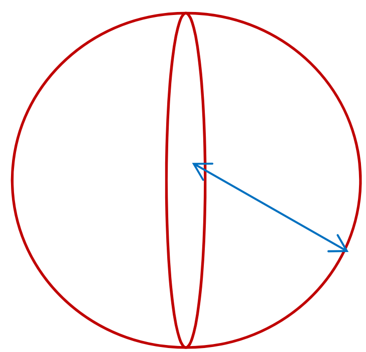
$$Z_{S^2}(\theta^i, \xi^i)\Big|_{CY3} \sim \mathrm{Vol}_{CY3}(\rho) + \zeta(3) Euler_{CY3} + \sum e^{-n\cdot 4\pi\xi}$$

α' exact "volume"

what does it compute ?

$(A_\mu, \lambda_\alpha, \sigma, D)^a$ with theta angles θ^i

$(X, \psi_\alpha, F)^I$ & FI constants ζ^i for U(1)'s



$R^2 \sim 2\pi\alpha' \Rightarrow 1$

$$Z_{S^2}(\theta^i, \xi^i)\Big|_{CY3} \sim \mathrm{Vol}_{CY3}(\rho) + \zeta(3) Euler_{CY3} + \sum e^{-n\cdot 4\pi\xi}$$

α' exact "volume"

in some simple cases, mirror symmetry can compute these quantities

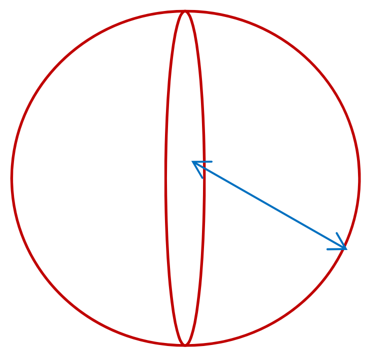
what does it compute ?

$$e^{-K(\rho^i,\bar{\rho}^i)} \quad = \quad {}_R\langle\bar{0}|0\rangle_R$$

Cecotti, Vafa 1991

for examples known from mirror computations with $\rho^i = \theta^i + i2\pi\xi^i$

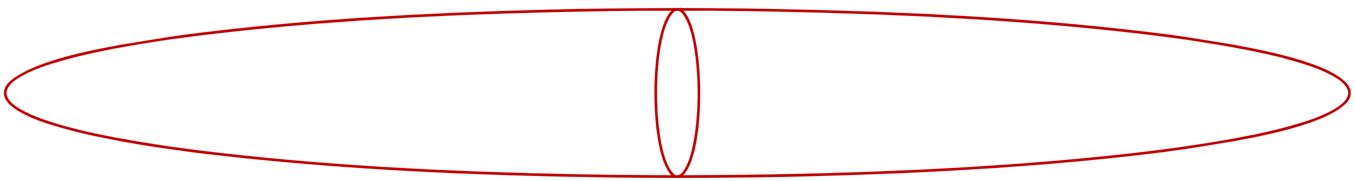
but why so?



$$Z_{S^2}(\theta^i,\xi^i) = \int [dA\cdots] e^{-S_{S^2}^{GLSM}}$$

Hans Jockers, Vijay Kumar, Joshua M. Lapan, David R. Morrison, Mauricio Romo 1208.6244 [hep-th]

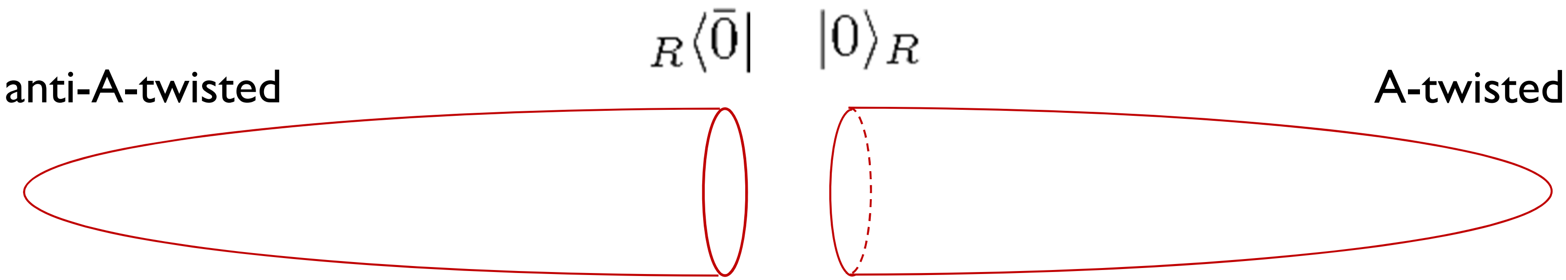
Gauged Linear Sigma Models on squashed S^2



$$Z_{\text{squashed } S^2}(\theta^i, \xi^i) = Z_{S^2}(\theta^i, \xi^i)$$

Jaume Gomis, Sungjay Lee 1210.6022 [hep-th]

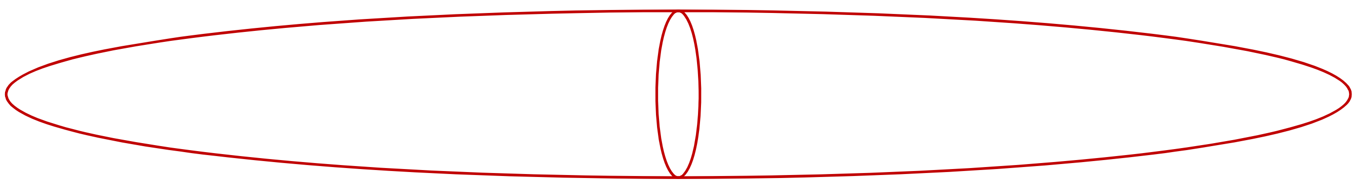
Gauged Linear Sigma Models on squashed S^2



$$Z_{\text{infinitely squashed } S^2}(\theta^i, \xi^i) \quad = \quad {}_R\langle\bar{0}|0\rangle_R$$

Jaume Gomis, Sungjay Lee 1210.6022 [hep-th]

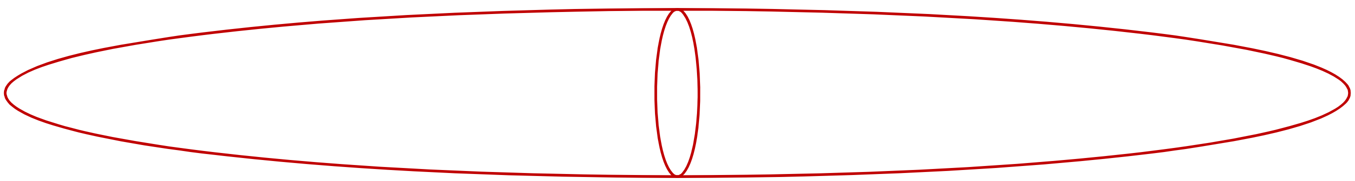
Gauged Linear Sigma Models on squashed S^2



$$Z_{\text{infinitely squashed } S^2}(\theta^i, \xi^i) = {}_R\langle\bar{0}|0\rangle_R = e^{-K(\rho^i,\bar{\rho}^i)}\Big|_{\rho^i=\theta^i+i2\pi\xi^i}$$

Jaume Gomis, Sungjay Lee 1210.6022 [hep-th]

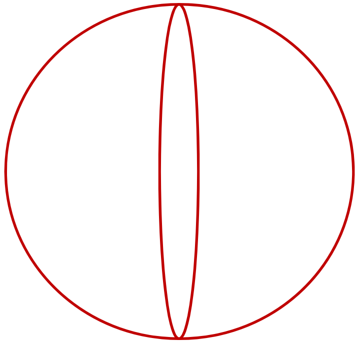
Gauged Linear Sigma Models on squashed S^2



$$Z_{\text{squashed } S^2}(\theta^i, \xi^i) = Z_{S^2}(\theta^i, \xi^i)$$

Jaume Gomis, Sungjay Lee 1210.6022 [hep-th]

Gauged Linear Sigma Models on S^2 computes α'exact kinetic Lagrangian for 4D physics

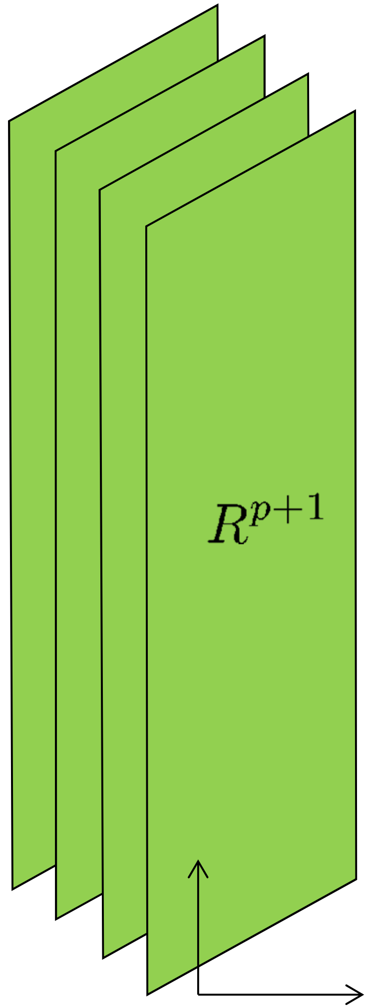


$$Z_{S^2}(\theta^i, \xi^i) = \; _R\langle\bar{0}|0\rangle_R = e^{-K(\rho^i,\bar{\rho}^i)}\Big|_{\rho^i=\theta^i+i2\pi\xi^i}$$

Jaume Gomis, Sungjay Lee 1210.6022 [hep-th]

anomaly inflow & characterstic classes on worldvolume

k Dp's



maximally SUSY U(k) Yang-Mills +
Chern-Simons couplings to RR-fields

$$- \quad \mu_{\rm p} \int dx^{p+1} e^{-\Phi} \sqrt{-h} \times k$$

$$- \quad (\pi\alpha')^2 \mu_{\rm p} \int dx^{p+1} e^{-\Phi} \sqrt{-h}\ {\rm tr}_{k\times\bar{k}} (h^{-1}\mathcal{F})^2$$

$$+ \quad \mu_{\rm p} \int \sum_{n=0}^{[p/2+1]} C_{p-2n+1} \wedge {\rm tr}_k\, e^{2\pi\alpha' \mathcal{F}} \wedge \cdots$$

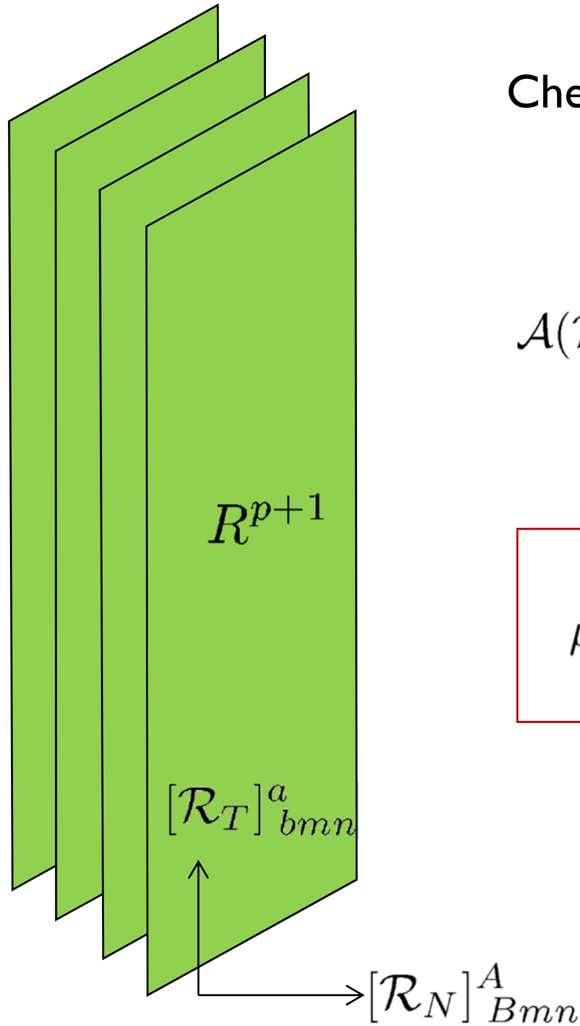
$$+ \quad \cdots$$

$$\mu_{\rm p} = \frac{2\pi}{(4\pi^2\alpha')^{(p+1)/2}}$$

anomaly inflow & characterstic classes on worldvolume

assumed to be Spin

k Dp's



Chern-Simons couplings to RR-fields

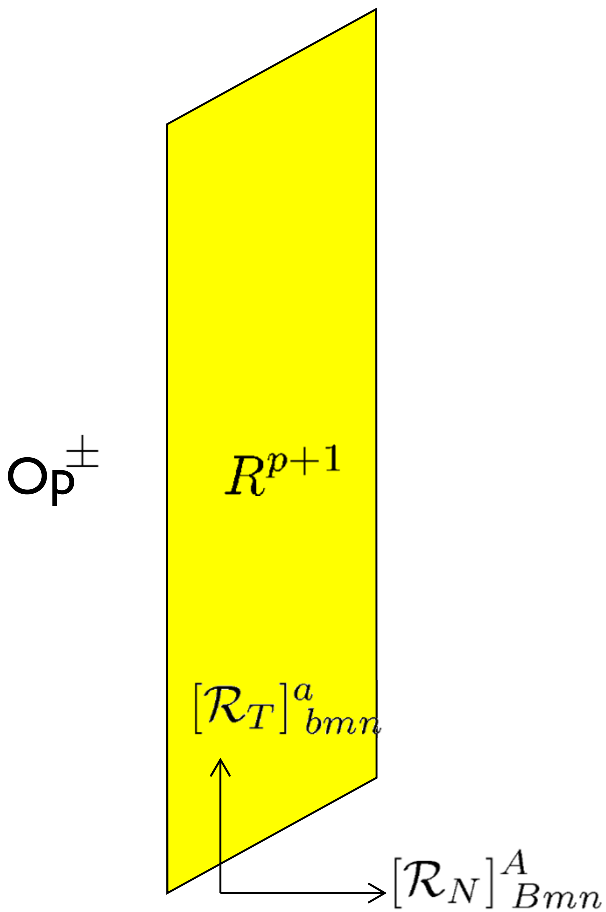
$$\mathcal{A}(\mathcal{R}) = \sqrt{\det{}_{SO}\left(\frac{\mathcal{R}/2}{\sinh(\mathcal{R}/2)}\right)}$$

$$\mu_{\mathrm{p}} \int \sum_{n=0}^{[p/2+1]} C_{p-2n+1} \wedge \mathrm{tr}_k\, e^{\mathcal{F}} \wedge \sqrt{\frac{\mathcal{A}(\mathcal{R}_T)}{\mathcal{A}(\mathcal{R}_N)}}$$

$$\mu_{\mathrm{p}} = \frac{1}{(2\pi)^{(p-1)/2}}$$

anomaly inflow & characterstic classes on worldvolume

assumed to be Spin



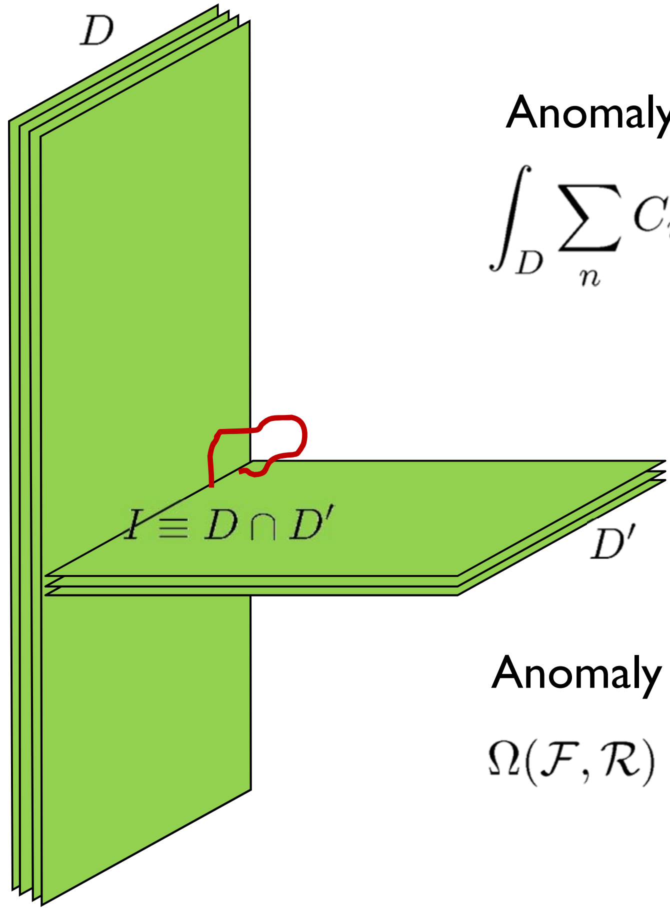
Chern-Simons couplings to RR-fields

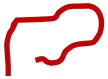
$$\mathcal{L}(\mathcal{R}) = \sqrt{\det_{SO}\left(\frac{\mathcal{R}}{\tanh(\mathcal{R})}\right)}$$

$$\pm\mu_{\rm p} 2^{p-4} \int \sum_{n=0}^{[p/2+1]} C_{p-2n+1} \wedge \sqrt{\frac{\mathcal{L}(\mathcal{R}_T/4)}{\mathcal{L}(\mathcal{R}_N/4)}}$$

$$\mu_{\rm p} = \frac{1}{(2\pi)^{(p-1)/2}}$$

anomaly inflow & characterstic classes on worldvolume



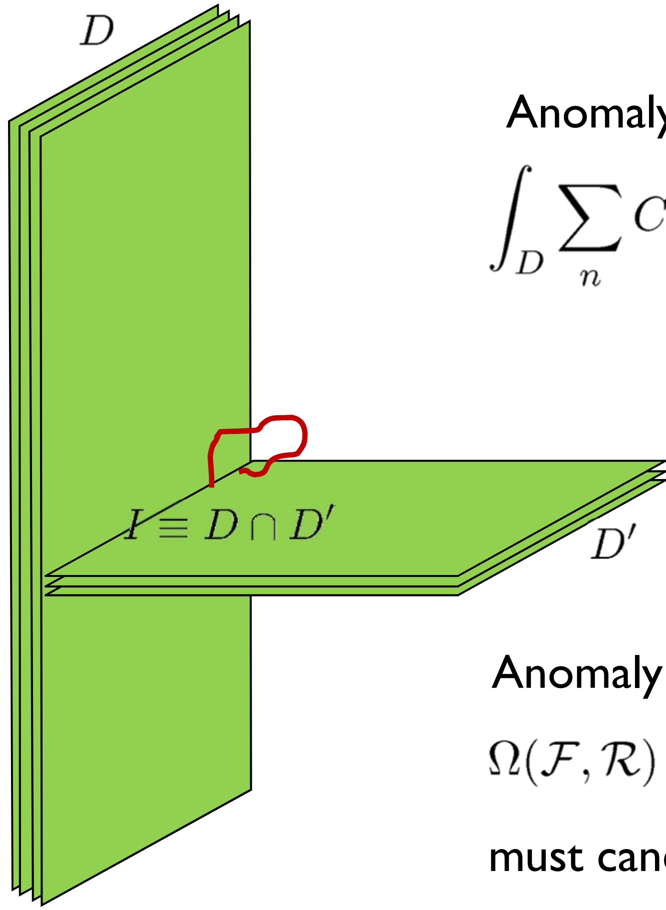
Anomaly Inflow + One-Loop Anomaly of  = 0

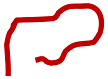
$$\int_D \sum_n C_{n+1} \wedge \Omega(\mathcal{F}, \mathcal{R}) + \int_{D'} \sum_m C_{m+1} \wedge \Omega(\mathcal{F}', \mathcal{R}')$$

Anomaly Inflow onto $I \equiv D \cap D'$

$$\Omega(\mathcal{F}, \mathcal{R}) \wedge \Omega(-\mathcal{F}', -\mathcal{R}') \wedge Euler(N \cap N')$$

anomaly inflow & characterstic classes on worldvolume

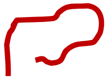


Anomaly Inflow + One-Loop Anomaly of  = 0

$$\int_D \sum_n C_{n+1} \wedge \Omega(\mathcal{F}, \mathcal{R}) + \int_{D'} \sum_m C_{m+1} \wedge \Omega(\mathcal{F}', \mathcal{R}')$$

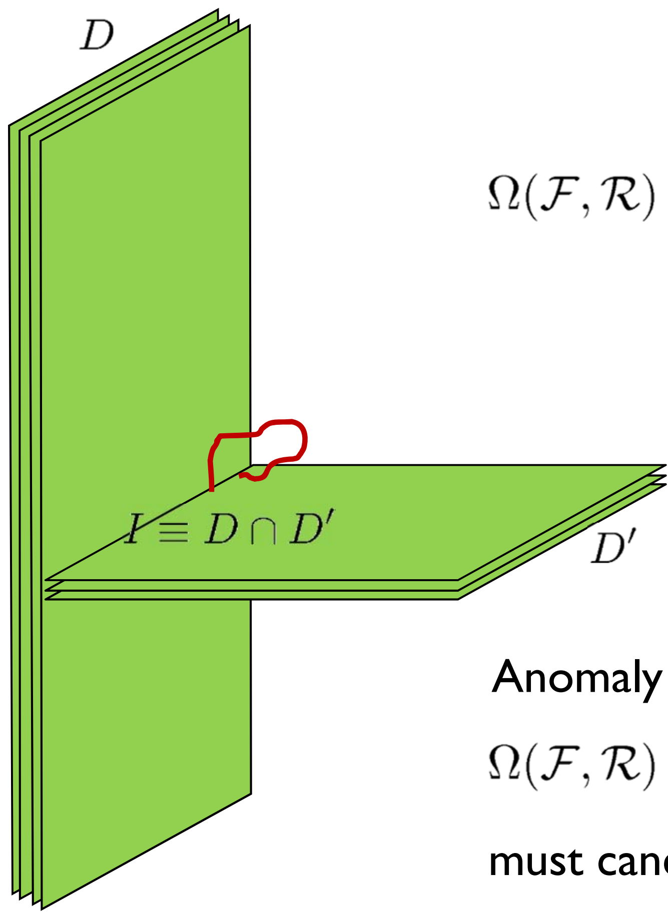
Anomaly Inflow onto $I \equiv D \cap D'$

$$\Omega(\mathcal{F}, \mathcal{R}) \wedge \Omega(-\mathcal{F}', -\mathcal{R}') \wedge Euler(N \cap N')$$

must cancel one-loop anomaly from 

$$-\mathrm{tr}_{k \otimes \bar{k}'} e^{\mathcal{F} \oplus \mathcal{F}'} \wedge \mathcal{A}(\mathcal{R}_I) \wedge [ch_{S^+} - ch_{S^-}](N \cap N')$$

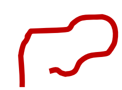
anomaly inflow & characterstic classes on worldvolume



$$\Omega(\mathcal{F},\mathcal{R}) = \mathrm{tr}_k\, e^{\mathcal{F}} \wedge \sqrt{\frac{\mathcal{A}(\mathcal{R}_T)}{\mathcal{A}(\mathcal{R}_N)}}$$

Anomaly Inflow onto $I \equiv D \cap D'$

$$\Omega(\mathcal{F},\mathcal{R}) \wedge \Omega(-\mathcal{F}',-\mathcal{R}') \wedge Euler(N \cap N')$$

must cancel one-loop anomaly from 

$$-\mathrm{tr}_{k\otimes\bar{k}'}\, e^{\mathcal{F}\oplus\mathcal{F}'} \wedge \mathcal{A}(\mathcal{R}_I) \wedge [ch_{S^+} - ch_{S^-}](N \cap N')$$

1984 Green, Schwarz

D-brane inflow for 32D9/O9$^-$ ← anomaly cancelation for type I SO(32)

1996 Green, Harvey, Moore

I-brane inflow for gravitational anomaly

1997 Cheung, Yin

D-brane & I-brane inflow for gravitational & axial anomaly

fixed an important & extremely subtle factor of ½ in CS coupling

1997 Dasgupta, Jatkar, Mukhi /1998 Morales, Scrucca, Serone

O^- orientifolds

1997-1999 Minasian, Moore / 1999 Freed, Witten

additional factor of a half-line-bundle when the D-brane worldvolume is $Spin^c$

2010 Distler, Freed, Moore

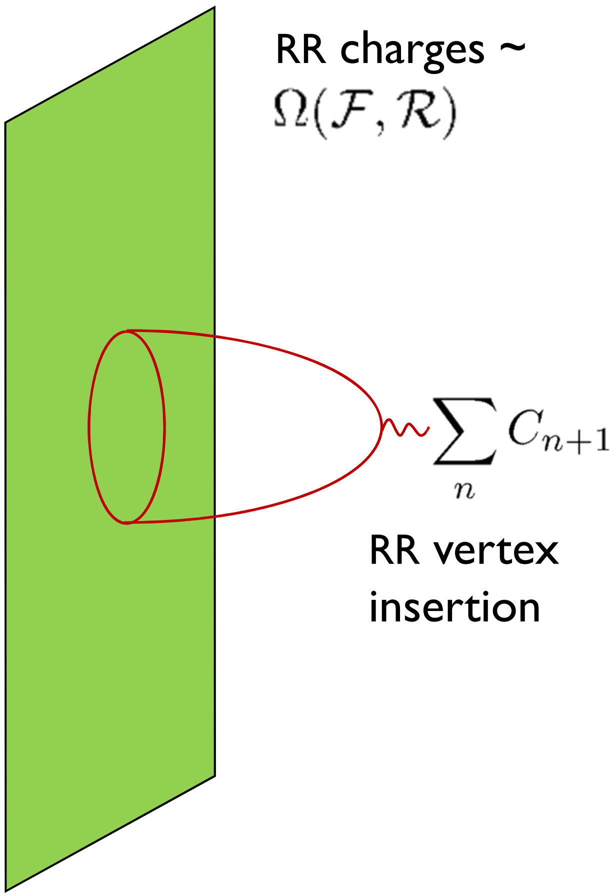
O^+ orientifolds

2012 Heeyeon Kim, P.Y.

inflow onto self-dual worldvolumes with/without O-planes

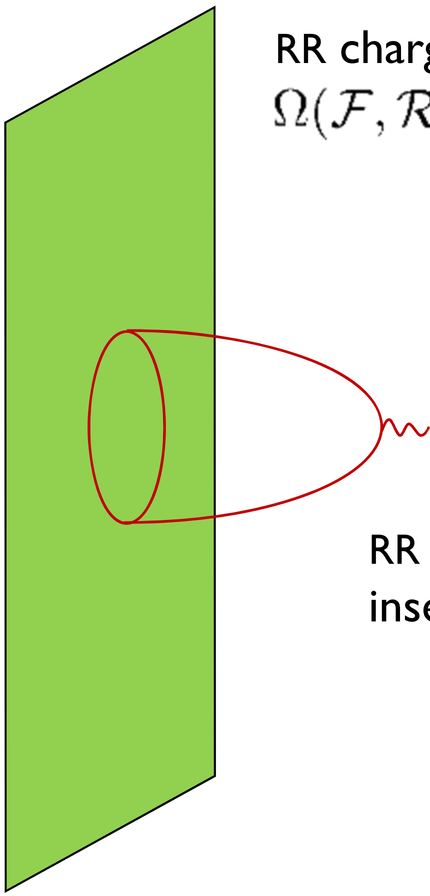
relation to string worldsheet

large volume (= small α') limits of the disk amplitudes



relation to string worldsheet

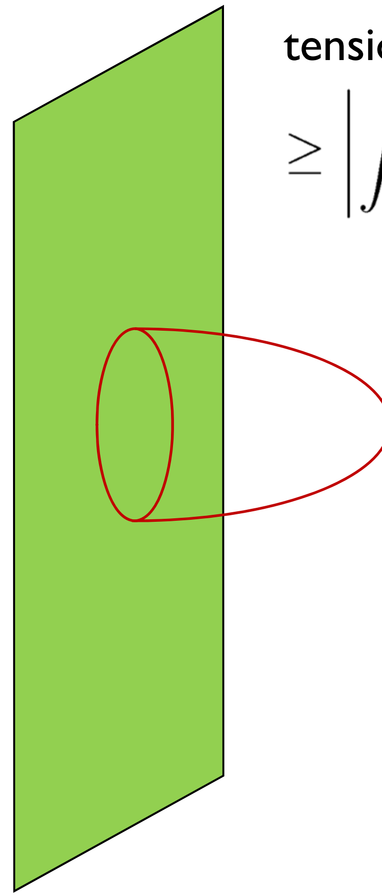
large volume (= small α') limits of the disk amplitudes



RR charges ~
$\Omega(\mathcal{F},\mathcal{R})$

$\sum_n C_{n+1}$

RR vertex
insertion

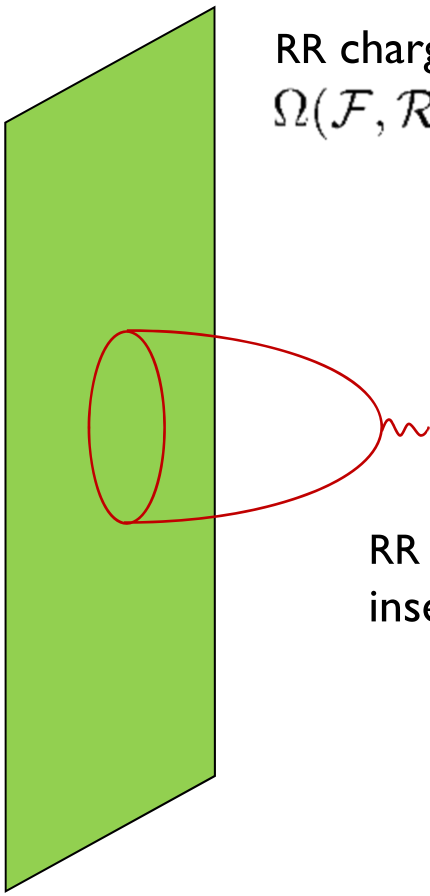


tension/mass ~

$$\geq \left| \int e^{B-iJ} \wedge \Omega(\mathcal{F},\mathcal{R}) \right|$$

central charges

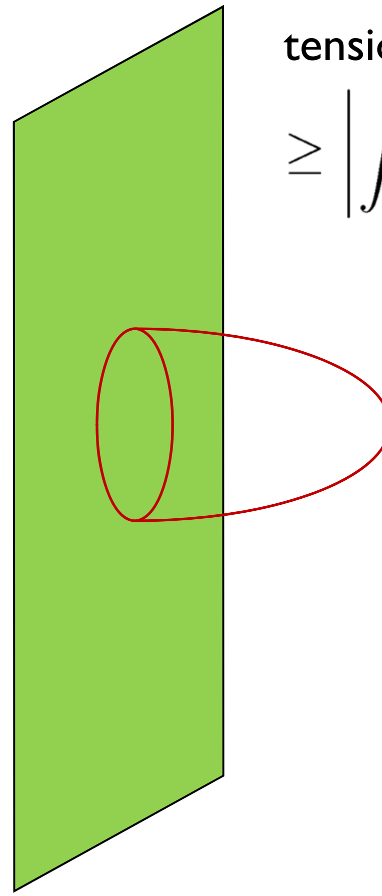
α' exact disk amplitudes ?



RR charges ~
$\Omega(\mathcal{F},\mathcal{R})$

$$\sum_n C_{n+1}$$

RR vertex
insertion



tension/mass ~

$$\geq \left| \int e^{B-iJ} \wedge \Omega(\mathcal{F},\mathcal{R}) \right|$$

central charges

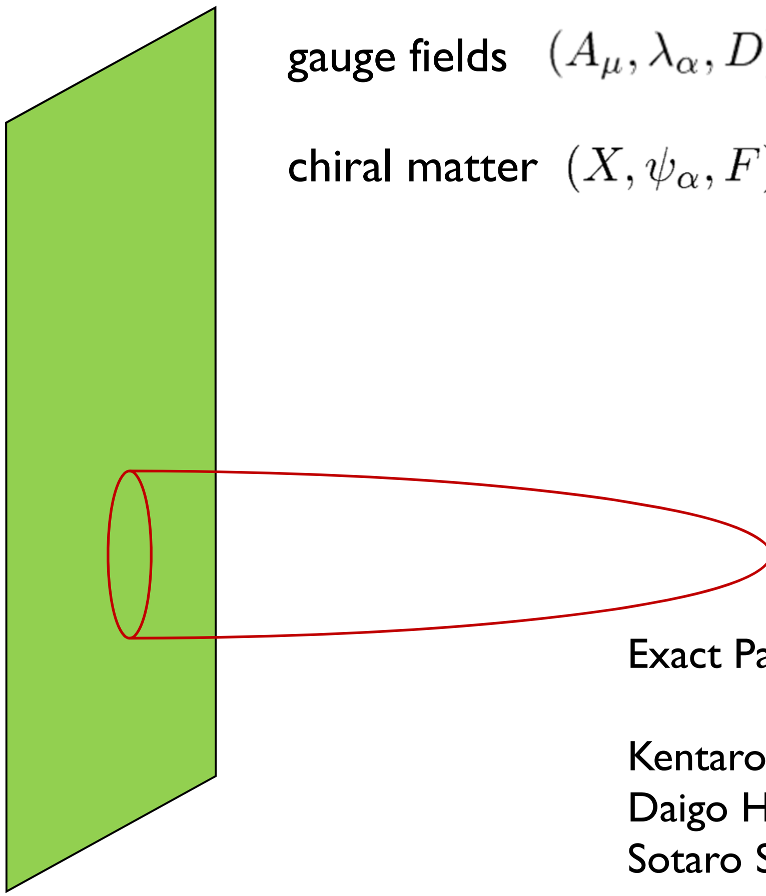
Gauged Linear Sigma Models on elongated D^2

UV field content → N=1 D=4 gauge theory dimensionally reduced to N=(2,2) D=2

D-branes

gauge fields $(A_\mu, \lambda_\alpha, D)^a$

chiral matter $(X, \psi_\alpha, F)^I$



Exact Partition Functions on D^2 :

Kentaro Hori, Mauricio Romo 1308.2438 [hep-th]
Daigo Honda, Takuya Okuda 1308.2217 [hep-th]
Sotaro Sugishita, Seiji Terashima 1308.1973 [hep-th]

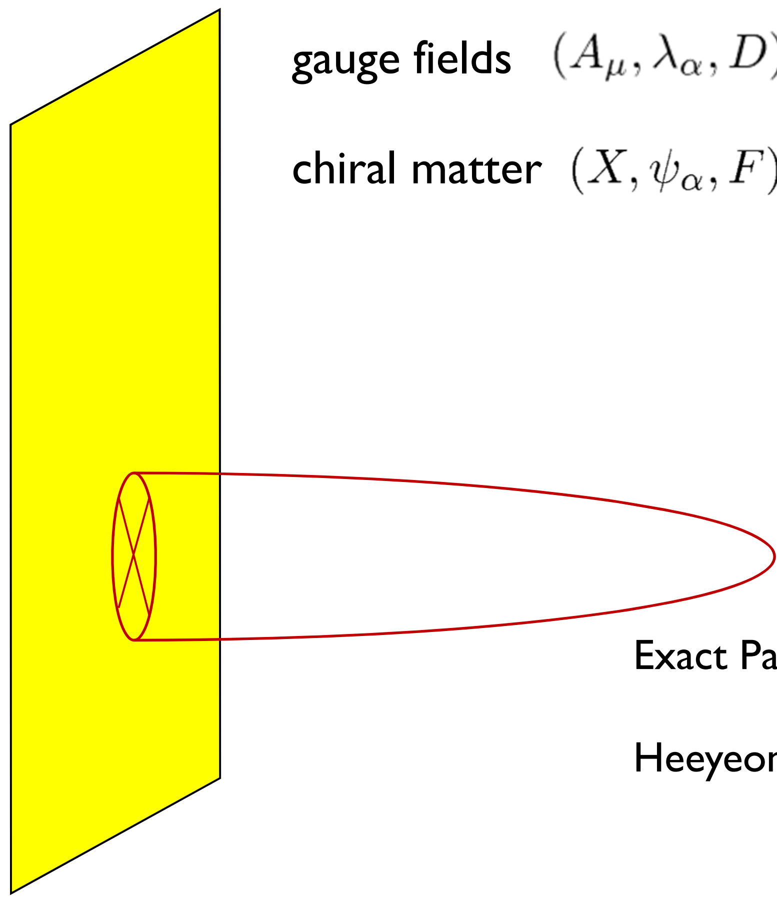
Gauged Linear Sigma Models on elongated $RP^2 = S^2/Z_2$

UV field content → N=1 D=4 gauge theory dimensionally reduced to N=(2,2) D=2

gauge fields $(A_\mu, \lambda_\alpha, D)^a$

chiral matter $(X, \psi_\alpha, F)^I$

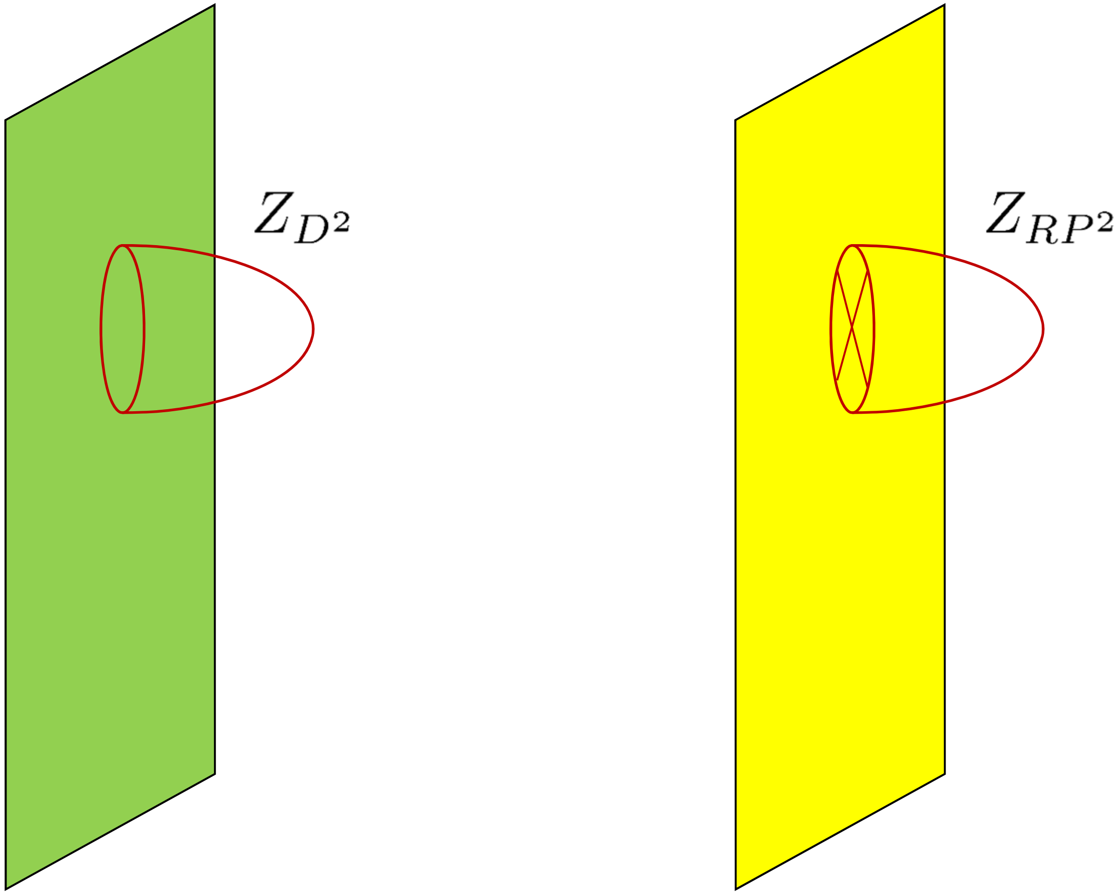
O-planes



Exact Partition Functions on S^2/Z_2 :

Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

what do they compute ?



the canonical example : CY(N-2) hypersurface in CP(N-1)

$$G_N(X) = 0 \qquad \{X^{1,\ldots,N}\}//U(1)$$

$$(A_\mu, \lambda_\alpha, \sigma, D)$$

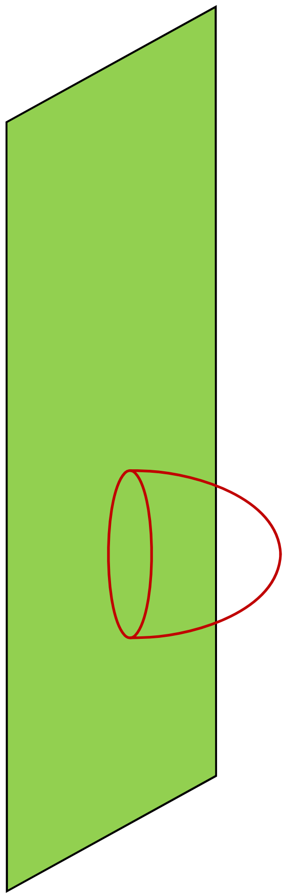
$$(X, \psi_\alpha, F)^{I=1,\cdots,N}_{Q=+1}$$

$$(P, \chi_\alpha, G)_{Q=-N}$$

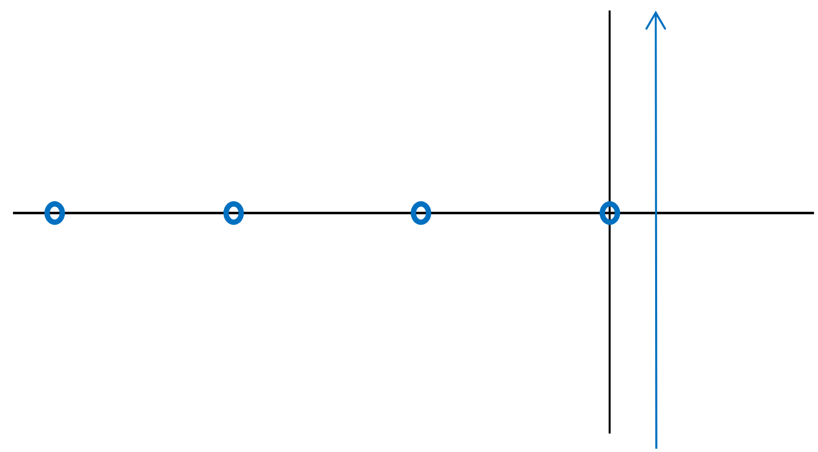
with superpotential : $W(X; P) = P \cdot G_N(X)$

the canonical example : CY(N-2) hypersurface in CP(N-1)

D-brane that wraps
the entire CY(N-2)



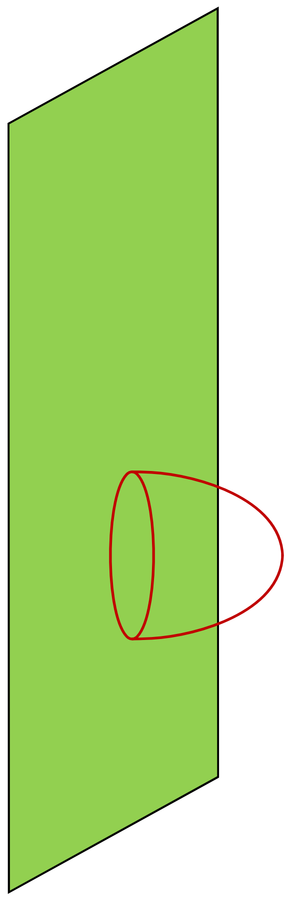
$$Z_{D^2} \sim \int_{0^+ - i\infty}^{0^+ + i\infty} \frac{d\epsilon}{2\pi i} \, e^{2\pi\xi\epsilon - i\theta\epsilon} \times \frac{\Gamma(\epsilon)^N}{\Gamma(N\epsilon)}$$



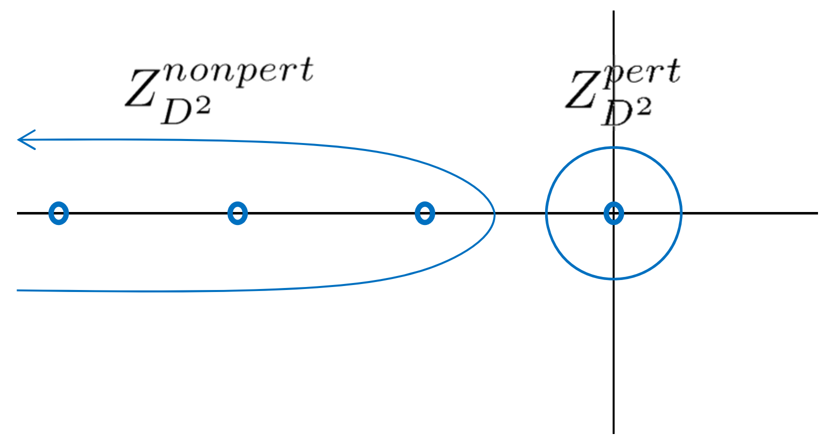
Kentaro Hori, Mauricio Romo 1308.2438 [hep-th]

the canonical example : CY(N-2) hypersurface in CP(N-1)

D-brane that wraps
the entire CY(N-2)



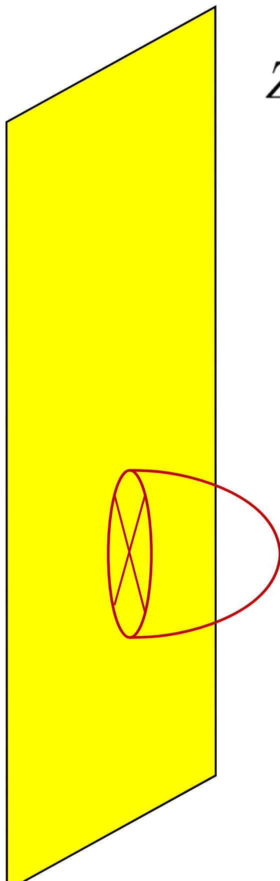
$$Z_{D^2} \sim \int_{0^+-i\infty}^{0^++i\infty} \frac{d\epsilon}{2\pi i}\, e^{2\pi\xi\epsilon - i\theta\epsilon} \times \frac{\Gamma(\epsilon)^N}{\Gamma(N\epsilon)}$$



Kentaro Hori, Mauricio Romo 1308.2438 [hep-th]

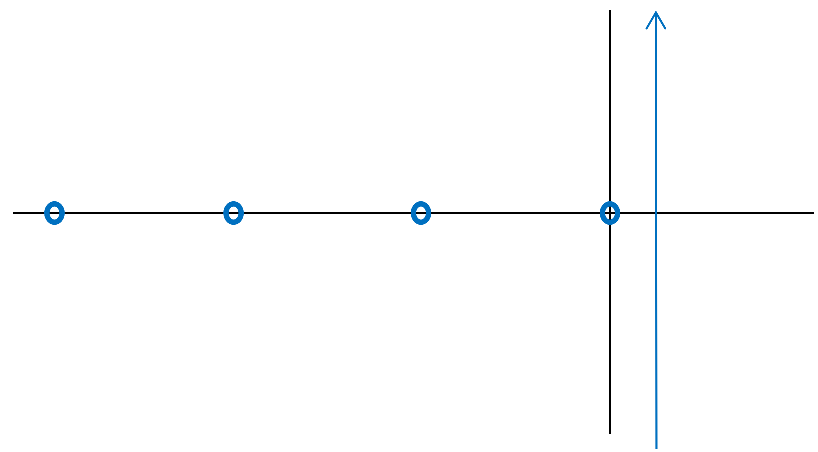
the canonical example : CY(N-2) hypersurface in CP(N-1)

D-plane that wraps
the entire CY(N-2)



$$Z_{RP^2} \sim \int_{0^+ - i\infty}^{0^+ + i\infty} \frac{d\epsilon}{2\pi i} e^{2\pi\xi\epsilon} \left[\frac{\Gamma\left(\frac{\epsilon}{2}\right)\Gamma\left(-\frac{\epsilon}{2}\right)}{\Gamma(-\epsilon)}\right]^N \cdot \left[\frac{\Gamma\left(\frac{1-N\epsilon}{2}\right)\Gamma\left(-\frac{1-N\epsilon}{2}\right)}{\Gamma(-1+N\epsilon)}\right]$$

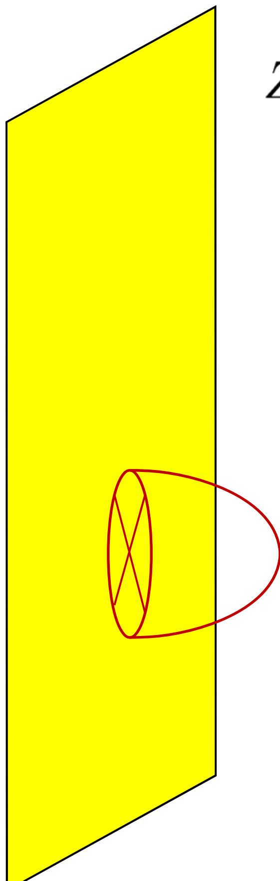
$$\pm \int_{0^+ - i\infty}^{0^+ + i\infty} \frac{d\epsilon}{2\pi i} e^{2\pi\xi\epsilon} \left[\frac{\Gamma(\epsilon)/\epsilon}{\Gamma\left(\frac{\epsilon}{2}\right)\Gamma\left(-\frac{\epsilon}{2}\right)}\right]^N \cdot \left[\frac{\Gamma(1-N\epsilon)/(1-N\epsilon)}{\Gamma\left(\frac{1-N\epsilon}{2}\right)\Gamma\left(-\frac{1-N\epsilon}{2}\right)}\right]$$



Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

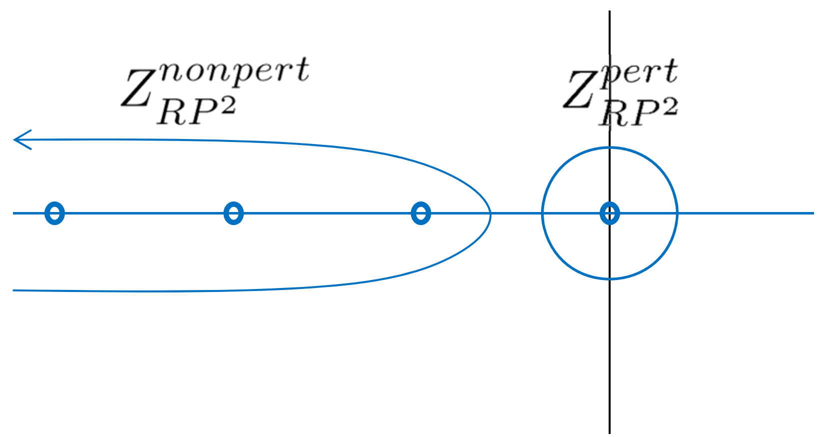
the canonical example : CY(N-2) hypersurface in CP(N-1)

D-plane that wraps
the entire CY(N-2)



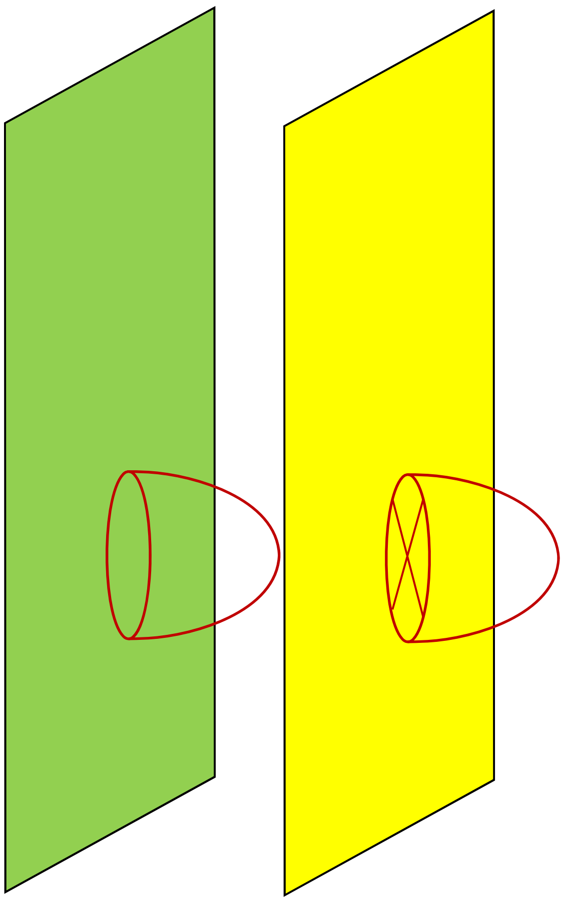
$$Z_{RP^2} \sim \int_{0^+ - i\infty}^{0^+ + i\infty} \frac{d\epsilon}{2\pi i} e^{2\pi\xi\epsilon} \left[\frac{\Gamma\left(\frac{\epsilon}{2}\right)\Gamma\left(-\frac{\epsilon}{2}\right)}{\Gamma(-\epsilon)}\right]^N \cdot \left[\frac{\Gamma\left(\frac{1-N\epsilon}{2}\right)\Gamma\left(-\frac{1-N\epsilon}{2}\right)}{\Gamma(-1+N\epsilon)}\right]$$

$$\pm \int_{0^+ - i\infty}^{0^+ + i\infty} \frac{d\epsilon}{2\pi i} e^{2\pi\xi\epsilon} \left[\frac{\Gamma(\epsilon)/\epsilon}{\Gamma\left(\frac{\epsilon}{2}\right)\Gamma\left(-\frac{\epsilon}{2}\right)}\right]^N \cdot \left[\frac{\Gamma(1-N\epsilon)/(1-N\epsilon)}{\Gamma\left(\frac{1-N\epsilon}{2}\right)\Gamma\left(-\frac{1-N\epsilon}{2}\right)}\right]$$



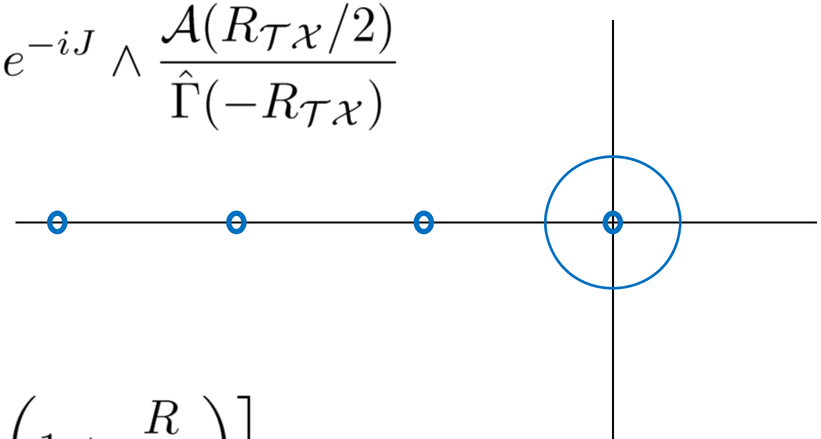
Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

geomerical form of $Z^{pert}_{D^2}$ for worldvolume = Calabi-Yau $\mathcal{X}$



$$Z^{pert}_{D^2} \sim \int_{\mathcal{X}} e^{B-iJ} \wedge \hat{\Gamma}(R_{\mathcal{T}\mathcal{X}})$$

$$Z^{pert}_{RP^2} \sim \pm \int_{\mathcal{X}} e^{-iJ} \wedge \frac{\mathcal{A}(R_{\mathcal{T}\mathcal{X}}/2)}{\hat{\Gamma}(-R_{\mathcal{T}\mathcal{X}})}$$

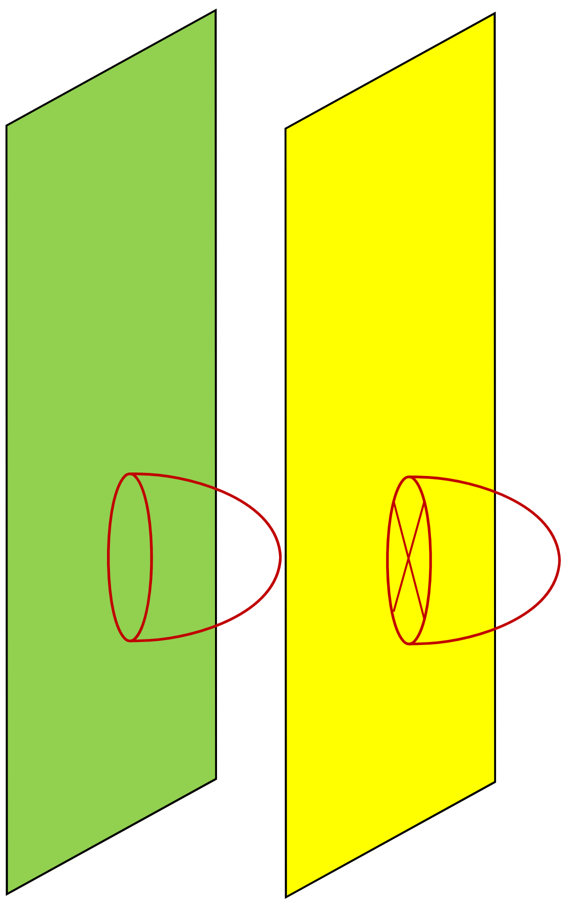


$$\hat{\Gamma}(R) \equiv \det\left[\Gamma\left(1+\frac{R}{2\pi i}\right)\right]$$

$$= \sqrt{\mathcal{A}(R)} \wedge e^{i\sum_{k\geq 0} \frac{(-1)^k (2k)!\zeta(2k+1)}{(2\pi)^{2k}} ch_{2k+1}(R)}$$

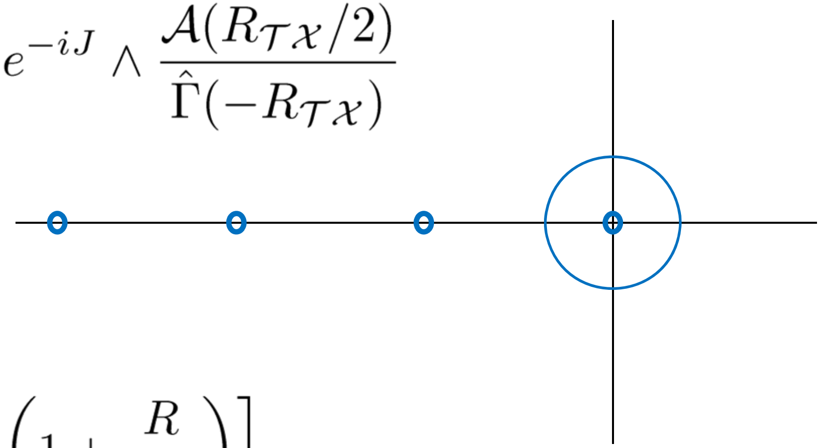
Kentaro Hori, Mauricio Romo 1308.2438 [hep-th]
Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

geomerical form of $Z^{pert}_{D^2}$ for worldvolume = Calabi-Yau $\mathcal{X}$



$$Z^{pert}_{D^2} \sim \int_{\mathcal{X}} e^{B-iJ} \wedge \hat{\Gamma}(R_{T\mathcal{X}})$$

$$Z^{pert}_{RP^2} \sim \pm \int_{\mathcal{X}} e^{-iJ} \wedge \frac{\mathcal{A}(R_{T\mathcal{X}}/2)}{\hat{\Gamma}(-R_{T\mathcal{X}})}$$

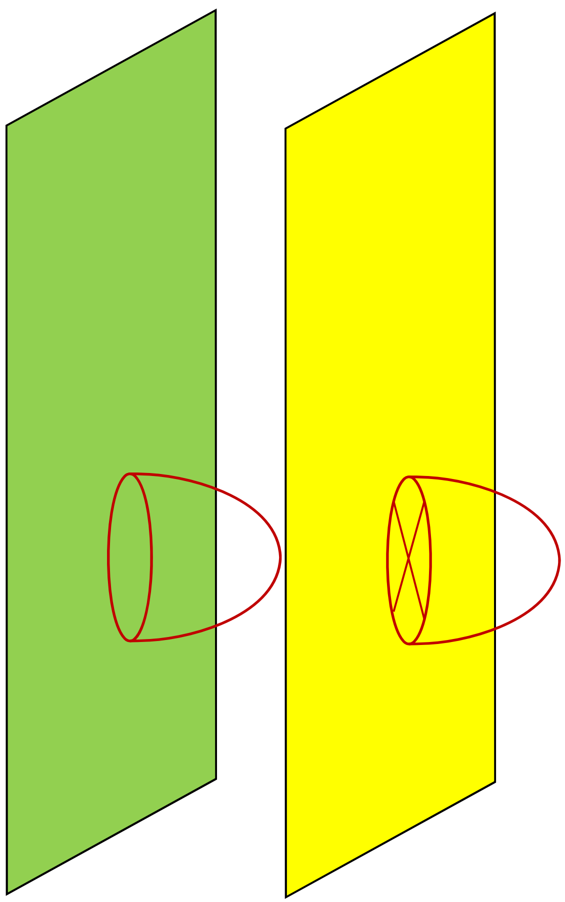


$$\hat{\Gamma}(R) \equiv \det\left[\Gamma\left(1+\frac{R}{2\pi i}\right)\right]$$

$$\mathcal{A}(R) = \hat{\Gamma}(R)\hat{\Gamma}(-R)$$

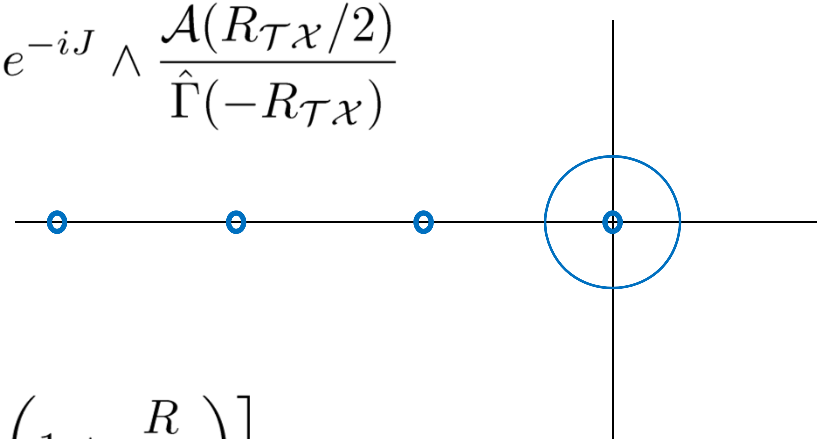
Kentaro Hori, Mauricio Romo 1308.2438 [hep-th]
Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

geomerical form of $Z^{pert}_{D^2}$ for worldvolume = Calabi-Yau $\mathcal{X}$



$$Z^{pert}_{D^2} \sim \int_{\mathcal{X}} e^{B-iJ} \wedge \hat{\Gamma}(R_{\mathcal{T}\mathcal{X}})$$

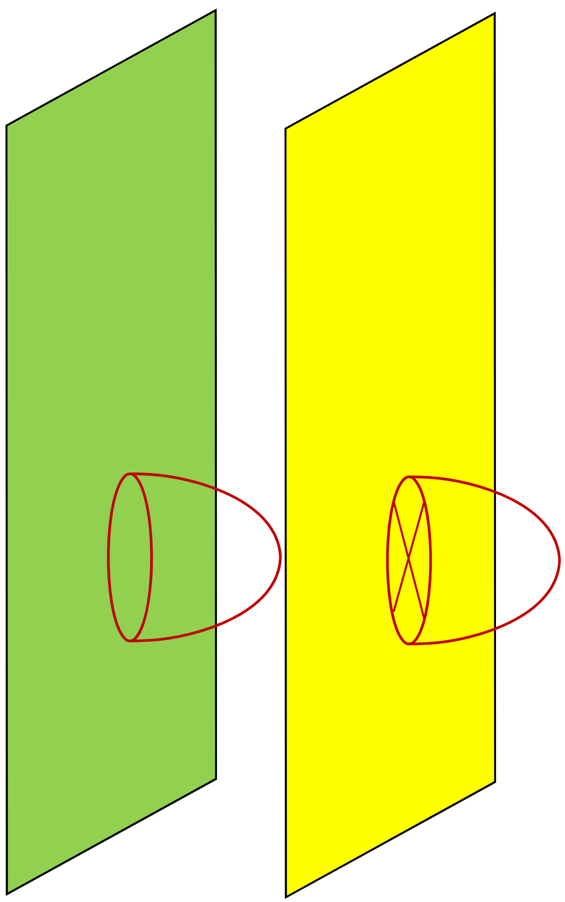
$$Z^{pert}_{RP^2} \sim \pm \int_{\mathcal{X}} e^{-iJ} \wedge \frac{\mathcal{A}(R_{\mathcal{T}\mathcal{X}}/2)}{\hat{\Gamma}(-R_{\mathcal{T}\mathcal{X}})}$$



$$\hat{\Gamma}(R) \equiv \det \left[\Gamma \left(1 + \frac{R}{2\pi i} \right) \right]$$

importance of the Gamma class foretold via mirror symmetry in
A. Libgober, [math.AG/9803119]
H. Iritani. [math.AG/0712.2204]
Katzarkov, M. Kontsevich and T. Pantev, [math.AG/0806.0107]

worldvolume M < Calabi-Yau $\mathcal{X}$

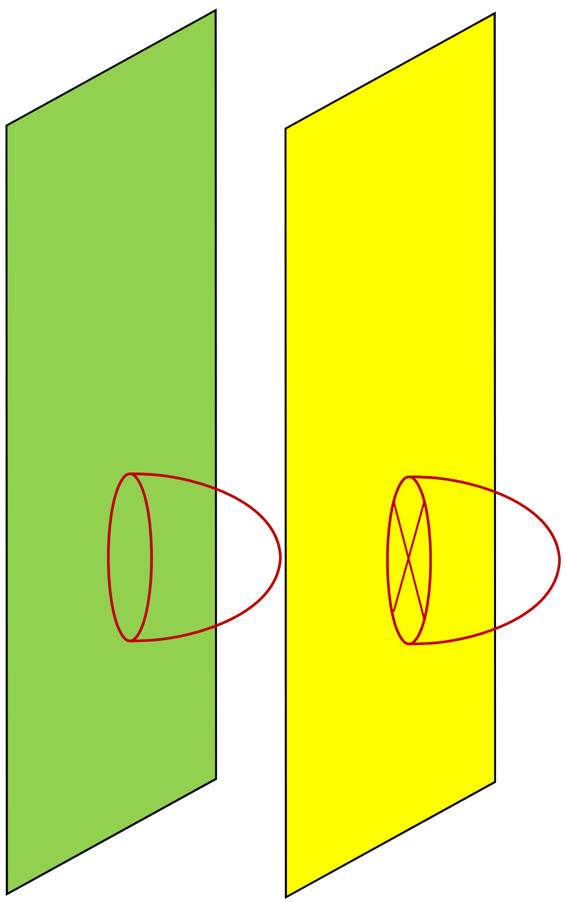


$$Z_{D^2}^{pert} \sim \int_M e^{B-iJ} \wedge \frac{\hat{\Gamma}(R_{\mathcal{T}M})}{\hat{\Gamma}(-R_{\mathcal{N}M})}$$

$$Z_{RP^2}^{pert} \sim \pm 2^{\#} \int_M e^{-iJ} \wedge \frac{\mathcal{A}(R_{\mathcal{T}M}/2)}{\mathcal{A}(R_{\mathcal{N}M}/2)} \wedge \frac{\hat{\Gamma}(R_{\mathcal{N}M})}{\hat{\Gamma}(-R_{\mathcal{T}M})}$$

Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

the Gamma class



$$Z^{pert}_{D^2} \sim \int_M e^{B-iJ} \wedge \frac{\hat{\Gamma}(R_{\mathcal{T}M})}{\hat{\Gamma}(-R_{\mathcal{N}M})}$$

$$Z^{pert}_{RP^2} \sim \pm 2^{\#} \int_M e^{-iJ} \wedge \frac{\mathcal{A}(R_{\mathcal{T}M}/2)}{\mathcal{A}(R_{\mathcal{N}M}/2)} \wedge \frac{\hat{\Gamma}(R_{\mathcal{N}M})}{\hat{\Gamma}(-R_{\mathcal{T}M})}$$

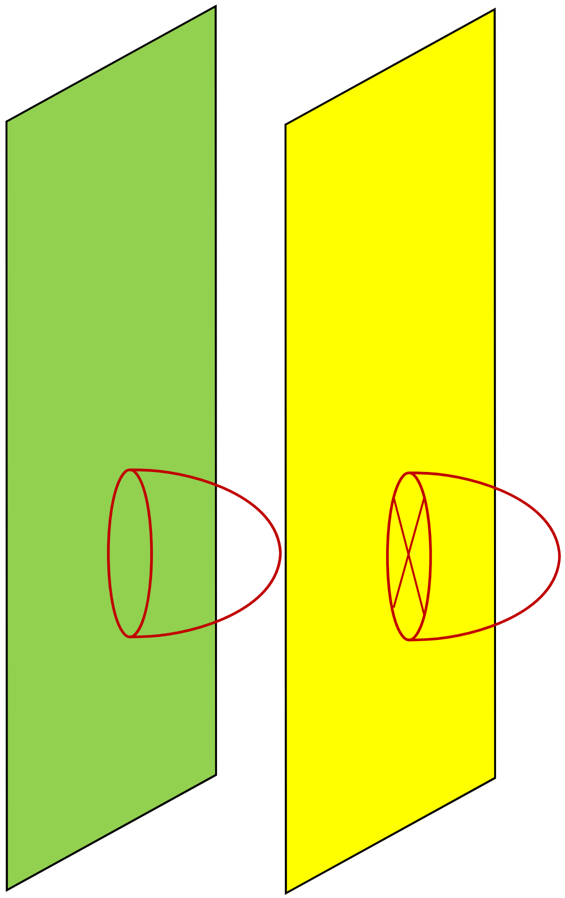
vs.
central charges via
the anomaly inflow

$$\sim \int_M e^{B-iJ} \wedge \sqrt{\frac{\mathcal{A}(R_{\mathcal{T}M})}{\mathcal{A}(R_{\mathcal{N}M})}}$$

$$\sim \pm 2^{\#} \int_M e^{-iJ} \wedge \sqrt{\frac{\mathcal{L}(R_{\mathcal{T}M}/4)}{\mathcal{L}(R_{\mathcal{N}M}/4)}}$$

Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

the Gamma class



$$Z^{pert}_{D^2} \sim \int_M e^{B-iJ} \wedge \frac{\hat{\Gamma}(R_{\mathcal{T}M})}{\hat{\Gamma}(-R_{\mathcal{N}M})}$$

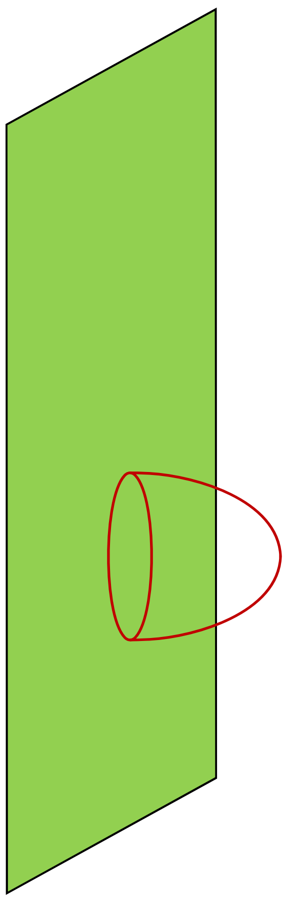
$$Z^{pert}_{RP^2} \sim \pm 2^{\#} \int_M e^{-iJ} \wedge \frac{\mathcal{A}(R_{\mathcal{T}M}/2)}{\mathcal{A}(R_{\mathcal{N}M}/2)} \wedge \frac{\hat{\Gamma}(R_{\mathcal{N}M})}{\hat{\Gamma}(-R_{\mathcal{T}M})}$$

vs.
central charges via
the anomaly inflow

$$\sqrt{\mathcal{A}(R)} \quad \rightarrow \quad \hat{\Gamma}(\pm R)$$

Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

the Gamma class α'-corrects the volume



$$Z_{D^2}^{pert} \sim \int_M e^{B-iJ} \wedge \frac{\hat{\Gamma}(R_{\mathcal{T}M})}{\hat{\Gamma}(-R_{\mathcal{N}M})}$$

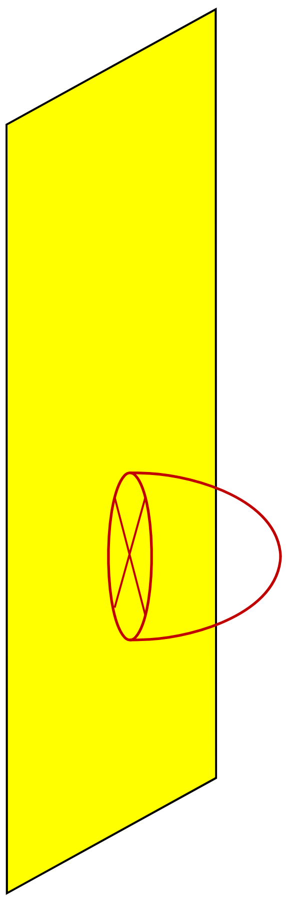
$$= \int_M e^{B-iJ} \wedge \sqrt{\frac{\mathcal{A}(R_{\mathcal{T}M})}{\mathcal{A}(R_{\mathcal{N}M})}} \wedge \sqrt{\frac{\hat{\Gamma}(R_{\mathcal{T}\mathcal{X}})}{\hat{\Gamma}(-R_{\mathcal{T}\mathcal{X}})}}$$

$$= \int_M \boxed{e^{B-iJ} \wedge \sqrt{\frac{\hat{\Gamma}(R_{\mathcal{T}\mathcal{X}})}{\hat{\Gamma}(-R_{\mathcal{T}\mathcal{X}})}}} \wedge \sqrt{\frac{\mathcal{A}(R_{\mathcal{T}M})}{\mathcal{A}(R_{\mathcal{N}M})}}$$

$$= \left[e^{B-iJ}\right]_{\alpha'\text{-exact}}$$

Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

the Gamma class α'-corrects the volume



$$Z^{pert}_{RP^2} \sim \pm 2^{\#} \int_M e^{-iJ} \wedge \frac{\mathcal{A}(R_{\mathcal{T}M}/2)}{\mathcal{A}(R_{\mathcal{N}M}/2)} \wedge \frac{\hat{\Gamma}(R_{\mathcal{N}M})}{\hat{\Gamma}(-R_{\mathcal{T}M})}$$

$$= \pm 2^{\#} \int_M e^{-iJ} \wedge \sqrt{\frac{\mathcal{L}(R_{\mathcal{T}M}/4)}{\mathcal{L}(R_{\mathcal{N}M}/4)}} \wedge \sqrt{\frac{\hat{\Gamma}(R_{\mathcal{T}X})}{\hat{\Gamma}(-R_{\mathcal{T}X})}}$$

$$= \pm 2^{\#} \int_M \boxed{e^{-iJ} \wedge \sqrt{\frac{\hat{\Gamma}(R_{\mathcal{T}X})}{\hat{\Gamma}(-R_{\mathcal{T}X})}}} \wedge \sqrt{\frac{\mathcal{L}(R_{\mathcal{T}M}/4)}{\mathcal{L}(R_{\mathcal{N}M})/4}}$$

$$= \left[e^{-iJ}\right]_{\alpha'\text{-exact}}$$

Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

the Gamma class α'-corrects the volume

$$\left[e^{-iJ}\right]_{\alpha'\text{-exact}}$$

$$= e^{-iJ} \wedge \sqrt{\frac{\hat{\Gamma}(R_{\mathcal{T}\mathcal{X}})}{\hat{\Gamma}(-R_{\mathcal{T}\mathcal{X}})}}$$

$$= \exp\left(-iJ + i\sum_{k\geq 1}\frac{(-1)^k(2k)!\zeta(2k+1)}{(2\pi)^{2k}}ch_{2k+1}(\mathcal{T}\mathcal{X})\right)$$

Heeyeon Kim, Sungjay Lee, P.Y. 1310.4505 [hep-th]

summary & outlook

exact GLSM partition functions on S^2, RP^2, hemisphere

α' exact moduli space metric for Kaehler moduli of CY

α' exact central charges for D/O wrapped on B-cycles

appearance of Gamma class & α' exact quantum volume
→ anomaly inflow intact

with already exact complex moduli space and D/O on A-cycle,
this offers a comprehensive and direct method of computing
stringy corrections to part of L_{4D} exactly

issues associated with Spin^c worldvolume resolved partially

systematic answer to unbroken spacetime supersymmetry unaddressed