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Cosmology of the Higgs Field

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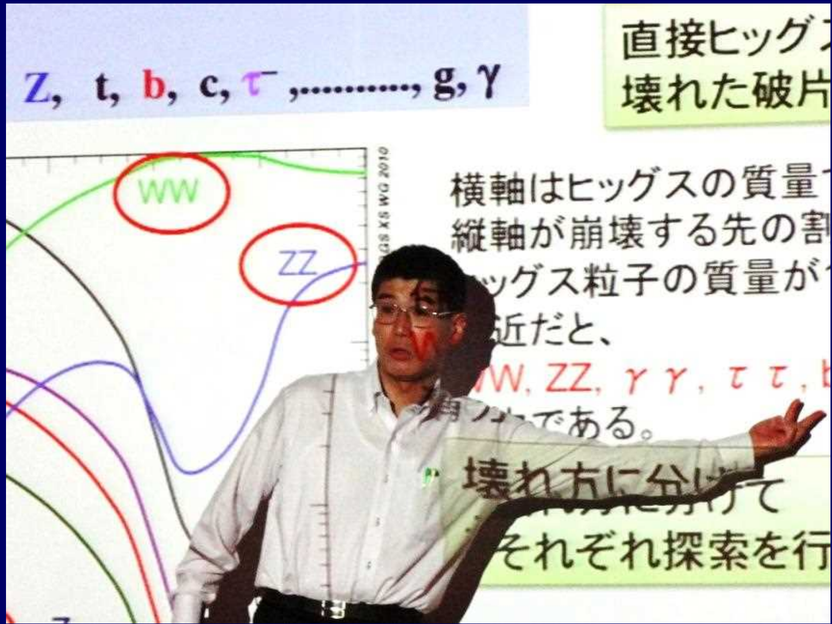
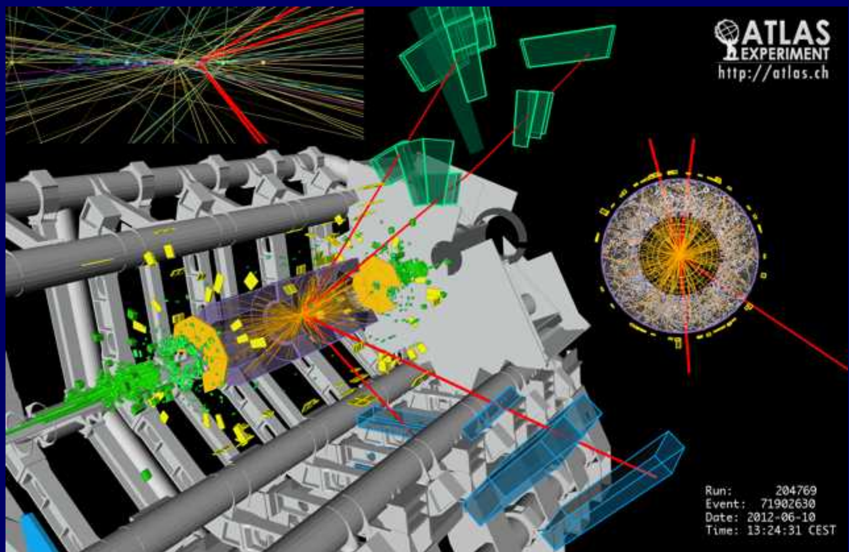
横山順一

The SM Higgs particle was finally discovered !? 2012/7/4



Professor Shoji Asai at the press conference together with his view graph on July 4.



A candidate event from a Higgs particle observed by ATLAS (© ATLAS experiment, by courtesy of S. Asai.)

ATLAS

$$m_h \cong 126.5\mathrm{GeV} \quad 5\sigma$$

$$H \to \gamma\gamma$$

$$H \to ZZ \to 4leptons$$

6% human resources,
10% $¥ from Japan

CMS

$$m_h = 125.3 \pm 0.6\mathrm{GeV} \quad 4.9\sigma$$

$$H \to \gamma\gamma$$

$$H \to ZZ \to 4l$$

$$H \to WW \to 2l2\nu$$

http://aappsbulletin.org/

The discovery of a scalar particle/field has a great impact not only to particle physics but also to cosmology, since scalar fields play very important roles to drive inflation in the early Universe and generate cosmological perturbations.

Here we discuss Cosmology of the Higgs Field in several distinct cases:

I. The Higgs field is NOT the inflaton but a subdominant field during inflation.

Kunimitsu & JY Phys.Rev. D86 (2012) 083541

II. The Higgs field itself is responsible for inflation in the early Universe.

Kamada, Kobayashi, Kunimitsu, Yamaguchi & JY 1309.7410
Kamada, Kobayashi, Yamaguchi & JY Phys.Rev. D86 (2012) 023504
Kamada, Kobayashi, Yamaguchi & JY Phys.Rev. D83 (2011) 083515

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I. The case Higgs field is NOT the "Inflaton."

Whatever the inflation model is, the Higgs field exists in the Standard Model and plays some roles.

A well known peculiar property of a practically massless field in the de Sitter (exponentially expanding) spacetime: (Bunchi & Davis 78, Vilenkin & Ford 82…)

$$\langle \varphi(\boldsymbol{x},t)^2 \rangle = \left(\frac{H}{2\pi}\right)^2 Ht \approx$$ Brownian motion with step $\pm\frac{H}{2\pi}$ and interval H^{-1}

In each Hubble time H^{-1}, quantum fluctuations with an amplitude $\delta\varphi \approx \pm\frac{H}{2\pi}$ and the initial wavelength $\lambda \approx H^{-1}$ is generated and stretched by inflation continuously.

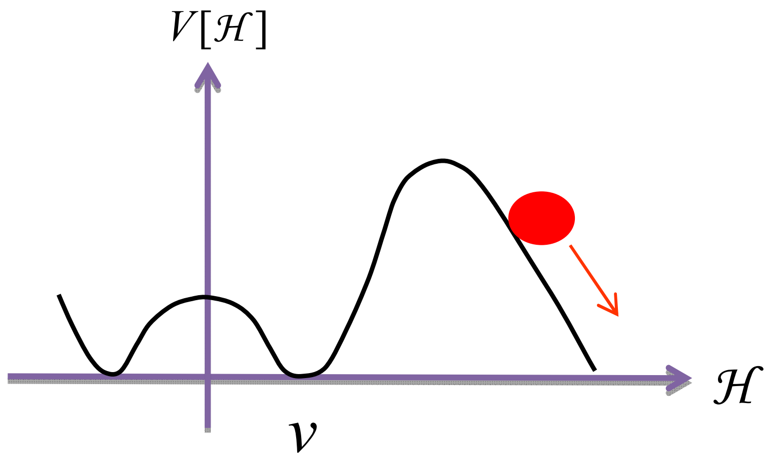
During inflation, short-wave quantum fluctuations are continuously generated for the Higgs field and its wave length is stretched by inflation to yield non-vanishing expectation values. ⇒ **Higgs Condensation**

The most dramatic case

If the self coupling $\lambda(\mu)$ becomes negative at high energy or high field value, the potential becomes unstable.

$$V[\varphi] \approx \frac{\lambda(\varphi)}{4}\varphi^4$$

φ : real and neutral component of the Higgs field



Our Universe cannot be reproduced in such a case.

(Espinosa et al 2008)

- We consider the case λ is positive (and constant) in the regime of our interest. $V[\varphi]=\frac{\lambda(\mu)}{4}\varphi^4 \cong \frac{\lambda}{4}\varphi^4, \quad \lambda = O(10^{-2})>0.$

- The behavior of a scalar field with such a potential during inflation has been studied using the stochastic inflation method.

(Starobinsky and JY 1994)

- Decompose the scalar field as

$$\varphi(\boldsymbol{x},t)=\overline{\varphi}(\boldsymbol{x},t)+\int\frac{d^3k}{(2\pi)^{3/2}}\theta(k-\varepsilon a(t)H)\left[a_{\boldsymbol{k}}\varphi_{\boldsymbol{k}}(t)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}+a_{\boldsymbol{k}}^{\dagger}\varphi_{\boldsymbol{k}}^{*}(t)e^{i\boldsymbol{k}\cdot\boldsymbol{x}}\right]$$

coarse-grained mode
long wavelength

short-wave quantum fluctuations

ε : a small parameter

act as a stochastic noise
due to cosmic expansion

Langevin eq. for long wave mode $\dot{\overline{\varphi}}(\boldsymbol{x},t)=-\frac{1}{3H}V'(\overline{\varphi})\boxed{+f(\boldsymbol{x},t)}$ stochastic noise term

$$\langle f(\boldsymbol{x}_1,t_1)f(\boldsymbol{x}_2,t_2)\rangle=\frac{H^3}{4\pi^2}\delta(t_1-t_2)j_0(\varepsilon a(t)H|\boldsymbol{x}_1-\boldsymbol{x}_2|) \qquad j_0(z)=\frac{\sin z}{z}$$

★ We can derive a Fokker-Planck equation for the one-point (one-domain) probability distribution function (PDF) $\rho_1[\overline{\varphi}(\boldsymbol{x},t)=\varphi] \equiv \rho_1[\varphi(\boldsymbol{x},t)] = \rho_1(\varphi,t)$

$$\frac{\partial \rho_1[\varphi(\mathbf{x},t)]}{\partial t} = \frac{1}{3H}\frac{\partial}{\partial \varphi}\{V'[\varphi(\mathbf{x},t)]\rho_1[\varphi(\mathbf{x},t)]\} + \frac{H^3}{8\pi^2}\frac{\partial^2 \rho_1[\varphi(\mathbf{x},t)]}{\partial \varphi^2} \equiv \Gamma_\varphi \rho_1[\varphi(\mathbf{x},t)].$$

★ Its generic solution can be expanded as

$$\rho_1(\varphi,t) = \exp\left(-\frac{4\pi^2 V(\varphi)}{3H^4}\right)\sum_{n=0}^{\infty} a_n \Phi_n(\varphi) e^{-\Lambda_n(t-t_0)}$$

$\Phi_n(\varphi)$ is the complete orthonormal set of eigenfunctions of the Schrödinger-type eq.

$$\left[-\frac{1}{2}\frac{\partial^2}{\partial \varphi^2} + W(\varphi)\right]\Phi_n(\varphi) = \frac{4\pi^2 \Lambda_n}{H^3}\Phi_n(\varphi),$$

$$W(\varphi) \equiv \frac{1}{2}\left[v'(\varphi)^2 - v''(\varphi)\right],$$

$$v(\varphi) \equiv \frac{4\pi^2}{3H^4}V(\varphi).$$

★ If $N \equiv \int_{-\infty}^{\infty} e^{-2v(\varphi)} d\varphi$ is finite, we find $\Lambda_0 = 0$ and the equilibrium PDF exists.

$$\rho_1(\varphi,t) = \rho_{1\mathrm{eq}}(\varphi) + \exp\left(-\frac{4\pi^2 V(\varphi)}{3H^4}\right)\sum_{n=1}^{\infty} a_n \Phi_n(\varphi) e^{-\Lambda_n(t-t_0)},$$

Each solution relaxes to the equilibrium PDF at late time which has the de Sitter invariance.

★For $V(\varphi)=\frac{\lambda}{4}\varphi^4$ we find an equilibrium PDF corresponding to $\Lambda_0=0$ as

$$\rho_{1\mathrm{eq}}(\varphi)=\left(\frac{32\pi^2\lambda}{3}\right)^{\frac{1}{4}}\frac{1}{\Gamma(\frac{1}{4})H}\exp\left(-\frac{2\pi^2\lambda\varphi^4}{3H^4}\right)$$

★First few eigenvalues (numerically obtained)

$$\Lambda_0=0,\qquad \underbrace{\Lambda_1=1.36859\sqrt{\frac{\lambda}{24\pi^2}}H}_{\sim 10^{-2}H},\qquad \Lambda_2=4.4537\sqrt{\frac{\lambda}{24\pi^2}}H,$$

★Equilibrium expectation values obtained using $\rho_{1\mathrm{eq}}(\varphi)$

$$\langle\varphi^2\rangle=\sqrt{\frac{3}{2\pi^2}}\frac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}\frac{H^2}{\sqrt{\lambda}}\simeq 0.132\frac{H^2}{\sqrt{\lambda}}=1.32\tilde{\lambda}^{-\frac{1}{2}}H^2\ \approx H^2$$

$$m_{\mathrm{eff}}^2\equiv\lambda\langle\varphi^2\rangle\simeq 1.32\times 10^{-2}\tilde{\lambda}^{\frac{1}{2}}H^2 \ll H^2$$

$$\langle V(\varphi)\rangle=\frac{\lambda}{4}\langle\varphi^4\rangle=\frac{3H^4}{32\pi^2}\simeq 9.50\times 10^{-3}H^4 \quad \ll H^4$$

where $\tilde{\lambda}\equiv 10^2\lambda\sim 1.$

Higgs condensation

Properties of the Higgs condensation

★ Two point equilibrium temporal correlation function can be well approximated by

$$G(t_1 - t_2) \equiv \langle \varphi(\mathbf{x}, t_1)\varphi(\mathbf{x}, t_2)\rangle \simeq \langle \varphi^2 \rangle e^{-\Lambda_1 |t_1 - t_2|} \quad \text{for} \quad |t_1 - t_2| \gtrsim H^{-1}.$$

★ The correlation time defined by $G(t_c) = \frac{1}{2}G(0)$ is given by $t_c \simeq 76.2\tilde{\lambda}^{-\frac{1}{2}} H^{-1}.$

★ The spatial correlation function can be evaluated from it using the de Sitter invariance.

$$G(r) \equiv \langle \varphi(\mathbf{x}_1, t)\varphi(\mathbf{x}_2, t)\rangle \simeq \langle \varphi^2 \rangle (Ha(t)r)^{-\frac{2\Lambda_1}{H}} \qquad r \equiv |\mathbf{x}_1 - \mathbf{x}_2|.$$

cf de Sitter invariant separation

$$z(x_1, x_2) = \cosh H(t_1 - t_2) - \frac{H^2}{2} a_0^2 e^{Ht_1 + Ht_2} |\mathbf{x}_1 - \mathbf{x}_2|^2 \qquad a(t) = a_0 e^{Ht}$$

★ The spatial correlation length defined by $G(r_c) = \frac{1}{2}G(0)$ therefore reads

$$a(t) r_c = \frac{H^{-1}}{2} e^{\frac{H}{2\Lambda_1}} \simeq \frac{H^{-1}}{2} e^{38.1\tilde{\lambda}^{-\frac{1}{2}}}$$

Exponentially large !

So we can treat it as a homogeneous field when we discuss its effect on reheating.

★ The energy density of the Higgs condensation remains constant until the Hubble parameter decreases to $m_{\rm eff} \simeq 0.115\tilde{\lambda}^{1/4} H_{\rm inf}$.
After this epoch it decreases in proportion to $a^{-4}(t)$ since it has a quartic potential.

$$\rho_{\rm cond} = \frac{3H_{\rm inf}^4}{32\pi^2} \simeq 9.50 \times 10^{-3} H_{\rm inf}^4 \qquad \propto a^{-4}(t)$$

★ At this time we find $\rho_{tot} = 3M_G^2 m_{\rm eff}^2 \simeq 3.96\tilde{\lambda}^{\frac{1}{2}} M_G^2 H_{\rm inf}^2 \quad \left(M_G \equiv m_{Pl}/\sqrt{8\pi}\right)$

namely, $\rho_{tot} \gg \rho_{\rm cond}$.

★ So the Higgs condensation or its decay products NEVER contributes to the total energy density appreciably in the usual inflation models, because

$\rho_{tot} \propto a^{-3}(t)$ in the field oscillation regime before reheating,

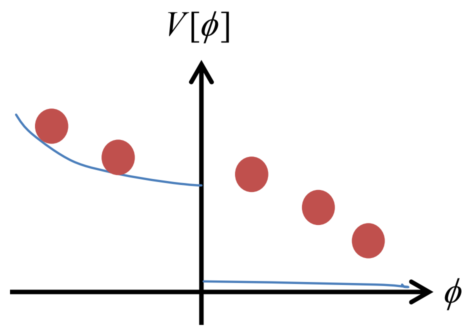
$\rho_{tot} \propto a^{-4}(t)$ after reheating.

★ The story is completely different in inflation modes where the total energy density decreases more rapidly than $a^{-4}(t)$ after inflation.

★ In some models, inflation may end abruptly without being followed by its coherent field oscillation and reheating proceeds only through gravitational particle creation.

★ Such inflation models include k-inflation, (original) G-inflation, and quintessential inflation models.

★ In such models the Higgs condensation contributes to the total energy density appreciably in the end.



Quintessential inflation

(Peebles & Vilenkin 99)

A simple k-inflation example (Armendariz-Picon, Damour & Mukhanov 99)

$$\mathcal{L} = K_1(\phi_{\rm inf})X + K_2(\phi_{\rm inf})X^2, \quad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi_{\rm inf}\partial_\nu\phi_{\rm inf}$$

$\phi_{\rm inf}$: inflaton

★ If K_1 and K_2 are constants with opposite sign, the kinetic function has an attractor solution $X = -\frac{K_1}{2K_2} > 0.$

★ Then the energy density and pressure are given by

$$\rho = 2X\frac{\partial\mathcal{L}}{\partial X} - \mathcal{L} = -P = \text{constant},$$

➡ inflation with $H_{\rm inf}^2 = \dfrac{K_1^2}{12M_G^2K_2}$

★ k-inflation ends when K_1 and K_2 both becomes positive. Then the kinetic energy starts to redshift quickly and only the first term of the Lagrangian becomes relevant.

$K_1 K_2 < 0$	k-inflation	$a(t) \propto e^{Ht}$	
$K_1 K_2 > 0$	Free scalar field	$a(t) \propto t^{1/3}$	kination

If this change occurs abruptly,
numerical calculation shows that this transition occurs within Hubble time.

★ Gravitational particle production takes place due to this rapid change of the expansion law.

Vacuum state in de Sitter space is different from that in the power-law expanding Universe.

$$\langle 0_{dS} | a^{\dagger}_{powerlaw\mathbf{k}} a_{powerlaw\mathbf{k}} | 0_{dS} \rangle \neq 0$$

★ The energy density of a massless boson created this way is given by

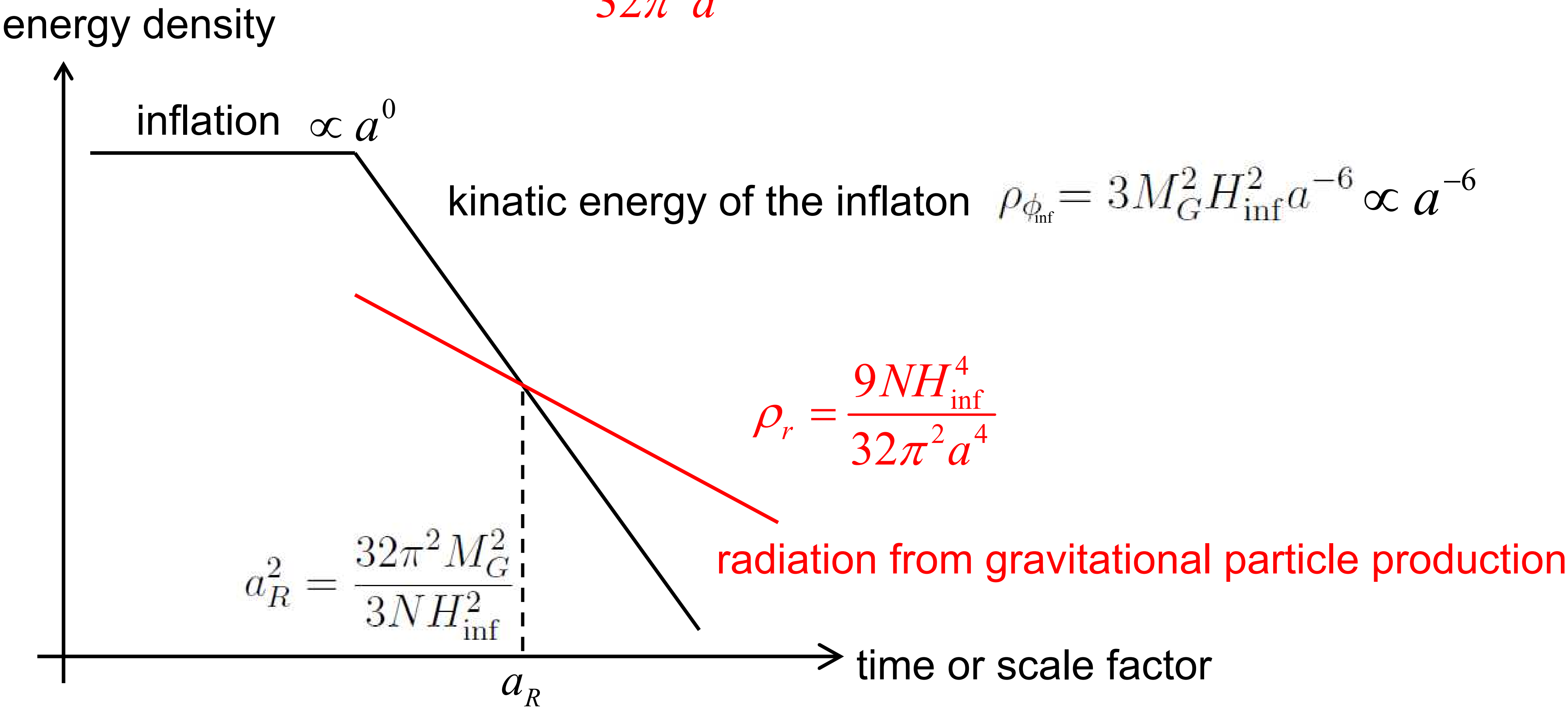
$$\rho_r \cong \frac{9H_{\rm inf}^4}{32\pi^2 a^4}$$

(*cf* Ford 87)

taking $a = 1$ at the end of inflation.

★ Let us assume there are effectively N such modes and neglect the logarithmic factor.

$$\rho_r = \frac{9NH_{\rm inf}^4}{32\pi^2 a^4}$$



$$T_R = \frac{3N^{\frac{3}{4}}}{(32\pi^2)^{\frac{3}{4}}}\left(\frac{30}{\pi^2 g_*}\right)^{\frac{1}{4}}\frac{H_{\rm inf}^2}{M_G} \simeq 3.9\times 10^6 N^{\frac{3}{4}}\left(\frac{g_*}{106.75}\right)^{-\frac{1}{4}}\left(\frac{r}{0.01}\right)\mathrm{GeV}.$$

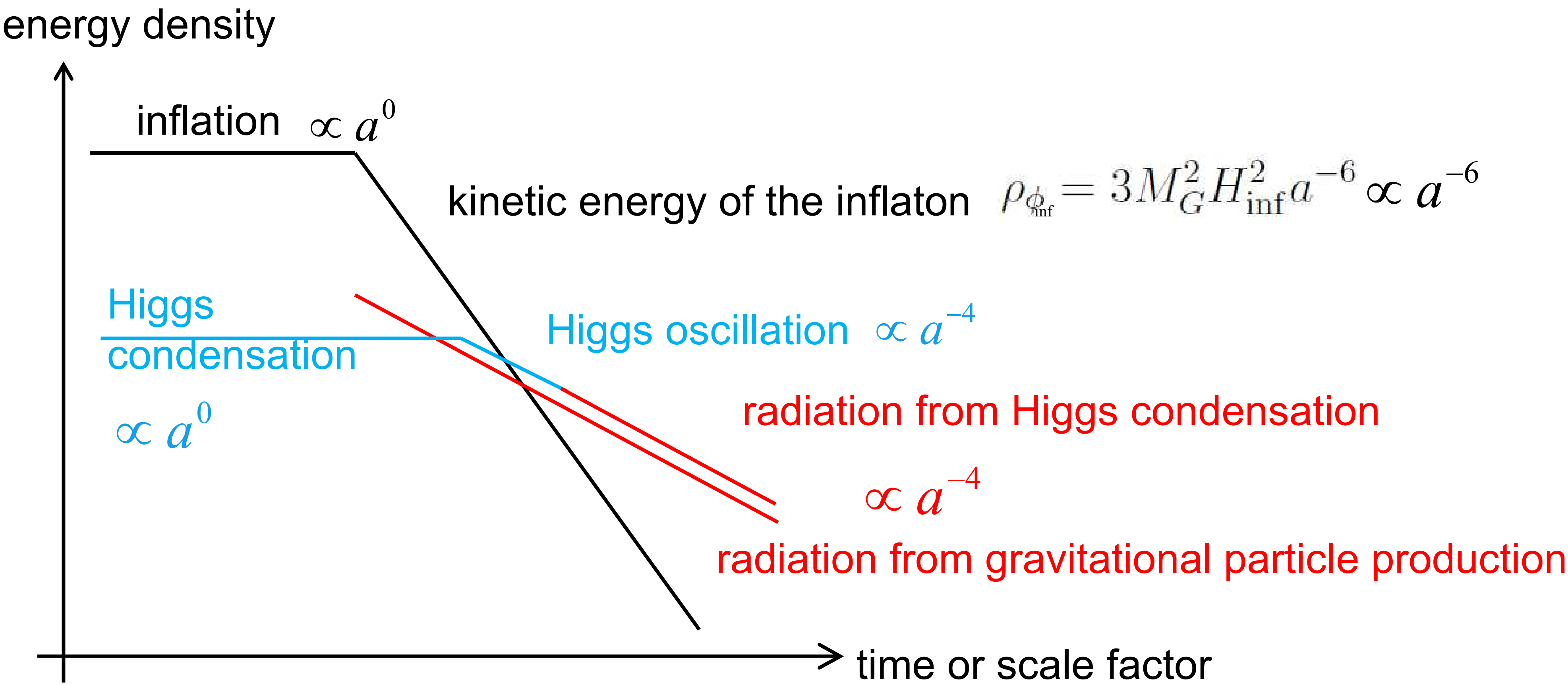
$r = 0.01\left(\frac{H_{\rm inf}}{2.4\times 10^{13}\mathrm{GeV}}\right)^2$ is the tensor-scalar ratio.

We incorporate the Higgs condensation. When it starts oscillation $@H(t) = m_{\rm eff}$

$$\rho_{\phi_{\rm inf}} = 3M_G^2 m_{\rm eff}^2 \simeq 3.96\tilde{\lambda}^{\frac{1}{2}} M_G^2 H_{\rm inf}^2$$

$$\rho_r = \frac{9NH_{\rm inf}^4}{32\pi^2}\left(\frac{m_{\rm eff}}{H_{\rm inf}}\right)^{\frac{4}{3}} \simeq 1.59\times 10^{-3} N H_{\rm inf}^4$$

$$\rho_{\rm cond} = \frac{3H_{\rm inf}^4}{32\pi^2} \simeq 9.50\times 10^{-3} H_{\rm inf}^4$$



★ If we try to estimate the reheat temperature from the equality $\rho_\phi = \rho_{\rm cond}$ taking the Higgs condensation into account, we find

$$T_R = 1.8 \times 10^7 \left(\frac{g_*}{106.75}\right)^{-\frac{1}{4}} \left(\frac{r}{0.01}\right) {\rm GeV}$$

which is to be compared with the previous estimate

$$T_R \simeq 3.9 \times 10^6 N^{\frac{3}{4}} \left(\frac{g_*}{106.75}\right)^{-\frac{1}{4}} \left(\frac{r}{0.01}\right) {\rm GeV}.$$

★ Thus the Higgs condensate or its decay product contributes to the total energy density appreciably.

⬇

Its fluctuation can be important.

Higgs condensation as a curvaton

(Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 02)

- Here we analyze properties of fluctuations of the Higgs condensation.

- A power-law spatial correlation function like

$$G(r) \equiv \langle \varphi(\mathbf{x}_1, t)\varphi(\mathbf{x}_2, t)\rangle \simeq \langle \varphi^2 \rangle (Ha(t)r)^{-\frac{2\Lambda_1}{H}}$$

can be obtained from a power law power spectrum $P(k) \equiv |\varphi_k|^2 \equiv Ak^n$ making use of the formula

$$G(R) = \int P(k) e^{-i\mathbf{k}\cdot\mathbf{R}} \frac{d^3k}{(2\pi)^3} = AR^{-n-3}\frac{\Gamma(n+2)}{2\pi^2}\sin\left[(n+2)\frac{\pi}{2}\right]$$

- We find $n = -3 + \dfrac{2\Lambda_1}{H_{\text{inf}}}$ and $A \simeq \dfrac{4\pi^2\Lambda_1}{H_{\text{inf}}}\sqrt{\dfrac{3}{2\pi^2\lambda}}\dfrac{\Gamma(\frac{3}{4})}{\Gamma(\frac{1}{4})}H_{\text{inf}}^{2-\frac{2\Lambda_1}{H}}$

$$\boxed{P(k) = |\varphi_k|^2 \simeq 0.462\frac{H_{\text{inf}}^2}{k^3}\left(\frac{k}{H}\right)^{\frac{2\Lambda_1}{H_{\text{inf}}}} \approx \frac{H_{\text{inf}}^2}{2k^3}} \qquad \because 2\Lambda_1 \ll H_{\text{inf}}$$

It practically behaves as a massless free field.

★The power spectrum of (potential) energy density fluctuation is given by

$$\Xi_{\Delta_h}(r) \equiv \left\langle \frac{\delta\rho_h(r)}{\rho_h}\frac{\delta\rho_h(0)}{\rho_h} \right\rangle = \frac{1}{\langle V\rangle^2}\left[\langle V(r)V(0)\rangle - \langle V(0)\rangle^2\right]$$

$$\simeq \frac{1}{\langle V\rangle^2}\left[\langle V^2\rangle - \langle V\rangle^2\right] e^{-\Lambda_1 t_*} = 4(Har)^{-\frac{2\Lambda_1}{H}} \equiv \int P_V(k) e^{-i\mathbf{k}\cdot\mathbf{R}} \frac{d^3k}{(2\pi)^3},$$

$$P_V(k) \approx 14\sqrt{\lambda}k^{-3} \qquad t_* \equiv \tfrac{2}{H}\ln(Har)$$

★ Amplitude of fractional density fluctuation on scale $r = \dfrac{2\pi}{k}$

$$\mathcal{P}_V(k) \equiv \frac{4\pi k^3}{(2\pi)^3} P_V(k) = 0.71\sqrt{\lambda} = 0.071\tilde{\lambda}^{\frac{1}{2}} = O(0.1)$$

★ Its contribution to the curvature perturbation ζ is unacceptably large.

$$\varsigma \approx \frac{\rho_{cond}}{\rho_{tot}}\left(c\frac{\delta\rho_{cond}}{\rho_{cond}} - c'\frac{\delta H_{osc}}{H_{osc}} \right)\Bigg|_{osc} \sim \frac{\rho_{cond}}{\rho_{tot}} \mathcal{P}_V^{1/2}(k) \gg 10^{-5}$$

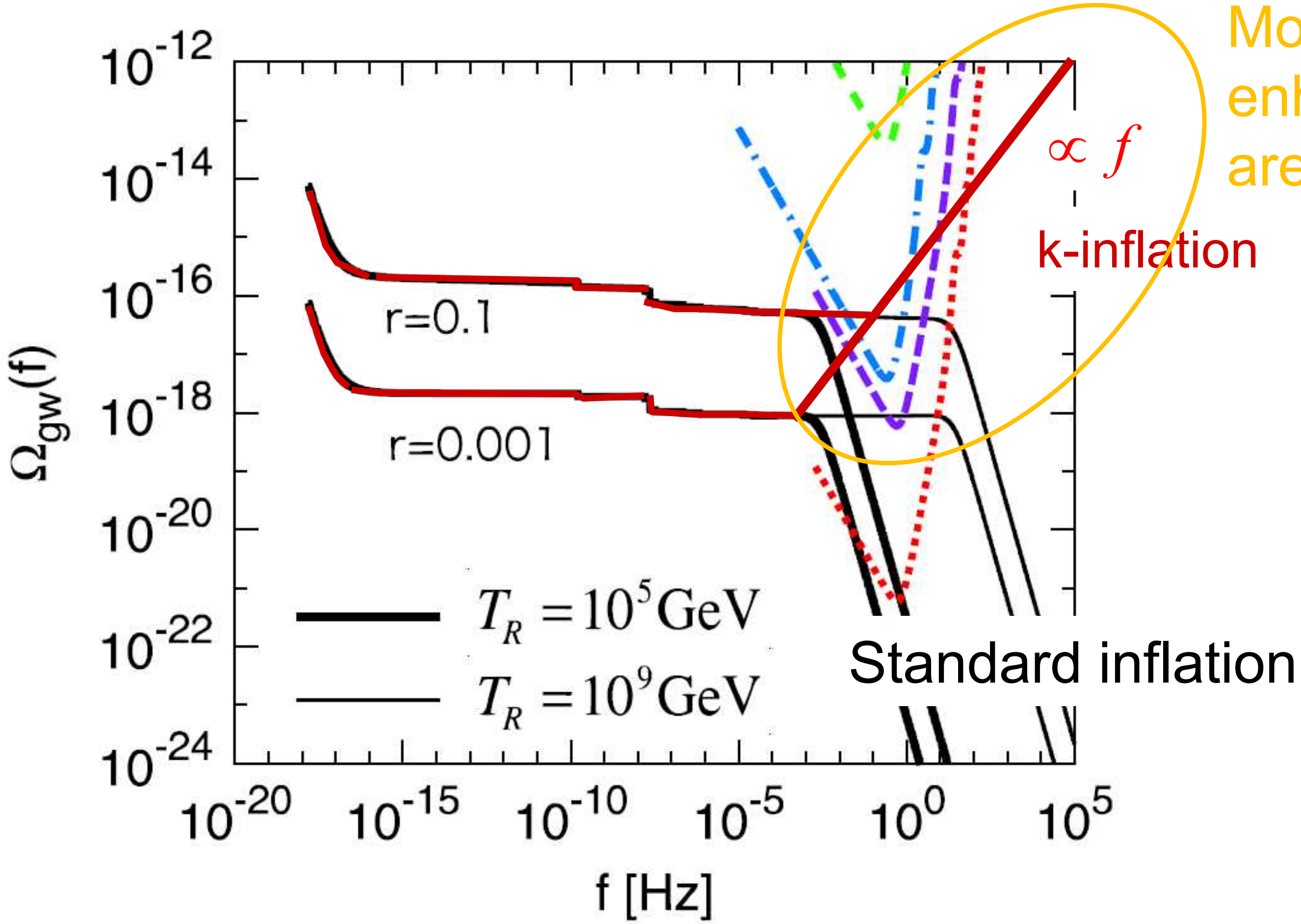
can be O(1)

Hubble parameter at the onset of oscillation $H_{osc} \approx m_{eff} = \sqrt{\lambda}\varphi$

(Kawasaki, Kobayashi & Takahashi 11)

Implication of the Higgs Condensation

- Inflation models followed by kination regime with gravitational reheating is incompatible with the Higgs condensation.



Models with such an enhanced spectrum are ruled out.

Stochastic gravitational wave background from inflation

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II. The case Higgs field is the "Inflaton."

The SM Higgs Boson as the Inflaton.

Easy to preach　　Difficult in practice

Tree level action of the SM Higgs field

$$S_0 = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - |D_\mu \mathcal{H}|^2 - \lambda(|\mathcal{H}|^2 - v^2)^2\right]$$

Taking ${}^t\mathcal{H} = (0,\ v + \phi/\sqrt{2})$ with ϕ being a real scalar field, we find

$$S_0 = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}\phi^4\right]$$

for $\phi \gg v$.

This theory is the same as that proposed by Linde in 1983 to drive chaotic inflation at $\phi \gg M_{Pl}$.

$m_H = \sqrt{2\lambda}v,$ $\quad v = 246$ GeV, $m_H \cong 126\text{GeV}$ $\Rightarrow$ $\lambda \approx 0.13$ at low energy scale

In order to realize $\frac{\delta T}{T} = 10^{-5}$ we must have $\lambda \simeq 10^{-13}$.

The Higgs potential is too steep as it is.

★ Square amplitude of curvature perturbation on scale $r = \frac{2\pi}{k}$

$$\mathcal{P}_\varsigma(k) \equiv \frac{4\pi k^3}{(2\pi)^3}|\varsigma_k|^2 = \frac{H^2}{8\pi^2 M_G^2 c_s \varepsilon_H} \sim \left(\frac{\delta T}{T}\right)^2$$

★ The spectral index of the curvature perturbation is given by

$$\boxed{n_s - 1 \equiv \frac{d\ln \mathcal{P}_\varsigma(k)}{d\ln k} = -2\varepsilon_H - \eta_H - s}$$

$$\varepsilon_H \equiv -\frac{\dot{H}}{H^2} \qquad s \equiv \frac{\dot{c}_s}{Hc_s}, \quad \eta_H \equiv \frac{\dot{\varepsilon}_H}{H\varepsilon_H}$$

★ In potential-driven inflation models

$$\boxed{n_s - 1 = -6\varepsilon_V + 2\eta_V}$$

$$\varepsilon_V \equiv \frac{M_G^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta_V \equiv M_G^2 \frac{V''}{V}$$

Models of Higgs inflation

The Higgs potential is too steep as it is.
Several remedies have been proposed to effectively flatten the potential.

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial_\mu \phi)^2 - \frac{\lambda}{4}\phi^4 \quad + \Delta\mathcal{L} \qquad M_{Pl} = M_G$$

1 The most traditional one (original idea: Spokoiny 1984
Cervantes-Cota and Dehnen 1995, ….)

introduce a large and negative nonminimal coupling to scalar curvature *R*

$$\Delta \mathcal{L} = -\frac{\xi}{2}\phi^2 R \qquad \xi \sim -10^5$$

$$M_{Pl}^2 \to M_{Pl}^2 + |\xi|\phi^2$$

Effectively flatten the spectrum

$$\mathcal{P}_\varsigma(k) \equiv \frac{4\pi k^3}{(2\pi)^3}|\varsigma_k|^2 = \frac{4\pi k^3}{(2\pi)^3}\left|\frac{v_k}{z}\right|^2 = \frac{H^2}{8\pi^2 M_{Pl}^2 c_s \varepsilon_H}$$

2 New Higgs inflation (Germani & Kehagias 2010)

Introduce a coupling to the Einstein tensor in the kinetic term.

$$-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \rightarrow -\frac{1}{2}\left(g^{\mu\nu}-w^2G^{\mu\nu}\right)\partial_\mu\phi\partial_\nu\phi \qquad w \equiv \frac{1}{M^2} = const$$

During inflation $w^2G^{00} \cong 3\frac{H^2}{M^2}$.

If $H \gg M$, the normalization of the scalar field changes.

$\tilde{\phi} \equiv 3^{\frac{1}{4}}\sqrt{\frac{H}{M}}\phi$ is the canonically normalized scalar field.

$V[\phi] = \frac{\lambda}{4}\phi^4 \rightarrow V[\tilde{\phi}] = \frac{\lambda}{12}\frac{M^2}{H^2}\tilde{\phi}^4$ effectively reducing the self coupling to an acceptable level.

(Actual dynamics is somewhat more complicated.)

3: Running kinetic inflation (Takahashi 2010, Nakayama & Takahashi 2010)

Introduce a field dependence to the kinetic term.

$$-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \to -\frac{1}{2}f(\phi)\partial_\mu\phi\partial_\nu\phi \qquad\qquad f(v)=1$$

Canonical normalization of the scalar field changes the potential effectively.

4 Higgs G-inflation

(Kamada, Kobayashi, Yamaguchi & JY 2011)

$$\mathcal{L}_\phi = X - V(\phi) - g(\phi) X \Box \phi,$$

$$X = -\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Galileon-type coupling

Background equations of motion

$$3M_{Pl}^2 H^2 = \left[1 - gH\dot{\phi}(6-\alpha)\right] X + V(\phi)$$

$$M_{Pl}^2 \dot{H} = -\left[1 - gH\dot{\phi}(3+\eta-\alpha)\right] X$$

$$\left[3 - \eta - gH\dot{\phi}(9 - 3\varepsilon - 6\eta + 2\eta\alpha)\right] H\dot{\phi} + (1+2\beta)V'(\phi) = 0$$

Slow-roll parameters $\varepsilon = -\frac{\dot{H}}{H^2}, \quad \eta = -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad \alpha = \frac{g'\dot{\phi}}{gH}, \quad \beta = \frac{g'' X^2}{V'(\phi)}.$

For $|\varepsilon|, |\mu|, |\alpha|, |\beta| \ll 1$ we find slow-roll EOMs

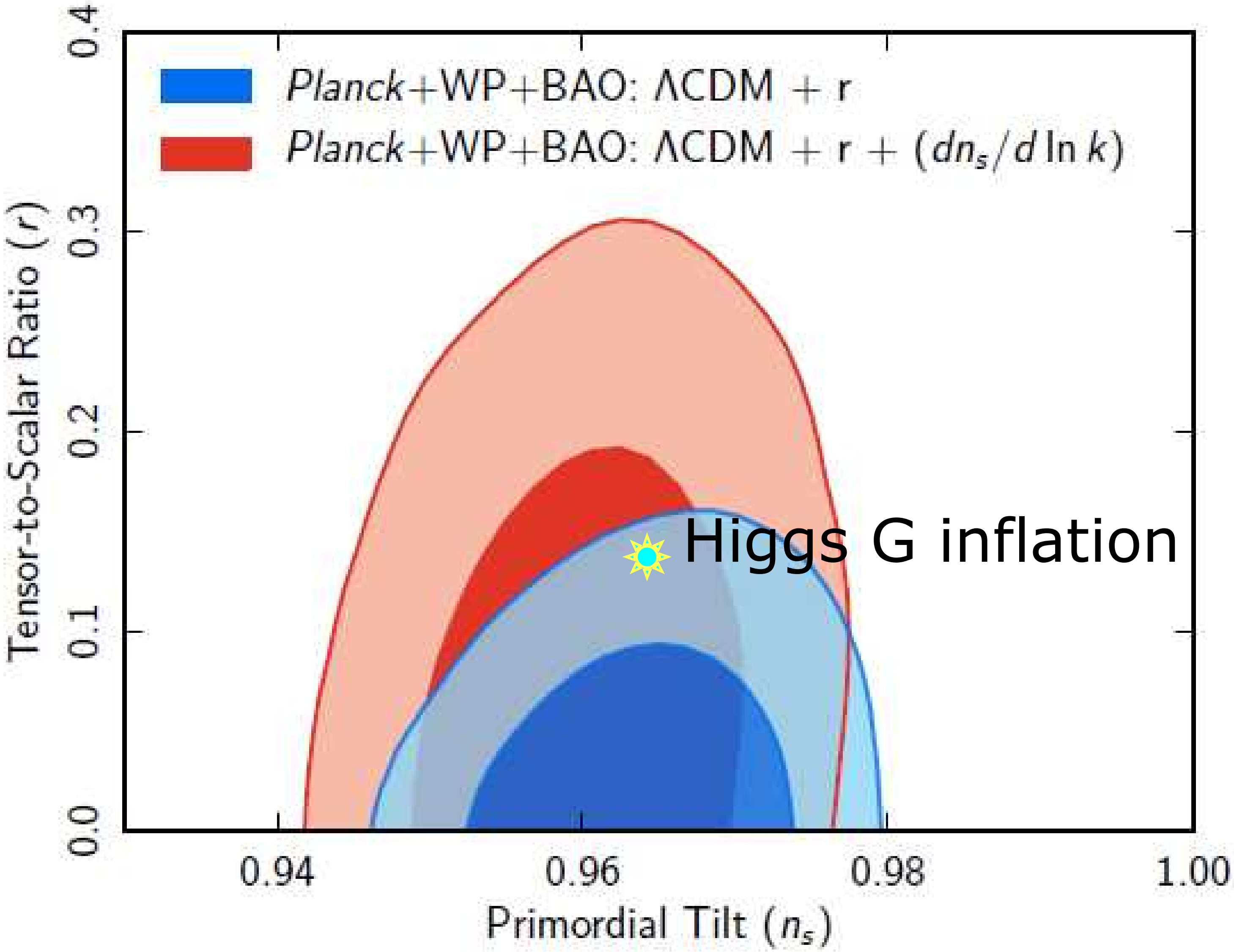
$$3H\dot{\phi}\left(1 - 3gH\dot{\phi}\right) + V'(\phi) \simeq 0 \qquad 3M_{Pl}^2 H^2 \simeq V(\phi)$$

This extra friction term enhances inflation and makes it possible to drive inflation by a standard Higgs field.

$$\mathcal{L}_\phi = X - \frac{\lambda}{4}\phi^4 - \frac{X\phi\Box\phi}{M^4}$$

$$\mathcal{P}_\varsigma(k) = 2.4\times10^{-9} \, @\, k = 0.002\mathrm{Mpc}^{-1} (\mathcal{N}_{\mathrm{COBE}} = 60)$$

$$\boxed{M \simeq 4.7\times10^{-6}\lambda^{-\frac{1}{4}} M_{Pl} \simeq 10^{13}\,\mathrm{GeV}.}$$



Higgs inflation models as variants of Generalized G-inflation

In fact, all these models can be described in the context of the Generalized G-inflation model seamlessly, which is the most general single-field inflation model with 2nd order field equations.

Inflation from a gravity + scalar system $S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

These Lagrangians are obtained by covariantizing the Galileon theory.

The Galileon (Nicolis, Rattazzi, & Trincherini 2009)

Higher derivative theory with a Galilean shift symmetry

$\partial_\mu \phi \to \partial_\mu \phi + \mathrm{const}_\mu$ in the flat spacetime.

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = (\nabla\phi)^2$$

$$\mathcal{L}_3 = (\nabla\phi)^2 \Box\phi$$

nonreleativistic limit of 4D probe brane action in 5D theory (de Rham & Tolley 2010)

$$\mathcal{L}_4 = (\nabla\phi)^2 \left[2(\Box\phi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2 \right]$$

$$\mathcal{L}_5 = (\nabla\phi)^2 \left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3 \right]$$

$$(\nabla_\mu\nabla_\nu\phi)^2 = \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\phi$$

$$(\nabla_\mu\nabla_\nu\phi)^3 = \nabla_\mu\nabla_\nu\phi\nabla^\nu\nabla^\lambda\phi\nabla_\lambda\nabla^\mu\phi$$

$\Rightarrow$ Field equation contains derivatives up to second-order at most.

Galilean symmetry exists only in flat spacetime.

Covariantization: Generalized Galileon

(Deffayet, Deswer, & Esposito-Farese, 2009)
(Deffayet, Esposito-Farese, & Vikman, 2009)

no longer Galilean invariant
but field equations remain of second-order.

Generic theory with second-order field eqs.

(Deffayet, Gao, Steer, Zahariade, 2011)

$$\mathcal{L}_2 = \boxed{K(\phi, X)}$$

$$\mathcal{L}_3 = -\boxed{G_3(\phi, X)}\Box\phi$$

$$\mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = \boxed{G_5(\phi, X)}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

4 arbitrary functions of ϕ and $X \equiv -\frac{1}{2}(\partial\phi)^2$

$$G_{iX} \equiv \frac{\partial G_i}{\partial X}$$

This theory includes the Einstein action with $G_4 \supset M_{Pl}^2/2$
as well as a nonminimal coupling with $G_4 \supset -\xi\phi^2/2$.

Gravity is naturally included by construction.

In fact the most general scalar+gravity theory that yields second-order field eqs. was discovered by Horndeski already in 1974. (recently revisited by Charmousis et al. 2011)

$$\mathcal{L}_H = \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1\nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \left(\tfrac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\right)\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi + \kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma}\Big]$$

$$+\delta^{\alpha\beta}_{\mu\nu}\Big[F R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\Big] - 6\left(F_\phi - X\kappa_8\right)\Box\phi + \kappa_9$$

with $\delta^{\alpha\beta\gamma}_{\mu\nu\sigma} = 3!\,\delta^{[\alpha}_{\mu}\delta^{\beta}_{\nu}\delta^{\gamma]}_{\sigma}$ $\quad F_X = 2\left(\kappa_3 + 2X\kappa_{3X} - \kappa_{1\phi}\right)$ $\quad \kappa_1,\kappa_3,\kappa_8,\kappa_9(\phi, X)$

We have found that the Generalized Galileon is equivalent to Horndeski theory by the following identification.

$$K = \kappa_9 + 4X\int^X dX'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right),$$

$$G_3 = 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X dX'(\kappa_8 - 2\kappa_{3\phi}),$$

$$G_4 = 2F - 4X\kappa_3,$$

$$G_5 = -4\kappa_1,$$

(Kobayashi, Yamaguchi & JY 2011)

Generalized G-inflation

Inflation from a gravity + scalar system $S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$G_4 \supset M_{Pl}^2/2$ gives the Einstein action

$$\mathcal{L}_4 = \boxed{G_4(\phi, X)}R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

This theory includes

- potential-driven inflation models $K(\phi, X) = X - V[\phi]$
- k-inflation model $K(\phi, X) = K(X)$
- Higgs inflation model $-\xi R\phi^2/2 \Leftarrow G_4 \supset -\xi\phi^2/2$
- New Higgs inflation model $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \Leftarrow G_5 \propto \phi$
- G-inflation model $K(\phi, X) - G(\phi, X)\Box\phi$

Generalized G-inflation

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This theory includes

Generalized G-inflation is a framework to study the most general single-field inflation model with second-order field equations.

G-inflation model $K(\phi, X) - G(\phi, X)\Box\phi$

In fact, all these models can be described in the context of the Generalized G-inflation model seamlessly, which is the most general single-field inflation model with 2nd order field equations.

Inflation from a gravity + scalar system $S=\sum_{i=2}^{5}\int \mathcal{L}_i\sqrt{-g}\,d^4x$

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]$$

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- New Higgs inflation model $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \Leftarrow G_5 \propto \phi$
- G-inflation model $K(\phi, X) - G(\phi, X)\Box\phi$

$$\mathcal{L}_2 = K(\phi, X),$$

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$$G_{iX} = \partial G_i / \partial X$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \tfrac{1}{6}G_{5X}[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]$$

We consider potential-driven inflation in the generalized G-inflation context. So we expand

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots,$$
$$G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots.$$

and neglect higher orders in X.

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}(\partial_\mu\phi)^2 - \frac{\lambda}{4}\phi^4 \quad +\Delta\mathcal{L}$$

$$\Delta\mathcal{L} = \kappa\phi^{2n}X \text{ (running kinetic inflation)},$$

$$\Delta\mathcal{L} = \frac{\phi}{M^4}X\Box\phi \text{ (Higgs G-inflation)},$$

$$\Delta\mathcal{L} = -\frac{\xi}{2}\phi^2 R \text{ (nonminimal Higgs inflation)},$$

$$\Delta\mathcal{L} = \frac{1}{2\mu^2}[XR + (\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2]$$

(new Higgs inflation).

$$\mathcal{K}(\phi) = 1 + \kappa\phi^{2n},$$

$$h_3(\phi) = \frac{\phi}{M^4},$$

$$g_4(\phi) = \frac{M_{\rm Pl}^2}{2} - \frac{\xi}{2}\phi^2,$$

$$h_4(\phi) = \frac{1}{2\mu^2},$$

$$h_5(\phi) = 0.$$

in the generalized G-inflation

Yet another new model:
Running Einstein Inflation

simplest case $\quad h_5(\phi) = \frac{\phi}{\Lambda}$

We analyze all these possibilities on equal footing, characterizing Higgs inflation by five functions $\phi,\ \mathcal{K},\ g_4,\ h_3,\ h_4,\ h_5$ and potential.

Potential-driven slow-roll inflation

$$\epsilon := -\frac{\dot{H}}{H^2} \ll 1, \qquad \eta := -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1, \quad \delta := \frac{\dot{g}_4}{Hg_4} \ll 1, \qquad \alpha_2 := \frac{\dot{\mathcal{K}}}{H\mathcal{K}} \ll 1,$$

$$\alpha_i := \frac{\dot{h}_i}{Hh_i} \ll 1 \qquad (i = 3, 4, 5). \qquad\qquad \dot{\delta}/H\delta,\ \dot{\alpha}_i/H\alpha_i \ll 1\ (i = 2, 3, 4, 5).$$

Slow-roll field equation

$$3HJ \simeq -V' + 12H^2 g_4' \quad \text{w/} \quad J \simeq \mathcal{K}\dot{\phi} + 3h_3H\dot{\phi}^2 + 6h_4H^2\dot{\phi} + 3h_5H^3\dot{\phi}^2$$

Gravitational field equations

$$6g_4H^2 \simeq V, \qquad\qquad -4g_4\dot{H} + 2g_4'\dot{\phi}H \simeq \dot{\phi}J.$$

Cosmological Perturbations

Tensor perturbation

$$S_T^{(2)} = \frac{1}{8}\int dt d^3x a^3\left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right],$$

$$\mathcal{F}_T := 2[G_4 - X(\ddot{\phi}G_{5X} + G_{5\phi})],$$

$$\mathcal{G}_T := 2[G_4 - 2XG_{4X} - X(H\dot{\phi}G_{5X} - G_{5\phi})].$$

$$\boxed{\mathcal{P}_T = 8\gamma_T \frac{\mathcal{G}_T^{1/2}}{\mathcal{F}_T^{3/2}}\frac{H^2}{4\pi^2}}$$

$$n_T = 3 - 2\nu_T \qquad \nu_T := \frac{3 - \epsilon + g_T}{2 - 2\epsilon - f_T + g_T}$$

$$f_T := \frac{\dot{\mathcal{F}}_T}{H\mathcal{F}_T} \qquad g_T := \frac{\dot{\mathcal{G}}_T}{H\mathcal{G}_T}$$

Scalar perturbations

$$S_S^{(2)} = \int dt d^3x a^3 \left[\mathcal{G}_S \dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2} (\vec{\nabla}\zeta)^2 \right]$$

$$\mathcal{F}_S := \frac{1}{a}\frac{d}{dt}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T, \qquad \mathcal{G}_S := \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T,$$

$$\begin{aligned}\Sigma := {} & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} + 6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 + 6[H^2(7XG_{4X} \\ & + 16X^2G_{4XX} + 4X^3G_{4XXX}) - H\dot{\phi}(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX})] + 30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\ & + 4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X(6G_{5\phi} + 9XG_{5\phi X} + 2X^2G_{5\phi XX}),\end{aligned}$$

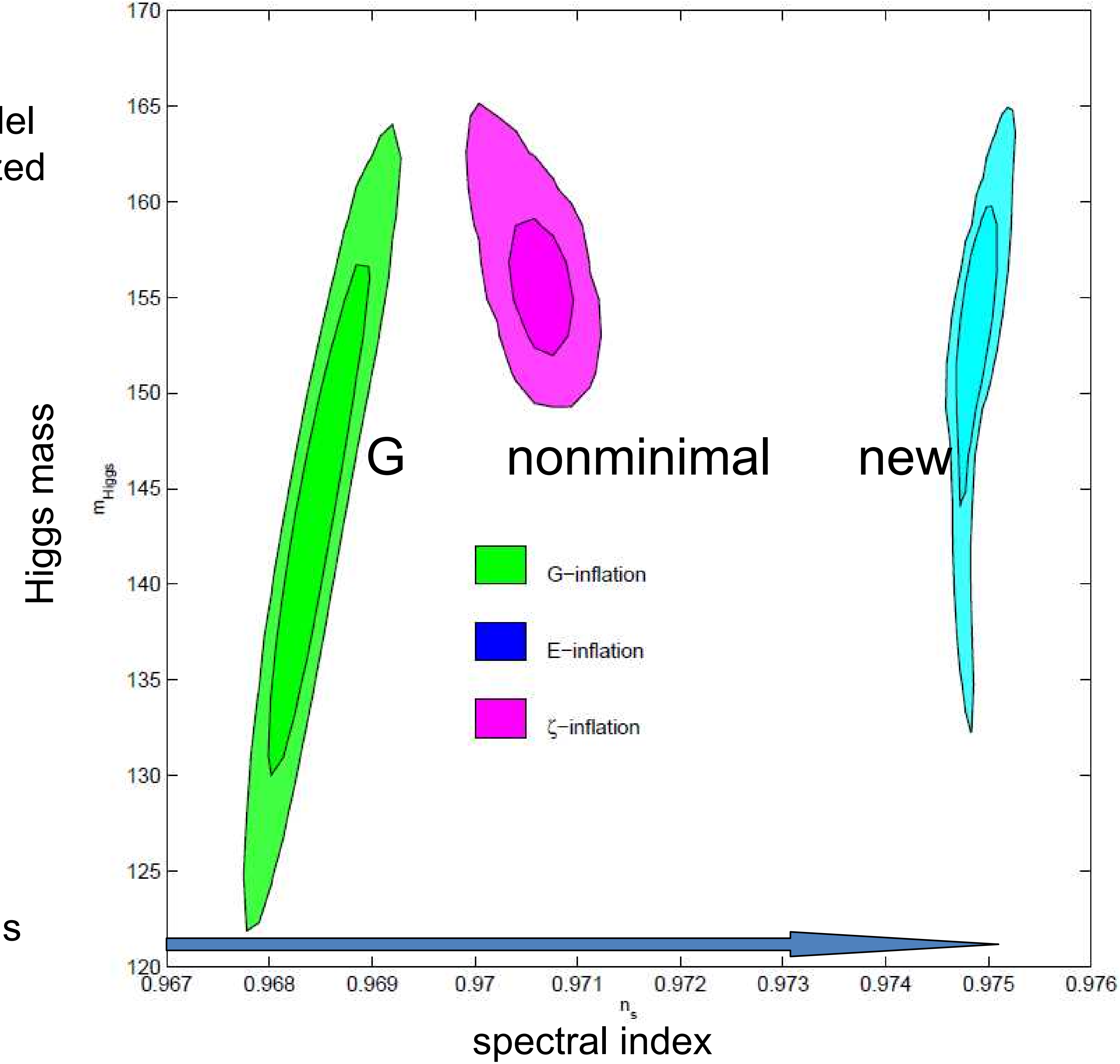
$$\begin{aligned}\Theta := {} & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} - 8HX^2G_{4XX} \\ & + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} - H^2\dot{\phi}(5XG_{5X} + 2X^2G_{5XX}) + 2HX(3G_{5\phi} + 2XG_{5\phi X}).\end{aligned}$$

$$\boxed{\mathcal{P}_\zeta = \frac{\gamma_S}{2}\frac{\mathcal{G}_S^{1/2}}{\mathcal{F}_S^{3/2}}\frac{H^2}{4\pi^2}}$$

$$n_s - 1 = 3 - 2\nu_S$$

$$\nu_S := \frac{3 - \epsilon + g_S}{2 - 2\epsilon - f_S + g_S}$$

So far each model has been analyzed separately.



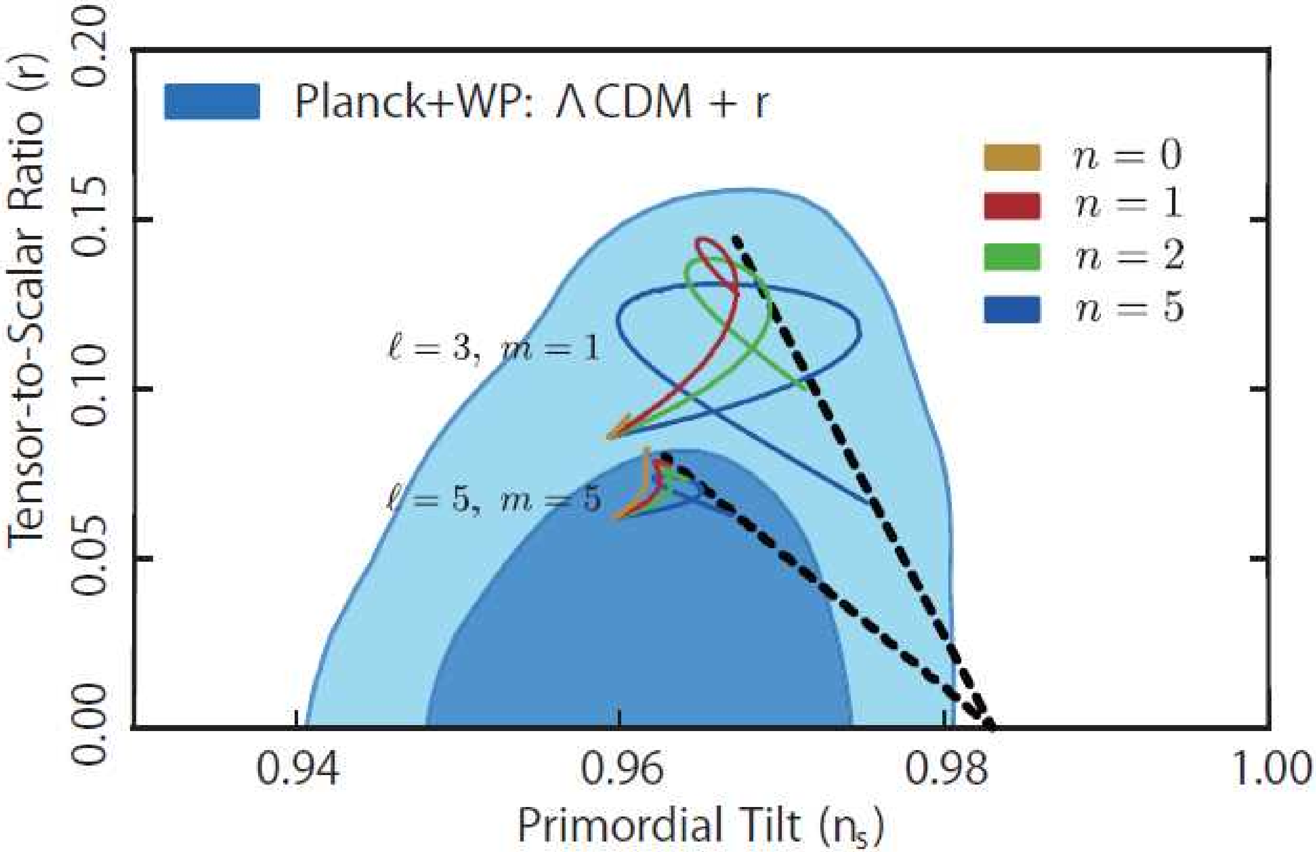
MCMC analysis
By Popa 2011

We can fill the gap using the generalized Higgs inflation

In fact, as for Higgs G-inflation, it was recently found that inclusion of higher order kinetic term X^{ℓ} $(\ell \geq 2)$ is required to make the theory well behaved in the field oscillation regime after inflation, which makes it possible to give various predictions for (n_s, r) .

(cf Ohashi & Tsujikawa 2012)

$$S = \int d^4x \sqrt{-g}\left[\frac{M_P^2}{2} + X + \frac{X^\ell}{\ell \widetilde{M}^{4(\ell-1)}} - \frac{1}{4}\lambda\varphi^4 + \frac{\varphi^{2n+1}X^m}{M^{2n+4m}}\Box\varphi\right]$$



(Kunimitsu et al 2013 in preparation)

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唯以正直接萬人
至誠依神期勝利

鑑三

Conclusion

The Higgs self coupling must remain positive to well above the scale of inflation.

Inflation followed by a kination regime with gravitational reheating should be ruled out.

Higgs inflation is possible and subject to observational tests in the context of the Generalized G inflation.

Conclusion

Higgs field plays interesting roles in cosmology.


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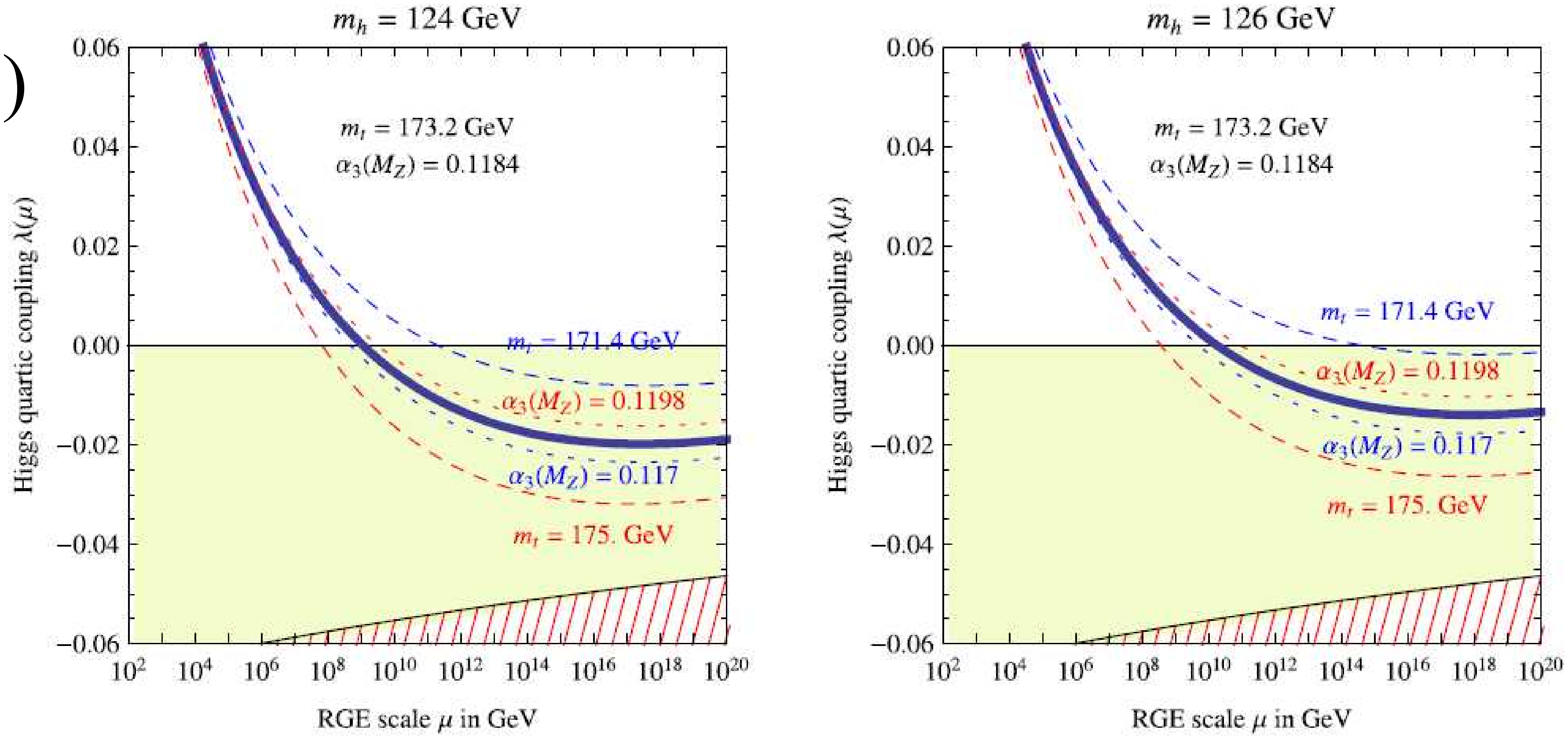
Stability of the SM Higgs field

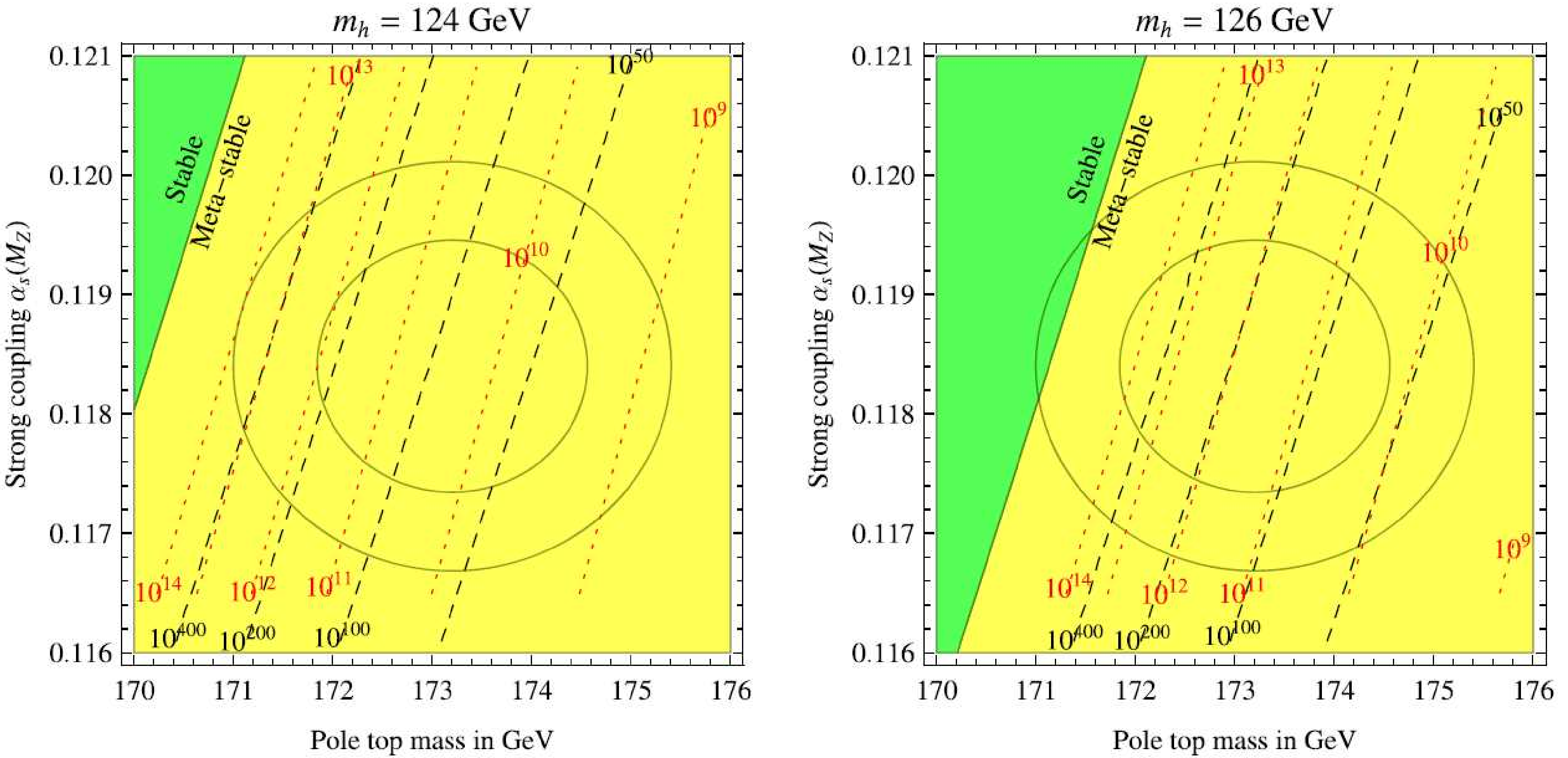
Just after the CERN seminars in 2011/12 announcing “hints of the Higgs boson” an analysis of the renormalized Higgs potential was done for m=124～126GeV.

$\lambda(\mu)$



Elias-Miro et al Phys Lett B709(2012)222

$$S_0 = \int d^4x \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - |D_\mu \mathcal{H}|^2 - \lambda(|\mathcal{H}|^2 - v^2)^2\right]$$



For $m_H = 126\text{GeV}$ there is a finite parameter region within 2σ where the Higgs potential is stable, or its self coupling remain positive even after renormalization.

All the Higgs inflation models are unified in the Generalized G-inflation.

Open issue:
Quantum corrections in the presence
of these new interaction terms.

$$\mathcal{L}_2 = K(\phi, X),$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi,$$

$$G_{iX} = \partial G_i/\partial X$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2],$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \tfrac{1}{6}G_{5X}[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]$$

We consider potential-driven inflation in the generalized G-inflation context.
So we expand $K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots,$

$$G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots.$$

and neglect higher orders in X.

We find the following identities. (t.d.) ＝total derivative

$$g_3(\phi)\Box\phi = 2g_3'X + (\text{t.d.}),$$

$$g_5(\phi)G^{\mu\nu}\nabla_\mu\nabla_\nu\phi = -g_5'[XR + (\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] + 3g_5''X\Box\phi - 2g_5'''X^2 + (\text{t.d.}),$$

As a result we can set $g_3 = 0 = g_5$ in the Lagrangian without loss of generality.

$$g_4 \equiv g$$