Quantum approach to the thermal resonant leptogenesis

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• Introduction

Baryon asymmetry $Y_B \sim 10^{-10}$

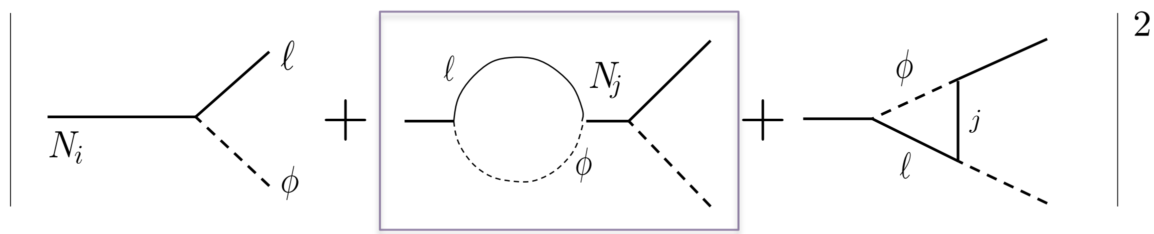
Leptogenesis ⟵ SM + RH neutrino N ⟶ Seesaw mechanism, Tiny neutrino mass

Davidson-Ibarra bound : $M \gtrsim 10^9 \mathrm{GeV}$

Resonant Leptogenesis can evade DI bound. :Pilaftsis(1997)

⟶ **TeV scale** Majorana mass $M_2 - M_1 \simeq \Gamma_N$

$|\mathcal{M}_{N_i \to \ell\phi}|^2 =$



Enhancement of CP-asymmetry

• Introduction

CP-violating parameter

$$\varepsilon_i = \frac{\Im(Y^\dagger Y)^2_{ij}}{(Y^\dagger Y)_{ii}(Y^\dagger Y)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + R_{ij}^2}$$

regulator

$$R_{ij} = M_i\Gamma_j$$

: Pilaftsis(1997)

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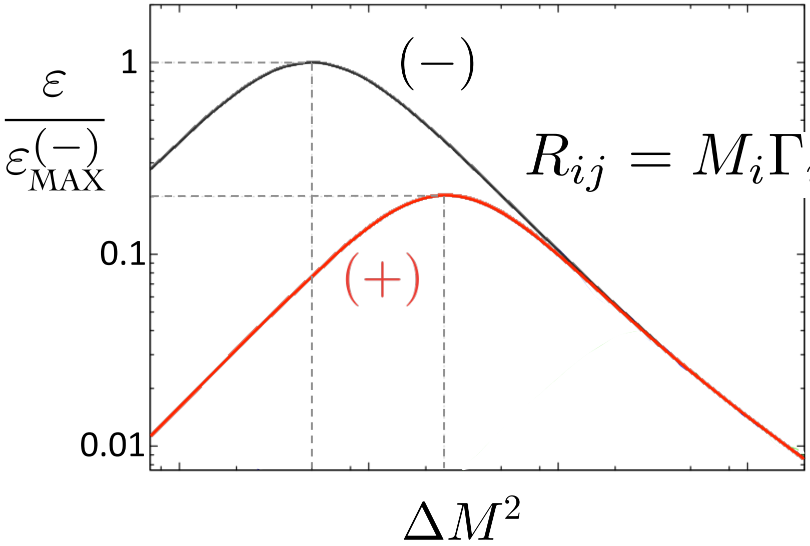

$$R_{ij} = M_i\Gamma_i \mp M_j\Gamma_j$$

− : Plumacher, Buchmuller(1998)

+ : Garny, et al. (2011)

Non-equilibrium QFT

Under the initial non-equilibrium cond.
Without the expansion of the Universe

• Introduction

CP-violating parameter

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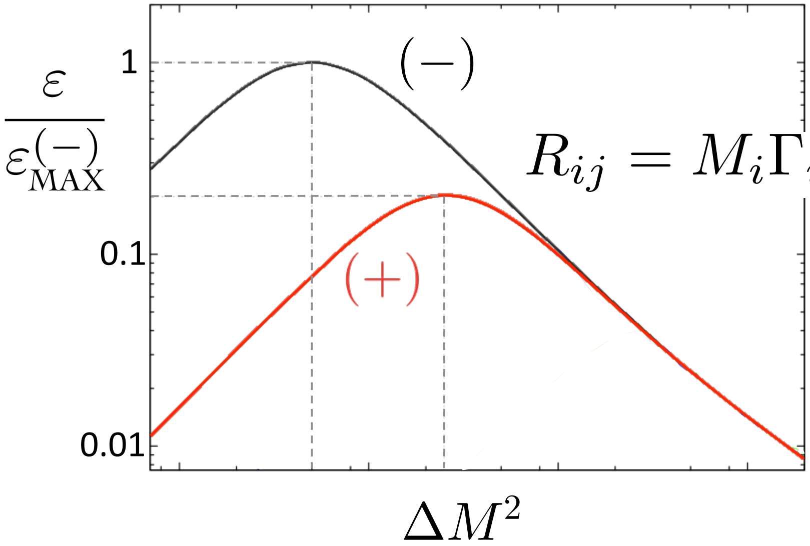

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Non-equilibrium QFT

Under the initial non-equilibrium cond.
Without the expansion of the Universe

Is this true in the Thermal leptogenesis ? → **Yes.**

- The (Classical) Boltzmann eq. for the lepton number

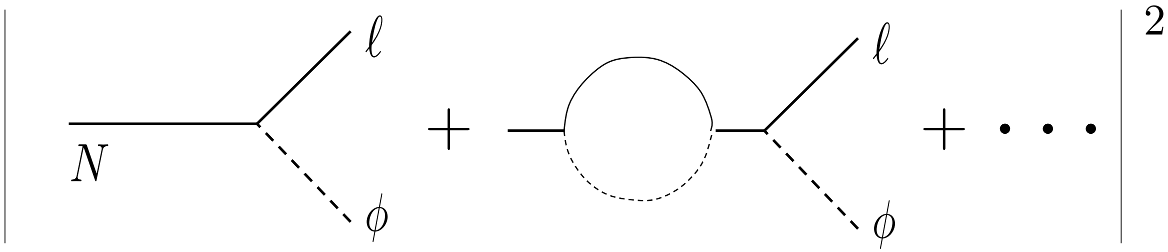
$$\frac{dn_L}{dt} + 3Hn_L = \sum_{i=1,2} \int d\Pi_{N_i,\ell,\phi} (2\pi)^4 \delta^{(4)}(p+k-q) \times \{ |\mathcal{M}|^2_{N_i \to \ell\phi} (1-f_\ell)(1+f_\phi) f_{N_i} - |\mathcal{M}|^2_{\ell\phi \to N_i} (1-f_{N_i}) f_\ell f_\phi \} - [\ CP\ conjugate\ ]$$

Gain term

Loss term

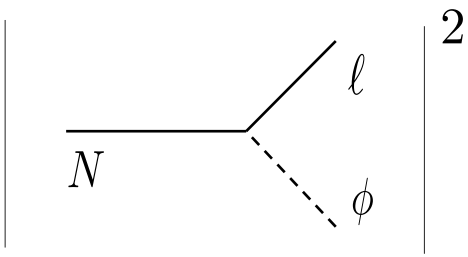
CP-violating Parameter $\varepsilon \neq 0$

$|\mathcal{M}|^2 =$



Resummation

$=$



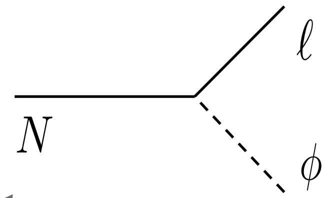
Mass eigenstate

RH neutrinos decay with flavor oscillation

$\mathcal{E}$ is enhanced
in the resonant case : $M_2 - M_1 \simeq \Gamma_N$

- Issues in the Boltzmann eq. for lepton number

S-matrix is calculated in **equilibrium** QFT

$|\mathcal{M}|^2 = \Big|$  $\Big|^2$

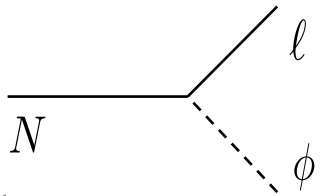
Mass eigenstate

- **On-shell** external line

  "**Classical**" particle picture before the decay

- Issues in the Boltzmann eq. for lepton number

S-matrix is calculated in **equilibrium** QFT

$$|\mathcal{M}|^2 = \Bigg|$$  $$\Bigg|^2$$

Mass eigenstate

- **On-shell** external line

"**Classical**" particle picture before the decay

Local collision

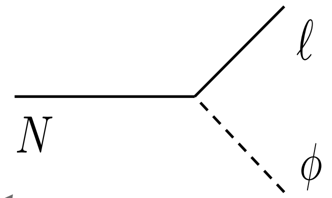
+

Propagation

- Issues in the Boltzmann eq. for lepton number

S-matrix is calculated
in **equilibrium** QFT

$$|\mathcal{M}|^2 = \left|\quad\right|^2$$



Mass eigenstate

- **On-shell** external line

“**Classical**” particle picture
before the decay

In reality

Non-equilibrium situation

(No asymptotic state)

If a counterpart of $|\mathcal{M}|^2$ exists,
it should involve

- **Off-shell** external line

Quantum coherence
before the decay

Kadanoff-Baym equations

. from Non-equilibrium QFT

- Review of the Kadanoff-Baym equations

Kadanoff-Baym eq. : the self-consistent eq. for full propagators

Spectrum of the system

$$(i\partial\!\!\!/_x - M)G_R(x,y) - \int d^4z\ \Pi_R(x,z)G_R(z,y) = -\delta^4(x-y)$$

$$(i\partial\!\!\!/_x - M)G_{\lessgtr}(x,y) - \int d^4z\ \Pi_R(x,z)G_{\lessgtr}(z,y) = \int d^4z\ \Pi_{\lessgtr}(x,z)G_A(z,y)$$

State of the system
➡ Distribution func. f

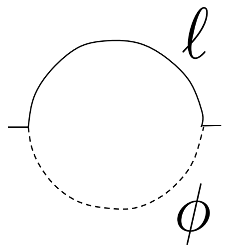
• Review of the Kadanoff-Baym equations

Kadanoff-Baym eq. : the self-consistent eq. for full propagators

Spectrum of the system

self-energy ➡ • Correction to mass • Width

$$(i\partial\!\!\!/_x - M)G_R(x,y) - \int d^4z\ \Pi_R(x,z)G_R(z,y) = -\delta^4(x-y)$$



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self-energy ➡
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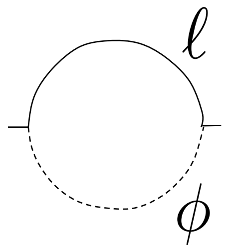
- <u>Width</u>



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State of the system
➡ Distribution func. f

$$G_>(x^0 = y^0; \mathbf{q}) \ \sim \langle a_\mathbf{q} a_\mathbf{q}^\dagger \rangle \ \Rightarrow \ 1 - f_\mathbf{q}$$

$$G_<(x^0 = y^0; \mathbf{q}) \ \sim \langle a_\mathbf{q}^\dagger a_\mathbf{q} \rangle \ \Rightarrow \ f_\mathbf{q}$$

Deviation from equilibrium

➡ Δf

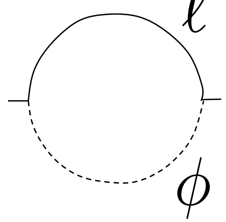
Change of distribution func.

- Review of the Kadanoff-Baym equations

Kadanoff-Baym eq. : the self-consistent eq. for full propagators

Spectrum of the system

self-energy →
- Correction to mass
- Width



$$(i\partial\!\!\!/_x - M)G_R(x,y) - \int d^4z\, \Pi_R(x,z)G_R(z,y) = -\delta^4(x-y)$$

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State of the system
→ Distribution func. f

Flavor oscillation
with degenerate masses

$$G_>(x^0 = y^0; \mathbf{q}) \sim \langle a_\mathbf{q} a_\mathbf{q}^\dagger \rangle \rightarrow 1 - f_\mathbf{q}$$

$$G_<(x^0 = y^0; \mathbf{q}) \sim \langle a_\mathbf{q}^\dagger a_\mathbf{q} \rangle \rightarrow f_\mathbf{q}$$

??

Deviation from equilibrium
→ Δf ??
Change of distribution func.

- Review of the Kadanoff-Baym equations

Kadanoff-Baym eq. : the self-consistent eq. for full propagators

Spectrum of the system

$$(i\partial\!\!\!/_x - M)G_R(x,y) - \int d^4z\ \Pi_R(x,z)G_R(z,y) = -\delta^4(x-y)$$

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source term

State of the system
➡ Distribution func. f

MEMORY INTEGRAL involves the flavor oscillation completely.

➡ $$G_{\lessgtr}(x,y) = -\int d^4u d^4v G_R(x,u)\Pi_{\lessgtr}(u,v)G_A(v,y)$$

In the following, we'll see that

- KB eq. for the SM **lepton** is reduced to
  the Boltzmann-like eq. for the lepton number.

↓

- CP-violating parameter $\mathcal{E}$ is obtained
  from the counterpart of $|\mathcal{M}|^2$.

Advantage of the MEMORY INTEGRAL
in the KB eq. for **RH neutrino**

- The evolution eq. for the lepton number

S : lepton propagator

Δ: Higgs propagator

$$\frac{dn_L}{dt} + 3Hn_L$$

K.B.eq. for the SM **lepton**

$$= 2(Y^\dagger Y)_{ji} \int_{-\infty}^{x^0} dz^4 \sqrt{-g} \Big[-\mathrm{tr}\{G^{ij}_{<}(x,z) S_{>}(z,x)\}\Delta_{>}(z,x)$$

$$+\mathrm{tr}\{G^{ij}_{>}(x,z) S_{<}(z,x)\}\Delta_{<}(z,x) \ + \cdots \Big]$$

Yukawa coupling

Time integral contributes only around $t = x^0$ because the SM gauge interaction is very rapid.

- The evolution eq. for the lepton number

$$\frac{dn_L}{dt} + 3Hn_L$$

K.B.eq. for the SM **lepton**

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$$+\mathrm{tr}\{G^{ij}_>(x,z) S_<(z,x)\} \Delta_<(z,x) + \cdots \Big]$$

??

$1 - f_\ell$ $1 + f_\phi$ Gain term

f_ℓ f_ϕ Loss term

Time integral contributes only around $t = x^0$
because the SM gauge interaction is very rapid.

$\sim$ Local Boltzmann collision term
of the (inverse) Decay

- The evolution eq. for the lepton number

$$\frac{dn_L}{dt} + 3Hn_L$$

$G_{<(t=x^0 \simeq z^0)} \Rightarrow f_N \,,\, \mathcal{E}$

K.B.eq. for the SM **lepton**

$$= 2(Y^\dagger Y)_{ji} \int_{-\infty}^{x^0} dz^4 \sqrt{-g} \Big[-\mathrm{tr}\{G_<^{ij}(x,z) S_>(z,x)\} \Delta_>(z,x) + \mathrm{tr}\{G_>^{ij}(x,z) S_<(z,x)\} \Delta_<(z,x) + \cdots \Big]$$

Gain term: $1 - f_\ell$, $1 + f_\phi$

Loss term: f_ℓ, f_ϕ

Time integral contributes only around $t = x^0$ because the SM gauge interaction is very rapid.

$\sim$ Local Boltzmann collision term of the (inverse) Decay

Concrete expression of $G_\gtrless$

$\Updownarrow$

Counterpart of $|\mathcal{M}|^2$

Correct value of $\mathcal{E}$

- Wightman propagator of the RH neutrino in the lepton number evolution eq.

$$G_{\lessgtr}(x,y) = -\int d^4u d^4v G_R(x,u)\Pi_{\lessgtr}(u,v)G_A(v,y)$$

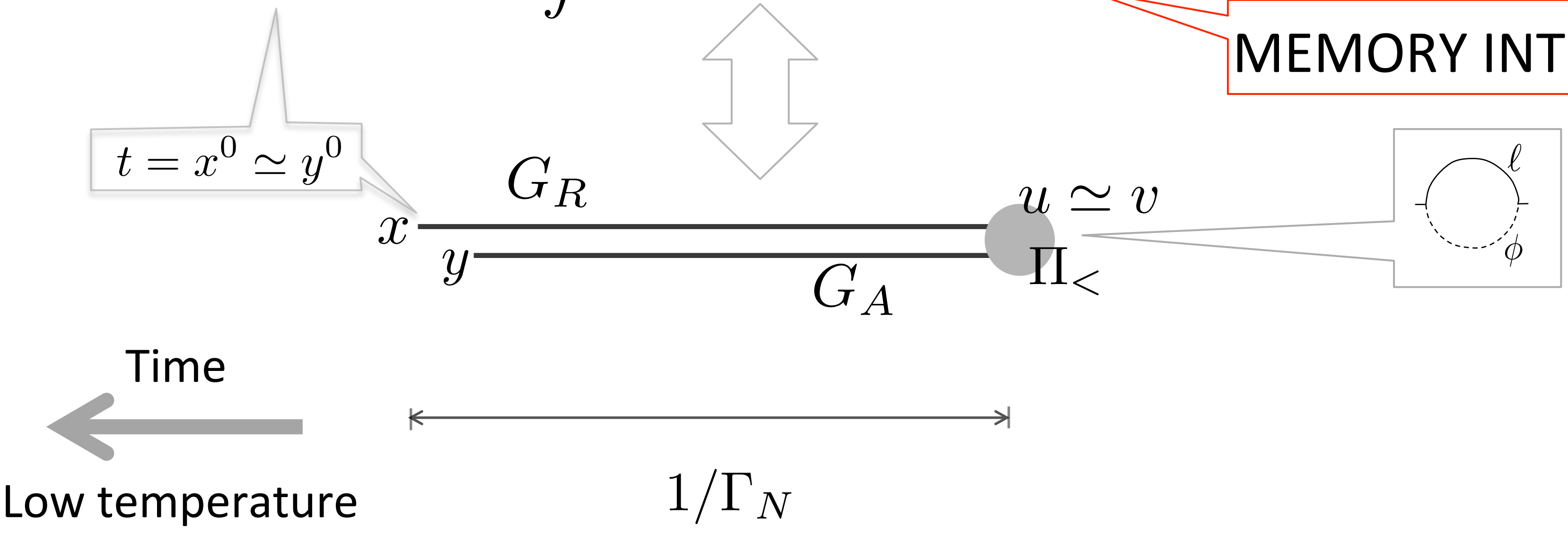
$t = x^0 \simeq y^0$

MEMORY INTEGRAL

- Wightman propagator of the RH neutrino in the lepton number evolution eq.

$$G_{\lessgtr}(x,y) = -\int d^4u\, d^4v\, G_R(x,u)\,\Pi_{\lessgtr}(u,v)\,G_A(v,y)$$

MEMORY INTEGRAL



Deviation from equilibrium

- Wightman propagator of the RH neutrino in the lepton number evolution eq.

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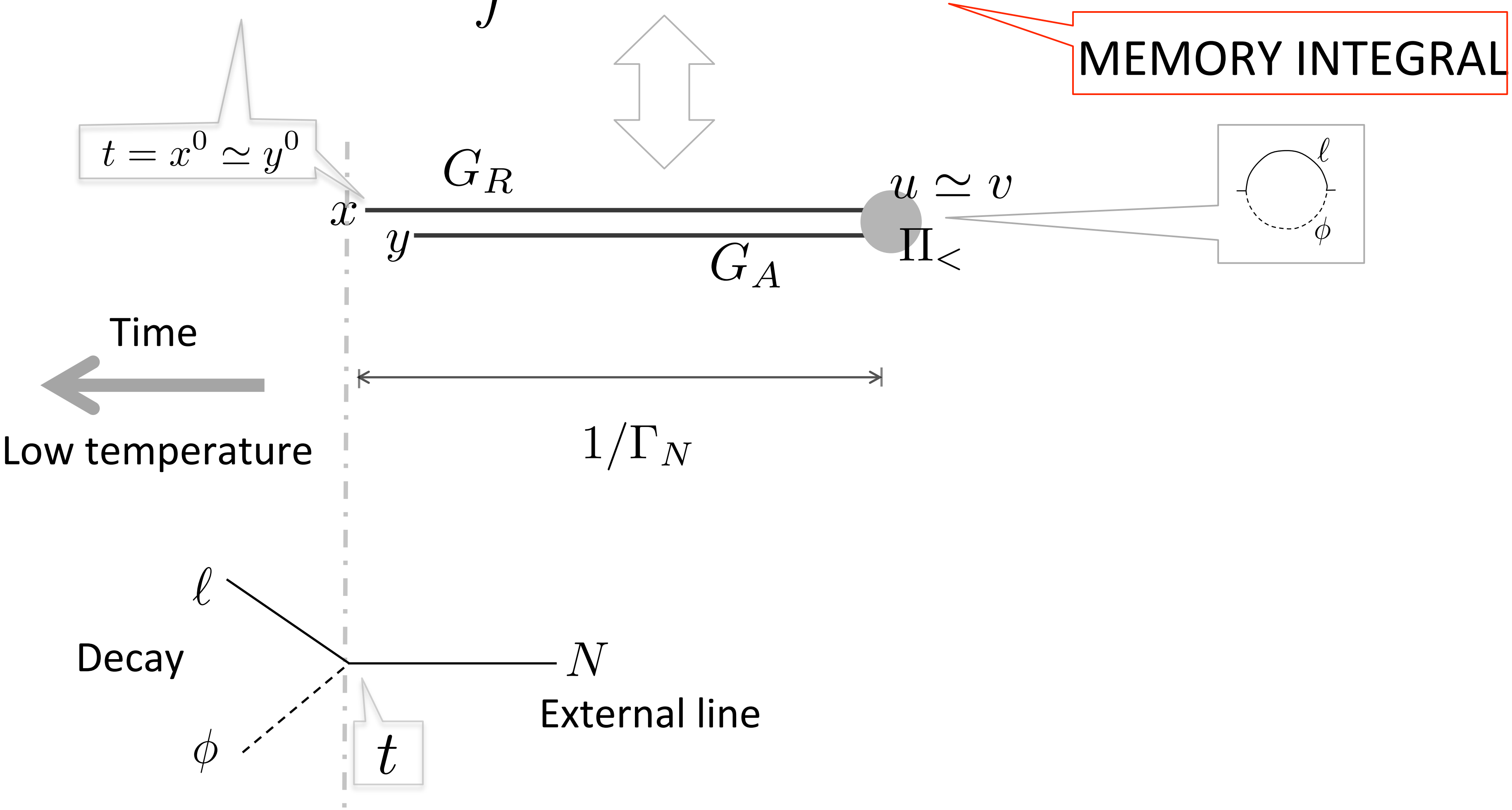

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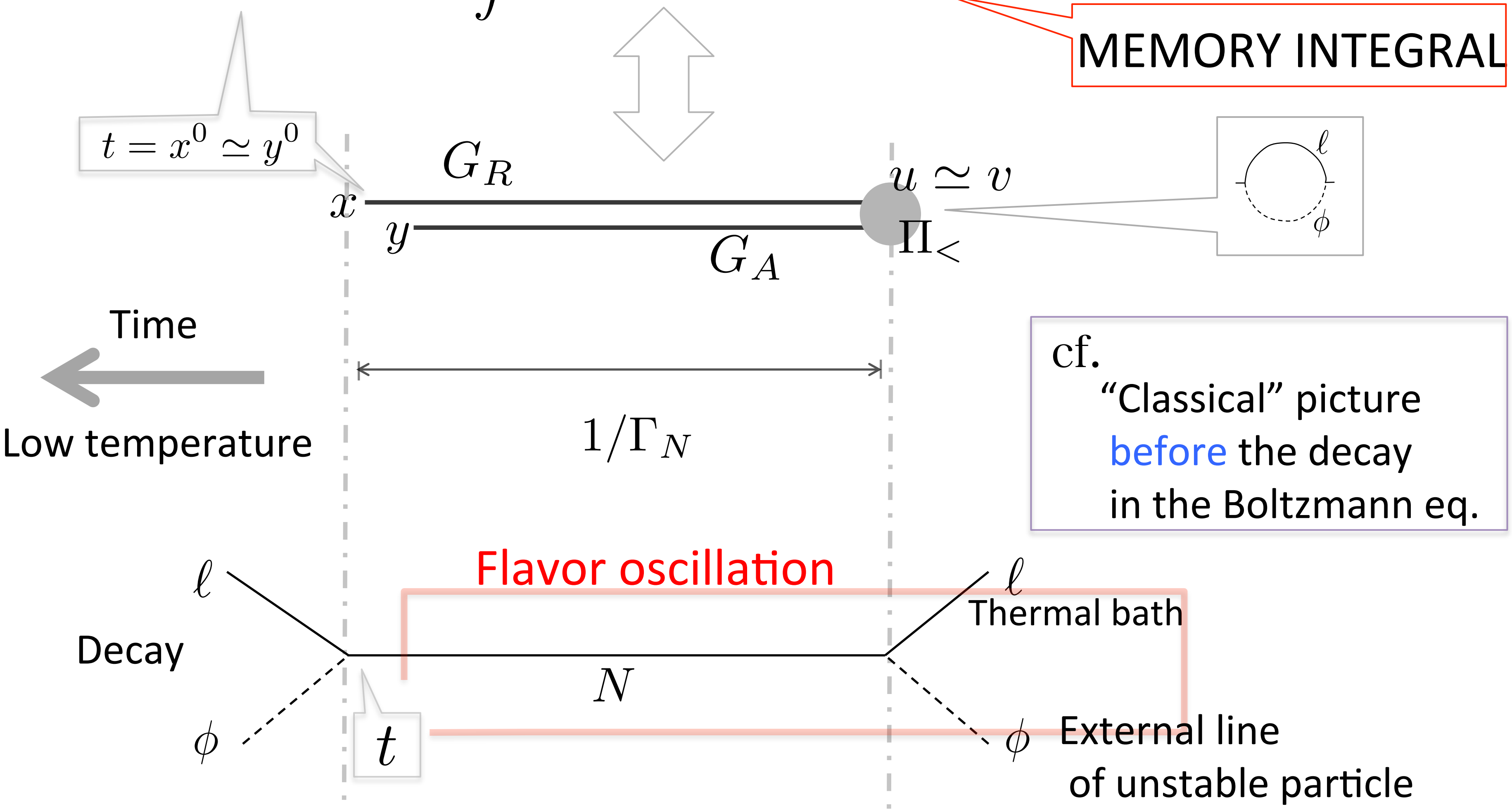
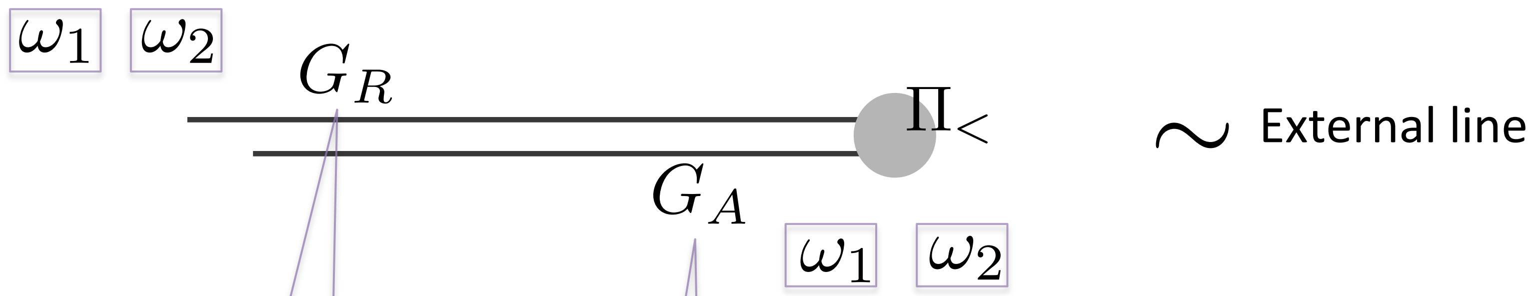
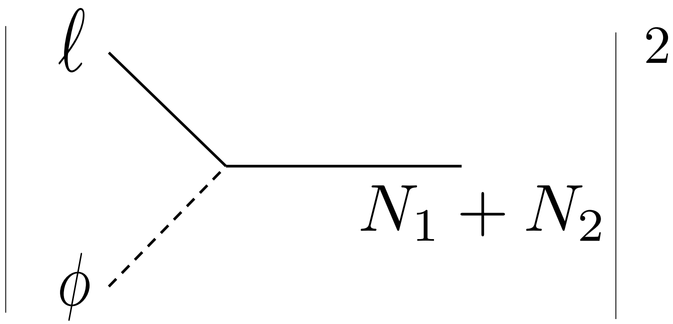


- CP-violating parameter ε from Wightman propagator



Each one is the superposition of the two different mass eigenstates.

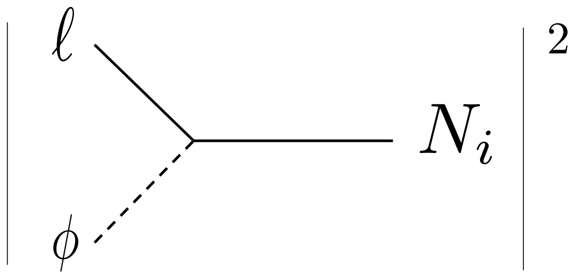
$\Leftrightarrow$



Decay of the superposition of the two different mass eigenstates

- CP-violating parameter ε from Wightman propagator

$\sum_{i=1,2}$

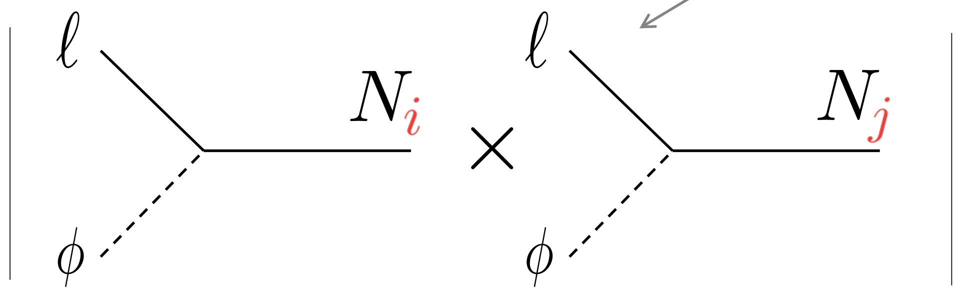


Decompose into

"Classical" and

"Quantum" contributions.

$+ \sum_{i \neq j}$



CP-violating parameter

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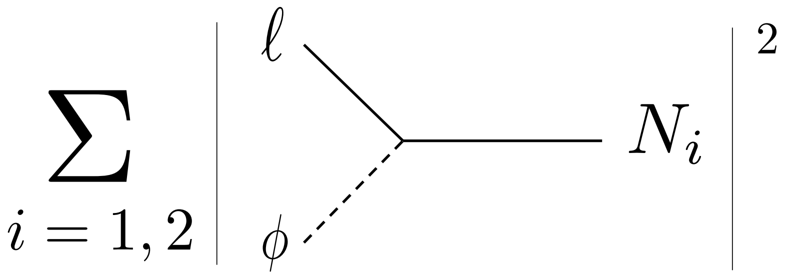
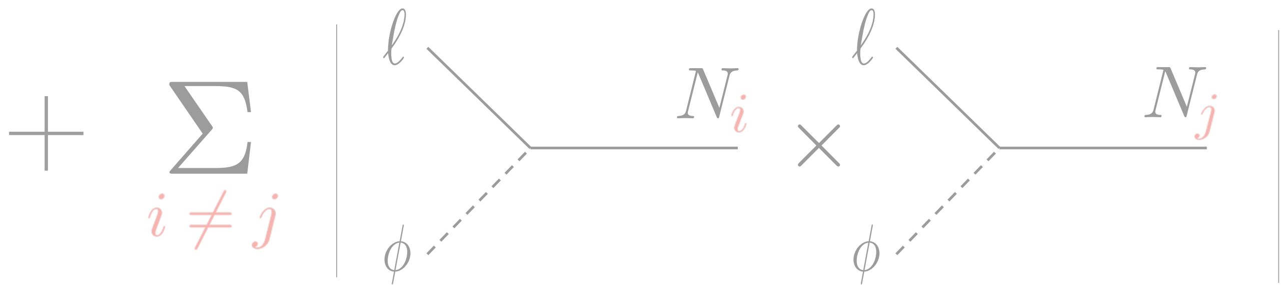


Neglecting the interference terms

regulator

$$R_{ij} = M_i\Gamma_i - M_j\Gamma_j$$

Boltzmann eq.



CP-violating parameter

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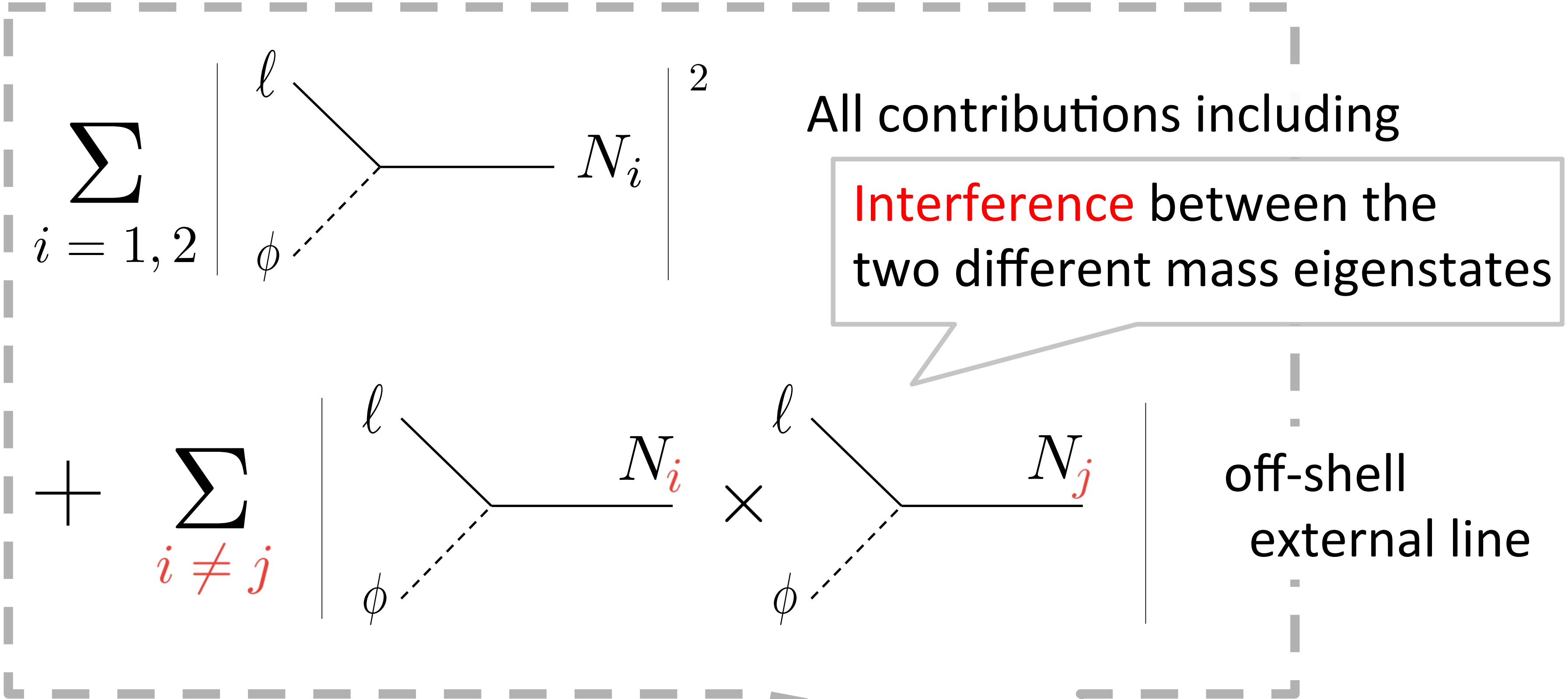

regulator

$$R_{ij} = M_i\Gamma_i - M_j\Gamma_j$$

All of quantum coherent effect

$$R_{ij} = M_i\Gamma_i + M_j\Gamma_j$$

- Final lepton asymmetry

$$Y_L = \frac{n_L}{s} = (\mathit{efficiency\ factor}) \times \mathcal{E}$$

Depending on
the magnitude of the Washout.

$$\frac{\mathcal{E}_{\mathrm{MAX}}(R_{ij} = M_i\Gamma_i + M_j\Gamma_j\ )}{\mathcal{E}_{\mathrm{MAX}}(R_{ij} = M_i\Gamma_j\ )} = \frac{1}{2}$$

➡ TeV scale leptogenesis is still possible.

To determine the precise value of the lower bound,
more detailed investigations are required.

- Summary

- In the expanding universe,
  the CP-violating parameter in the resonant leptogenesis
  is derived from Kaddanoff-Baym equation.

$$\varepsilon_i^{KB} = \frac{\Im(Y^\dagger Y)_{ij}^2}{(Y^\dagger Y)_{ii}(Y^\dagger Y)_{jj}} \frac{\Delta M_{ij}^2 M_i \Gamma_j}{(\Delta M_{ij}^2)^2 + (M_i \Gamma_i + M_j \Gamma_j)^2}$$

- The flavor oscillation (quantum effect)
  can be taken into account completely by the memory integral.

Applying this result to the models which need the resonant leptogenesis,
investigate whether there is the parameter region
to generate the correct baryon asymmetry.

Thank you for your attention.