

**Nucleon decay  
via dimension-6 operators  
in  
anomalous  $U(1)_A$  SUSY GUT models  
and  $E_6 \times SU(2)_F$  SUSY GUT model**

PASCOS 2013 @ NTU

based on arXiv:1307.7529 [hep-ph] and more  
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**1. Introduction and  
previous work**

**2.  $E_6 \times SU(2)_F$  Model**



# Grand Unified Theory

many advantages and several realistic models

How do we identify models as “the” grand unified theory?

## Model identification

$$\underline{\Lambda_{GUT} = 10^{12-16} GeV \gg \Lambda_{LHC} = 10^{3-4} GeV}$$

It is hard to identify models by accelerator.

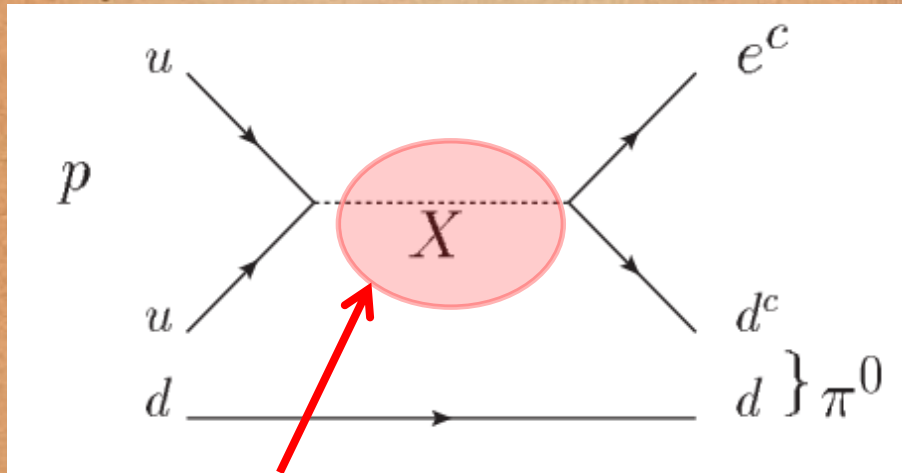
In this work, we try GUT model identification by

“the nucleon decay” (arXiv:1307.7529 [hep-ph]).

- phenomenon from GUT(bSM)
- expected to be observed in the near future

# The nucleon decay in SUSY GUT models

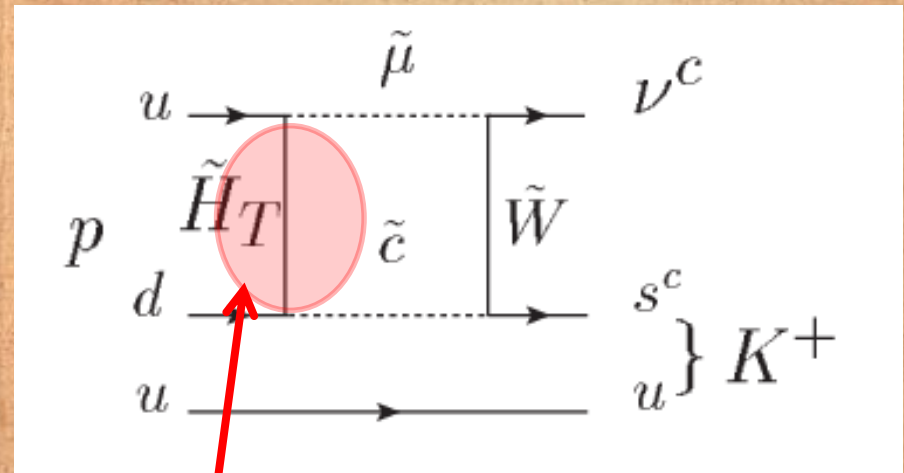
via dimension-6 operators  
the main decay mode



X-type( $X(3,2)$ ) gauge boson

$$p \rightarrow \pi^0 + e^c$$

via dimension-5 operators  
the main decay mode



triplet(colored) Higgsino

$$p \rightarrow K^+ + \nu^c$$

SUSY GUT	$\Lambda_{SUSY\ GUT} \sim 2 \times 10^{16}$ GeV
	$\tau_N \sim 10^{36}$ years ( $\propto \Lambda_{SUSY\ GUT}^4$ )

**weake suppression**

significant interaction

$$\tau_{p \rightarrow \pi^0 + e^c} \geq 1.3 \times 10^{34} \text{ @Super-Kamiokande}$$



# Anomalous $U(1)_A$ SUSY GUT

constructed under “natural assumptions”



- consider all operators which are allowed by symmetries
- consider effects of all higher-order operators
- operators' coefficients are order 1

Anomalous  $U(1)_A$  SUSY GUT models realize many features.

- doublet-triplet splitting
- gauge coupling unification
- realistic quark and lepton masses and mixings etc.

Nucleon decay is one of the most interesting predictions.

via dimension-6 operators

enhanced

Cabibbo angle

$$\Lambda_A \sim \lambda^{-a} \Lambda_{SUSY\ GUT} < \Lambda_{SUSY\ GUT}$$

GUT scale in anomalous  $U(1)_A$  SUSY GUT models

$$\Lambda_A \sim \lambda^{0.5} \Lambda_{SUSY\ GUT} = 10^{16} \text{ GeV}$$

via dimension-5 operators

Natural realization of doublet-triplet splitting **suppresses** the nucleon decay via dimension-5 operators.

The nucleon decay via dimension-6 operators is significant.

We study the nucleon decay via dimension-6 operators.



# Diagonalizing Matrix

$$\begin{aligned}\psi_{Li} Y_{ij} \psi_{Rj}^c &= (L_\psi^\dagger \psi_L)_i (L_\psi^T Y R_\psi)_{ij} (R_\psi^\dagger \psi_R^c)_j \\ &= \psi'_{Li} Y_{diag\ ij} \psi'_{Rj}^c\end{aligned}$$

$L_\psi, R_\psi$  : diagonalizing matrix

In the Standard Model (weak interaction)

$$U_{CKM} = L_u^\dagger L_d, \quad U_{MNS} = L_\nu^\dagger L_e$$

In nucleon decay operators (X type gauge int.)

Diagonalizing matrices appear directly.

Experimentally,

We cannot measure diagonalizing matrices directly.

Theoretically,

For example in the minimal SU(5) GUT model,

Yukawa matrices have few degrees of freedom.

➡ Diagonalizing matrices are strongly restricted.

$$\text{But, } Y_d = Y_e^T$$

To realize realistic quark and lepton masses and mixings,  
we introduce additional interactions.

➡ additional degrees of freedom

➡ Diagonalizing matrices have uncertainties.



In the previous work (arXiv:1307.7529 [hep-ph]),  
we try to identify GUT models by nucleon decay.

two important ratios

$$R_1 \equiv \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$$

To identify  
grand unification group

$$R_2 \equiv \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$$

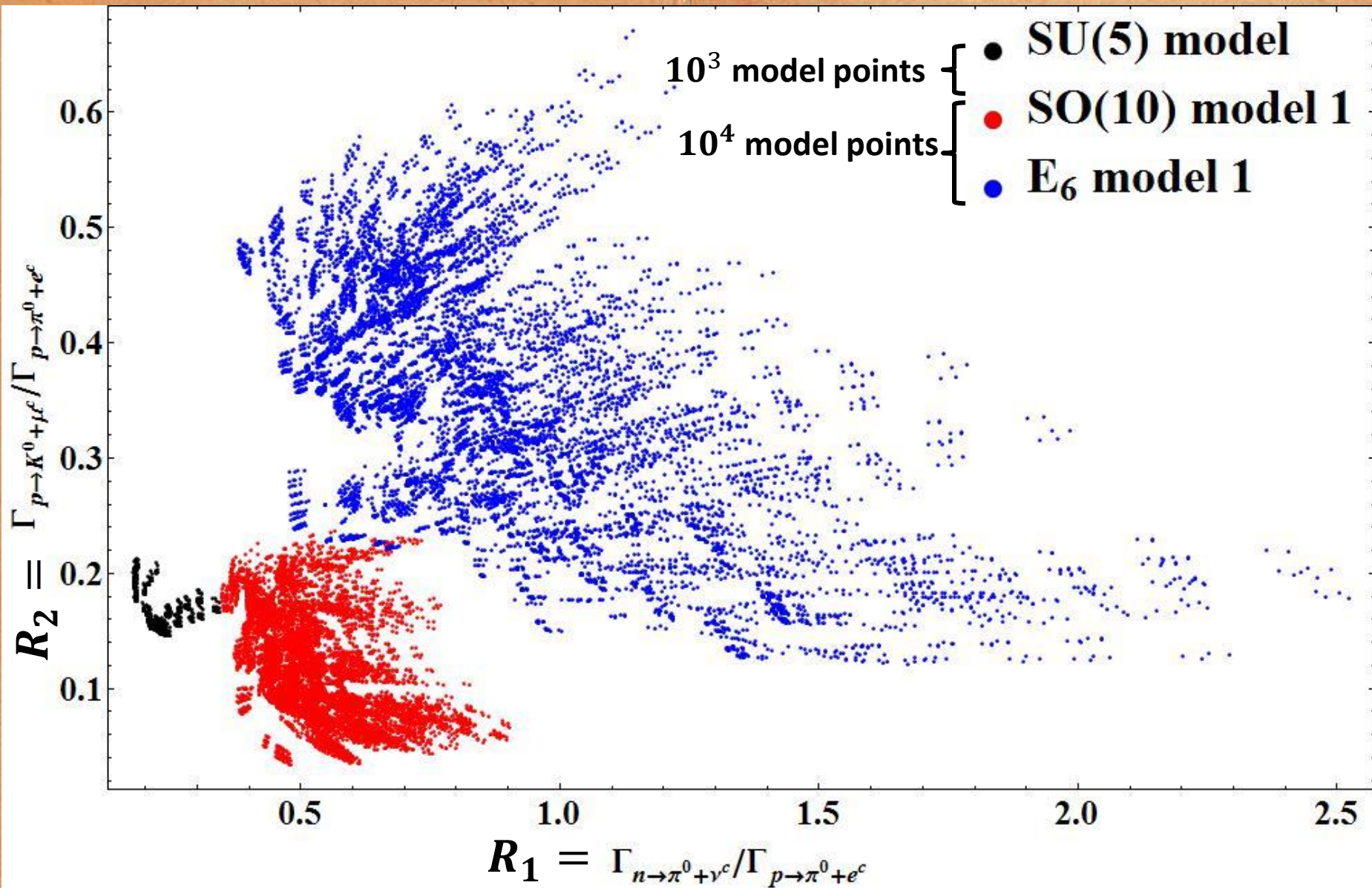
To identify  
Yukawa structure

$$\begin{aligned} \mathcal{L}_{eff} = & \frac{g_{GUT}^2}{M_X^2} \left\{ \overline{(e_{Ri}^c u_{Rj})} (\overline{u_{Lj}^c d_{Li}}) + \overline{(e_{Ri}^c u_{Rj})} (\overline{u_{Li}^c d_{Lj}}) \right. \\ & \left. + \overline{(e_{Li}^c u_{Lj})} (\overline{u_{Rj}^c d_{Ri}}) - \overline{(v_{Li}^c d_{Lj})} (\overline{u_{Rj}^c d_{Ri}}) \right\} \\ & + \frac{g_{GUT}^2}{M_{X'}^2} \left\{ \overline{(e_{Li}^c u_{Lj})} (\overline{u_{Ri}^c d_{Ri}}) - \overline{(v_{Li}^c d_{Lj})} (\overline{u_{Ri}^c d_{Rj}}) \right\} \\ & + \frac{g_{GUT}^2}{M_{X''}^2} \left\{ \overline{(E_{Li}^c u_{Lj})} (\overline{u_{Ri}^c D_{Bj}}) - \overline{(N_{Li}^c d_{Lj})} (\overline{u_{Ri}^c D_{Bj}}) \right\} \end{aligned}$$

SU(5) GUT

SO(10) GUT

E<sub>6</sub> GUT





**1. Introduction and  
previous work**

**2.  $E_6 \times SU(2)_F$  Model**

# $E_6 \times SU(2)_F$ Model

Yukawa structure at GUT scale is restricted.

27 dimensional representation for matter :  $\Psi$

$\Psi_a (a = 1,2) : SU(2)_F$  doublet for first- and second-generation matters

$\Psi_3 : SU(2)_F$  singlet for third-generation matters

## Features

- Realize realistic quark and lepton masses and mixings within restricted Yukawa structure
- Solve SUSY CP problem (Chromo-EDM constraint) by spontaneous CP violation mechanism
- Suppress Flavor Changing Neutral Current processes



in previous work

$$Y_u = \begin{pmatrix} y_{u11}\lambda^6 & y_{u12}\lambda^5 & y_{u13}\lambda^3 \\ y_{u21}\lambda^5 & y_{u22}\lambda^4 & y_{u23}\lambda^2 \\ y_{u31}\lambda^3 & y_{u32}\lambda^2 & y_{u33} \end{pmatrix}$$

$$Y_d = \begin{pmatrix} y_{d11}\lambda^6 & y_{d12}\lambda^{5.5} & y_{d13}\lambda^5 \\ y_{d21}\lambda^5 & y_{d22}\lambda^{4.5} & y_{d23}\lambda^4 \\ y_{d31}\lambda^3 & y_{d32}\lambda^{2.5} & y_{d33}\lambda^2 \end{pmatrix}$$

$$Y_e = \begin{pmatrix} y_{e11}\lambda^6 & y_{e12}\lambda^5 & y_{e13}\lambda^3 \\ y_{e21}\lambda^{5.5} & y_{e22}\lambda^{4.5} & y_{e23}\lambda^{2.5} \\ y_{e31}\lambda^5 & y_{e32}\lambda^4 & y_{e33}\lambda^2 \end{pmatrix}$$

$9 \times 3 = 27$   $O(1)$  parameters  
for masses and mixings

in  $E_6 \times SU(2)_F$  Model

$$Y_u = \begin{pmatrix} 0 & \frac{1}{3}y_{u12}\lambda^5 & 0 \\ -\frac{1}{3}y_{u12}\lambda^5 & y_{u22}\lambda^4 & y_{u23}\lambda^2 \\ 0 & y_{u23}\lambda^2 & y_{u33} \end{pmatrix}$$

$$Y_d = \begin{pmatrix} y_{d11}\lambda^6 & y_{d12}\lambda^{5.5} & \frac{1}{3}y_{d13}\lambda^5 \\ y_{d21}\lambda^5 & y_{d22}\lambda^{4.5} & y_{d23}\lambda^4 \\ y_{d31}\lambda^3 & y_{d32}\lambda^{2.5} & y_{d33}\lambda^2 \end{pmatrix}$$

$$Y_e = \begin{pmatrix} y_{d11}\lambda^6 & y_{e12}\lambda^5 & 0 \\ 0 & y_{e22}\lambda^{4.5} & y_{e23}\lambda^{2.5} \\ -y_{e12}\lambda^5 & y_{d23}\lambda^4 & y_{d33}\lambda^2 \end{pmatrix}$$

$4 + 9 + 3 = 16$   $O(1)$   
parameters for masses and  
mixings



$\theta_{ij}^{\psi_{L,R}}$ : mixing angle for diagonalizing matrix of  $\psi_{L,R}$

$$L_{\Psi}, R_{\Psi} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^{\Psi_{L,R}} & s_{23}^{\Psi_{L,R}} \\ 0 & -s_{23}^{\Psi_{L,R}} & c_{23}^{\Psi_{L,R}} \end{pmatrix} \begin{pmatrix} c_{13}^{\Psi_{L,R}} & 0 & s_{13}^{\Psi_{L,R}} \\ 0 & 1 & 0 \\ -s_{13}^{\Psi_{L,R}} & 0 & c_{13}^{\Psi_{L,R}} \end{pmatrix} \begin{pmatrix} c_{12}^{\Psi_{L,R}} & s_{12}^{\Psi_{L,R}} & 0 \\ -s_{12}^{\Psi_{L,R}} & c_{12}^{\Psi_{L,R}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$s_{ij}^{\Psi_{L,R}} \sim \theta_{ij}^{\Psi_{L,R}}, c_{ij}^{\Psi_{L,R}} \sim 1 \quad (\text{main order})$$

$$\theta_{13}^{uL} = 0 \quad \theta_{13}^{uR} = 0 \quad \theta_{13}^{eL} = 0$$

$$\theta_{23}^{uL} = \theta_{23}^{uR} \quad \theta_{23}^{dL} = \theta_{23}^{eR} \quad \theta_{12}^{eR} = \theta_{23}^{eL} \theta_{12}^{eL}$$

$$|\theta_{12}^{uL}| = |\theta_{12}^{uR}| = \sqrt{m_u/m_c} \quad \frac{m_{\mu}}{m_{\tau}} = -\frac{\theta_{13}^{eR}}{\theta_{12}^{eL}}$$

$$\frac{\theta_{13}^{eL} \theta_{13}^{eR} m_{\tau} - \theta_{13}^{dL} \theta_{13}^{dR} m_b + \theta_{12}^{eL} \theta_{12}^{eR} m_{\mu} - \theta_{12}^{dL} \theta_{12}^{dR} m_s}{m_d - m_e} = 1$$

$$m_b = m_{\tau}$$

11 conditions @ GUT scale



# Yukawa matrix 1-loop renormalization group equation

$$\frac{d}{dt} \mathbf{Y}_{u,d,e} = \frac{1}{16\pi^2} \beta_{\mathbf{Y}_{u,d,e}}^{(1)} \quad \text{in MSSM}$$

$$\beta_{\mathbf{Y}_u}^{(1)} = \mathbf{Y}_u \left\{ 3\text{Tr}(\mathbf{Y}_u \mathbf{Y}_u^\dagger) + 3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_d^\dagger \mathbf{Y}_d - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 \right\}$$

$$\beta_{\mathbf{Y}_d}^{(1)} = \mathbf{Y}_d \left\{ \text{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) + 3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_u^\dagger \mathbf{Y}_u - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 \right\}$$

$$\beta_{\mathbf{Y}_e}^{(1)} = \mathbf{Y}_e \left\{ \text{Tr}(3\mathbf{Y}_d \mathbf{Y}_d^\dagger + \mathbf{Y}_e \mathbf{Y}_e^\dagger) + 3\mathbf{Y}_e^\dagger \mathbf{Y}_e - 3g_2^2 - \frac{9}{5}g_1^2 \right\}$$



$$\theta_{13}^{uL} = \text{negligible} \quad \theta_{13}^{uR} = \text{negligible} \quad \theta_{13}^{eL} = \text{negligible}$$

$$\theta_{23}^{uL} = \theta_{23}^{uR} \quad \theta_{23}^{dL} = \theta_{23}^{eR} \quad \theta_{12}^{eR} = \theta_{23}^{eL} \theta_{12}^{eL}$$

$$|\theta_{12}^{uL}| = |\theta_{12}^{uR}| = \sqrt{m_u/m_c} \quad \frac{m_\mu}{m_\tau} = -\frac{\theta_{13}^{eR}}{\theta_{12}^{eL}}$$

~~$$\frac{\theta_{13}^{eL} \theta_{13}^{eR} m_\tau - \theta_{13}^{dL} \theta_{13}^{dR} m_b + \theta_{12}^{eL} \theta_{12}^{eR} m_\mu - \theta_{12}^{dL} \theta_{12}^{dR} m_s}{m_d - m_e} = 1$$~~

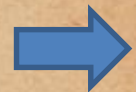
9 conditions for mixing angles @ low energy scale



Parameters for real diagonalizing matrix :

3 parameters (mixing angle) for each matrix

7 diagonalizing matrices :  $L_u, L_d, L_e, L_\nu, R_u, R_d, R_e$

 21 parameters

To realize

$$U_{CKM} = L_u^\dagger L_d, U_{MNS} = L_\nu^\dagger L_e$$

we use 6 parameters.

$E_6$  GUT model in previous work :

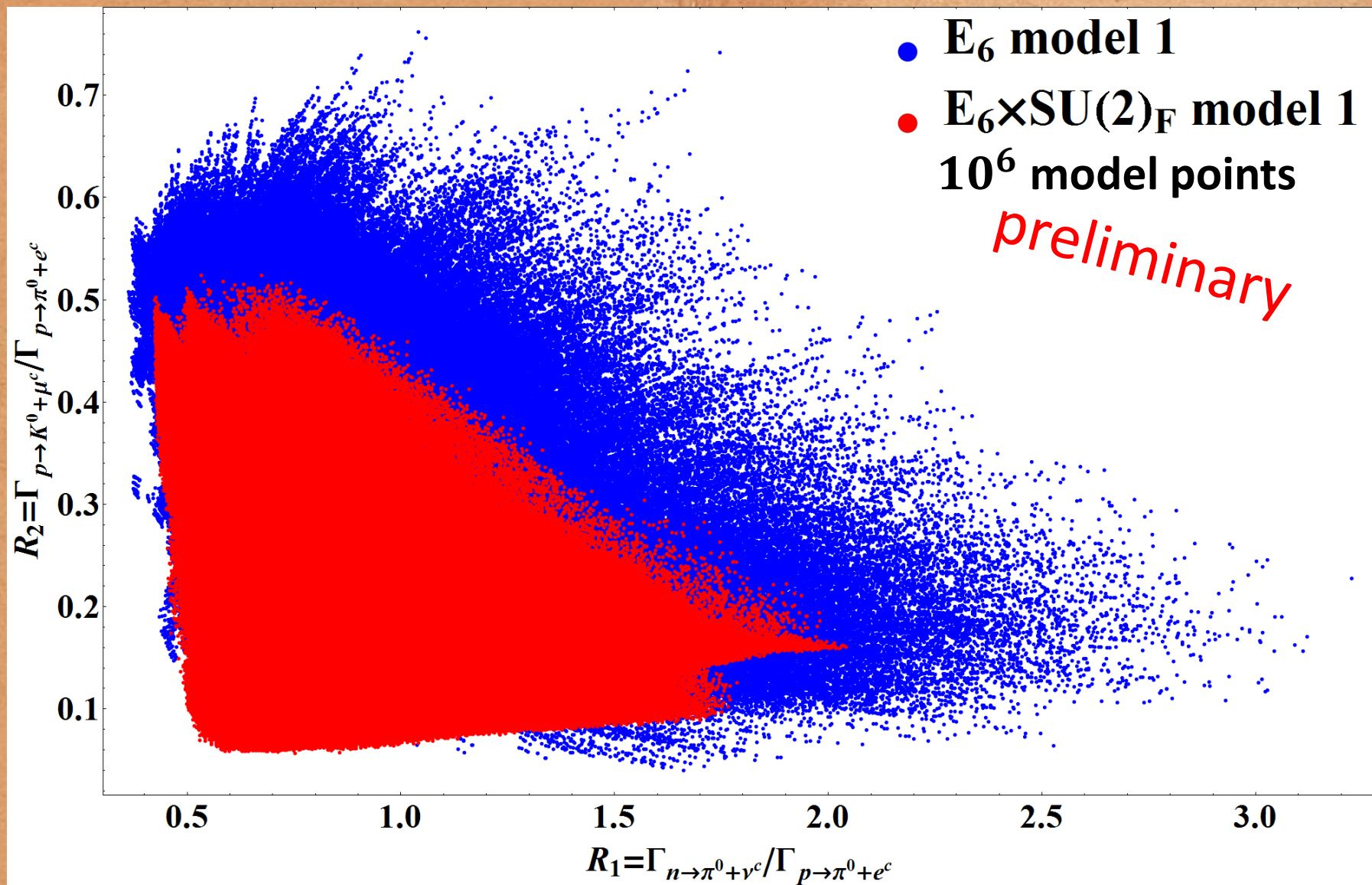
$$21 - 6 = 15 \text{ parameters}$$

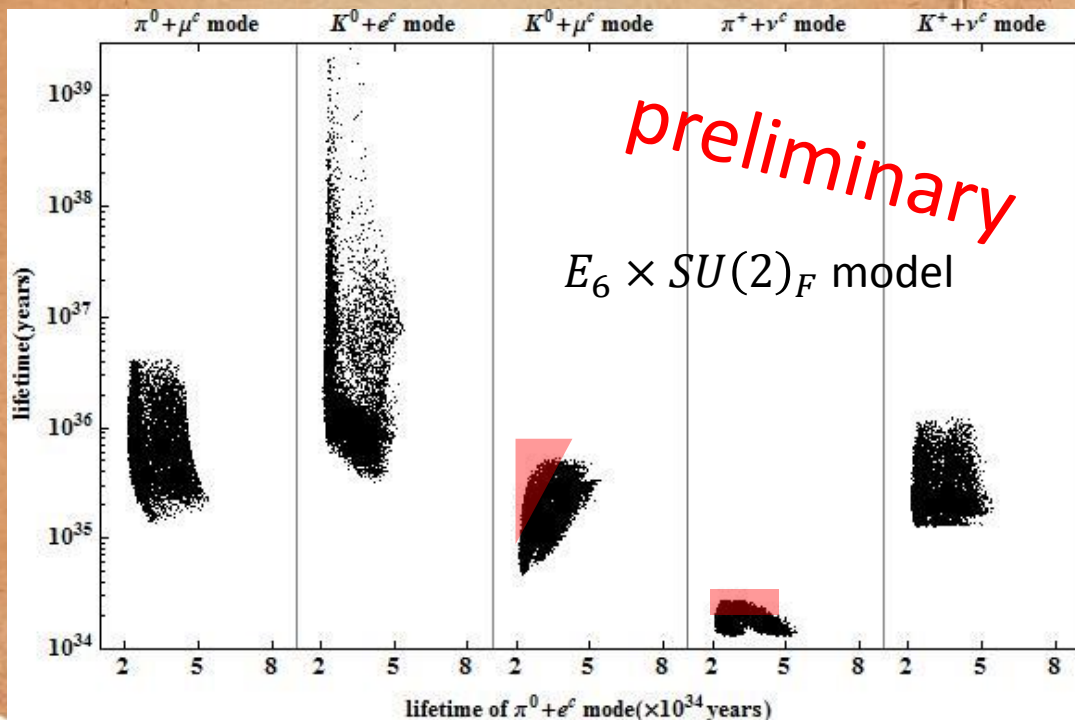
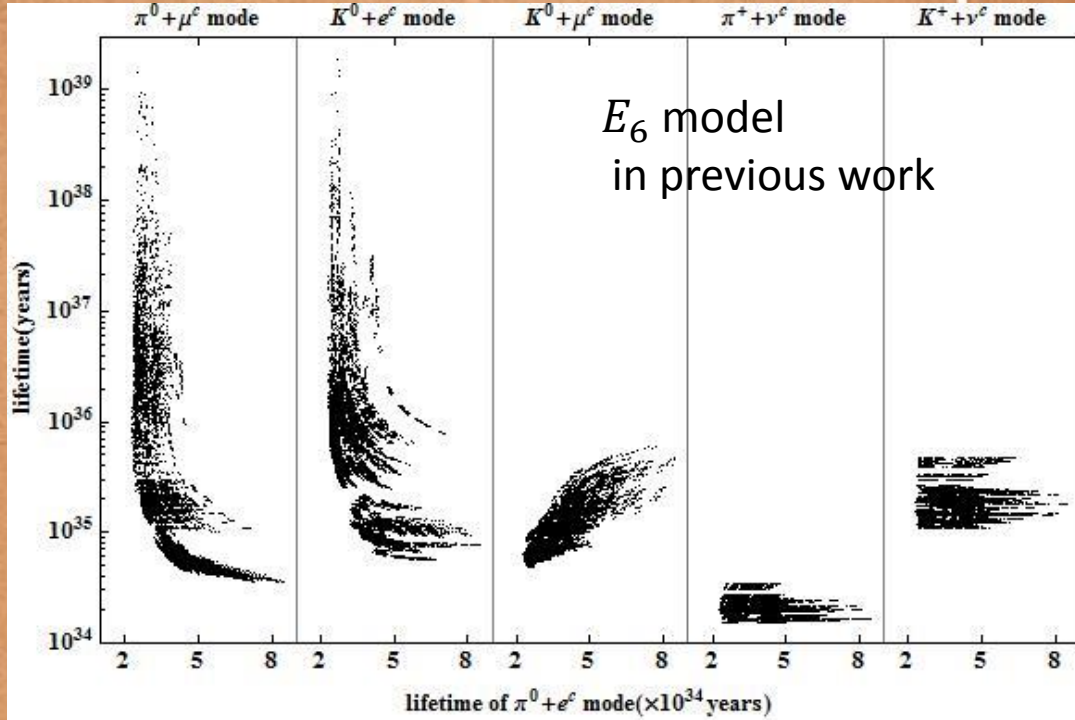
In this work we use 9 conditions.

$E_6 \times SU(2)_F$  GUT model :

$$21 - 6 - 9 = 6 \text{ parameters (+ 1 sign)}$$







In previous  $E_6$  model

$$L_{e12} \sim \lambda^{0.5} \sim 0.5,$$

$$L_{e13} \sim \lambda^3 \sim 0.01$$



In  $E_6 \times SU(2)_F$  model

$$L_{e12} \sim 0.39 < \lambda^{0.5}, L_{e13} = 0$$

**Small mixing  
from first-generation  
charged lepton**

$$\tau_{p \rightarrow \pi^0 + e^c} \geq 1.3 \times 10^{34} \text{ years}$$

@Super-Kamiokande



$$\tau_{p \rightarrow \pi^0 + e^c} \geq 1.3 \times 10^{35} \text{ years}$$

@Hyper-Kamiokande(planning)



# Summary

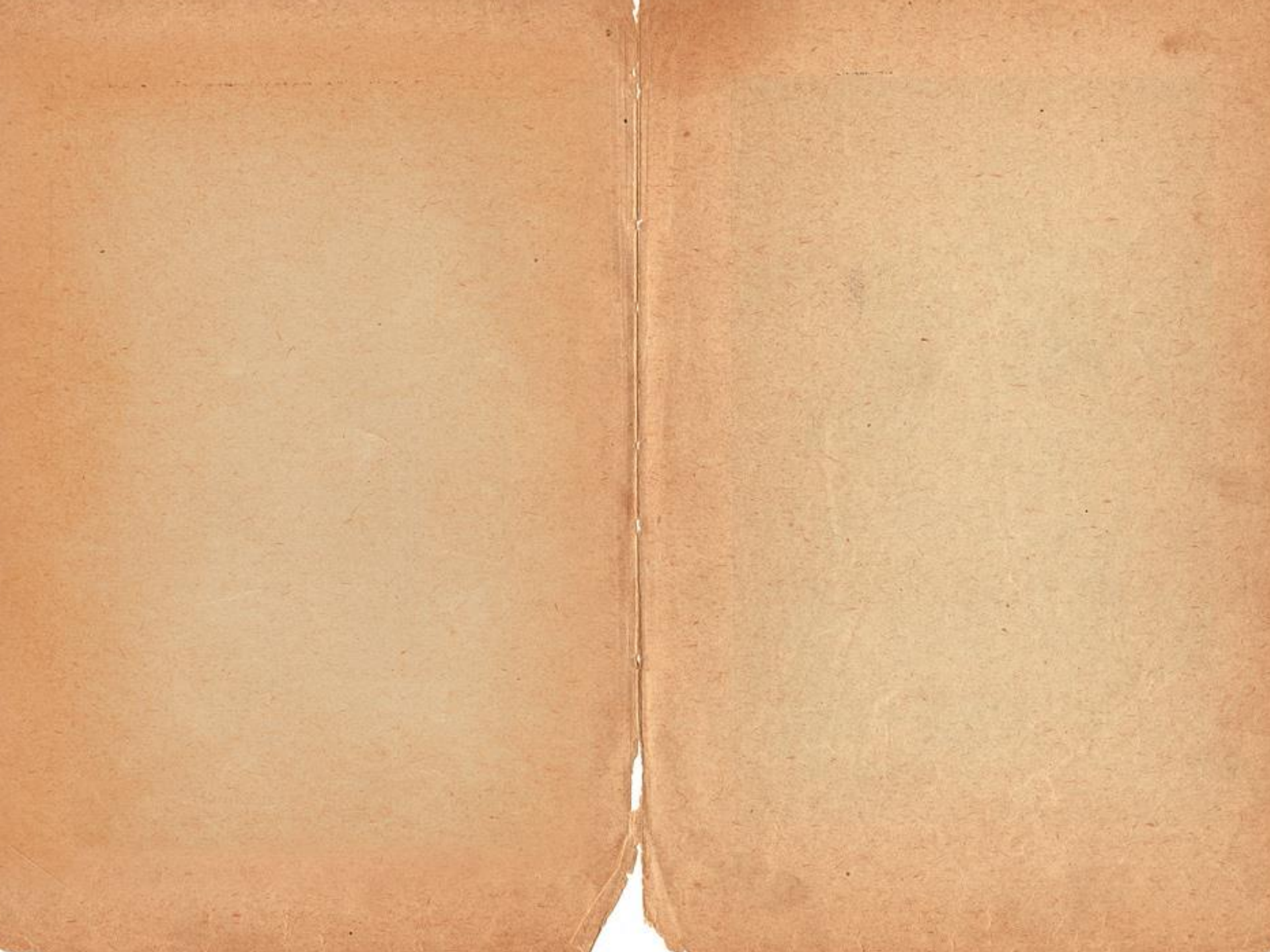
- We calculate nucleon lifetimes.  
Especially, we pay attention to uncertainties of diagonalizing matrices.
- Flavor symmetry restrict Yukawa structure and diagonalizing matrices, therefore we can reduce uncertainties of diagonalizing matrices.
- In many model points  $R_1$  and  $R_2$  tend to be smaller than these in the  $E_6$  model of previous work by conditions for mixing angles .





Thank you for your attention.





**Back up slide**



# Why do we use $n \rightarrow \pi^0 + \nu^c$ ?

$$\Gamma_{n \rightarrow \pi^0 + \nu^c} / \Gamma_{p \rightarrow \pi^0 + e^c}$$

## not $p \rightarrow \pi^+ + \nu^c$ ?

- sensitivity

$$N \rightarrow \nu \pi$$

Partial mean life  
( $10^{30}$  years)

$$> 112 (n), > 25 (p)$$

PDG(2012)

- form factor

$$\langle \pi^0 | (ud)_{\Gamma} u_{\Gamma'} | p \rangle = \langle \pi^0 | (du)_{\Gamma} d_{\Gamma'} | n \rangle$$

We can **cancel form factor and uncertainty of that!!**

# $\bar{5}$ rep. mixing

In the minimal SO(10) GUT model, all SM matters and  $\nu_R$  in each generation belong to only 16 rep.

➔ Diagonalizing matrices for each matter are same at the GUT scale.

➔ It is hard to realize realistic fermion masses and mixings in the minimal SO(10) GUT model.

## SO(10) GUT with 10 rep.

$$\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

diagonalizing matrix for each matter that belong to 16 rep. in SO(10).

➔ for  $\bar{5}$  rep.

belong to 16 rep.

belong to 10 rep.

$$\begin{pmatrix} \bar{5}_{\psi_1} \\ \bar{5}_{\psi_2} \\ \bar{5}_{\psi_3} \\ \bar{5}_T \end{pmatrix}$$

mixed

➔ for 10 rep.

$$\begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

diagonalizing matrix for each matter that belong to 10 rep. in SU(5).  
CKM type

$$\begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}$$

diagonalizing matrix for each matter that belong to  $\bar{5}$  rep. in SU(5).  
MNS type



# dependence for anti-electron mode

$$\begin{pmatrix} A_{11} & A_{12}\lambda \\ A_{21}\lambda & A_{22} \end{pmatrix}$$

$A_{ij}$ : order 1

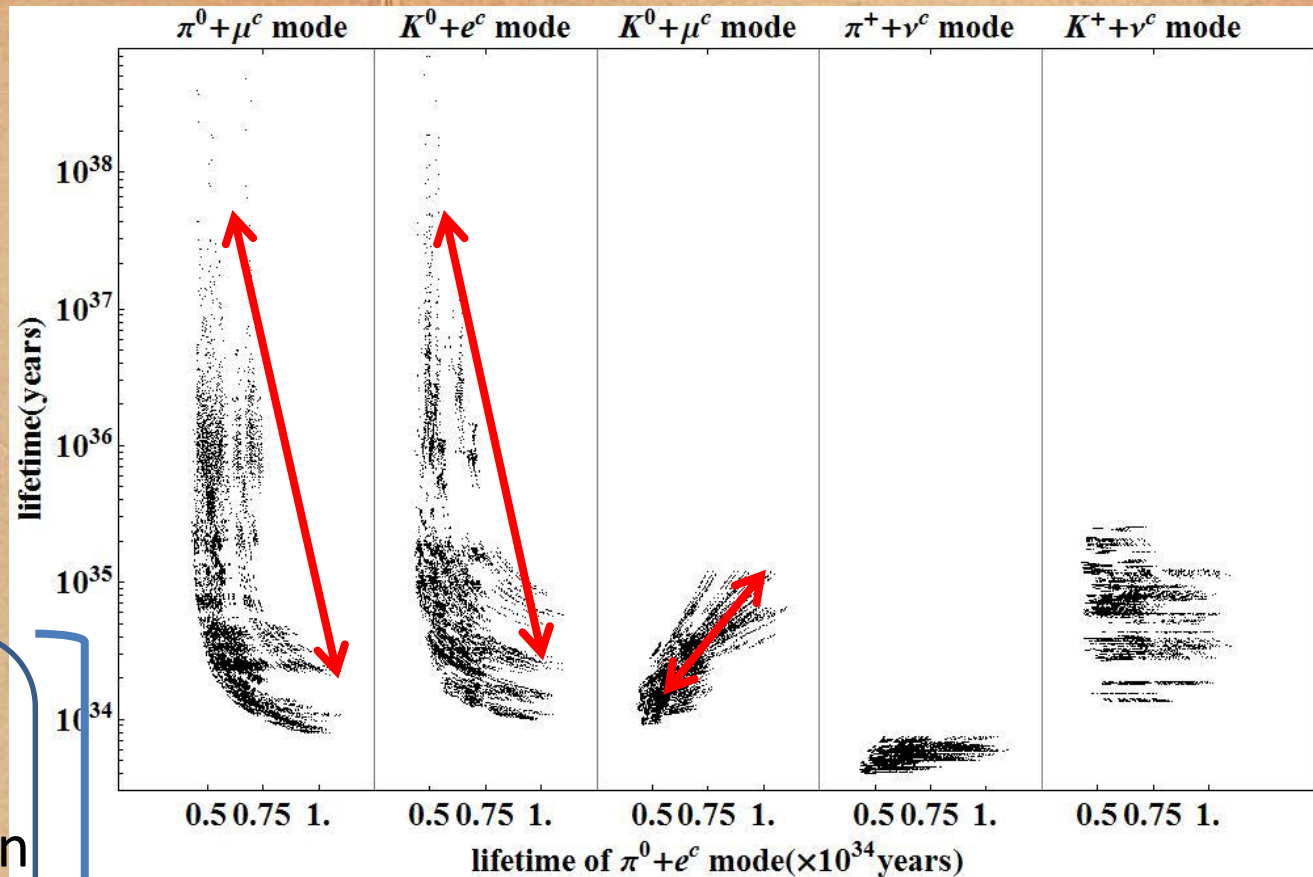
uncertainty

large off diagonal  
element

↓ unitary condition

small diagonal  
element

= large mixing

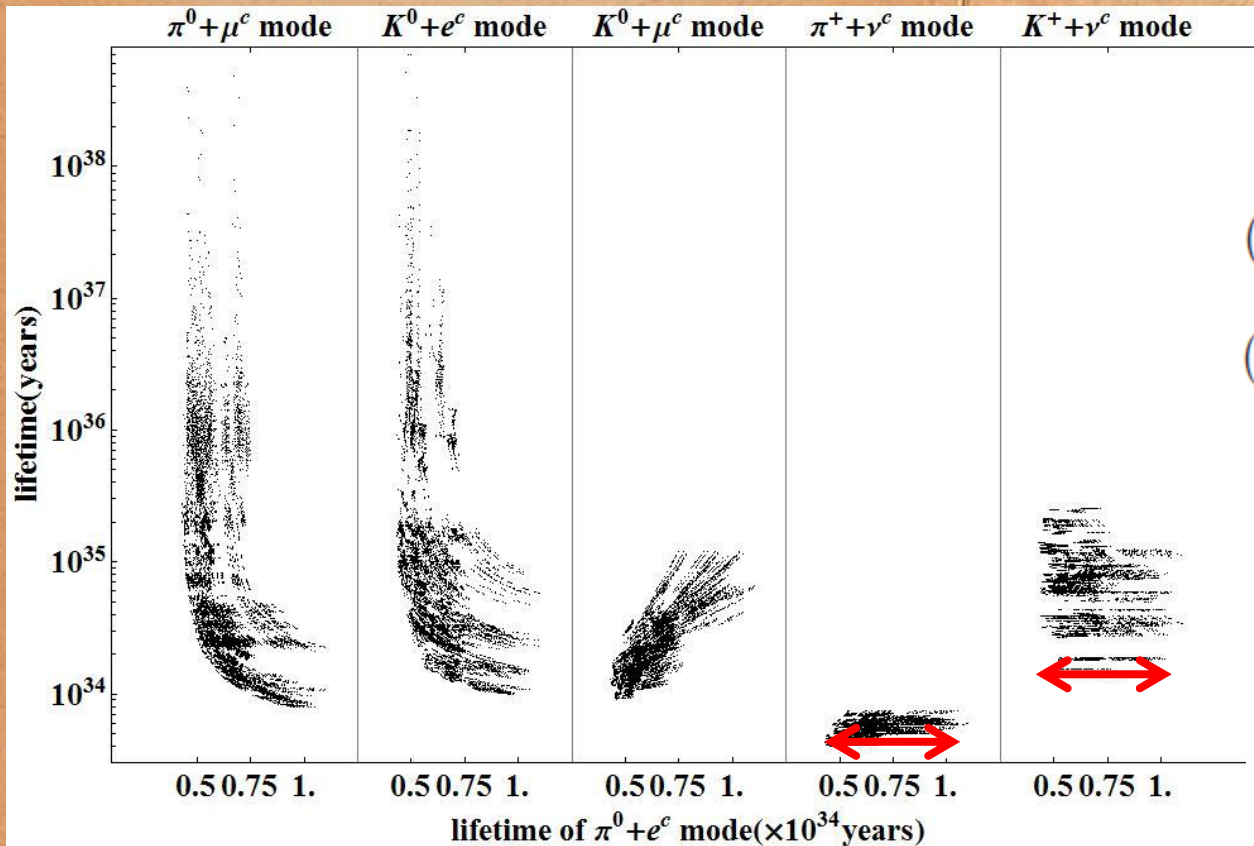


Lifetime of  $p \rightarrow \pi^0 + e^c, K^0 + \mu^c$  mode become **long**.



Lifetime of  $p \rightarrow \pi^0 + \mu^c, K^0 + e^c$  mode become **short**.

# dependence for anti-neutrino mode



nucleon to  
anti-electron operator

$$(\epsilon^{\alpha\beta\gamma} \bar{u}_{R\gamma}^c \gamma^\mu u_{L\beta}) (\bar{e}_R^+ \gamma_\mu d_{L\alpha})$$

$$(\epsilon^{\alpha\beta\gamma} \bar{u}_{R\gamma}^c \gamma^\mu u_{L\beta}) (\bar{e}_L^+ \gamma_\mu d_{R\alpha})$$

$$L_u, R_e, L_e, L_d, R_d$$

nucleon to  
anti-neutrino operator

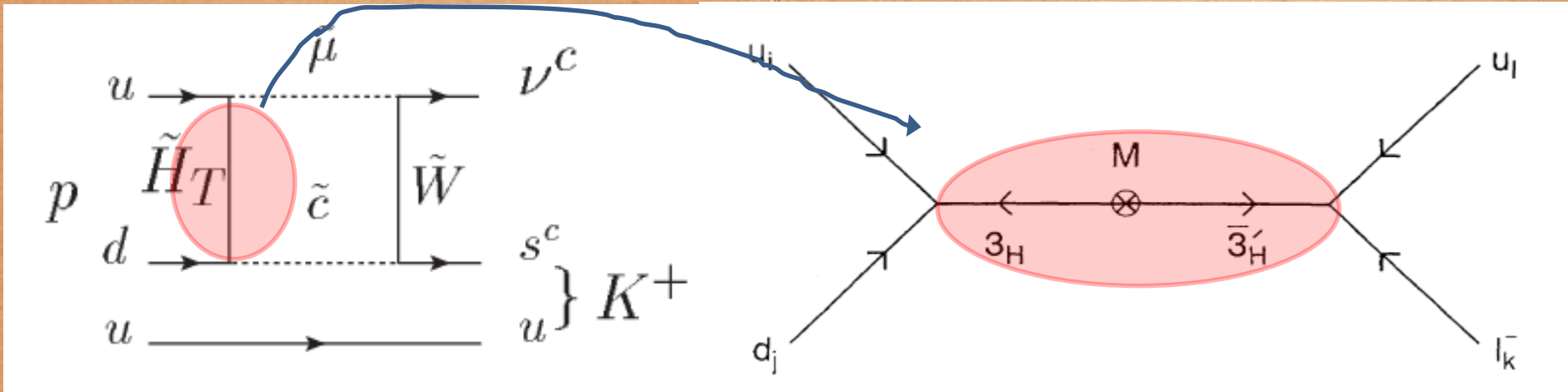
$$(\epsilon^{\alpha\beta\gamma} \bar{u}_{R\gamma}^c \gamma^\mu d_{L\beta}) (\bar{\nu}_L^c \gamma_\mu d_{R\alpha})$$

$$L_\nu, L_d, R_d$$

$$R_u = 1_{3 \times 3}$$



# suppression of dim 5 effective int.



mass matrix of triple Higgs

$$(3_H \quad 3_{H'}) \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \bar{3}_H \\ \bar{3}_{H'} \end{pmatrix}$$

effective colored Higgs mass

$$m_c^{eff} \sim m^2 / M$$

In anomalous  $U(1)_A$  SUSY GUT models,

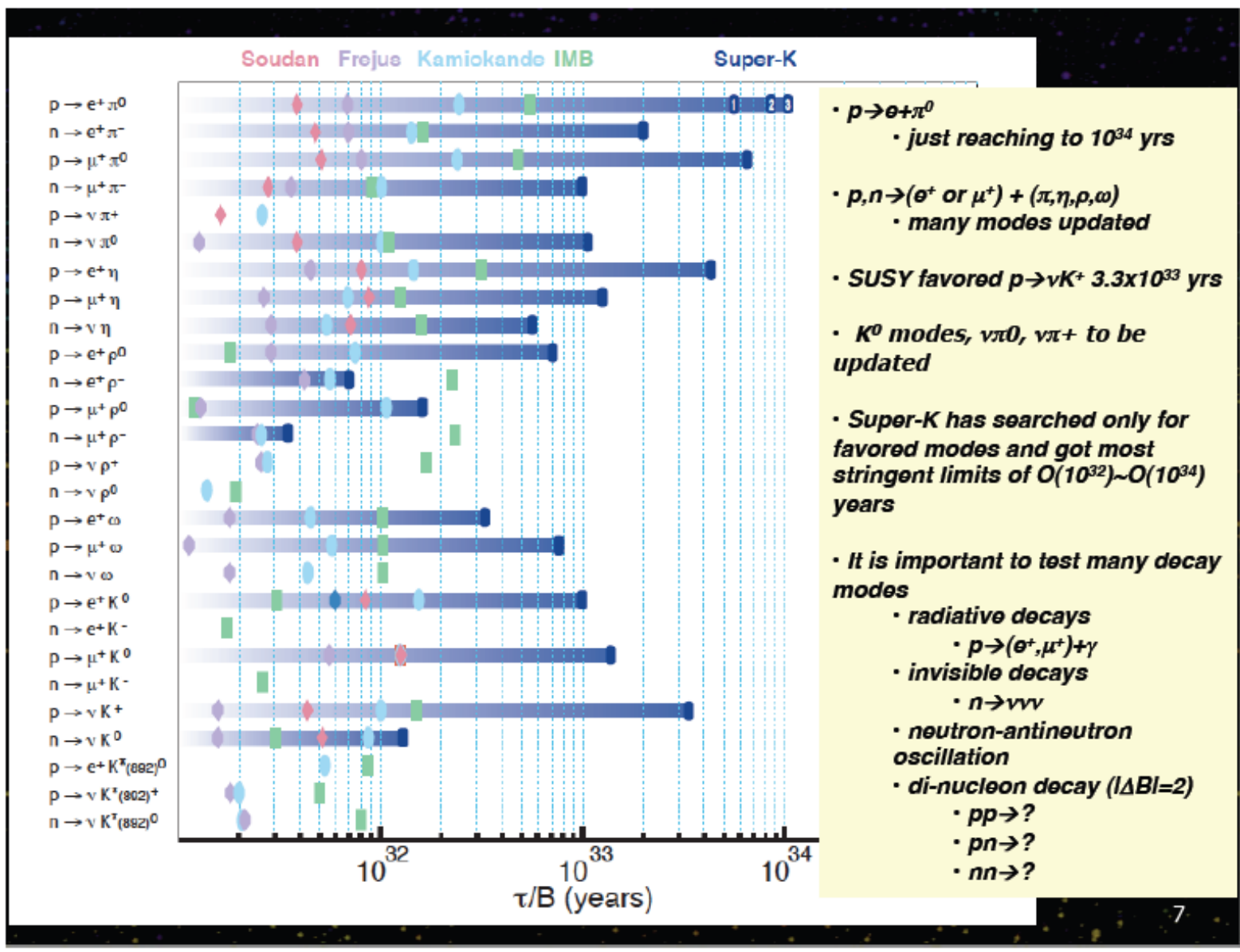
we can realize  $m_c^{eff} > 10^{18} GeV$ .



strongly suppressed

forbidden

# lower limit of nucleon lifetime





future lower limit of nucleon lifetime

@ Hyper-Kamiokande (10years running)

Mode	Sensitivity (90% CL)	Current limit
$p \rightarrow e^+ \pi^0$	$13 \times 10^{34}$ years	$1.3 \times 10^{34}$ years
$p \rightarrow \mu^+ \pi^0$	$9.0 \times 10^{34}$	$1.1 \times 10^{34}$
$p \rightarrow e^+ \eta^0$	$5.0 \times 10^{34}$	$0.42 \times 10^{34}$
$p \rightarrow \mu^+ \eta^0$	$3.0 \times 10^{34}$	$0.13 \times 10^{34}$
$p \rightarrow e^+ \rho^0$	$1.0 \times 10^{34}$	$0.07 \times 10^{34}$
$p \rightarrow \mu^+ \rho^0$	$0.37 \times 10^{34}$	$0.02 \times 10^{34}$
$p \rightarrow e^+ \omega^0$	$0.84 \times 10^{34}$	$0.03 \times 10^{34}$
$p \rightarrow \mu^+ \omega^0$	$0.88 \times 10^{34}$	$0.08 \times 10^{34}$
$n \rightarrow e^+ \pi^-$	$3.8 \times 10^{34}$	$0.20 \times 10^{34}$
$n \rightarrow \mu^+ \pi^-$	$2.9 \times 10^{34}$	$0.10 \times 10^{34}$
$p \rightarrow \bar{\nu} K^+$	$2.5 \times 10^{34}$	$0.40 \times 10^{34}$

$$Y_u = \begin{pmatrix} 0 & \frac{1}{3}y_{u12}\lambda^5 & 0 \\ -\frac{1}{3}y_{u12}\lambda^5 & y_{u22}\lambda^4 & y_{u23}\lambda^2 \\ 0 & y_{u23}\lambda^2 & y_{u33} \end{pmatrix}$$

origin of  $\frac{1}{3}$

$$\Psi^a A \Psi_a H$$

$H$ : fundamental representation Higgs

$A$ : adjoint Higgs  $\langle A \rangle \propto Q_{B-L}$

$\frac{1}{3}$  is B-L charge of quarks.