Parity Violating Hydrodynamics from Gravity

Shou-Huang Dai

Leung Center for Cosmology and Particle Astrophysics, NTU

PASCOS 2013

Based on arXiv: 1206.0850
with JW Chen, NE Lee and D Maity

Relativistic Hydrodynamics

- a long-wavelength effective theory for scales >> $\ell_{\rm mfp}$, with the hydrodynamic variables: u^μ, T , μ.

- local equilibrium is assumed at the scale ~ $\ell_{\rm mfp}$.

- Governed by the conservation laws

$$\nabla_\mu T^{\mu\nu} = F^{\nu\lambda}_{ext} J_\lambda \ , \qquad \nabla_\mu J^\mu = 0 \ .$$

with the following constitutive equations….

Relativistic Hydrodynamics

- The constitutive equations: (Poincare symm., *parity-even*)

$$T^{\mu\nu} = \rho\, u^\mu\, u^\nu + P\Delta^{\mu\nu}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$\Delta^{\mu\nu} u_\nu = 0$$

$$u^\mu u^\mu = -1$$

$$J^\mu = n_0\, u^\mu$$

Where we will be in the *Landau frame*

Relativistic Hydrodynamics

- The constitutive equations: (Poincare symm., *parity-even*)

$$T^{\mu\nu} = \rho\, u^{\mu}\, u^{\nu} + P\Delta^{\mu\nu} - 2\,\eta\,\sigma^{\mu\nu} - \Delta^{\mu\nu}\,\zeta\,\theta + \ldots$$

Shear Viscosity — bulk Viscosity

$$J^{\mu} = n_0\, u^{\mu} + \sigma\, E^{\mu} - \kappa\, \Delta^{\mu\nu}\nabla_{\nu}\frac{\mu}{T} + \ldots$$

electric conductivity — thermal Viscosity

Where

$$\sigma^{\mu\nu} = \Delta^{\mu\alpha}\Delta^{\nu\beta}\nabla_{(\alpha}u_{\beta)} - \frac{\Delta^{\mu\nu}}{d-1}\theta \qquad \text{(Shear)}$$

$$\theta = \Delta^{\mu\nu}\nabla_{\mu}u_{\nu} \qquad \text{(Expansion)}$$

Relativistic Hydrodynamics

- The constitutive equations: (Poincare symm., *parity-odd*)

$$T^{\mu\nu} = \rho\, u^\mu\, u^\nu + P\Delta^{\mu\nu} - 2\,\eta\,\sigma^{\mu\nu} - \Delta^{\mu\nu}\,\zeta\,\theta - (\widetilde{\zeta}_A \Omega + \widetilde{\zeta}_B B)\Delta^{\mu\nu} - \widetilde{\eta}_A \widetilde{\sigma}^{\mu\nu}$$

$$J^\mu = n_0\, u^\mu + \sigma\, E^\mu - \kappa\,\Delta^{\mu\nu}\nabla_\nu \frac{\mu}{T}$$

$$+\widetilde{\sigma}\epsilon^{\mu\nu\rho}u_\nu E_\rho + \widetilde{\kappa}\epsilon^{\mu\nu\rho}u_\nu \nabla_\rho \frac{\mu}{T} + \widetilde{\xi}\epsilon^{\mu\nu\rho}u_\nu \nabla_\rho T$$

Where

$$\begin{aligned} \Omega &= -\epsilon^{\mu\nu\rho}u_\mu \nabla_\nu u_\rho, && \text{(vorticity)} \\ \widetilde{\sigma}^{\mu\nu} &= \frac{1}{2}(\epsilon^{\mu\alpha\rho}u_\alpha \sigma_\rho{}^\nu + \epsilon^{\nu\alpha\rho}u_\alpha \sigma_\rho{}^\mu), && \text{(parity odd shear)} \\ B &= -\frac{1}{2}\epsilon^{\mu\nu\rho}u_\mu F^{ext}_{\nu\rho}, && \text{(external magnetic field)} \end{aligned}$$

Relativistic Hydrodynamics

- The constitutive equations: (Poincare symm., *parity-odd*)

$$T^{\mu\nu} = \rho\, u^\mu\, u^\nu + P\Delta^{\mu\nu} - 2\,\eta\,\sigma^{\mu\nu} - \Delta^{\mu\nu}\,\zeta\,\theta - (\widetilde{\zeta}_A\Omega + \widetilde{\zeta}_B B)\Delta^{\mu\nu} - \widetilde{\eta}_A\widetilde{\sigma}^{\mu\nu}$$

Curl Viscosity — Magnetic Viscosity — Hall Viscosity

$$J^\mu = n_0\, u^\mu + \sigma\, E^\mu - \kappa\,\Delta^{\mu\nu}\nabla_\nu \frac{\mu}{T}$$

$$+\widetilde{\sigma}\epsilon^{\mu\nu\rho}u_\nu E_\rho + \widetilde{\kappa}\epsilon^{\mu\nu\rho}u_\nu\nabla_\rho\frac{\mu}{T} + \widetilde{\xi}\epsilon^{\mu\nu\rho}u_\nu\nabla_\rho T$$

Hall Conductivity — thermal Hall Conductivity — heat Hall Conductivity

Where

$$\Omega = -\epsilon^{\mu\nu\rho}u_\mu\nabla_\nu u_\rho, \qquad \text{(vorticity)}$$

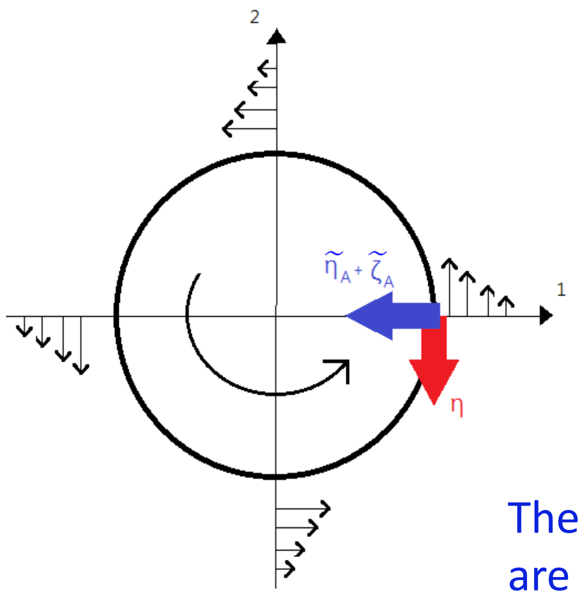
$$\widetilde{\sigma}^{\mu\nu} = \frac{1}{2}(\epsilon^{\mu\alpha\rho}u_\alpha\sigma_\rho{}^\nu + \epsilon^{\nu\alpha\rho}u_\alpha\sigma_\rho{}^\mu), \qquad \text{(parity odd shear)}$$

$$B = -\frac{1}{2}\epsilon^{\mu\nu\rho}u_\mu F^{ext}_{\nu\rho}, \qquad \text{(external magnetic field)}$$

Relativistic Hydrodynamics

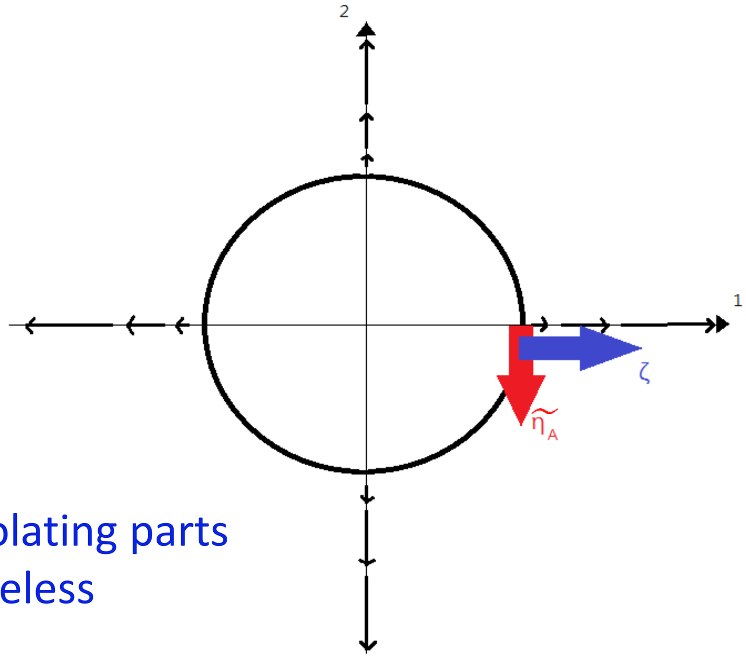
In (2+1)-dimensions: $-(\widetilde{\zeta}_A \Omega + \widetilde{\zeta}_B B)\Delta^{\mu\nu} - \widetilde{\eta}_A \widetilde{\sigma}^{\mu\nu},$

Curl Viscosity — Magnetic Viscosity — Hall Viscosity



Shear, Hall, and Curl

The parity violating parts are dissipativeless



Bulk and Hall

Chen, Lee, Maity, Wen (1110.0793)

Relativistic Hydrodynamics

- The constitute equations are written down according to the underlying symmetry (Poincare, parity), and are subject to the condition of non-negative entropy production $\nabla_\mu \mathcal{J_S}^\mu \geq 0.$
- Thermodynamic parameters ϵ, ρ, T, μ satisfy the thermodynamic relation:

$$dP = s\, dT + \rho\, d\mu + \frac{\partial P}{\partial B} dB + \frac{\partial P}{\partial \Omega} d\Omega\,,$$

$$\epsilon + P = sT + \rho\mu\,.$$

- For weak interacting system, the transport coefficients can be calculated from microscopic theory.
- For strong interacting system: AdS/CFT, or Gravity/Hydrodynamics

Fluid/Gravity Correspondence

- A long-wavelength application of AdS/CFT.
- An AdS bulk solution is dual to a strongly-coupled boundary fluid (i.e. energy-momentum conservation $+$ constitutive eqn. of $T^{\mu\nu}$)

Bhattacharyya, Hubeny, Minwalla, Rangamani (0712.2456)

Fluid/Gravity Correspondence

- Our bulk gravity in (3+1)-dimensions

$$\mathcal{L} = \frac{1}{16\pi G_N}\left(R + \frac{6}{L^2}\right) - \frac{1}{4}F^2 - \frac{1}{2}(\partial\theta)^2 - (\frac{1}{2}m^2\theta^2 + \frac{1}{4}c\,\theta^4) + \frac{\lambda}{4}\theta\,\tilde{F}F - \frac{\lambda}{4}\theta\,\tilde{R}R,$$

$$\tilde{R}R = \tilde{R}^M{}_N{}^{PQ}\,R^N{}_{MPQ}\,, \qquad \tilde{R}^M{}_N{}^{PQ} := \frac{1}{2}\epsilon^{PQRS}R^M{}_{NRS},$$

$$\tilde{F}F = \tilde{F}^{MN}F_{MN}\,, \qquad \tilde{F}^{MN} := \frac{1}{2}\epsilon^{MNPQ}F_{PQ}\,.$$

(Set c=0.5)

- For calculability, we take the probe limit for the pseudo scalar θ:

$$\theta \to \lambda\theta\,, \qquad V(\theta) \to \lambda V(\theta)\,, \qquad \lambda \to 0\,,$$

such that θ dynamics is of order $O(\lambda^1)$ and decouples from $O(\lambda^0)$.

Fluid/Gravity Correspondence

d-dim AdS Boundary

(d+1)-dim AdS Bulk

$$T^{\mu\nu} = \rho\, u^\mu\, u^\nu + P\Delta^{\mu\nu}$$

$$J^\mu = n_0\, u^\mu$$

$\Longleftrightarrow$

Charged boosted black brane with uniform M, Q, u^μ, $A^\mu{}_{ext}$.

Fluid/Gravity Correspondence

d-dim AdS Boundary

(d+1)-dim AdS Bulk

$$T^{\mu\nu} = \rho\, u^\mu\, u^\nu + P\Delta^{\mu\nu}$$

$$J^\mu = n_0\, u^\mu$$

⟺

Charged boosted black brane with uniform M, Q, u^μ, $A^\mu{}_{ext}$.

At $O(\lambda^1)$, the bulk pseudo scalar behaves asymptotically as

$$\theta = \frac{\theta_0}{r^{\Delta_-}} + \frac{\langle\mathcal{O}\rangle^{(0)}}{r^{\Delta_+}} + \cdots, \qquad \Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} + m^2}\,.$$

AdS/CFT dictionary: θ_0 is identify as the source for the dual boundary operator $\langle\mathcal{O}\rangle^{(0)}$ is the v.e.v. of the dual operator .

The parity of the boundary fluid can be broken by pseudo scalar order ($\theta_0 = 0$), or pseudo scalar source ($\theta_0 \neq 0$).

Fluid/Gravity Correspondence

(2+1)-dim AdS Boundary

(3+1)-dim AdS Bulk

$$T^{\mu\nu} = \rho\, u^\mu\, u^\nu + P\Delta^{\mu\nu}$$
$$-2\,\eta\,\sigma^{\mu\nu} - \Delta^{\mu\nu}\,\zeta\,\theta$$
$$-(\widetilde{\zeta}_A \Omega + \widetilde{\zeta}_B B)\Delta^{\mu\nu} - \widetilde{\eta}_A \widetilde{\sigma}^{\mu\nu}$$

$$J^\mu = n_0\, u^\mu$$
$$+\sigma\, E^\mu - \kappa\,\Delta^{\mu\nu}\nabla_\nu \frac{\mu}{T}$$
$$+\widetilde{\sigma}\epsilon^{\mu\nu\rho} u_\nu E_\rho + \widetilde{\kappa}\epsilon^{\mu\nu\rho} u_\nu \nabla_\rho \frac{\mu}{T}$$
$$+\widetilde{\xi}\epsilon^{\mu\nu\rho} u_\nu \nabla_\rho T$$

$\Longleftrightarrow$

Charged boosted black brane with slow varying M, Q, u^μ and $A^\mu{}_{ext}$. (Schematically)

Fluid/Gravity Correspondence

- The slow varying black brane parameters M, Q, u^μ and $A^\mu{}_{ext}$ allow derivative expansions, e.g. $M(x^\nu) = M_0 + x^\nu \partial_\nu M$, and subsequently gives rise to derivative expansion in $g_{\mu\nu}$, A^μ, and θ.

$$ds^2 = ds^{2\ (0)} + \epsilon\,(\text{der. exp.'s in } x^\mu),$$

$$A = A^{(0)} + \epsilon\,(\text{der. exp.'s in } x^\mu),$$

$$\theta = \theta^{(0)} + \epsilon\,(\text{der. exp.'s in } x^\mu).$$

Fluid/Gravity Correspondence

- The slow varying black brane parameters M, Q, u^{μ} and $A^{\mu}{}_{ext}$ allow derivative expansions, e.g. $M(x^{\nu}) = M_0 + x^{\nu}\partial_{\nu}M$, and subsequently gives rise to derivative expansion in $g_{\mu\nu}$, A^{μ}, and θ.

- However, in order to solve the e.o.m.'s at $O(\varepsilon)$, we need to introduce perturbation (or, correction) anzatz $g_{\mu\nu}{}^{(1)}(r)$, $A^{\mu(1)}(r)$, and $\theta^{(1)}(r)$ at $O(\varepsilon)$, which are to be solved.

$$ds^2 = ds^{2\,(0)} + \epsilon\,(\text{der. exp.'s in } x^{\mu}) + \epsilon\, ds^{2\,(1)}(r),$$

$$A = A^{(0)} + \epsilon\,(\text{der. exp.'s in } x^{\mu}) + \epsilon\, A^{(1)}(r),$$

$$\theta = \theta^{(0)} + \epsilon\,(\text{der. exp.'s in } x^{\mu}) + \epsilon\, \theta^{(1)}(r).$$

Fluid/Gravity Correspondence

- Perturbations are solved asymptotically by assuming *renormalizable b.c.* on the boundary and *regularity* on the horizon.

- The boundary fluid constitutive equations are reproduced via the standard AdS/CFT:

$$\langle T_{\mu\nu}\rangle = \lim_{r\to\infty} r^3 g_{\mu\nu}\,, \qquad \langle J_\mu\rangle = \lim_{r\to\infty} \frac{1}{\sqrt{-g}} F^r{}_\mu\,.$$

$$\begin{aligned} T^{\mu\nu} &= \epsilon_0 u^\mu u^\nu + (P_0 - \zeta\nabla_\lambda u^\lambda - \widetilde{\zeta}_A\Omega - \widetilde{\zeta}_B B)\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} - \widetilde{\eta}_A\widetilde{\sigma}^{\mu\nu}, \\ J^\mu &= \rho u^\mu + \sigma E^\mu - \kappa\Delta^{\mu\nu}\nabla_\nu\frac{\mu}{T} + \widetilde{\sigma}\epsilon^{\mu\nu\rho}u_\nu E_\rho + \widetilde{\kappa}\epsilon^{\mu\nu\rho}u_\nu\nabla_\rho\frac{\mu}{T} + \widetilde{\xi}\epsilon^{\mu\nu\rho}u_\nu\nabla_\rho T. \end{aligned}$$

Parity Violating Hydrodynamics

- Analytic results:

Shear viscosity: $$\frac{\eta}{s} = \frac{1}{4\pi}$$

Hall viscosity: $$\frac{\widetilde{\eta}_A}{s} = -\frac{r_H^2}{8\pi}\frac{f'(r_H)\theta'(r_H)}{H(r_H)^2}$$

electric conductivity $$\sigma = \left(1 - \frac{4Q^2}{3M}\frac{1}{r_H}\right)^2 = \left(\frac{4\pi r_H^2 T}{3M}\right)^2$$

Thermal conductivity $$\kappa = \sigma T = \frac{1}{4\pi}\left(1 - \frac{4Q^2}{3M}\frac{1}{r_H}\right)^2\left(\frac{3M}{r_H^2} - \frac{4Q^2}{r_H^3}\right)$$

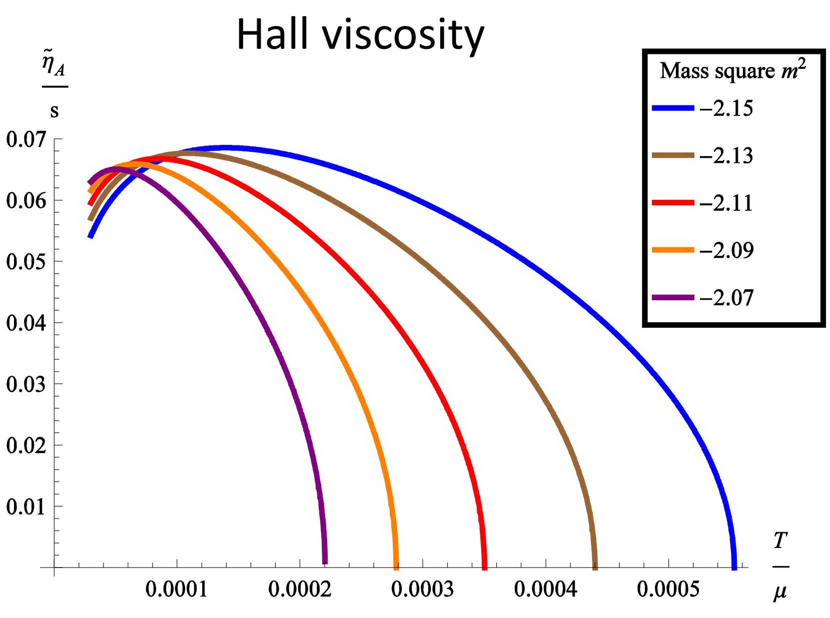
- Other transport coefficients are computed numerically.

NB: in the comiving frame $u^\mu = (1,0,0)$, $\quad -r^2 f = g_{tt}, \quad 2H = g_{tr}$

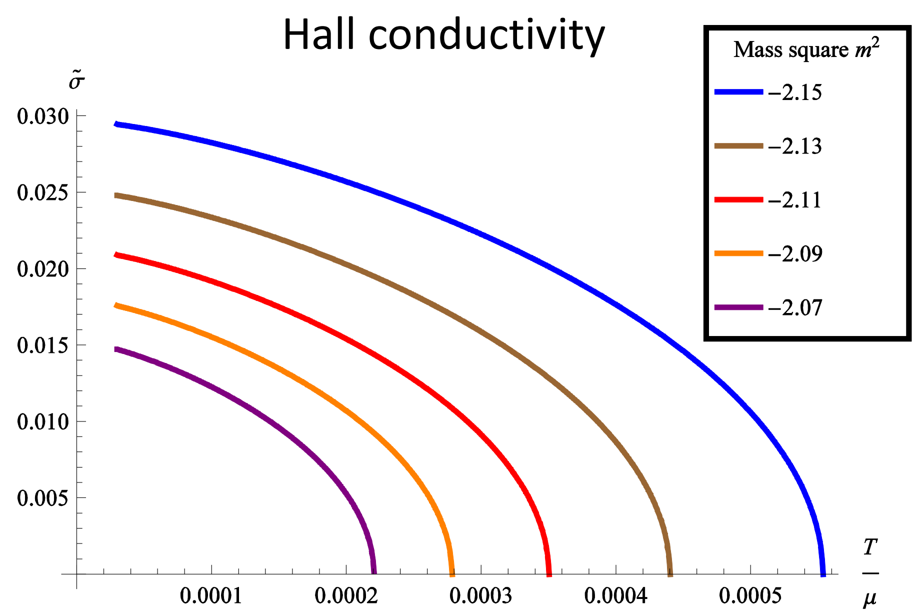
Parity Violating Hydrodynamics

- If the boundary parity is broken by the pseudo scalar order (SSB):

$$\zeta = 0, \quad \widetilde{\zeta}_A = 0, \quad \widetilde{\zeta}_B = 0$$



Critical exponent = 0.5

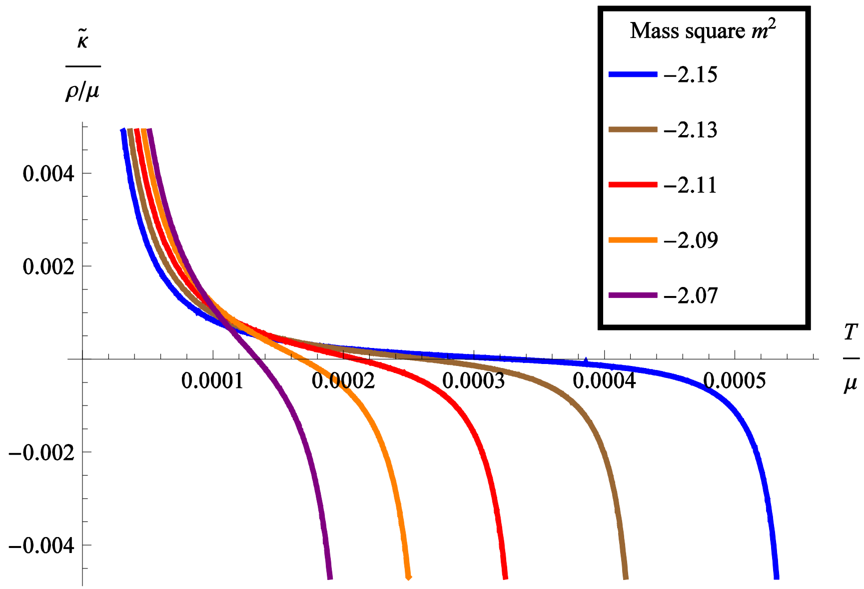


Critical exponent = 0.5

Parity Violating Hydrodynamics

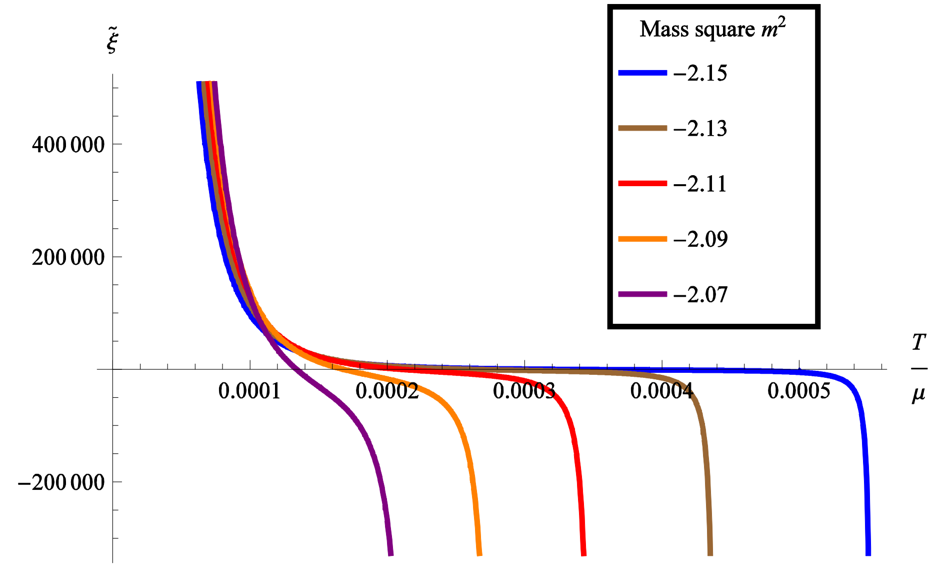
- If the boundary parity is broken by the pseudo scalar order (SSB):

Thermal Hall conductivity



Critical exponent = -1.5

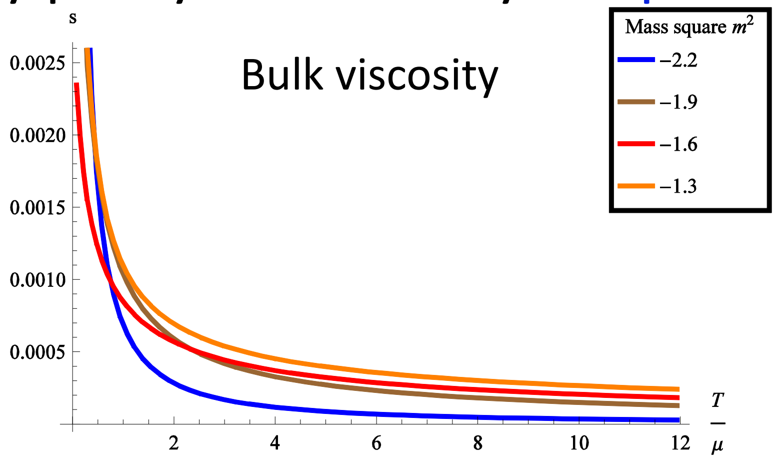
Heat Hall conductivity

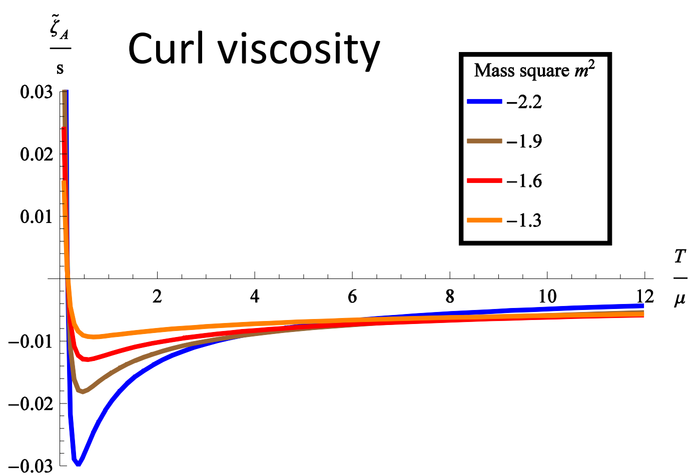


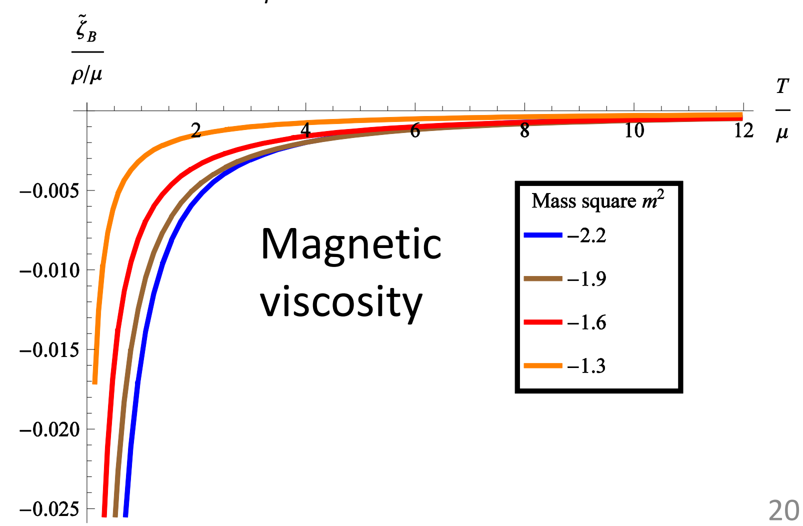
Critical exponent = -1.5

Parity Violating Hydrodynamics

- If the boundary parity is broken by the pseudo scalar source:



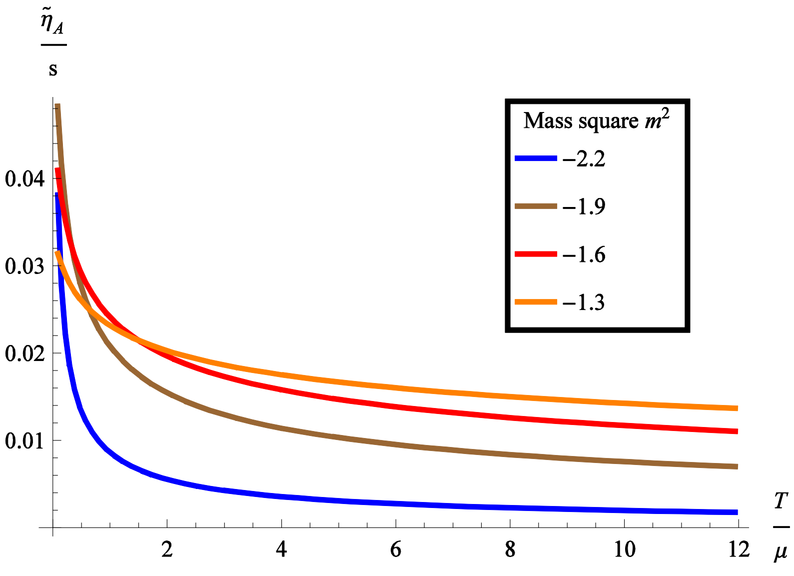




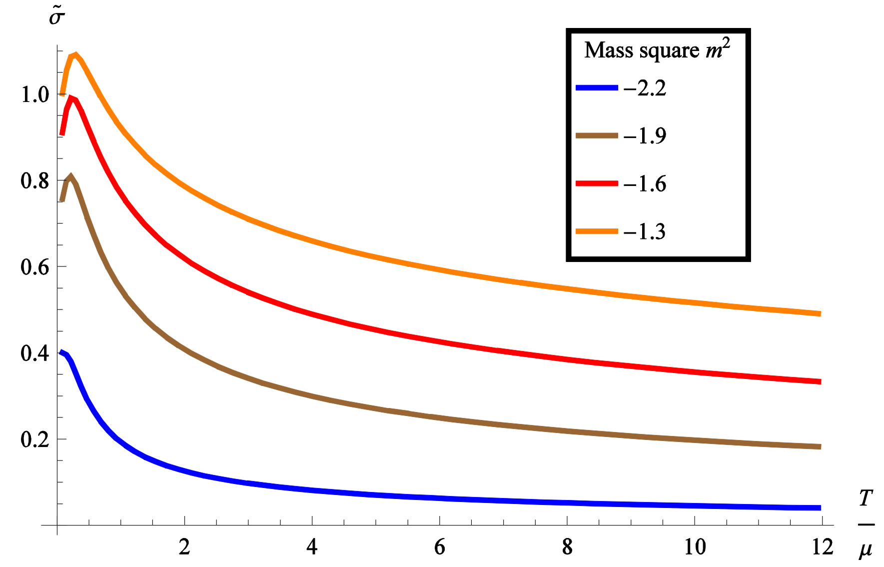
Parity Violating Hydrodynamics

- If the boundary parity is broken by the pseudo scalar source:

Hall viscosity



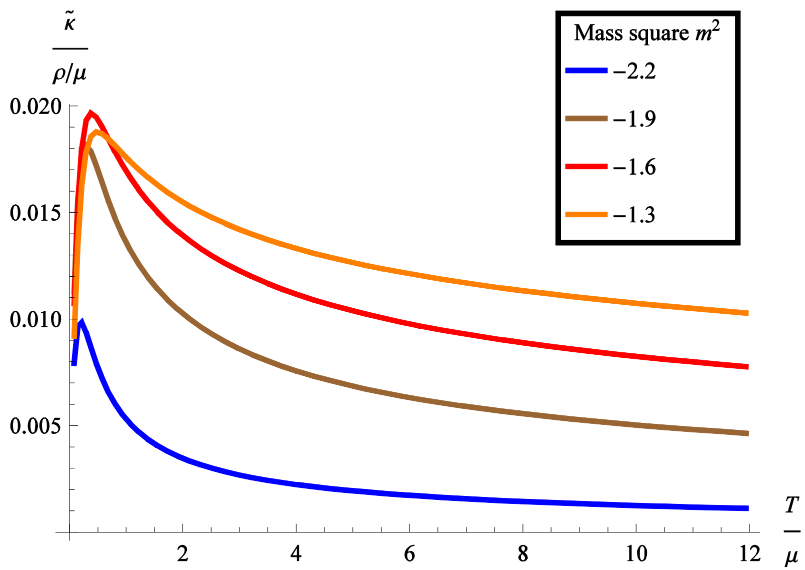
Hall conductivity



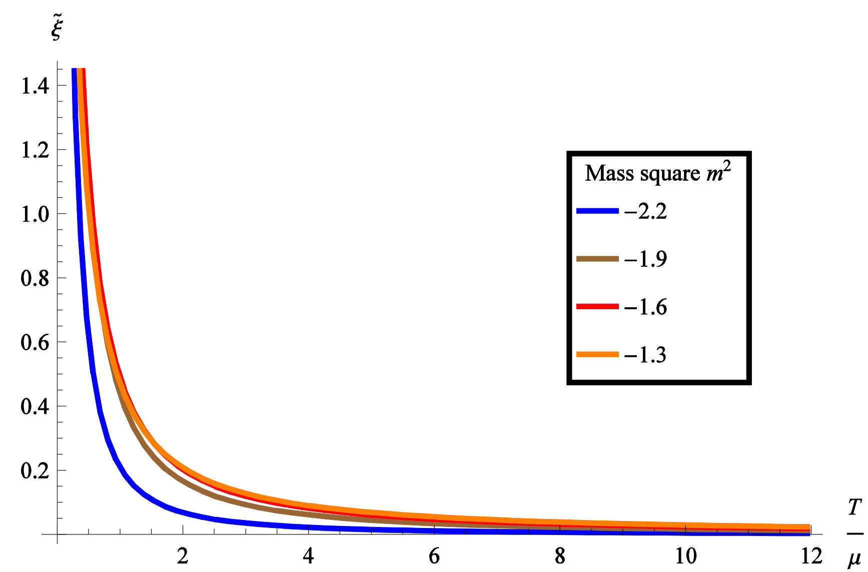
Parity Violating Hydrodynamics

- If the boundary parity is broken by the pseudo scalar source:

Thermal Hall viscosity



Heat Hall conductivity



Discussion

- In our model, all kinds of "Hall" transport coefficients are present without the background magnetic fields. This is because the parity is broken by the pseudo scalar θ, rather than by the external B fields. This is reflected in that such transport coefficients are given in terms of θ.

- Gravity knows about the entropy constraint and the thermodynamic relations!

Jensen et al.(1112.4498)

Thank You

Backup Slides

- Boosted black brane

$$\begin{aligned} ds^2 &= -2\,H(r,M,Q)\,u_\mu dx^\mu dr - r^2 f(r,M,Q)\,u_\mu u_\nu dx^\mu dx^\nu + r^2\Delta_{\mu\nu}dx^\mu dx^\nu , \\ A &= \left[A(r,M,Q)\,u_\mu + A^{ext}_\mu\right] dx^\mu \ , \qquad \theta \ = \ \theta(r,M,Q) \end{aligned}$$

- The background at the probe limit

$$\begin{aligned} H(r,M,Q) &= 1, \qquad\qquad A(r,M,Q) \ = \ -\frac{2Q}{r}, \\ f(r,M,Q) &= 1-\frac{M}{r^3}+\frac{Q^2}{r^4}, \qquad A^{ext}_\mu \ = \ (A^{ext}_v, A^{ext}_x, A^{ext}_y) = \text{constant} \ , \end{aligned}$$

- Perturbation ansatz at $O(\varepsilon)$

$$\begin{aligned} ds^{2\,(1)} &= r^2k(r)\,dv^2 + 2Hh(r)\,dvdr + 2r^2j_i(r)\,dvdx^i - r^2h(r)\,dx^idx^i + r^2\alpha_{ij}(r)\,dx^idx^j , \\ A^{(1)} &= a_v(r)\,dv + a_i(r)\,dx^i \ , \qquad\qquad \theta^{(1)} \ = \ \varphi(r)\,. \end{aligned}$$

Backup Slides

- All together:

$$
\begin{aligned}
ds^2 &= ds^{2\,(0)} \\
&+ \epsilon\left[-r^2\delta f\, dv^2 + 2\,\delta H\, dvdr - 2\,r^2(1-f(r))\,\delta\beta^i\, dvdx^i - 2\,H(r)\,\delta\beta^i\, drdx^i\right] \\
&+ \epsilon\, ds^{2\,(1)}, \\
A &= A^{(0)} + \epsilon\,(-\delta A\, dv + A(r)\,\delta\beta^i\, dx^i + A^{(1)}), \\
\theta &= \theta^{(0)} + \epsilon\,(\delta\theta + \theta^{(1)}).
\end{aligned}
$$

AdS/CFT

$$\left\langle e^{\int d^d x \phi_0(x^\mu)\mathcal{O}(x^\mu)} \right\rangle_{CFT} = \mathcal{Z}_{gravity} \left[\phi(x^\mu, r)\big|_{r=\infty} = \phi_0(x^\mu) \right]$$

d-dim AdS Boundary		(d+1)-dim AdS Bulk
Gauge Field Theory		Gravity
Strong coupling		Weak Coupling
Field theory T	⟺	Black Hole T
Conformal Symmetry *SO(2, d-1)*		Isometry *SO(2, d-1)*
$\mathcal{O}$		ϕ