## Generalised geometry of unified theories

Hadi Godazgar

DAMTP University of Cambridge

November, 22 2013

PASCOS 2013

## Motivation

Great deal of research and interest in incorporating matter and gravity into a simple single framework: unified theory.

Has lead to significant progress in our understanding of viable theoretical frameworks for studying nature.

Archetypal example: Kaluza-Klein theory.

Attempt to unify gravity and electromagnetism by embedding both theories in a purely geometric five-dimensional spacetime.

Important framework for understanding unification: describing complicated lower dimensional theories using simpler higher dimensional ones.

Main issue in unification: reconciliation of matter degrees of freedom with gravitational ones within a single framework, including an understanding of how various gauge freedoms come about.

Study D = 11 supergravity as simple example of a unified theory.

Bonus: gain better understanding of Kaluza-Klein theory.

# D = 11 supergravity

Eleven is the maximum dimension in which one can define a consistent supergravity.

D = 11 supergravity thought to be unique. It contains:

 $E_M{}^A$ ,  $A_{MNP}$ ,  $\Psi_M$ 

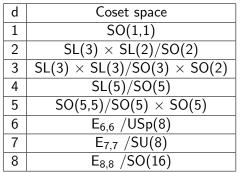
```
related by SUSY transformations
```

```
\delta(bosons) = fermions, \quad \delta(fermions) = bosons,
```

which leave the action unchanged.

 Advantage of supersymmetric theories: can work with SUSY transformations—much simpler than equations of motion.

- Reduction of 11-dimensional supergravity on a torus gives rise to symmetries [Cremmer Julia 1978, Julia 1981]
- Reduction on d-torus gives following symmetry groups



 de Wit and Nicolai [1986] speculate that these duality groups are present in the fundamental theory

### Generalised geometry

►

- Consider a split of the 11 dimensions into a *D*-dimensional piece and *d*-dimensional piece
- Try to make E<sub>d</sub> duality explicit in the d-dimensional piece without assuming Killing vectors.
- ► This can be done by extending *d*-dimensional tangent space → generalised geometry [Hitchen 2003, Gualtieri 2004]

$$T(M) \longrightarrow T(M) \oplus \Lambda^2 T^*(M) \oplus \Lambda^5 T^*(M) \oplus (\Lambda T^*(M) \Lambda^7 T^*(M))$$

[Hull, Pacheco, Waldram, Coimbra, Strickland-Constable, Grana ...]

 Can also extend d-dimensional space
[West, Hillmann, Berman, Perry, HG, Jeon, Lee, Park, M Godazgar, Thompson, Cederwall, Kleinschmidt, Musaev, Malek, Aldazabal, Grana, Marques, Rosabal, Blair, Hohm, Samtleben ...]

- Can write d dimensional part of D = 11 supergravity in terms of generalised metric that is valued in E<sub>d</sub> and unifies metric and matter degrees of freedom
- Diffeomorphisms and gauge transformations of gauge potentials unified in generalised Lie transformations of generalised geomtery
- Question: How do these structures arise in the whole theory?

[with Mahdi Godazgar and Hermann Nicolai] Aim:

- move away from a description in which matter fields reside on some geometric background
- reformulation in which all fields are on the same footing leading to enlarged symmetries: generalised geometry

Strategy:

- analyse SUSY transformations
- find structures constructed from matter and gravitational degrees of freedom leading to enlargement of symmetry

### Observation:

- must give up manifest Lorentz invariance and spacetime covariance
- must consider dualisation of fields

Work in de Wit-Nicolai formalism: split 11 dimensions to 4+7. Decompose fields in terms of this splitting, e.g. use SO(10, 1) to set

$$E_M{}^A = \begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^m e_m{}^a \\ 0 & e_m{}^a \end{pmatrix}$$

Introduce six-form dual  $A_{(6)}$  using equation of motion of 3-form potential  $A_{(3)}$ :

$$d \star F_{(4)} + \dots = 0$$
  
$$d(\star F_{(4)} + \dots) = 0$$
  
$$\star F_{(4)} + \dots = dA_{(6)}.$$

Can determine SUSY transformation of  $A_{(6)}$ .

▶ Remain equivalent to full *D* = 11 theory—no truncation assumed.

#### Results:

 All bosonic degrees of freedom are assembled into E<sub>7</sub> objects—enlargement of symmetry.

One unified SUSY transformation:

$$\delta \mathcal{B}_{\mu}{}^{\mathcal{I}} = \mathcal{V}_{AB}^{\mathcal{I}} \text{ (fermions)}$$
  
$$\delta \mathcal{V}_{AB}^{\mathcal{I}} = \Sigma_{ABCD} \mathcal{V}^{\mathcal{I}CD}$$

 Components of E<sub>7</sub> vielbein V<sup>I</sup><sub>AB</sub> satisfy generalised vielbein postulates.

## Generalised vielbein postulates (GVPs)

In Riemannian geometry, vielbein postulate is a differential constraint satisfied by the vielbein

$$\nabla_{\mu} e_{\nu}{}^{\alpha} = \partial_{\mu} e_{\nu}{}^{\alpha} - \Gamma^{\rho}_{\mu\nu} e_{\rho}{}^{\alpha} + \omega_{\mu}{}^{\alpha}{}_{\beta} e_{\nu}{}^{\beta} = 0.$$

 GVPs have rich and beautiful structure: matter and gravitational degrees of freedom packaged into E<sub>7</sub> connections.

In addition there appear connections not gauge invariant under gauge transformations of 3 and 6-forms, e.g.

$$\Xi_{p|mn|q} = \partial_p A_{mnq} - \frac{1}{4!} F_{pmnq}.$$

Remarkably,

$$\Xi_{[p|mn|q]}=0.$$

Cf.  $\Gamma^{\rho}_{[\mu\nu]} = 0$  for torsion-free affine connection.

### Relation to 4d theories

This framework provides a direct relation to four dimensions.

- ► Given any solution of 4d theory can obtain *full* D = 11 solution!
- Discover novel features of Kaluza-Klein theory.
- GVPs provide deeper understanding of consistency of 4d theory.

# Testability!!

### Outlook

▶ ...

Promises to be a very fruitful program of research.

- understanding of precise relations between 4d theories and D = 11 theory.
- ▶ is D = 11 theory unique? Implications for string/M-theory?
- extending geometry—a generalised geometry?
- other dimensions: new structures expected.

Ultimately, what is the geometry of unified theory?