Generalised geometry of unified theories

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November, 22 2013

PASCOS 2013

Motivation

- Great deal of research and interest in incorporating matter and gravity into a simple single framework: unified theory.

Has lead to significant progress in our understanding of viable theoretical frameworks for studying nature.

- Archetypal example: Kaluza-Klein theory.

Attempt to unify gravity and electromagnetism by embedding both theories in a purely geometric five-dimensional spacetime.

Important framework for understanding unification: describing complicated lower dimensional theories using simpler higher dimensional ones.

Main issue in unification: reconciliation of matter degrees of freedom with gravitational ones within a single framework, including an understanding of how various gauge freedoms come about.

- Study $D = 11$ supergravity as simple example of a unified theory.

Bonus: gain better understanding of Kaluza-Klein theory.

$D = 11$ supergravity

Eleven is the maximum dimension in which one can define a consistent supergravity.

$D = 11$ supergravity thought to be unique. It contains:

$$E_M{}^A, \; A_{MNP}, \; \Psi_M$$

related by SUSY transformations

$$\delta(\text{bosons}) = \text{fermions}, \quad \delta(\text{fermions}) = \text{bosons},$$

which leave the action unchanged.

- Advantage of supersymmetric theories: can work with SUSY transformations—much simpler than equations of motion.

- Reduction of 11-dimensional supergravity on a torus gives rise to symmetries [Cremmer Julia 1978, Julia 1981]
- Reduction on d-torus gives following symmetry groups

d	Coset space
1	SO(1,1)
2	SL(3) $\times$ SL(2)/SO(2)
3	SL(3) $\times$ SL(3)/SO(3) $\times$ SO(2)
4	SL(5)/SO(5)
5	SO(5,5)/SO(5) $\times$ SO(5)
6	$E_{6,6}$ /USp(8)
7	$E_{7,7}$ /SU(8)
8	$E_{8,8}$ /SO(16)

- de Wit and Nicolai [1986] speculate that these duality groups are present in the fundamental theory

Generalised geometry

- Consider a split of the 11 dimensions into a D-dimensional piece and d-dimensional piece
- Try to make E_d duality explicit in the d-dimensional piece without assuming Killing vectors.
- This can be done by extending d-dimensional tangent space $\longrightarrow$ generalised geometry [Hitchen 2003, Gualtieri 2004]
- $$T(M) \longrightarrow T(M) \oplus \Lambda^2 T^*(M) \oplus \Lambda^5 T^*(M) \oplus (\Lambda T^*(M) \Lambda^7 T^*(M))$$

  [Hull, Pacheco, Waldram, Coimbra, Strickland-Constable, Grana ...]
- Can also extend d-dimensional space [West, Hillmann, Berman, Perry, HG, Jeon, Lee, Park, M Godazgar, Thompson, Cederwall, Kleinschmidt, Musaev, Malek, Aldazabal, Grana, Marques, Rosabal, Blair, Hohm, Samtleben ...]

- Can write d dimensional part of $D = 11$ supergravity in terms of generalised metric that is valued in E_d and unifies metric and matter degrees of freedom
- Diffeomorphisms and gauge transformations of gauge potentials unified in generalised Lie transformations of generalised geomtery
- Question: How do these structures arise in the whole theory?

[with Mahdi Godazgar and Hermann Nicolai]

Aim:

- move away from a description in which matter fields reside on some geometric background
- reformulation in which all fields are on the same footing leading to enlarged symmetries: generalised geometry

Strategy:

- analyse SUSY transformations
- find structures constructed from matter and gravitational degrees of freedom leading to enlargement of symmetry

Observation:

- must give up manifest Lorentz invariance and spacetime covariance
- must consider dualisation of fields

Work in de Wit-Nicolai formalism: split 11 dimensions to 4+7.

Decompose fields in terms of this splitting, e.g. use $SO(10,1)$ to set

$$E_M{}^A = \begin{pmatrix} e_\mu{}^\alpha & B_\mu{}^m e_m{}^a \\ 0 & e_m{}^a \end{pmatrix}.$$

Introduce six-form dual $A_{(6)}$ using equation of motion of 3-form potential $A_{(3)}$:

$$d \star F_{(4)} + \cdots = 0$$
$$d(\star F_{(4)} + \dots) = 0$$
$$\star F_{(4)} + \cdots = dA_{(6)}.$$

Can determine SUSY transformation of $A_{(6)}$.

- Remain equivalent to full $D = 11$ theory—no truncation assumed.

Results:

- All bosonic degrees of freedom are assembled into E_7 objects—enlargement of symmetry.

One unified SUSY transformation:

$$\delta \mathcal{B}_\mu{}^{\mathcal{I}} = \mathcal{V}^{\mathcal{I}}_{AB} \text{ (fermions)}$$
$$\delta \mathcal{V}^{\mathcal{I}}_{AB} = \Sigma_{ABCD} \mathcal{V}^{\mathcal{I}CD}$$

- Components of E_7 vielbein $\mathcal{V}^{\mathcal{I}}_{AB}$ satisfy generalised vielbein postulates.

Generalised vielbein postulates (GVPs)

In Riemannian geometry, vielbein postulate is a differential constraint satisfied by the vielbein

$$\nabla_\mu e_\nu{}^\alpha = \partial_\mu e_\nu{}^\alpha - \Gamma^\rho_{\mu\nu} e_\rho{}^\alpha + \omega_\mu{}^\alpha{}_\beta e_\nu{}^\beta = 0.$$

- GVPs have rich and beautiful structure: matter and gravitational degrees of freedom packaged into E_7 connections.

In addition there appear connections not gauge invariant under gauge transformations of 3 and 6-forms, e.g.

$$\Xi_{p|mn|q} = \partial_p A_{mnq} - \tfrac{1}{4!} F_{pmnq}.$$

Remarkably,

$$\Xi_{[p|mn|q]} = 0.$$

Cf. $\Gamma^\rho_{[\mu\nu]} = 0$ for torsion-free affine connection.

Relation to 4d theories

This framework provides a direct relation to four dimensions.

- Given any solution of 4d theory can obtain *full* $D = 11$ solution!
- Discover novel features of Kaluza-Klein theory.
- GVPs provide deeper understanding of consistency of 4d theory.

Testability!!

Outlook

Promises to be a very fruitful program of research.

- understanding of precise relations between 4d theories and $D = 11$ theory.
- is $D = 11$ theory unique? Implications for string/M-theory?
- extending geometry—a generalised geometry?
- other dimensions: new structures expected.
- ...

Ultimately, what is the geometry of unified theory?