Terminating black holes in quantum gravity

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Based on:
C. Bambi, D. Malafarina & L. Modesto, PRD 88 (2013) 044009
C. Bambi, D. Malafarina & L. Modesto, arXiv:1306.1668

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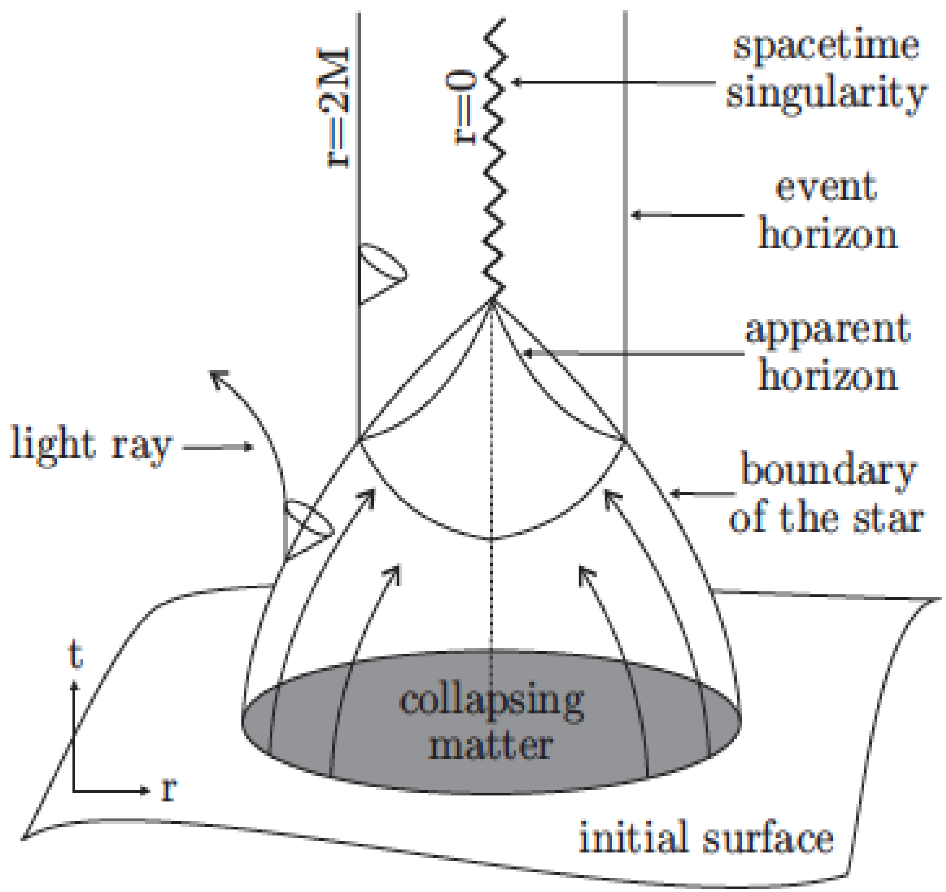
Gravitational Collapse in GR

- **The outcome of the collapse is a spacetime singularity. Assumptions: strong energy condition (rho+P and rho+3P non-negative), existence of global hyperbolicity, etc.**

- **Weak Cosmic Censorship Conjecture: singularities produced in the gravitational collapse must be hidden within black holes.**

- **Simplest example: Oppenheimer-Snyder model (1939), collapse of a homogeneous cloud of dust.**

Finkelstein Diagram (Oppenheimer-Snyder Model)



Space-time

Spherically symmetric interior describing a collapsing matter cloud in comoving coordinates

$$ds^2 = -e^{2\nu(t,r)}dt^2 + \frac{R'^2}{G(t,r)}dr^2 + R(t,r)^2 d\Omega^2$$

depends on 3 functions ν, G, R and matches smoothly across the boundary to a known exterior (generalized Vaidya or Schwarzschild).

Matter

The energy-momentum tensor is composed by

- Energy density: $\rho = -T^t_t$
- Radial pressure: $p_r = T^r_r$
- Tangential pressure: $p_\theta = T^\theta_\theta = T^\phi_\phi$

The metric components are related to the energy-momentum tensor via Einstein's field equations.

Einstein Equations

$$
\begin{aligned}
p_r &= -\frac{\dot{F}}{R^2 \dot{R}} \\
\rho &= \frac{F'}{R^2 R'} \\
\dot{G} &= 2\nu' \frac{\dot{R}}{R'} G \\
\nu' &= \frac{2(p_\theta - p_r)}{\rho + p_r} \frac{R'}{R} - \frac{p_r'}{\rho + p_r}
\end{aligned}
$$

Misner-Sharp mass F:

$$F = r^3 M(r,t) = R(1 - G + e^{-2\nu(r,v)} \dot{R}^2)$$

The first equation implies that for dust ($p_r = p_\theta = 0$) we must have $M = M(r)$.

In general we have 5 equations for the 7 unknown ν, G, R, ρ, p_r, p_θ, F.

For perfect fluid $p_r = p_\theta = p$ with an equation of state $p = p(\rho)$ the system is closed.

Scaling Factor

There is the gauge freedom to fix the scaling factor in the area function $R(r,t)$:

$$R = ra(r,t)$$

- initial time t_i: $a(r,t_i) = 1$
- singularity time t_s: $a(r,t_s) = 0$
- collapse: $\dot{a} < 0$

Due to its monotonic behaviour we can use a as a time coordinate.
With the choice of the scaling factor it diverges only at the singularity.

Trapped Surfaces

The apparent horizon is the surface that separates light rays directed outwards that are outgoing from those directed outwards that are ingoing. In vacuum it coincides with the event horizon.

$$1 - \frac{F}{R} = 1 - \frac{r^2 M}{a} = 0$$

Homogeneous collapse

Homogeneus if $\rho = \rho(t)$ and $p = p(t) \Rightarrow \quad M = M(t)$

$$\nu' = 0 \Rightarrow G = 1 + k$$

Marginally bound case: $k = 0$

Classical dust collapse

$$p = 0$$
$$\rho = \frac{3M_0}{a^3}$$
$$M = M_0$$
$$M_0 = a\dot{a}^2$$
$$a(t) = \left(1 - \frac{3}{2}\sqrt{M_0}t\right)^{2/3}$$

Classical radiation collapse

$$p = \frac{\rho}{3}$$
$$\rho = \frac{3M_0}{a^4}$$
$$M = M_0 a$$
$$M_0 = a^2\dot{a}^2$$
$$a(t) = (1 - 2\sqrt{M_0}t)^{1/2}$$

Classically leads to the formation of a black hole

Effective model

$$\rho_{\rm eff} = \rho + \rho_{\rm corr} = \rho + \alpha_1 \rho^2 + \alpha_2 \rho^3 + o(\rho^3) \ ,$$

with α_i depending upon a critical density $\rho_{\rm cr}$.

$$\rho_{\rm eff} = \rho \left(1 - \frac{\rho}{\rho_{\rm cr}}\right)^\gamma , \ \gamma \geq 1 \ .$$

Quantum dust collapse

$$p_{\rm eff} = -\gamma \frac{\rho^2}{\rho_{\rm cr}} \left(1 - \frac{\rho}{\rho_{\rm cr}}\right)^{\gamma - 1}$$

$$M_{\rm eff} = M_0 \left(1 - \frac{\rho}{\rho_{\rm cr}}\right)^\gamma$$

$$\dot{a}^2 = \frac{M_0}{a^{3\gamma+1}} \left(a^3 - a_{\rm cr}^3\right)^\gamma$$

$$a_{\rm cr}^3 = \frac{3M_0}{\rho_{\rm cr}}$$

$$a(t) = \left[a_{\rm cr}^3 + \left(\sqrt{1 - a_{\rm cr}^3} - \frac{3\sqrt{M_0}}{2} t\right)^2\right]^{1/3}$$

Quantum radiation collapse

$$p_{\rm eff} = \frac{\rho}{3} \left(1 - 5\gamma \frac{\rho}{\rho_{\rm cr}}\right) \left(1 - \frac{\rho}{\rho_{\rm cr}}\right)^{\gamma - 1}$$

$$M_{\rm eff} = \frac{M_0}{a} \left(1 - \frac{\rho}{\rho_{\rm cr}}\right)^\gamma$$

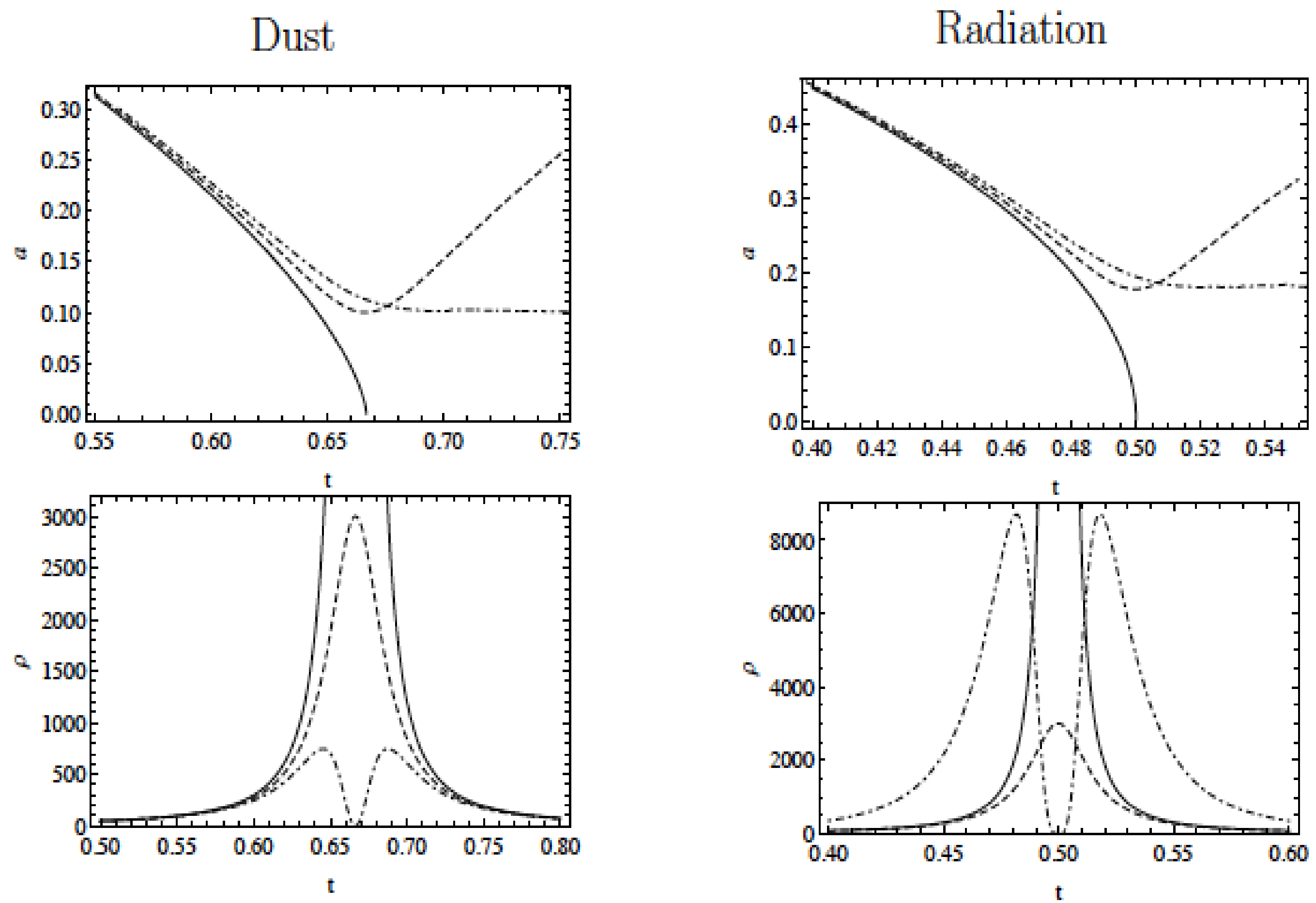
$$\dot{a}^2 = \frac{M_0}{a^{4\gamma+2}} (a^4 - a_{\rm cr}^4)^\gamma$$

$$a_{\rm cr}^4 = \frac{3M_0}{\rho_{\rm cr}}$$

$$a(t) = \left[a_{\rm cr}^4 + \left(\sqrt{1 - a_{\rm cr}^4} - 2\sqrt{M_0} t\right)^2\right]^{1/4}$$

Quantum corrections

The system reaches a critical scale at which ρ is finite and then bounces.



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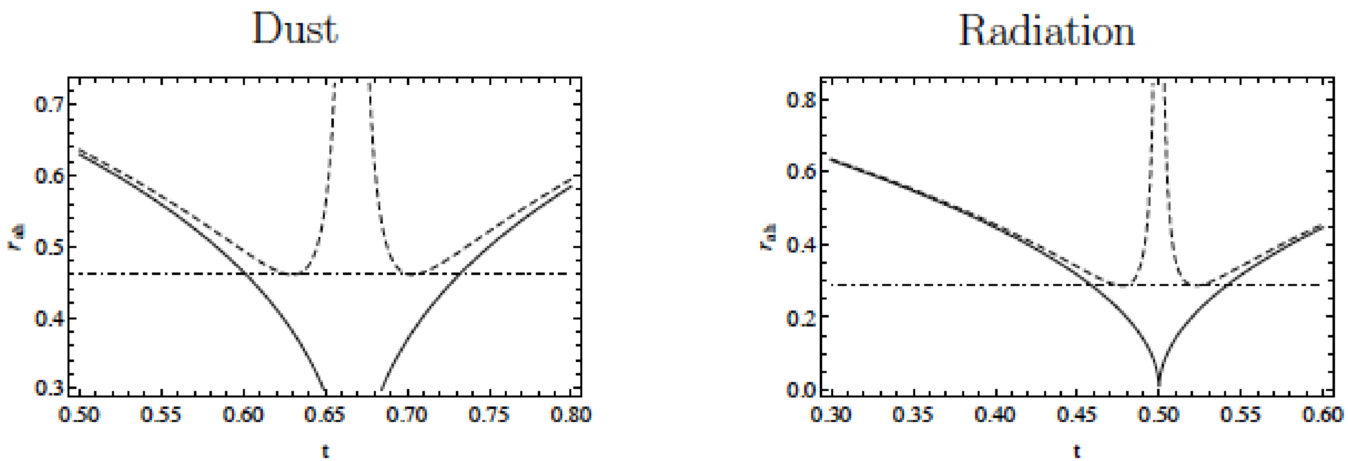
Apparent horizon

The equation for the apparent horizon becomes

$$r_{\rm ah}(t) = \frac{a^{n-1}}{\sqrt{M_0(a^n - a_{\rm cr}^n)}} \quad \text{with } n = 3 \text{ for dust and } n = 4 \text{ for radiation}$$



Minimum radius:

$$r_{\rm min} = 2^{4/3}\sqrt{\frac{a_{\rm cr}}{3M_0}} \text{ for dust and } r_{\rm min} = 3^{3/4}\frac{a_{\rm cr}}{\sqrt{2M_0}} \text{ for radiation}$$

Corrsponding to a minimal mass.

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Extension of Stelle's Quadratic Theory

- **Classical action:** $\mathcal{S} = \int d^4x \frac{2\sqrt{|g|}}{\kappa^2}\left[R - G_{\mu\nu}\frac{V(-\Box/\Lambda^2)^{-1} - 1}{\Box}R^{\mu\nu}\right]$
- **V(z) must have no poles in the whole complex domain (in order to ensure unitarity) and must exhibit at least logarithmic bahavior in the UV regime (to give super-renormalizability at the quantum level):**

$$V(z)^{-1} = |p_{\gamma+1}(z)|\; e^{\frac{1}{2}\left[\Gamma\left(0,p^2_{\gamma+1}(z)\right)+\gamma_E\right]},$$

$$V(z)^{-1} = e^{z^n} \quad n \in \mathbb{N}^+,$$

- **The theory is uniquely specified once the form factor is fixed, because the latter does not receive any renormalizability (the UV theory is dominated by the bare action). We have only the graviton pole.**
- **Since V(z) is an entire function, there are no ghosts and no tachyons, independently of the number of time derivatives present in the action**

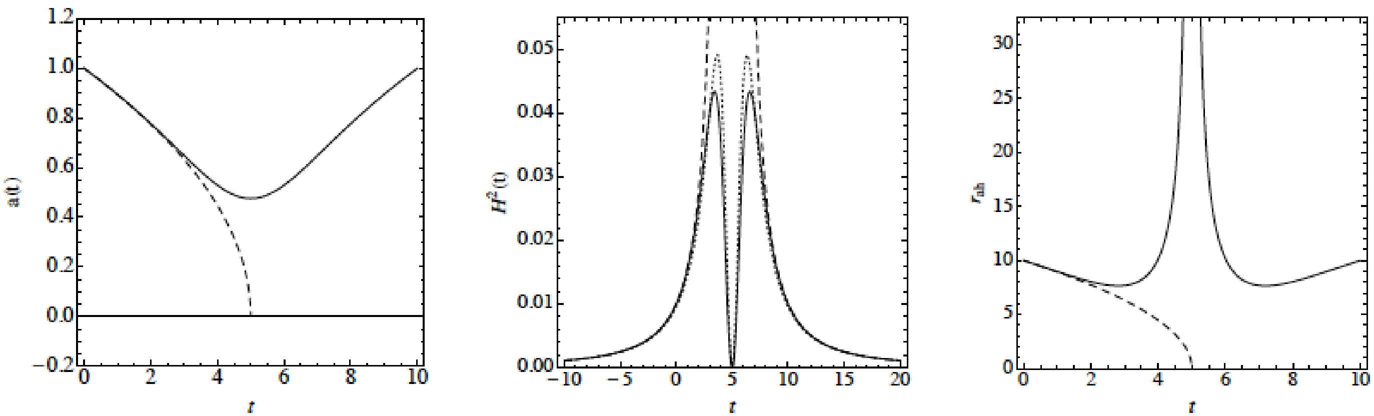
Homogeneous collapse

- **Dust**

$$a(t)^2 = \left|\frac{t_0 - t}{t_0}\right|^{\frac{4}{3}} \qquad a^2(t) = -\frac{2\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right){}_1F_1\left(-\frac{2}{3};\frac{1}{2};-\frac{(t_0-t)^2\Lambda^2}{4}\right)}{\Lambda^{4/3}\sqrt{3}\pi t_0^{4/3}}$$

- **Radiation**

$$a(t)^2 = \left|\frac{t_0 - t}{t_0}\right| \qquad a^2(t) = -\frac{2\Gamma\left(-\frac{2}{3}\right)\Gamma\left(\frac{4}{3}\right){}_1F_1\left(-\frac{2}{3};\frac{1}{2};-\frac{(t_0-t)^2\Lambda^2}{4}\right)}{\Lambda^{4/3}\sqrt{3}\pi t_0^{4/3}}$$



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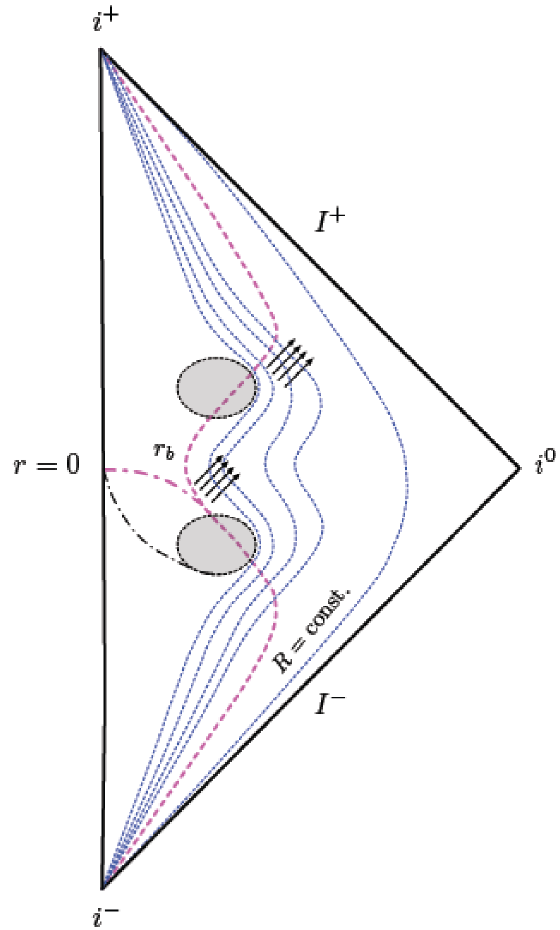
Penrose Diagram



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Conclusion

- **The classical singularity is replaced by a bounce, after which the cloud of matter expands**
- **It seems that black holes, strictly speaking, do not existe and the collapse only generates a temporary trapped surface**
- **Key-point is the Vaidya-like exterior solution. If we want to see everything in terms of an effective Einstein theory, we have an effective ingoing flux of negative energy that destroys the event horizon**
- **Another possibility: baby universe**

Thank you!