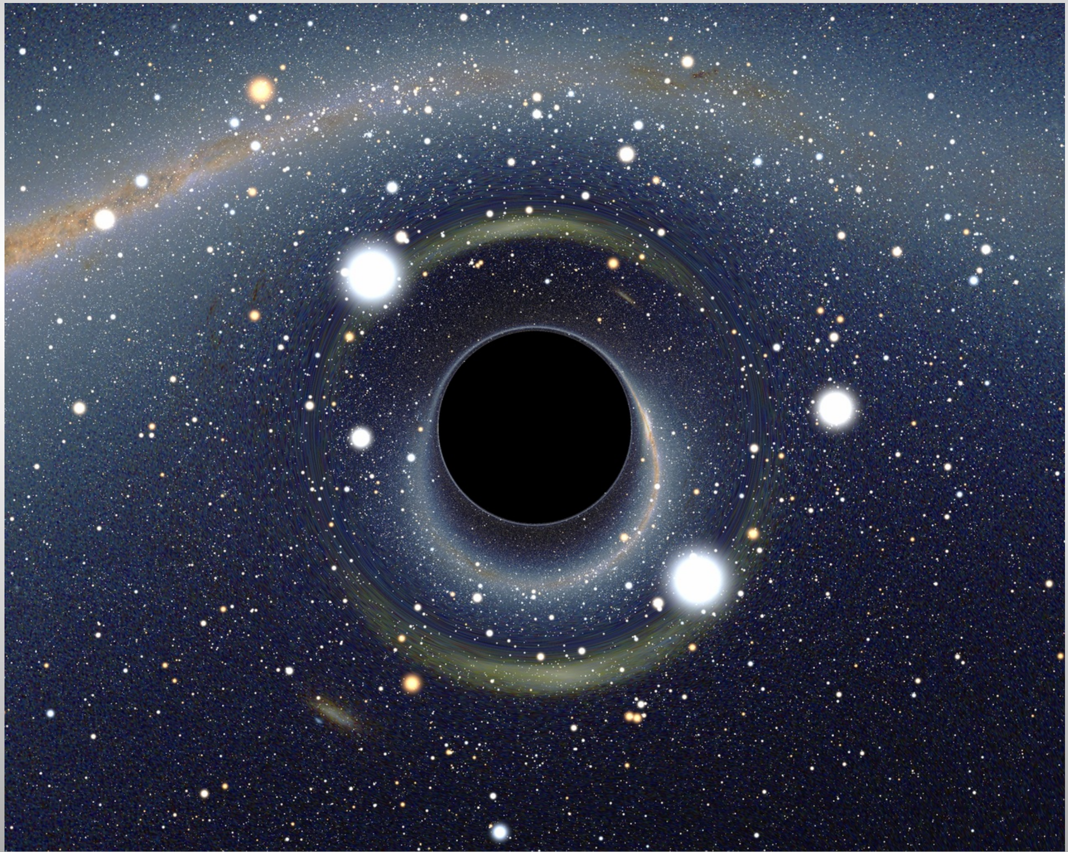
Durham University

Black Holes as Bubble Nucleation Sites



Ruth Gregory

Centre for Particle Theory

+ Ian Moss and Ben Withers

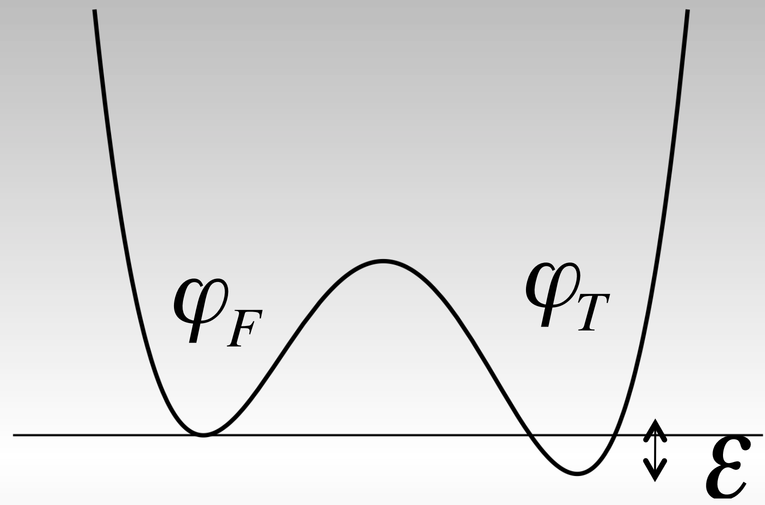
To appear

Overview

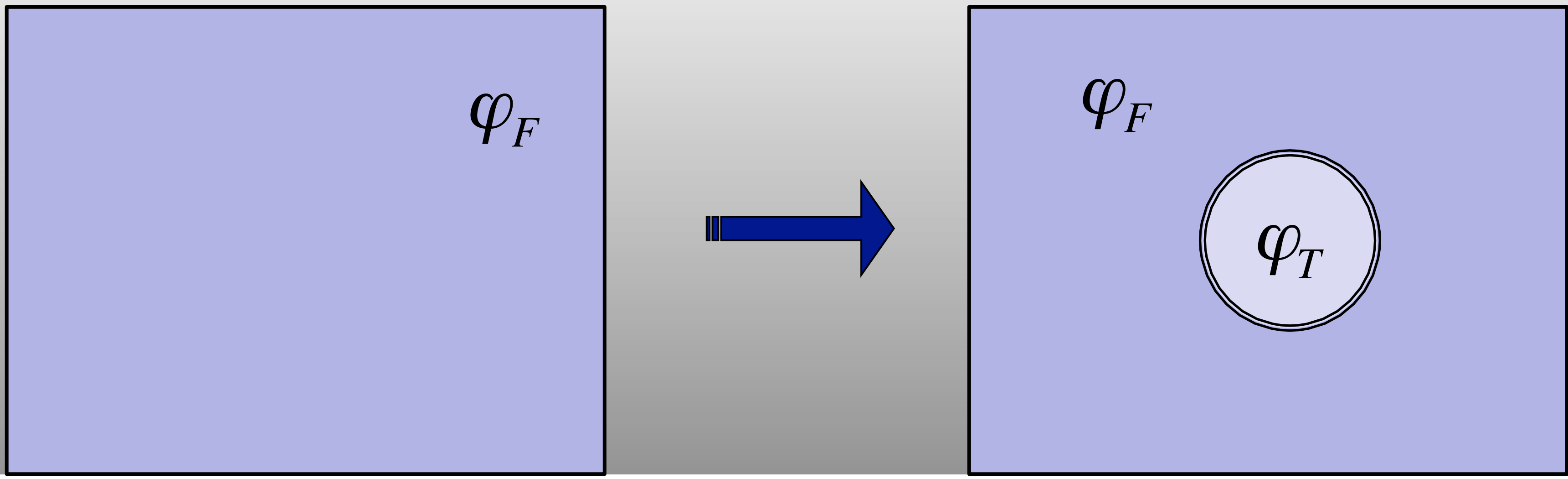
The Coleman de Luccia instanton started a trend of understanding more complex and physically realistic tunnelling scenarios, including gravity *and* nonlinear field theory.
CDL still the "gold standard" in computing probability of false vacuum decay, but –

? How dependent is amplitude on homogeneity?

Coleman



Original work of Coleman considered a field theory with false vacuum, showed that in limit of small energy difference (relative to barrier) transition modelled by a “thin wall” bubble.



Coleman

Amplitude determined by action of Euclidean tunneling solution: “The Bounce”

$$\mathcal{B} = \varepsilon \int d^4x\sqrt{g} - \sigma \int d^3x\sqrt{h}$$

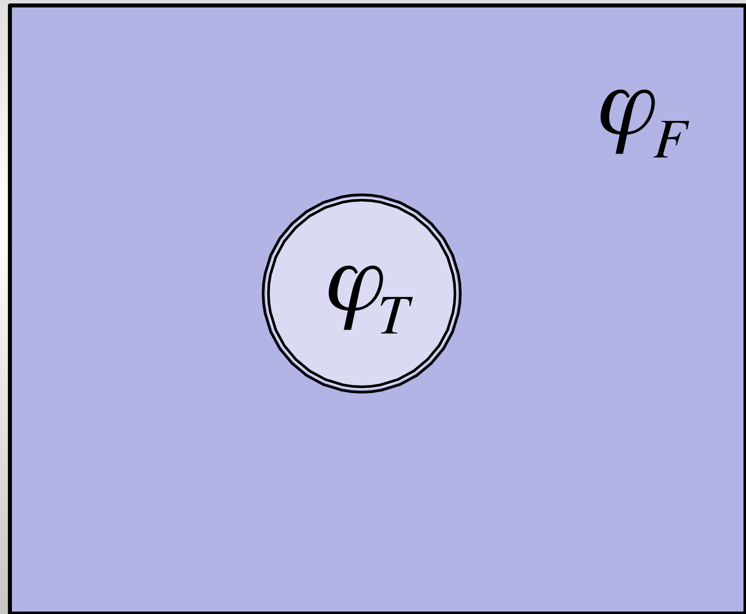
$$\sim \frac{\pi^2}{2}\varepsilon R^4 - 2\pi^2\sigma R^3$$

Stationarity wrt R:

$$R = \frac{3\sigma}{\varepsilon} \quad , \quad \mathcal{B} = \frac{27\pi^2\sigma^4}{2\varepsilon^3}$$

Tunneling amplitude:

$$\mathcal{P} \sim e^{-\mathcal{B}/\hbar}$$



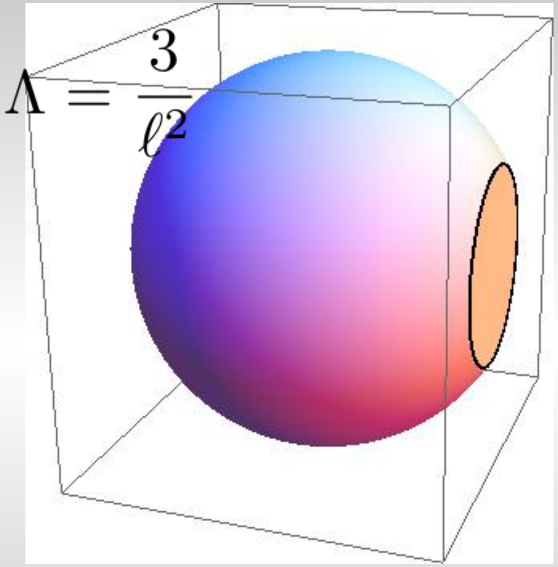
Coleman-de Luccia

The picture is very similar, but gravity is included.

o The bubble is now a solution of the Euclidean Einstein equations with a bubble of flat space separated from dS space by a thin wall.
o The wall radius is determined by the Israel junction conditions
o The action of the bounce is the difference of the action of this wall configuration and a pure de Sitter geometry.

CDL Action

The instanton looks like a truncated sphere:



$$ds^2_{\rm dS} = d\chi^2 + \ell^2 \sin^2 \frac{\chi}{\ell}\, d\Omega^2_{III} \quad , \quad S_{\rm dS} = -\frac{\ell^2 \pi}{G}$$

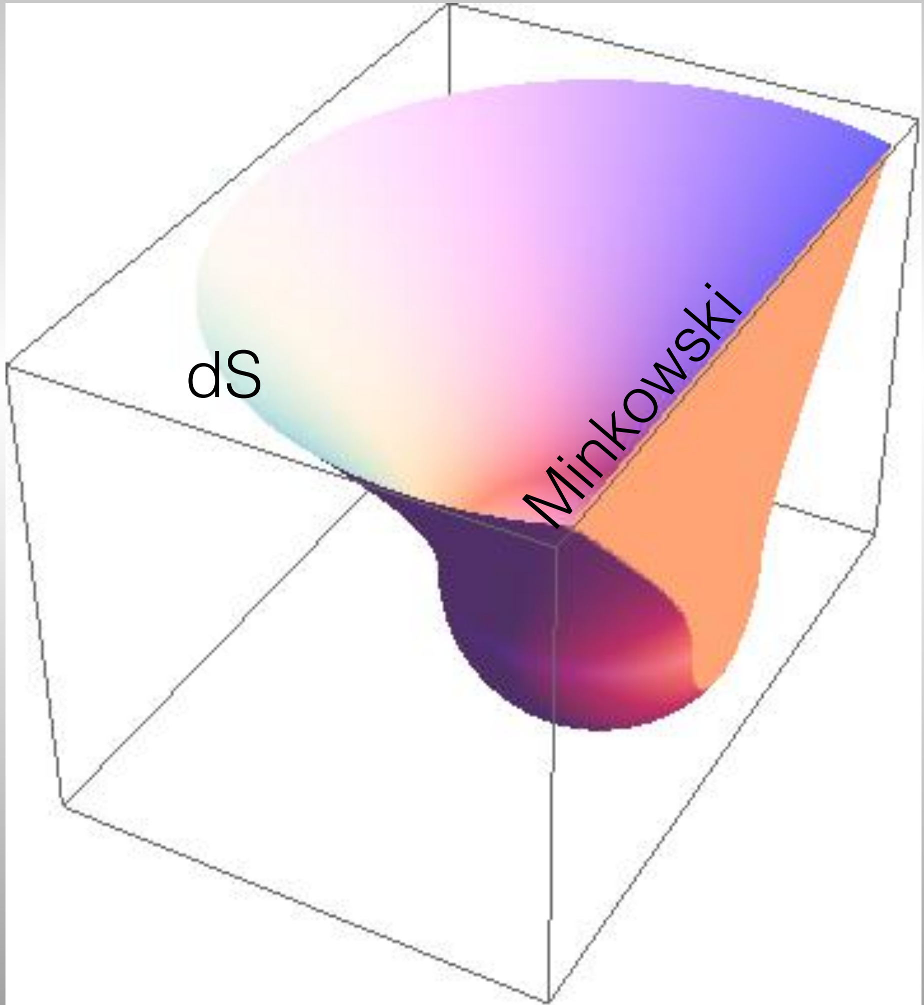
Israel conditions give truncation radius:

$$\frac{3}{\ell}\left(\cot \chi_0 - \csc \chi_0\right) = -4\pi G \sigma$$

hence bounce action:

$$\mathcal{B} = -\frac{\Lambda}{8\pi G}\int_{\rm int} d^4x\sqrt{g} - \frac{\sigma}{2}\int_{\mathcal{W}} d^3x\sqrt{h}$$
$$= \frac{\pi \ell^2}{4G}(1-\cos\chi_0)^2$$

Geometrical Picture



de Sitter space is represented by a hyperboloid (sphere) in 5D Lorentzian (Euclidean) spacetime. The instanton is often represented by joining the virtual Euclidean geometry to the real Lorentzian geometry across a surface of ‘time’ symmetry.

A more general look

In general, the wall separates two different regions of spacetime, which are solutions to Einstein equations:

$$ds^2 = f(r)dt^2 \pm \left[f^{-1}(r)dr^2 + r^2 d\mathbf{x}_\kappa^2\right]$$

$$f(r) = \kappa - \frac{\Lambda}{3}r^2 - \frac{2GM}{r}$$

The regions in general have different cosmological constants, and possibly a black hole mass.

Wall trajectories

Wall trajectory:

$$X^\mu = (t(\lambda), R(\lambda), \theta, \phi) \qquad g_{tt}\dot{t}^2 \pm g_{rr}\dot{R}^2 = 1$$

Israel junction conditions determine the equation of motion:

$$\Delta K_{ab} = -4\pi G \sigma h_{ab}$$

Inputting the form of the trajectory gives a Friedmann like equation for R:

Lorentz

Euclid

$$\pm \left(\frac{\dot{R}}{R}\right)^2 = \bar{\sigma}^2 - \frac{\bar{f}}{R^2} + \frac{(\Delta f)^2}{16\bar{\sigma}^2 R^4}$$

CDL Wall

Coleman-de Luccia has:

$$f_+ = 1 - \frac{r^2}{\ell^2}$$

$$f_- = 1$$

Static patch

Hence

$$\pm \left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{R^2} = \left(\bar{\sigma} + \frac{1}{4\bar{\sigma}\ell^2}\right)^2$$

$$\bar{\sigma} = 2\pi G \sigma$$

and

$$R(\lambda) = \gamma \cos \frac{\lambda}{\gamma}$$

$$t_-(\lambda) = \gamma \sin \frac{\lambda}{\gamma}$$

$$\sqrt{\ell^2 - \gamma^2}\, \tan \frac{t_+(\lambda)}{\ell} = \gamma \sin \frac{\lambda}{\gamma}$$

Periodicity *not* the same as static patch

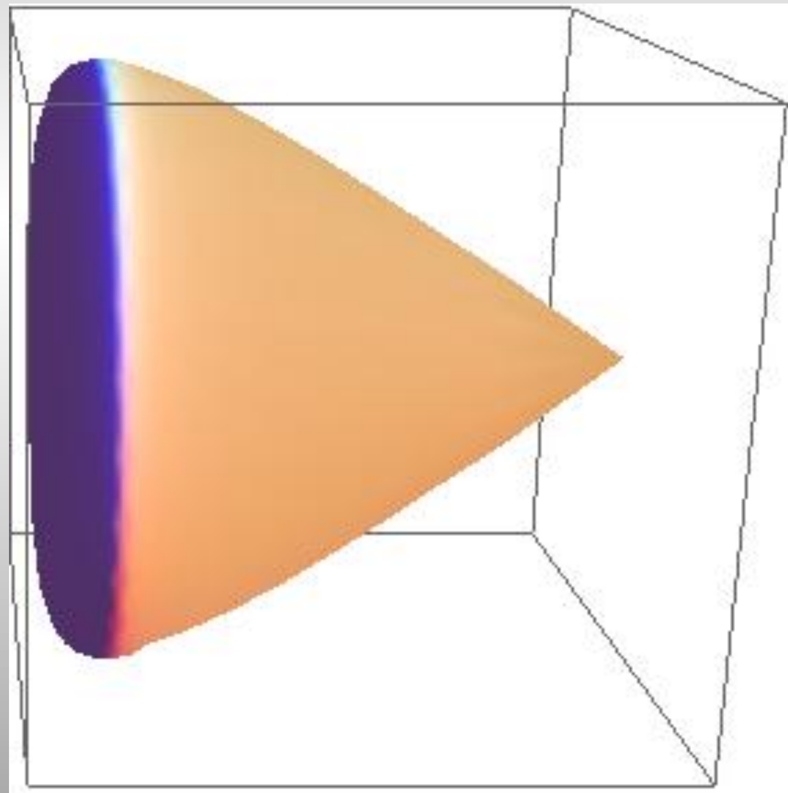
Hence conical deficit in bounce

Conical Deficits

Familiar in Euclidean sections, SdS has a deficit/excess on at least one horizon:

$$\Delta\tau = \frac{4\pi}{|f'(r_i)|}$$

$$\frac{\Delta\tau_h}{\Delta\tau_c} = \frac{r_c(1-3\frac{r_h^2}{\ell^2})}{r_h(3\frac{r_c^2}{\ell^2}-1)} \sim \frac{\ell}{2r_h} \quad (r_h \to 0)$$



Conical Actions

The conical deficit is produced by a delta function in the Ricci tensor (caveat – no transverse energy momentum, metric a product space) so can compute the action:

$$ds^2 = d\rho^2 + A^2(\rho)d\chi^2 + C^2(\rho)d\Omega_{II}^2$$

Smooth out A:

$$A'(0) = 1 \quad , \quad A'(\varepsilon) = (1-\delta)$$

$$\mathcal{R} = -\frac{2A''}{A} - \frac{4C''}{C} - \frac{4A'C'}{AC} + \frac{2(1-C'^2)}{C^2} \sim -\frac{2A''}{A} - \frac{8C_2}{C_0} + \frac{2}{C_0^2} + \mathcal{O}(\rho < \varepsilon)$$

$$\int d^4x\sqrt{g}\mathcal{R} \sim (4\pi C_0^2)4\pi[A'(0) - A'(\varepsilon)] + \mathcal{O}(\varepsilon) = (4\pi C_0^2)(4\pi\delta) + \mathcal{O}(\varepsilon)$$

SdS Action

Applying this to the SdS black hole now gives an interesting result. For a general periodicity:

$$S_{SdS} = \int d^4x\sqrt{g}(-R+2\Lambda) = -\frac{\pi(r_c^2+r_h^2)}{G} + \frac{\beta}{2G}\left[2\pi\frac{r_c^2}{\beta_c} + 2\pi\frac{r_h^2}{\beta_h} - \frac{r_c^3-r_h^3}{\ell^2}\right]$$

i.e. the result is independent of β

$$r_c^2 + r_h^2 + r_c r_h = \ell^2$$

$$\beta_i = \frac{4\pi r_i}{|1-3\frac{r_i^2}{\ell^2}|}$$

Back to walls

Adding in a wall adds in a contribution to the action:

$$-\int_{\mathcal{W}} \frac{\sigma}{2} - \frac{1}{16\pi G}\int_{\mathcal{W}} \left(f'_+\dot{\tau}_+ - f'_-\dot{\tau}_-\right)$$

So can compute the action of a bubble of Minkowski space inside SdS with a wall boundary:

$$\mathcal{B} = \frac{\pi r_h^2}{G} - \frac{\bar{\sigma}}{G}\int d\lambda R^2 - \frac{\beta M}{2} + \frac{1}{2\pi G\ell^2}\int d\lambda R^3\dot{\tau}_+$$

For CDL, M=0, and recover usual result.

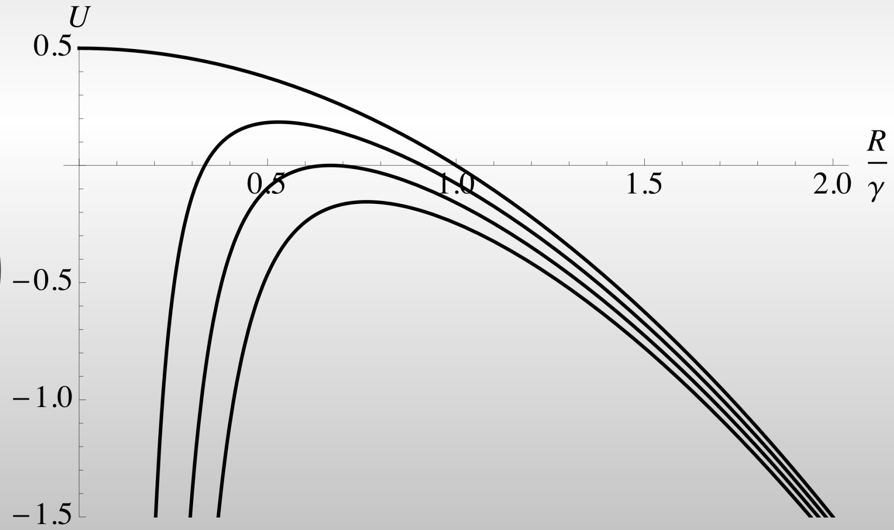
Adding a black hole

Can take a more general instanton with a Minkowski bubble inside SdS

$$\pm\left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{R^2} = \left(\bar{\sigma} + \frac{1}{4\bar{\sigma}\ell^2} + \frac{GM}{2\bar{\sigma}R^3}\right)^2$$

$$U(R) = \frac{1}{2}\left[1 - \left(\frac{R}{\gamma} + \kappa\frac{\gamma^2}{R^2}\right)^2\right] \qquad \left(\kappa = \frac{GM}{2\bar{\sigma}\gamma^2}\right)$$

$$R = R_* = \sqrt{\frac{3GM}{2\bar{\sigma}}} \quad (\kappa = \kappa_* = 4/27)$$



Find numerically (except for unstable static solution R_*)

Static solution

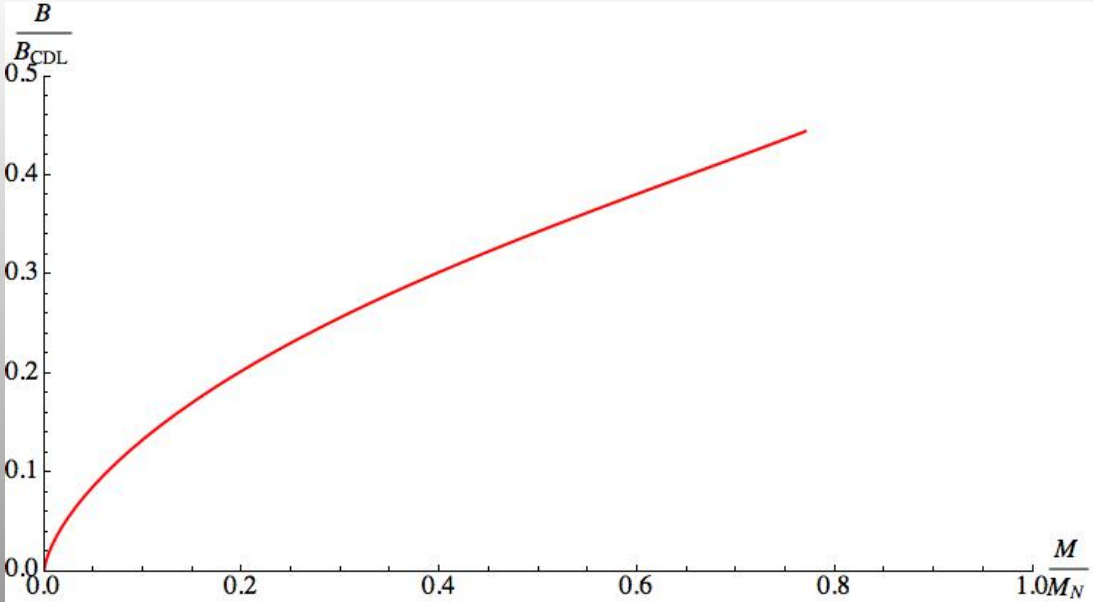
This is the one example where we can have no conical deficit (hence another cross-check).

$$R = R_* = \sqrt{\frac{3GM}{2\bar{\sigma}}}$$

Both methods give the bounce action:

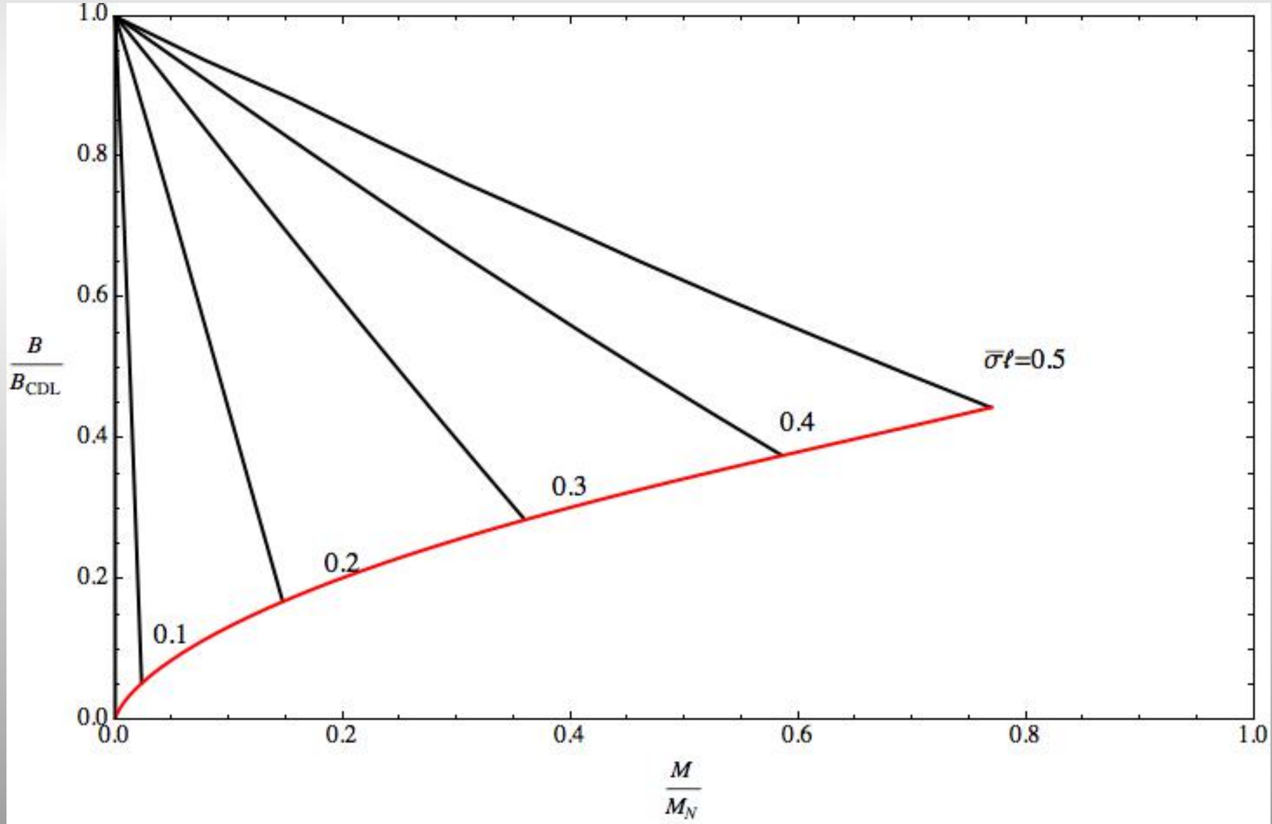
$$\mathcal{B}_* = \frac{\pi r_h^2}{G}$$

$$M_N = \frac{\ell}{3\sqrt{3}G}$$

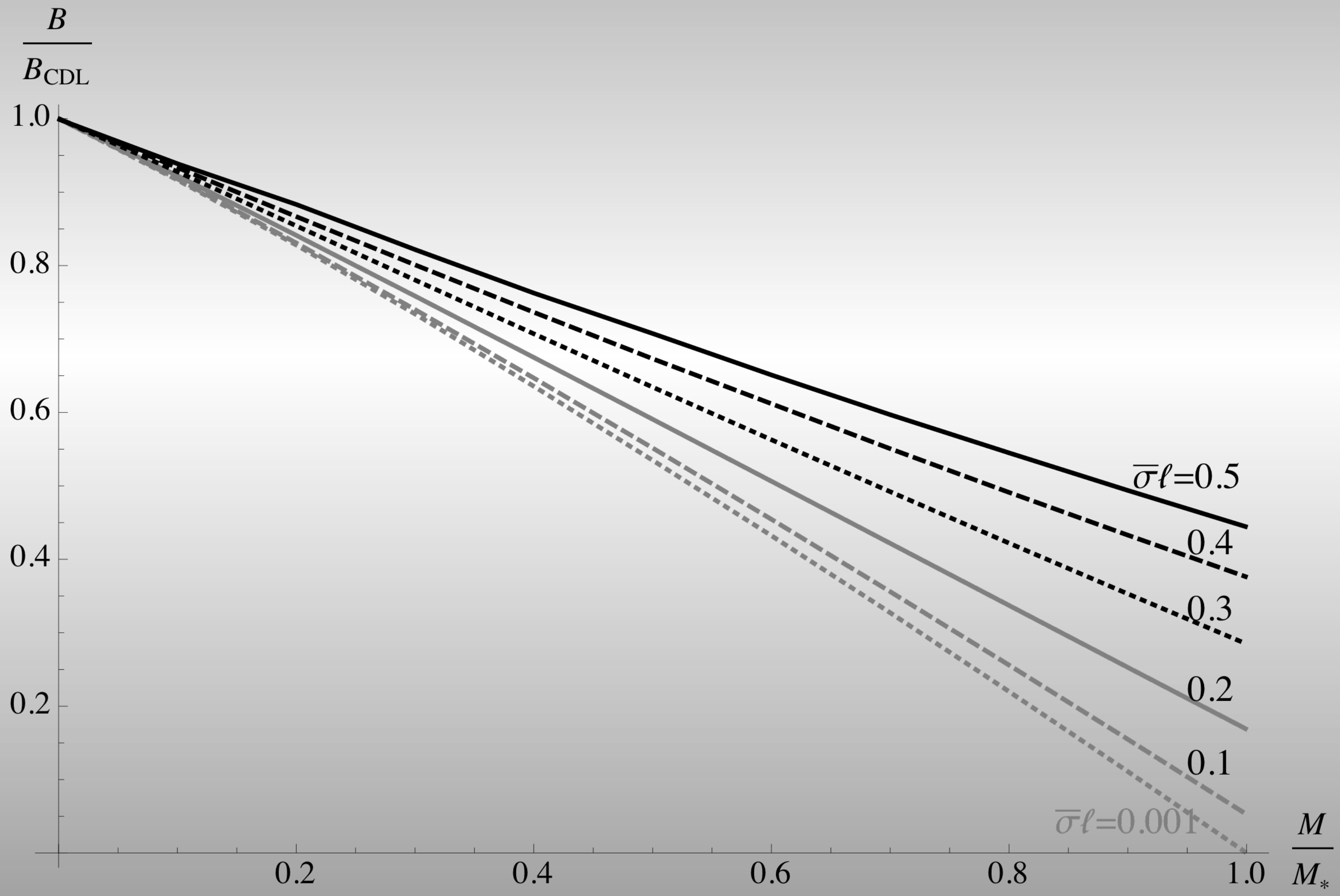


General Results

In general, have to construct the wall numerically, determine its periodicity, then perform the action integral.



or as a function of M/M*

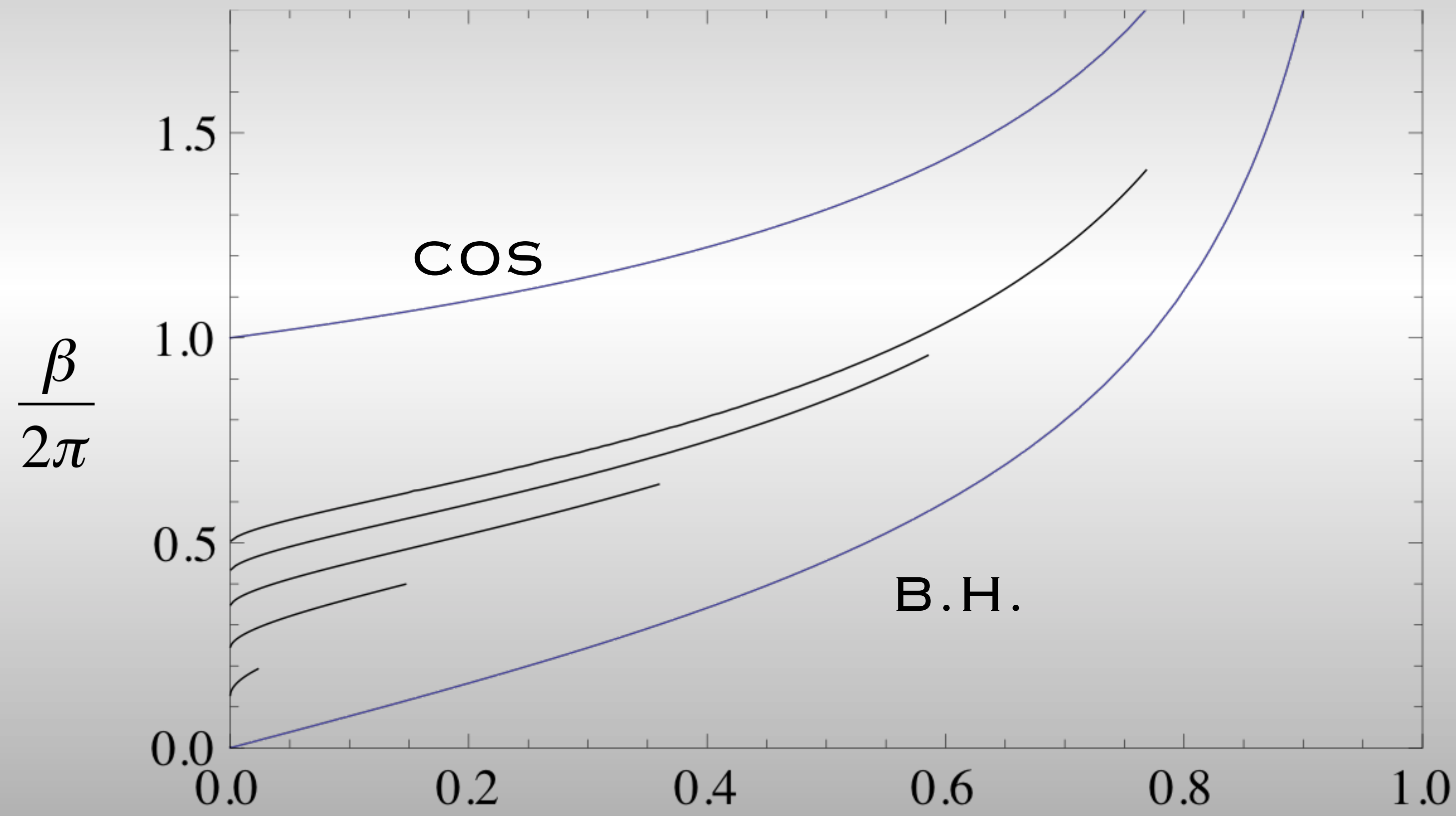


The bounce action clearly drops as we add in an inhomogeneity, the larger the black hole, the lower the action.

For light tension walls the effect is most pronounced.

The periodicity of the bubble is (approx) a fixed ratio of the cosmological to black hole periodicity – depending on $\sigma\ell$

Periodicity as a function of M/M_N



Tunneling v Evaporation:

Interesting to compare lifetime of universe v black hole:

For Hawking evaporation: $\Gamma_H \approx 3.6 \times 10^{-4}(G^2 M_*^3)^{-1}$

And tunneling: $$\Gamma_* \approx \left(\frac{2}{G}\right)^{1/2} e^{-4\pi G M_*^2}.$$

Ratio: $$\frac{\Gamma_*}{\Gamma_H} \approx 3.9 \times 10^3 (G M_*^2)^{3/2} e^{-4\pi G M_*^2}.$$

i.e. less than unity for black holes above the Planck mass.

Black holes in pure dS will evaporate before they seed decay, but if not quite dS (slow roll) accretion can dominate.

Summary

- Have shown how to compute the action of a singular instanton, verified for known or special cases.
- Tunneling amplitude significantly enhanced in the presence of a black hole – even more so if a regular source
- Evaporation beats tunneling in pure dS, however if in a slow roll background, accretion beats evaporation.