

Cosmological constant problem and lower-dimensional field theory, Liouville theory in de Sitter space

Yoji Koyama
(National Tsing-Hua U)

Collaborater

Takeo Inami (Chuo U), Yu Nakayama (Caltech), Mariko Suzuki (Shizuoka U)

20.11.2013, PASCOS2013,NTU

Contents

- 1 Introduction
- 2 Infrared screening effect
- 3 Field theory in lower-dim dS space
and a model for IR screening effect
- 4 Summary and outlook

This work in progress.

1. Introduction

- We consider the loop effect in deSitter space as an approach to the cosmological constant problem.
- The CC problem is a sort of hierarchy problem. We are interested in the possible infrared divergences present in the loop corrections in dS space and its interpretation for the purpose of finding the screening mechanism of the Λ in the infrared region.
- We propose a 2D toy model (Liouville +matter) for 4D gravity and matter in dS space.

dS field theory and cosmological constant

dS space has positive cosmological const. which is responsible for inflationary expansion of universe.

Poincare coordinate is convenient for the calculation of loop correction in dS space.

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 = a^2(\tau)(-d\tau^2 + d\vec{x}^2) \quad a(t) = e^{Ht}$$

$$\text{conformal time: } \tau = -H^{-1}e^{-Ht} \quad (-\infty < \tau < 0)$$

$$\text{Hubble parameter: } H \equiv \dot{a}/a$$

$$\text{Friedman eq. in D dim } R = D(D-1)H^2 = D\Lambda$$

dS symmetry $SO(D, 1) \rightarrow \Lambda = \text{constant in time.}$

dS invariant vacuum state exists : Bunch-Davies vacuum
(Euclidean vacuum)

If dS symmetry is broken $\rightarrow \Lambda = \Lambda(\tau)$

2. Infrared screening effect

Massless scalar in dS and IR divergence [Allen et al. 87, Kitamoto et al. 09]

massless minimally coupled scalar field in 4D dS space

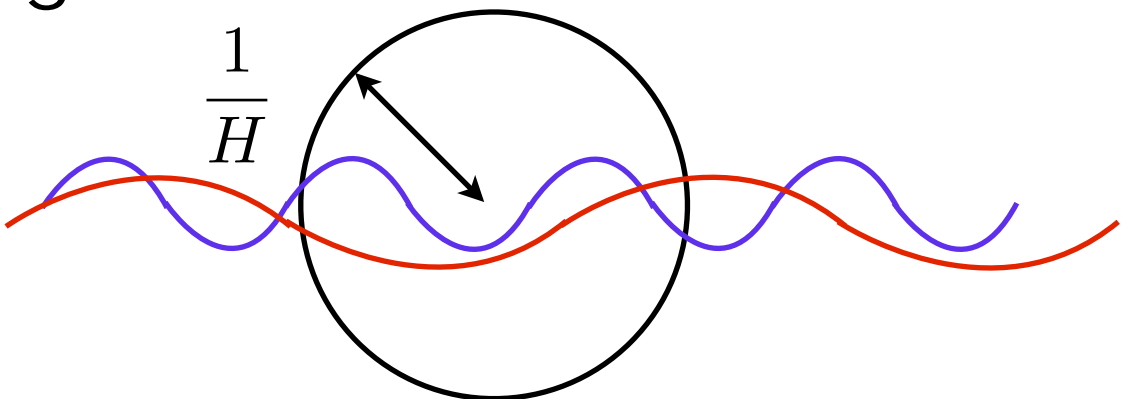
$$\phi(x) = \int \frac{d^3 k}{(2\pi)^3} (a_{\vec{k}} \varphi(x) + a_{\vec{k}}^\dagger \varphi^*(x)) \quad a_{\vec{k}} |BD\rangle = 0$$

$$\varphi(x) = (-H\tau) \frac{i}{\sqrt{2k}} \left(1 - \frac{i}{k\tau}\right) e^{-ik\tau} e^{i\vec{k}\cdot\vec{x}}$$

In the limit $k \rightarrow \infty$ it reproduces the mode function of Minkowski space.

For $k < |\tau|^{-1}$ ($K = k/a < H$) the second term is dominant.

Two point function has infrared divergence

$$\langle \phi(x)\phi(x) \rangle \sim \int_H^{\text{UV}} d^3 K \frac{1}{2K} + H^2 \int^H d^3 K \frac{1}{K^3}$$


If IR cutoff k_0 is introduced,

$$\begin{aligned}\langle \phi(x)\phi(x) \rangle &= UV \text{ const} + H^2 \int_{k_0/a}^H \frac{d^3 K}{(2\pi)^3} \frac{1}{2K^3} \\ &= UV \text{ const} + \frac{H^2}{4\pi^2} \ln a(\tau) + \frac{H^2}{4\pi^2} \ln \left(\frac{H}{k_0} \right)\end{aligned}$$

the two point function acquires the **time dependence**.

This IR log factor breaks the dS symmetry.

Transverse-traceless-synchronous graviton has the same IR divergence (in poincare coordinate).

In 4D dS space

The time dependences in the two point functions of massless scalar and graviton have been applied to the CC problem as a sort of screening mechanism.

- scalar loop

[Woodard et al. 02, Polyakov 82,08 Janssen et al. 08, Kitamoto et al. 09,10]

$$R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R + (\Lambda - \frac{\kappa}{4}T_{\rho}{}^{\rho})g_{\mu\nu} = 0 \quad \Lambda_{\text{eff}} = \Lambda - \frac{\kappa}{4}T_{\rho}{}^{\rho}$$

In perturbation theory, Λ_{eff} acquires the logarithmic time dependence (at most) but it increases in time.

In non-perturbative analysis, there is no time dependence in Λ_{eff} in ϕ^4 theory and non-linear sigma model. [Kitamoto et al. 09,10]

- graviton loop [Tsamis et al. 97,11 Higuchi et al 08,11]

There are several problems about gauge dof and existence of dS inv. vacuum for interacting graviton. This issue is still open.

3. Field theory on lower-dimensional dS space and a model for IR screening effect

We consider the lower-dimensional field theory in dS space for the IR screening effect.

To see IR quantum effects, we consider massless field theories.

In lower dimensions

- In flat space, IR divergence are stronger than in 4 dimension.
- Some models are solvable, so we may study effects in closed form.

cf: 2D Gravity Callan et al. (92)

What about in lower-dimensional dS space ?

For massless scalar field, we consider

1 Perturbative effects ϕ^4 theory (2 dim)

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{4!}\phi^4$$

ϕ^6 theory (3 dim)

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{\lambda}{6!}\phi^6$$

2 Non-perturbative effects Liouville type potential (2 dim)

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - e^{\lambda\phi}$$

Non linear sigma model (2 dim)

$$\mathcal{L} = \frac{1}{2g^2}g^{\mu\nu}(x)\partial_\mu\phi^a(x)\partial_\nu\phi^b(x)G_{ab}(\phi)$$

Liouville theory

The theory has a few remarkable properties.

1. It is solvable at quantum level in 2 D Minkowski space.

Liouville theory is solvable in flat space.

There are an infinite number of conserved quantities.

2. It has conformal invariance.

3. Liouville theory may be regarded as **2D quantum gravity**.

$$S_L = \frac{1}{4\pi} \int d^2x \sqrt{\hat{g}} \left(\frac{1}{2} \hat{g}^{ab} \partial_a \phi \partial_b \phi + \Lambda e^{\gamma \phi} + \dots \right)$$

2D model for IR screening effect

Using Liouville theory, we can evaluate the effects of gravitational loop and matter loop in 2D dS space.

$$S_{\text{L+matter}}[\phi, \chi] = \int d^2x \sqrt{\hat{g}} [\hat{g}^{ab} \partial_a \phi \partial_b \phi + Q \hat{R} \phi + \Lambda e^\phi + \hat{g}^{ab} \partial_a \chi \partial_b \chi - e^\phi V(\chi)]$$

ϕ : Liouville field χ : matter field

The first three terms are the 2D gravity action.

Last term describes the coupling of gravity and matter.

2 D toy model for 4 D gravity + matter

We are currently working on the matter loop effect.

Matter loop effect: Perturbative effect

For loop correction, we need propagator $i\Delta(x; x')$ on dS.

We consider massless scalar field in D dim.

$$i\Delta = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \left\{ \frac{1}{-\frac{D}{2} + 1} \frac{\Gamma(\frac{D}{2})}{\Gamma(1)} \left(\frac{y}{4}\right)^{-\frac{D}{2}+1} - \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} \pi \cot\left(\pi \frac{D}{2}\right) \right. \\ \left. - \sum_{n=1}^{\infty} \left[(\dots) \left(\frac{y}{4}\right)^{-\frac{D}{2}+1+n} - (\dots) \left(\frac{y}{4}\right)^n \right] + \frac{\Gamma(D-1)}{\Gamma(\frac{D}{2})} [\ln(a(\tau)a(\tau')) + 2 \ln(H/k_0)] \right\}$$

Onemli et al.(04), Janssen et al. (08)

UV div \rightarrow dim regularization, IR div \rightarrow cut off

de Sitter invariant distance

$$y \equiv \frac{-(\tau - \tau')^2 + (\vec{x} - \vec{x}')^2}{\tau\tau'}$$

Massless propagator is UV ($y \rightarrow 0$) div as well as IR div.

We see that dS space has logarithmic IR div in any dimensions.

To evaluate the effective cosmological constant. We calculate the VEV of the energy momentum tensor.

$$\begin{aligned} T_{\mu\nu} &\equiv \frac{-2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int \sqrt{-g} d^D x [g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi + 2V(\phi)] \\ &= \left(\delta_\mu^\rho \delta_\nu^\sigma - \frac{1}{2} \eta_{\mu\nu} \eta^{\rho\sigma} \right) \partial_\rho \phi \partial_\sigma \phi - g_{\mu\nu} (V(\phi)) \end{aligned}$$

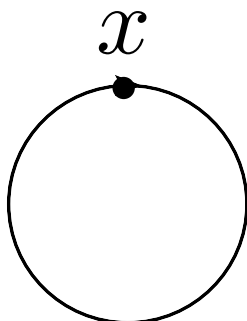
$$\Lambda_{\text{eff}} = \Lambda - \frac{\kappa}{D} \langle T_\rho{}^\rho \rangle$$

IR effects from kinetic term is weaker than potential term.

(effects in potential term) > (effects in kinetic term)

In 2D theory

$$i\Delta(D = 2) = -\frac{1}{4\pi} \ln\left(\frac{y}{4}\right) + \frac{1}{4\pi} \ln\left(\frac{H^2}{k_0^2} aa'\right)$$

1-loop vacuum energy $i\Delta(x; x')$ $\xrightarrow{\lim x' \rightarrow x}$ 

We have obtained $T_{\mu\nu}$.

Conformal anomaly calculation (point splitting) using propagator.

$$\langle T_{\mu\nu} \rangle = -\frac{R}{48\pi} g_{\mu\nu} \quad \text{Davis Fulling (76)}$$

$$\Lambda_{\text{eff}} = \Lambda + \frac{R}{48\pi} \quad \text{2D}$$

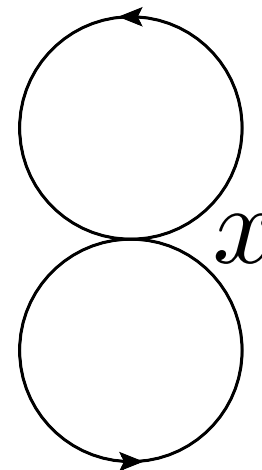
$$\Lambda_{\text{eff}} = \Lambda - \kappa \frac{3H^4}{32\pi^2} \quad \text{4D (KK) (11)}$$

Connections to Potential term's effect

ϕ^4 theory (2D)

$$\langle T_{\mu\nu}(x) \rangle = (\text{kinetic term}) - \underline{g_{\mu\nu} \frac{\lambda}{4!} \langle \phi^4 \rangle}$$

dominant contribution



$$\frac{\lambda}{4!} \langle \phi^4 \rangle = \frac{\lambda}{8} \langle \phi^2 \rangle^2$$

Here the VEV is taken with respect to free BD vacuum.

$$\langle \phi(x)^2 \rangle_{\text{ren}} \sim \alpha \log a(\tau) \quad \alpha : \text{numerical const.}$$

$$\langle \phi(x)^2 \rangle_{\text{ren}}^2 \sim \alpha^2 4(\log(a(\tau)))^2$$

$$(T_{\rho}{}^{\rho})_{\text{pot}} \sim -\frac{2\lambda}{8} \times \alpha^2 4(\log(a(\tau)))^2$$

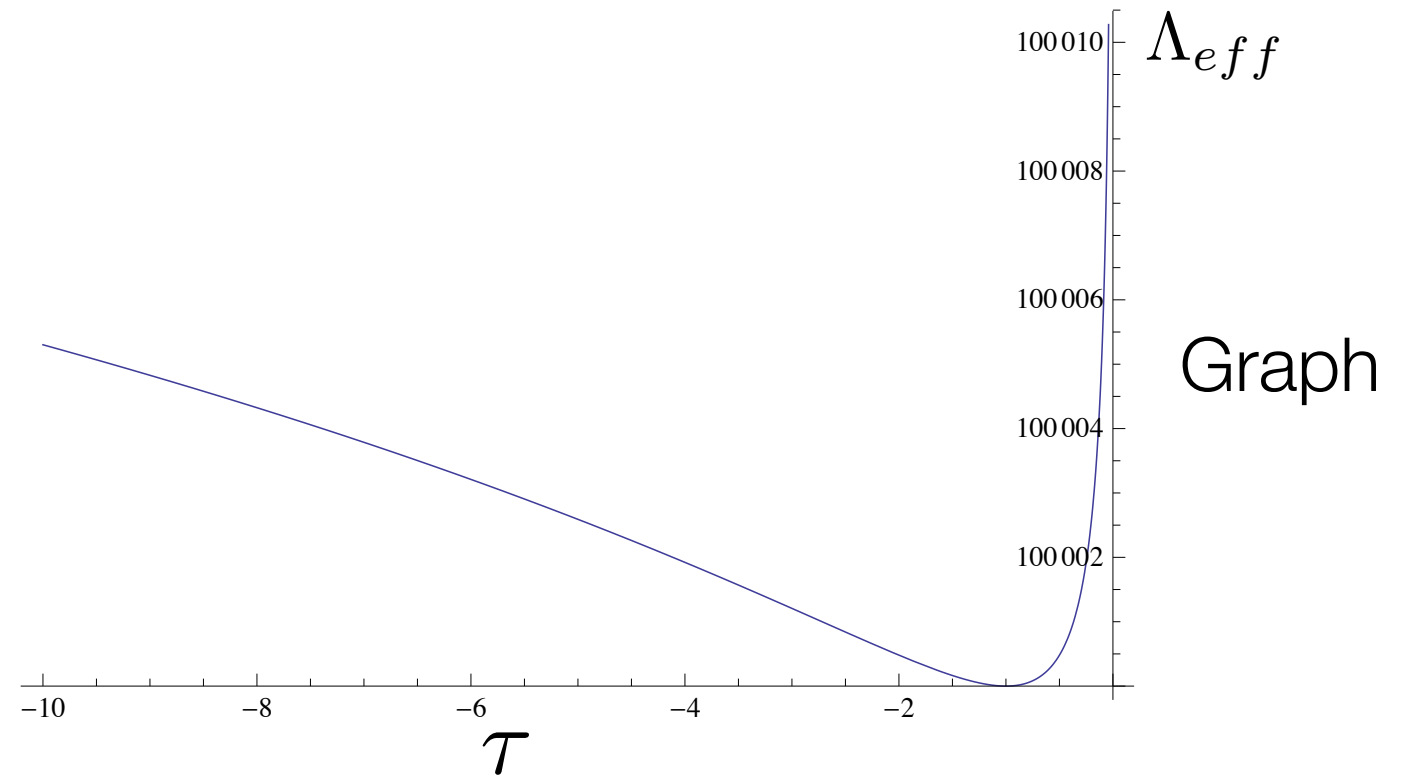
$$\therefore \Lambda_{\text{eff}} \sim \Lambda + \lambda \times \alpha^2 \left(\log\left(-\frac{1}{H\tau}\right) \right)^2$$

Effective cosmo const has time dependence.

ϕ^4 theory (2D)

$$\therefore \Lambda_{eff}$$

$$\sim \Lambda + \lambda \times \alpha^2 \left(\log\left(-\frac{1}{H\tau}\right) \right)^2$$



The loop correction is always positive and Λ_{eff} is still very large. In this model we do not see IR screening effect.

Liouville type potential

We evaluate the potential energy contribution.

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - e^{\lambda\phi}$$

$$\langle T_{\mu\nu} \rangle_{\text{pot}} = -g_{\mu\nu} \langle e^{\lambda\phi} \rangle$$

$$\langle e^{\lambda\phi} \rangle = e^{\left(\frac{1}{2}\lambda^2\langle\phi^2\rangle\right)}$$

$$\langle\phi^2\rangle \sim \langle\phi^2(x)\rangle_{\text{rem}} \sim \frac{1}{2\pi} \log a(\tau)$$

$$e^{\left(\frac{1}{2}\lambda^2\langle\phi^2\rangle\right)} \sim e^{\left(\frac{1}{2}\lambda^2\frac{1}{2\pi} \log a(\tau)\right)} \quad e^{\lambda\phi} \text{ obtained in a closed form.}$$
$$= \left(-\frac{1}{H\tau}\right)^{\frac{\lambda^2}{4\pi}}$$

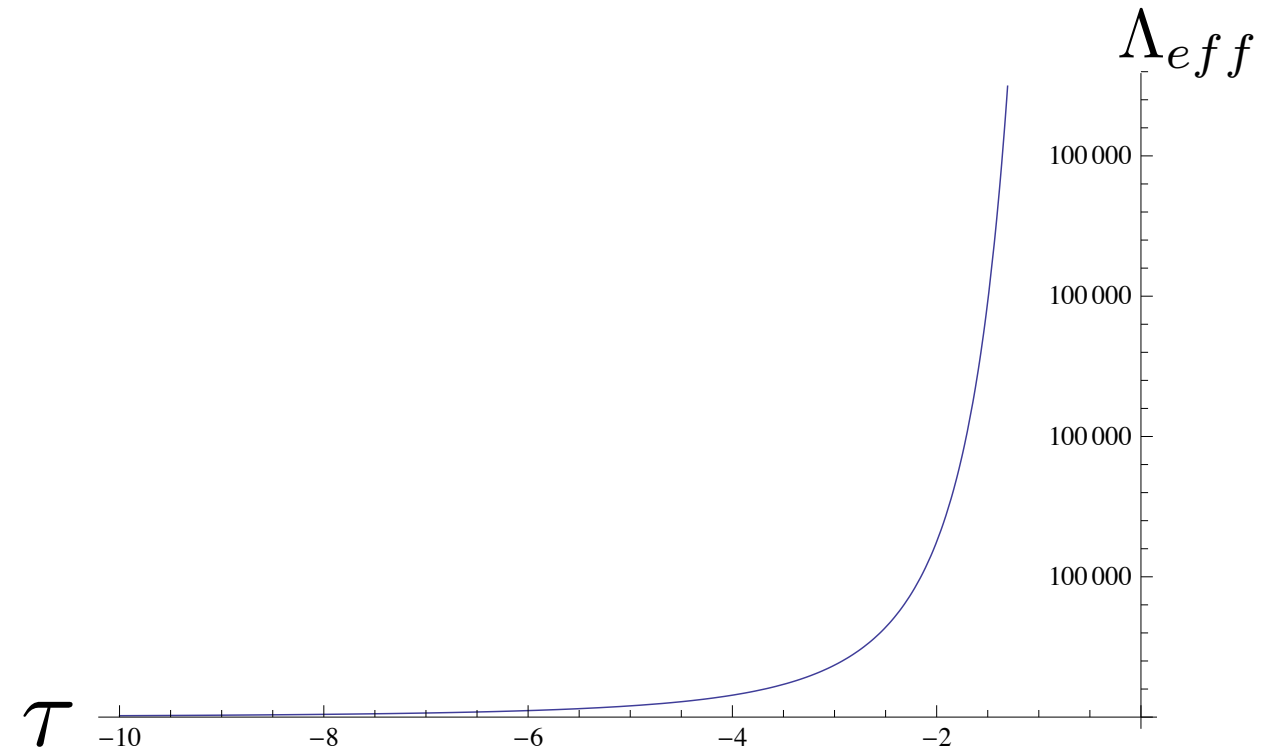
$$\therefore \Lambda_{\text{eff}} = \Lambda + \kappa\left(-\frac{1}{H\tau}\right)^{\frac{\lambda^2}{4\pi}}$$

The time dependence is different from perturbative theory.

$$\phi^4 \text{ theory } \Lambda_{\text{eff}} \sim \Lambda + \lambda \times \alpha^2 \left(\log\left(-\frac{1}{H\tau}\right)\right)^2$$

Liouville potential

$$\therefore \Lambda_{eff} = \Lambda + \kappa \left(-\frac{1}{H\tau} \right)^{\lambda^2/4\pi}$$



Quantum effect has a **power dependence** in τ .

However the effective cosmological const increases as τ approaches 0.

Contribution of kinetic term (free theory)

No time dependence appears

in 2D $T_{\mu}^{\mu} \neq 0$ Conformal anomaly

in 3D constant effect

in 4D constant effect (Kitamoto et al.) (11)

By differential, kinetic term's contribution is constant.

Contribution of potential term

There appears time dependence.

polynomial interaction

$$\log a(\tau)$$

exponential interaction

$$a(\tau)^{\frac{\lambda^2}{4\pi}}$$

But, it does not have IR screening effect on cosmological
const Λ_{eff} .

5. Summary and outlook

- We have constructed a good 2D toy model for 4D gravity and matter for IR screening effect of cosmological const. in dS space.
- Matter and Liouville loops for cosmological const Λ are being computed.
- For matter loop, we evaluated the VEV of EM tensor only in free vacuum. We now compute that in the interacting vacuum.
- Solvability of Liouville theory: We extend Liouville theory solvability from flat space to dS space. If it is solvable, we may calculate quantum corrections to cosmological constant in closed form.