Massive Gravity and Cosmology

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Based on works with: C. de Rham, A. Tolley;
and with G. D'Amico, L. Hui, D. Pirtskhalava

November 25, 2013

Motivation:

General Relativity (GR) is a very successful theory. For further tests of GR, good to have an alternative theory to compare with, and test both against the data. The Brans-Dicke theory was introduced for that purpose in 1960s

Cosmic acceleration: a new physical scale of dark energy, $10^{-33}\ eV$; this might be a scale where GR should be modified

Extension of GR by a mass term is arguably the best motivated modification. Yet, such an extension had been a problem up until recently. This problem – good enough motivation for a theorist to ask the questions: *what is the potential for gravity?*

A scalar field theory:

Kinetic term $= -(\partial_\mu \Phi)^2$

Mass+ potential (renormalisable) $= m^2\Phi^2 + \lambda_4\Phi^4$

Effective potential$= m^2\Phi^2 + \lambda_4\Phi^4 + \lambda_6\Phi^6 + \cdots$

The coefficients $\lambda_4, \lambda_6, \lambda_8, etc.$ need not take any special values (only need to satisfy some positivity constraints); they need not be tuned to each other for consistency of the effective field theory.

However, as we'll see in the potential for gravity all the polynomial terms have to be related, even in the classical theory!

The Potential for General Relativity:

$\sqrt{g}R \rightarrow$ Kinetic term$= \partial h \partial h$

$h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$:

$\sqrt{g} =$ simplest potential $= 1 + \frac{h}{2} + \frac{h^2}{8} - \frac{h_{\mu\nu}^2}{4} \cdots$

Coefficients in different orders are fixed to sum up into the root g

This potential does not change the number of degrees of freedom, it's not a mass term

Mass term for gravity: 5 degrees of freedom

Quadratic mass term$= b_1 h_{\mu\nu}^2 + b_2 h^2$

Fierz and Pauli 1939: $b_1 = -b_2$ to preserve unitarity

Fierz-Pauli mass term$= M_{\rm pl}^2 m^2 (h_{\mu\nu}^2 - h^2)$

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A nonlinear invariant form:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \to \quad g_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab} + H_{\mu\nu} \quad \textit{Siegel, '93}$$

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$$\mathcal{L}_{FP} = M_{\rm pl}^2 \sqrt{g} \left(R - m^2 (H_{\mu\nu}^2 - H^2) \right)$$

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$$\mathcal{L}_{FP} = M_{\rm pl}^2 \sqrt{g} \left(R - m^2 (H_{\mu\nu}^2 - H^2) \right)$$

However, this contains h^3 and higher terms that give rise to nonlinear instabilities (Boulware and Deser '72).

Can this be fixed by adding appropriate higher order terms?

$$\sqrt{g} \quad (R - m^2(H_{\mu\nu}^2 - H^2 + c_1 H_{\mu\nu}^3 + c_2 H H_{\mu\nu}^2 + c_3 H^3$$

$$+ \quad d_1 H_{\mu\nu}^4 + d_2 H H_{\mu\nu}^3 + d_3 H^2 H_{\mu\nu}^2 + d_4 (H_{\mu\nu}^2)^2 + d_5 H^4 + \cdots))$$

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This can be avoided for specific choice of the coefficients:
For $c_1 = 2c_3 + 1/2$, $c_2 = -3c_3 - 1/2$ no ghost in 3rd order; however, the instability is still present in the 4th order; choose the coefficients in the 4th order $O(H_{\mu\nu}^4)$ terms, and so on...

The BD ghost cancellation can be achieved order-by-order:
de Rham, GG ,'10.

The cancellation conditions are so powerful that the entire series can be summed up: *de Rham, GG, Tolley '11*.

GR Extended by Mass and Potential Terms

de Rham, GG, Tolley, '11

5 dof's in $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} f_{\alpha\nu}} \qquad f_{\alpha\nu} \equiv \partial_\alpha \phi^a \partial_\nu \phi^b \eta_{ab}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L}_{dRGT} = M_{\rm pl}^2 \sqrt{g} \left(R + m^2 \left(\mathcal{U}_2 + \alpha_3\, \mathcal{U}_3 + \alpha_4\, \mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]$$

Lagrangian Rewritten via Levi-Civita Symbols:

de Rham, GG, Heisenberg, Pirtskhalava '11 (decoupling limit)

$$\mathcal{L}_{dRGT} = M_{\rm pl}^2 \sqrt{g} \left(R + m^2 \left(\mathcal{U}_2 + \alpha_3 \, \mathcal{U}_3 + \alpha_4 \, \mathcal{U}_4 \right) \right)$$

$$\mathcal{U}_2 = \epsilon_{\mu\nu\alpha\beta} \epsilon^{\rho\sigma\alpha\beta} \mathcal{K}^\mu_\rho \mathcal{K}^\nu_\sigma$$

$$\mathcal{U}_3 = \epsilon_{\mu\nu\alpha\gamma} \epsilon^{\rho\sigma\beta\gamma} \mathcal{K}^\mu_\rho \mathcal{K}^\nu_\sigma \mathcal{K}^\alpha_\beta$$

$$\mathcal{U}_4 = \epsilon_{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \mathcal{K}^\mu_\alpha \mathcal{K}^\nu_\beta \mathcal{K}^\rho_\gamma \mathcal{K}^\sigma_\delta$$

In various dimensions: D=3 only two terms, in D=5 one more term, in D=6 two more terms, and so on

Hamiltonian construction: *Hassan, Rachel A. Rosen, '11,'12*

Another proof of 5 dof: *Mirbabayi, '12*

Minkowski background and massive graviton

$$g_{\mu\nu} = \eta_{\mu\nu}, \qquad \partial_\mu \phi^a = \delta^a_\mu$$

Symmetry breaking pattern

$$ISO(3,1)_{\mathrm{GCT}} \times ISO(3,1)_{\mathrm{INT}} \rightarrow ISO(3,1)_{\mathrm{DIAG}} \tag{1}$$

Linearized theory: 3 NG Bosons eaten up by the tensor field that becomes massive. The theory guarantees unitary 5 degrees of freedom on (nearly) Minkowski backgrounds.

Nonlinear interactions are such that there are 5 degrees of freedom on any background. However, there is no guarantee that some of these 5 degrees of freedom aren't bad on certain backgrounds, thus destabilizing those backgrounds.

Cosmological solutions: No flat FRW solution:

D'Amico, de Rham, Dubovsky, GG, Pirtskhalava, Tolley, '11

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad \phi^0(t) = f(t), \quad \phi^j(x) = x^j$$

Minisuperspace Lagrangian (for $\alpha_{3,4} = 0$):

$$L = 3M_{\rm pl}^2 \left(-a\dot{a}^2 + m^2(2a^3 - 3a^2 + a) - m^2\dot{f}(a^3 - a^2)\right)$$

$$\frac{d}{dt}(m^2(a^3 - a^2)) = 0$$

No cosmology if m is a constant.

Possible ways to proceed for the flat universe:

(1) Heterogeneous and/or anisotropic cosmologies

(2) Field dependent mass $m \to m(\sigma)$: FRW solutions re-emerge

Exception: Open FRW selfaccelerated universe:

Gumrukcuoglu, Lin, Mykohyama

Heterogeneous Solutions: Qualitative Picture

The Vainshtein radius for a domain of density ρ and size R

$$r_* = \left(\frac{\rho}{\rho_{co}}\right)^{1/3} R, \qquad \rho_{co} \equiv 3M_{\rm pl}^2 m^2$$

Within a patch of radius $1/m$, consider a typical Hubble volume, i.e., the volume enclosed by the sphere of radius

$$H^{-1} = \sqrt{\frac{3M_{\rm pl}^2}{\rho}}$$

This volume is in the Vainshtein regime, i.e., $r_* >> H^{-1}$, as long as

$$\rho >> \rho_{co}$$

Hence, should recover FRW with great accuracy for $\rho >> \rho_{co}$!

Heterogeneous solutions: Quantitative Picture

$$ds^2 = -dt^2 + C(t,r)dtdr + A(t,r)^2(dr^2 + r^2 d\Omega^2),$$
$$\phi^0 = f(t,r), \quad \phi^j(x) = g(t,r)\frac{x^j}{r}$$

Einstein's equation extended with the mass and potential terms:

$$G_{\mu\nu} = m^2 X_{\mu\nu} + 8\pi G_N T_{\mu\nu}$$

Early universe: in the first approximation neglect $m^2 X_{\mu\nu}$, get FRW.
In the obtained FRW background solve for ϕ^a's

$$m^2 \nabla^{\mu}_{g_{FRW}} X_{\mu\nu}(g_{FRW}, \phi^a) = 0$$

Find ϕ^a's, and calculate backreaction to make sure that $m^2 X_{\mu\nu} << 8\pi G_N T_{\mu\nu}$. This is the case for $\rho >> \rho_{co}$.
What about the case when $\rho \sim \rho_{co}$?

Selfacceleration and pseudo-homogeneous solutions: In the dec limit: *de Rham, GG, Heisenberg, Pirtskhalava*. Exact solution: *Koyama, Niz, Tasinato (1,2,3), M. Volkov; L. Berezhiani, et al; ...*

For instance, *Koyama-Niz-Tasinato* solution:

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2)$$

while, ϕ^0 and ϕ^ρ, are inhomogeneous functions of

$$\mathrm{arctanh}\left(\frac{\sinh(m\tau/2) + e^{m\tau/2}m^2\rho^2/8}{\cosh(m\tau/2) - e^{m\tau/2}m^2\rho^2/8}\right), \quad \rho e^{m\tau/2}$$

Selfacceleration with heterogeneous metric: *Gratia, Hu, Wyman 12*
Selfacceleration is a generic feature of this theory, however, vanishing of the kinetic terms for some of the 5 modes seems to be a common feature of these solutions.
Anisotropic solutions with stable fluctuations: Gumrukcuoglu, Lin, Mukohyama, '12.

Theory of Quasi-Dilaton: *D'Amico, GG, Hui, Pirtskhalava, '12*

Invariance of the action to rescaling of the reference frame coordinates ϕ^a w.r.t. the physical space coordinates, x^a, requires a field σ. In the Einstein frame:

$$\phi^a \to e^{\alpha}\ \phi^a, \qquad \sigma \to \sigma - \alpha M_{\rm Pl}$$

Hence we can construct the invariant action by replacing $\mathcal{K}$ by $\bar{K}$

$$\bar{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \sqrt{g^{\mu\alpha}\bar{f}_{\alpha\nu}} \qquad \bar{f}_{\alpha_\nu} = e^{2\sigma/M_{\rm Pl}}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

and adding the sigma kinetic term

$$\mathcal{L} = \mathcal{L}_{dRGT}\left(\mathcal{K} \to \bar{K}\right) - \omega\sqrt{g}(\partial\sigma)^2$$

and the term $\int d^4x\sqrt{-det\bar{f}}$ can also be added. In the Einstein frame σ does not couple to matter, but it does in the Jordan frame

Extended Quasi-Dilaton: *De Felice, Mukohyama, '13*

The quasidilaton Lagrangian is not the most general one consistent with the symmetries – derivative terms for σ, suppressed by a higher scale can also be included, retaining the symmetry

$$\phi^a \to e^{\alpha}\ \phi^a, \qquad \sigma \to \sigma - \alpha M_{\rm Pl}$$

Hence we can construct the invariant action by replacing $\mathcal{K}$ by $\tilde{K}$

$$\tilde{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \sqrt{g^{\mu\alpha}\tilde{f}_{\alpha\nu}} \qquad \tilde{f}_{\alpha\nu} = \bar{f}_{\alpha\nu} - \frac{\partial_\alpha\sigma\partial_\nu\sigma}{m^2 M_{\rm pl}^2}$$

and adding the sigma kinetic term

$$\mathcal{L} = \mathcal{L}_{dRGT}\left(\mathcal{K} \to \tilde{K}\right) - \omega\sqrt{g}(\partial\sigma)^2$$

and the term $\int d^4x\sqrt{-det\tilde{f}}$ can also be added. In the Einstein frame σ does not couple to matter, but it does in the Jordan frame

Cosmology of Quasi-Dilaton: Flat FRW Solutions, Extended Quasidilaton

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad \phi^0 = f(t), \quad \phi^i = x^i, \quad \sigma = \sigma(t)$$

Friedmann equation:

$$\begin{aligned} 3M_{\rm Pl}^2 H^2 &= \frac{\omega}{2}\dot{\sigma}^2 + \rho_m + \\ 3M_{\rm Pl}^2 m^2 & \left[c_0 + c_1 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right) + c_2 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^2 + c_3 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^3 \right] \end{aligned}$$

Constraint equation:

$$q_0 + q_1 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right) + q_2 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^2 + q_3 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^3 = \frac{k e^{-\sigma/M_{\rm Pl}}}{a^3}.$$

Particular Solutions for $k = 0$:

$$\left(\frac{e^{\sigma/M_{\rm Pl}}}{a}\right) = c, \quad \dot{\sigma} = M_{\rm Pl} H$$

Friedmann equation

$$(3 - \frac{\omega}{2}) M_{\rm Pl}^2 H^2 = \rho_m + 3 M_{\rm Pl}^2 m^2 \left[c_0 + c_1 c + c_2 c^2 + c_3 c^3\right]$$

Expansion like in ΛCDM. Constraint equation

$$q_0 + q_1 c + q_2 c^2 + q_3 c^3 = 0$$

Determine $f(t)$ from the sigma equation:

$$a\dot{f} = 1 + \frac{\omega}{3\kappa m^3}(3H^2 + \dot{H})$$

Small Perturbations:

The selfaccelerated solution in the Quasidilaton theory necessarily contains a ghost: De Felice, Gumrukcuoglu, Hinterbichler, Lin, Mukohyama, Trodden '13 & D'Amico, GG, Hui, Pirtskhalava '13

Extended Quasidilaton has a homogeneous and isotropic selfaccelerated solution with no ghosts! The first model of this kind De Felice, Mukohyama, '13; Mukohyama, '13; De Felice, Gumrukcuoglu, Mukohyama, '13.

Quasidilaton for $\omega > 6$ is similar to massive gravity (for $\omega \to \infty$ it reduces to massive gravity) and therefore has non-FRW solutions; their perturbations appear to be stable GG, Kimura, Pirtskhalava, in progress.

Open issues and outlook

I. Superluminality vs acausality

II. Strong coupling behavior

I. Superluminality: In the high energy limit, $E, p >> m$, the theory reduces to certain Galileons. Galileons in general are known to lead to superluminal phase and group velocities. For some parameter space there is no superuminality for massive gravity Galileons, at least for the spherically symmetric solutions due to specific nature of these theories:

$$-(\partial\pi)^2 + \frac{\pi\epsilon\epsilon\partial\partial\pi\partial\partial\pi}{m^2 M_{\rm pl}} + \frac{\pi\epsilon\epsilon\partial\partial\pi\partial\partial\pi\partial\partial\pi}{m^4 M_{\rm pl}^2}$$

(no cubic Galileon without the quartic one; special couplings to matter, superluminal solutions unstable, L. Berezhiani, G. Chkareuli, GG). However, in most of the extensions of massive gravity (quasidilaton, extended quasidilaton, bigravity), and for a generic parameter choice one finds superluminal phase and group velocities. Does this mean that these theories are acausal?

1. Chronology protection due to strong coupling Burrage, de Rham, L. Heisenberg, Tolley, '11.
2. (A)causality is determined by the front velocity, which is affected by the strong coupling regime. Work in progress

A well-known example of GR + QED: Drummond Hathrell, '80

$$\mathcal{L}_{\rm GR+QED} = M_{\rm pl}^2 eR + e\left(-\frac{1}{4}FF + \bar{\psi}(i\hat{D} - m_e)\psi\right)$$

A good effective theory below $M_{\rm pl}$ (other charged particles included in the standard way).

At energies below the electron mass $E, p << m_e$, via one loop vacuum polarization diagram one gets an effective theory

$$\mathcal{L}_{\rm eff} = M_{\rm pl}^2 e(R + c\frac{\alpha_{\rm em}}{m_e^2}RFF) - e\frac{1}{4}FF\cdots$$

Among the *RFF* terms is *RiemannFF* term that renormalizes the photon kinetic term in an external gravitational field (e.g., of the Earth), and gives superluminal phase and group velocities.
However, this does not mean that $\mathcal{L}_{\rm GR+QED}$ gives a acausal theory, in fact it gives a good causal effective theory below $M_{\rm pl}$.
Reconciliation – extensive discussions by Hollowood and Shore

II. Toward UV completion via vierbein formulation:

The square root structure in the massive gravity Lagrangian is indicative of a low energy theory. The vierbein formalism for massive gravity can get rid of that structure Hinterbichler and Rachel A. Rosen '12. Moreover, one can formulate a fully GCT and LLT invariant action for massive gravity in the vierbein formalism GG, Hinterbichler, Pirtskhalava, Yanwen Shang, '13

$$\mathcal{L}_2 \sim M_{\rm pl}^2 m^2\ \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd}\ e_\mu{}^a e_\nu{}^b k_\alpha{}^c k_\beta{}^d\ , \tag{2}$$

$$\mathcal{L}_3 \sim \alpha_3 M_{\rm pl}^2 m^2\ \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd}\ e_\mu{}^a k_\nu{}^b k_\alpha{}^c k_\beta{}^d\ , \tag{3}$$

$$\mathcal{L}_4 \sim \alpha_4 M_{\rm pl}^2 m^2\ \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd}\ k_\mu{}^a k_\nu{}^b k_\alpha{}^c k_\beta{}^d\ .$$

where $k_\mu{}^a \equiv e_\mu{}^a - \lambda^a{}_{\bar a}\partial_\mu\phi^{\bar a}$, and $\lambda^a_{\bar a}$ belongs to $SO(3,1)$.
The mass terms can be promoted to the locally $SL(4)$ symmetric structures by promoting λ's to $SL(4)$! Hence the mass terms can have a larger local symmetry group than the EH term does.

Conclusions:

- A classical theory that extends GR by the mass and potential term is available now; many questions of astrophysics and cosmology can be studies and comparisons can be made with GR as well as with data
- Generic cosmological solutions have no FRW symmetries, but can approximate well FRW cosmologies in the early universe
- Selfaccelerated solutions emerge as a generic feature; but some fluctuations loose kinetic terms for pseudo-homogeneous solutions.
- Dynamical mass theories differ – FRW solutions re-emerge. An example: Extended Quasi-dilaton with selfacceleration exhibits nonvanishing kinetic terms for all perturbations
- The GCT and LLT invariant first order formulations is a useful staring point for addressing the issue of the UV completion, and hence of superluminality vs. (a)causality.
- Other extensions: bi-gravity Hassan, R.A. Rosen '11, multigravity with very interesting structure Hinterbichler, R.A. Rosen, '12; mass-varying massive gravity Q-G Huang, Y-S Piao, S-Y Zhou '12