

String  
Cosmology

R. Branden-  
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Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

# String Gas Cosmology: Progress and Challenges

Robert Brandenberger  
McGill University

PASCOS 2013, November 22, 2013

# Outline

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- 1 Introduction
- 2 String Gas Cosmology
- 3 String Gas Cosmology and Structure Formation
- 4 Moduli Stabilization and Supersymmetry Breaking in SGC
- 5 Challenges for String Gas Cosmology
- 6 Conclusions

# Plan

String  
Cosmology

R. Branden-  
berger

**Introduction**

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- 1 **Introduction**
- 2 String Gas Cosmology
- 3 String Gas Cosmology and Structure Formation
- 4 Moduli Stabilization and Supersymmetry Breaking in SGC
- 5 Challenges for String Gas Cosmology
- 6 Conclusions

String  
Cosmology

R. Branden-  
berger

## Introduction

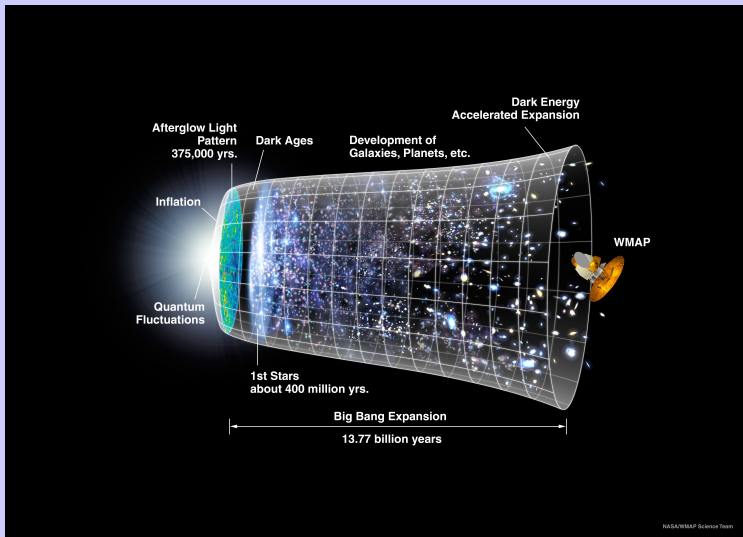
String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions



# Goals of Early Universe Cosmology

String  
Cosmology

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berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- Understand the **origin** and **very early evolution** of the universe.
- **Test** theory of the very early universe against **cosmological observations**.
- Make **predictions** for future observations.

# Current Paradigm for Early Universe Cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

The **Inflationary Universe Scenario** is the current paradigm of early universe cosmology.

Successes:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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Cosmology

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berger

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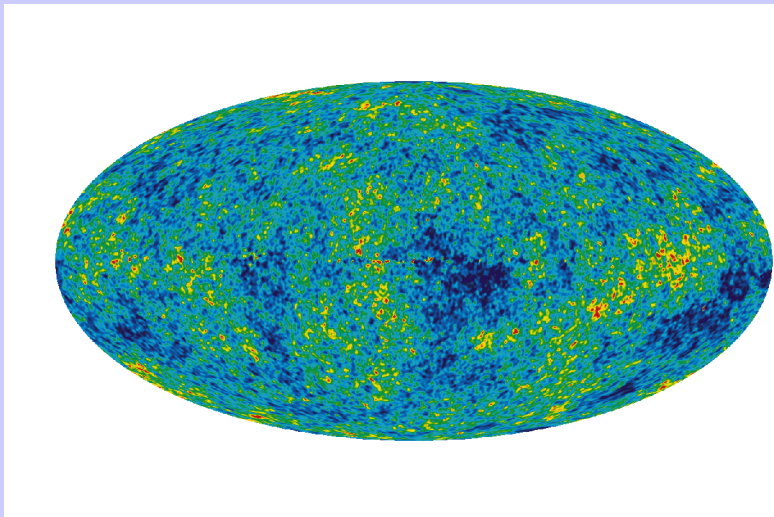
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Cosmology

Structure  
Formation

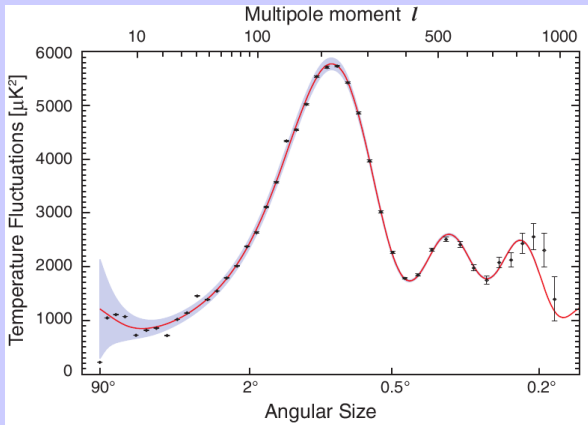
Moduli  
Stabilization

Challenges

Conclusions



Credit: NASA/WMAP Science Team



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# Deficiencies of the Inflationary Paradigm

String  
Cosmology

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berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- Does not eliminate **cosmological singularity**.
- → not a theory of the very early universe.
- Uses low energy field theory framework in a realm where this theory breaks down.
- **Trans-Planckian problem** for cosmological fluctuations.
- → analysis of cosmological fluctuations is based on incomplete physics.
- Not robust against our ignorance of what solves the cosmological constant problem.

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- **String Gas Cosmology** Toy model of the very early universe based on fundamental principles and symmetries of superstring theory (R.B. and C. Vafa, 1989).
- String Gas Cosmology yields a **structure formation mechanism** alternative to inflation (A. Nayeri, R.B. and C. Vafa, 2005).
- Problems of superstring cosmology such as the **moduli stabilization problem** have new solutions (S. Patil and R.B., 2004, R.B. et al 2012).
- But: significant **challenges** before we will have a **theory** of superstring cosmology.

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- **Superstring theory** give a completely different picture of the evolution of the very early universe than **particle theory**.
- **String Gas Cosmology** yields new theory of structure formation.
  - - consistent with current observations
  - - yielding predictions for future observations different from inflation
  - Corollary: alternatives to inflationary cosmology exist.

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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# Review of Inflationary Cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

## Context:

- General Relativity
- Scalar Field Matter

$$\text{Metric : } ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \quad (1)$$

## Inflation:

- phase with  $a(t) \sim e^{tH}$
- requires matter with  $p \sim -\rho$
- requires a slowly rolling scalar field  $\varphi$
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long



# Review of Inflationary Cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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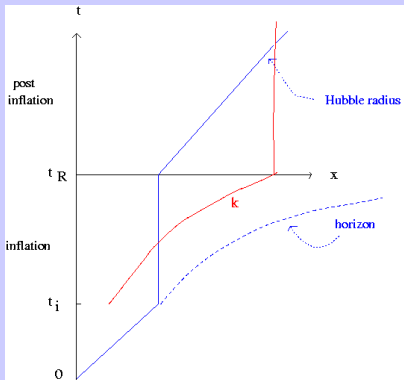
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# Review of Inflationary Cosmology II

## Space-time sketch of inflationary cosmology:



Note:

- $H = \frac{\dot{a}}{a}$
- curve labelled by  $k$ : wavelength of a fluctuation

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- inflation renders the universe large, homogeneous and spatially flat
- classical matter redshifts  $\rightarrow$  matter vacuum remains
- **quantum vacuum fluctuations: seeds for the observed structure** [Chibisov & Mukhanov, 1981]
- sub-Hubble  $\rightarrow$  locally causal

# Singularity Problem

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Cosmology

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berger

## Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- Standard cosmology: Penrose-Hawking theorems  $\rightarrow$  initial singularity  $\rightarrow$  incompleteness of the theory.
- Inflationary cosmology: In scalar field-driven inflationary models the initial singularity persists [Borde and Vilenkin]  $\rightarrow$  **incompleteness of the theory.**

# Trans-Planckian Problem

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Cosmology

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## Introduction

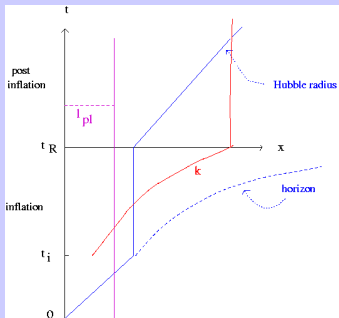
String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions



- **Success of inflation:** At early times scales are inside the Hubble radius  $\rightarrow$  causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than  $70H^{-1}$ , then  $\lambda_p(t) < l_{pl}$  at the beginning of inflation
- $\rightarrow$  new physics **MUST** enter into the calculation of the fluctuations.

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String  
Cosmology  
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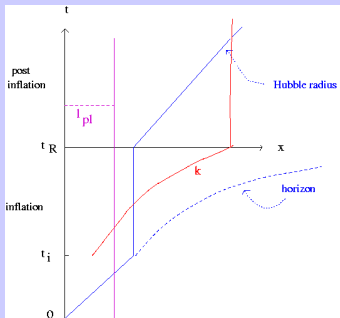
String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions



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# Applicability of GR

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically  $\eta \sim 10^{16} \text{GeV}$ .
- $\rightarrow \eta$  too close to  $m_{pl}$  to trust predictions made using GR.

# Zones of Ignorance

String  
Cosmology

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Introduction

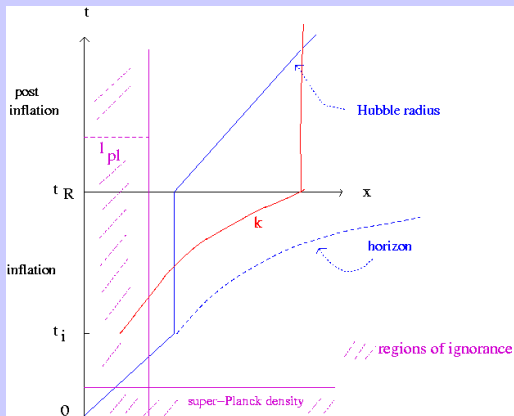
String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions





# Cosmological Constant Problem

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Cosmology

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berger

## Introduction

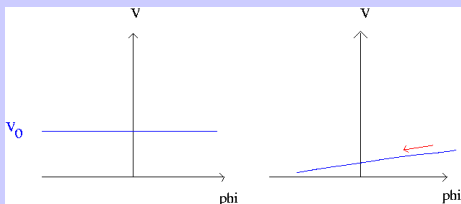
String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions



- Quantum vacuum energy does not gravitate.
- **Why should the almost constant  $V(\varphi)$  gravitate?**

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120} \quad (2)$$

# Plan

String  
Cosmology

R. Branden-  
berger

Introduction

**String Gas  
Cosmology**

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- 1 Introduction
- 2 String Gas Cosmology**
- 3 String Gas Cosmology and Structure Formation
- 4 Moduli Stabilization and Supersymmetry Breaking in SGC
- 5 Challenges for String Gas Cosmology
- 6 Conclusions

# Principles

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the **new symmetries** and **new degrees of freedom** which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- **New degrees of freedom:** string oscillatory modes
- Leads to a **maximal temperature** for a gas of strings, the Hagedorn temperature
- **New degrees of freedom:** string winding modes
- Leads to a **new symmetry:** physics at large  $R$  is equivalent to physics at small  $R$

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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# T-Duality

String  
Cosmology

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Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

## T-Duality

- Momentum modes:  $E_n = n/R$
- Winding modes:  $E_m = mR$
- Duality:  $R \rightarrow 1/R$   $(n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level  $\rightarrow$  existence of D-branes

# Adiabatic Considerations

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

String  
Cosmology

R. Branden-  
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Introduction

String Gas  
Cosmology

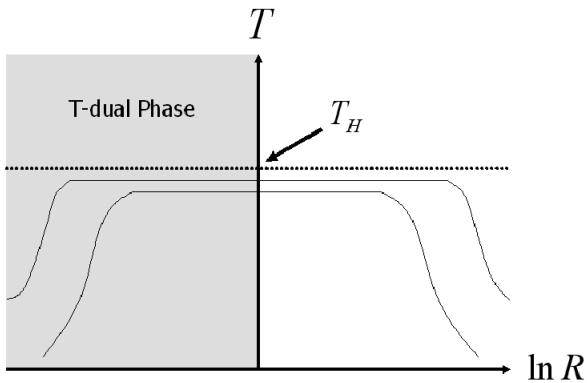
Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

## Temperature-size relation in string gas cosmology



# Singularity Problem in Standard and Inflationary Cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

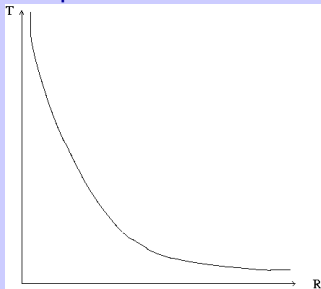
Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

## Temperature-size relation in standard cosmology





# Dynamics

String  
Cosmology

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berger

Introduction

String Gas  
Cosmology

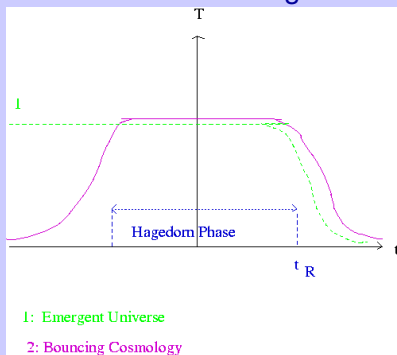
Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

Assume some action gives us  $R(t)$



# Dynamics II

String  
Cosmology

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Introduction

String Gas  
Cosmology

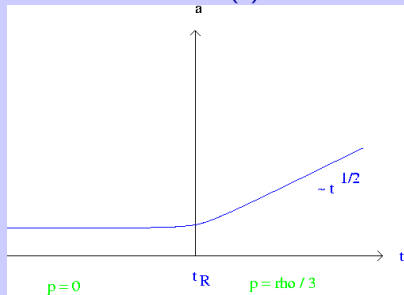
Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

We will thus consider the following background dynamics for the scale factor  $a(t)$ :



# Dimensionality of Space in SGC

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

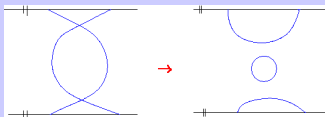
Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- Begin with all 9 spatial dimensions small, initial temperature close to  $T_H$   $\rightarrow$  winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



- Decay only possible in three large spatial dimensions.
- $\rightarrow$  dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

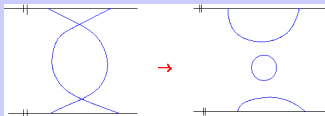
Structure  
Formation

Moduli  
Stabilization

Challenges

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Cosmology

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berger

Introduction

String Gas  
Cosmology

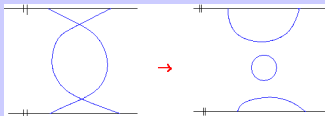
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Formation

Moduli  
Stabilization

Challenges

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions

- 1 Introduction
- 2 String Gas Cosmology
- 3 String Gas Cosmology and Structure Formation**
- 4 Moduli Stabilization and Supersymmetry Breaking in SGC
- 5 Challenges for String Gas Cosmology
- 6 Conclusions

# Theory of Cosmological Perturbations: Basics

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of **matter** → large-scale structure
- Fluctuations of **metric** → CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- Fluctuations are small today on large scales
- → fluctuations were very small in the early universe
- → can use **linear perturbation theory**

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

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# Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep.* 215:203 (1992)

## Step 1: Metric including fluctuations

$$ds^2 = a^2[(1 + 2\Phi)d\eta^2 - (1 - 2\Phi)d\mathbf{x}^2] \quad (3)$$

$$\varphi = \varphi_0 + \delta\varphi \quad (4)$$

Note:  $\Phi$  and  $\delta\varphi$  related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

$$S^{(2)} = \frac{1}{2} \int d^4x ((v')^2 - v_{,i}v^{,i} + \frac{z''}{z}v^2) \quad (5)$$

$$v = a(\delta\varphi + \frac{z}{a}\Phi) \quad (6)$$

$$z = a\frac{\varphi_0'}{\mathcal{H}} \quad (7)$$

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

# Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep.* 215:203 (1992)

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$$v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0 \quad (8)$$

Features:

- **oscillations** on sub-Hubble scales
- **squeezing** on super-Hubble scales  $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1} \quad (9)$$

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# Structure formation in inflationary cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

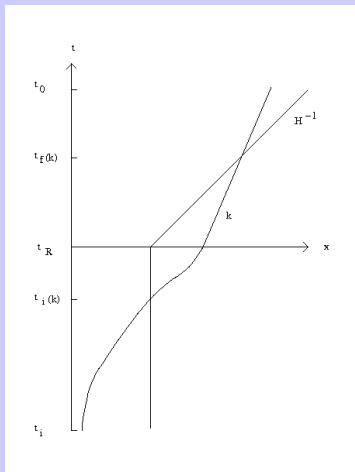
String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions



**N.B.** Perturbations originate as quantum vacuum fluctuations.

# Background for string gas cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

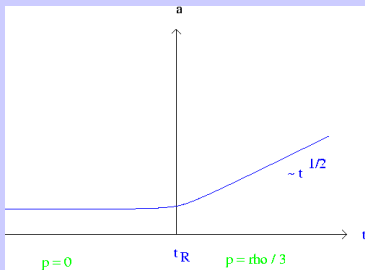
String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions



# Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett.* 97:021302 (2006)

String  
Cosmology

R. Branden-  
berger

Introduction

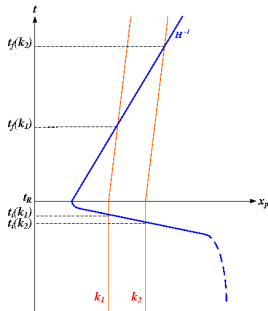
String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions



N.B. Perturbations originate as thermal string gas fluctuations.



# Method

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed  $k$ , convert the matter fluctuations to metric fluctuations at Hubble radius crossing  $t = t_i(k)$
- Evolve the metric fluctuations for  $t > t_i(k)$  using the usual theory of cosmological perturbations

# Extracting the Metric Fluctuations

String  
Cosmology

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Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta) \left( (1 + 2\Phi) d\eta^2 - [(1 - 2\Phi)\delta_{ij} + h_{ij}] dx^i dx^j \right). \quad (10)$$

Inserting into the perturbed Einstein equations yields

$$\langle |\Phi(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0_0(k) \delta T^0_0(k) \rangle, \quad (11)$$

$$\langle |h(k)|^2 \rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i_j(k) \delta T^i_j(k) \rangle. \quad (12)$$

# Power Spectrum of Cosmological Perturbations

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions

Key ingredient: For **thermal fluctuations**:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V. \quad (13)$$

Key ingredient: For **string thermodynamics** in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)}. \quad (14)$$

# Power Spectrum of Cosmological Perturbations

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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## Power spectrum of cosmological fluctuations

$$P_{\Phi}(k) = 8G^2 k^{-1} \langle |\delta\rho(k)|^2 \rangle \quad (15)$$

$$= 8G^2 k^2 \langle (\delta M)^2 \rangle_R \quad (16)$$

$$= 8G^2 k^{-4} \langle (\delta\rho)^2 \rangle_R \quad (17)$$

$$= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \quad (18)$$

Key features:

- **scale-invariant** like for inflation
- **slight red tilt** like for inflation

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# Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett.* (2007)

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

$$P_h(k) = 16\pi^2 G^2 k^{-1} \langle |T_{ij}(k)|^2 \rangle \quad (19)$$

$$= 16\pi^2 G^2 k^{-4} \langle |T_{ij}(R)|^2 \rangle \quad (20)$$

$$\sim 16\pi^2 G^2 \frac{T}{\ell_s^3} (1 - T/T_H) \quad (21)$$

Key ingredient for **string thermodynamics**

$$\langle |T_{ij}(R)|^2 \rangle \sim \frac{T}{\ell_s^3 R^4} (1 - T/T_H) \quad (22)$$

Key features:

- scale-invariant (like for inflation)
- slight blue tilt (unlike for inflation)

# Spectrum of Gravitational Waves

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String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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# Requirements

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions

- 1. static Hagedorn phase (including static dilaton) → new physics required.
- 2.  $C_V(R) \sim R^2$  obtained from a thermal gas of strings provided there are winding modes which dominate.
- 3. Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

# Requirements

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

**Structure  
Formation**

Moduli  
Stabilization

Challenges

Conclusions

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# Plan

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

**Moduli  
Stabilization**

Challenges

Conclusions

- 1 Introduction
- 2 String Gas Cosmology
- 3 String Gas Cosmology and Structure Formation
- 4 Moduli Stabilization and Supersymmetry Breaking in SGC**
- 5 Challenges for String Gas Cosmology
- 6 Conclusions

# Moduli Stabilization in SGC

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

## Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow V_{\text{eff}}(R)$  has a minimum at a finite value of  $R$ ,  $\rightarrow R_{\text{min}}$
- in heterotic string theory there are **enhanced symmetry states** containing both momentum and winding which are massless at  $R_{\text{min}}$
- $\rightarrow V_{\text{eff}}(R_{\text{min}}) = 0$
- $\rightarrow$  **size moduli stabilized** in Einstein gravity background

# Moduli Stabilization in SGC

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

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# Moduli Stabilization in SGC II

String  
Cosmology

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berger

Introduction

String Gas  
Cosmology

Structure  
Formation

**Moduli  
Stabilization**

Challenges

Conclusions

## Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

- enhanced symmetry states
- → harmonic oscillator potential for  $\theta$
- → **shape moduli stabilized**

# Dilaton stabilization in SGC

R. Danos, A. Frey and R.B., 2008

String  
Cosmology

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Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- The only remaining modulus is the dilaton
- Make use of **gaugino condensation** to give the dilaton a potential with a unique minimum
- → dilaton is stabilized
- Context Perturbative  $E_8 \times E_8$  superstring theory.
- Hidden sector gauge group becomes strongly coupled at a scale  $\mu$ .
- At this scale gaugino condensation sets in.
- NB: Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008]

# Supersymmetry Breaking in SGC

S. Mishra, W. Xue, R.B. and U. Yajnik, 2012

String  
Cosmology

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Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- Gaugino condensation scale  $\mu$ .
- Gravitino mass  $m_{3/2} \sim \frac{\mu^3}{M_4^2}$
- Supersymmetry breaking scale given by  $M_S^2 \sim \frac{\mu^3}{M_4}$
- TeV scale gravitino mass implies high scale supersymmetry breaking.
- NB: consistent with moduli stabiliation.



# Plan

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

**Challenges**

Conclusions

- 1 Introduction
- 2 String Gas Cosmology
- 3 String Gas Cosmology and Structure Formation
- 4 Moduli Stabilization and Supersymmetry Breaking in SGC
- 5 Challenges for String Gas Cosmology**
- 6 Conclusions

# Challenges for String Gas Cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

**Challenges**

Conclusions

- **Size** and **Entropy** problems not solved.
- **Dynamics** of Hagedorn phase not known.
- - Einstein gravity and dilaton gravity do not apply in this regime.

# Challenges for String Gas Cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

**Challenges**

Conclusions

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# Challenges for String Gas Cosmology

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

**Challenges**

Conclusions

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# Possible Approaches

String  
Cosmology

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berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

**Challenges**

Conclusions

- **Double Field Theory** as a way of obtaining dynamics of the Hagedorn phase?
- **Emergent geometry** as in matrix theory to replace the Hagedorn phase?
- Bottom line: we need a better understanding of non-perturbative string theory.

# Plan

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

**Conclusions**

- 1 Introduction
- 2 String Gas Cosmology
- 3 String Gas Cosmology and Structure Formation
- 4 Moduli Stabilization and Supersymmetry Breaking in SGC
- 5 Challenges for String Gas Cosmology
- 6 **Conclusions**

# Conclusions

String  
Cosmology

R. Branden-  
berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

Conclusions

- **String Gas Cosmology**: Model of cosmology of the very early universe based on new degrees of freedom and new symmetries of superstring theory.
- String Theory leads to a very different picture for the very early universe than point particle theories.
- SGC → **nonsingular cosmology**
- SGC → natural explanation of the number of large spatial dimensions.

# Conclusions II

String  
Cosmology

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Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

**Conclusions**

- SGC → **new scenario of structure formation**
- Scale invariant spectrum of cosmological fluctuations (like in inflationary cosmology).
- **Spectrum of gravitational waves has a small blue tilt** (unlike in inflationary cosmology).
- **String Theory testable** through cosmological observations.



# Conclusions II

String  
Cosmology

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berger

Introduction

String Gas  
Cosmology

Structure  
Formation

Moduli  
Stabilization

Challenges

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