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SUSY spectra of magnetized brane models

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To appear with Hiroyuki Abe and Junichiro Kawamura

Introduction

- Higher dimensional U(N) SYM theories

  - EFTs of D-branes

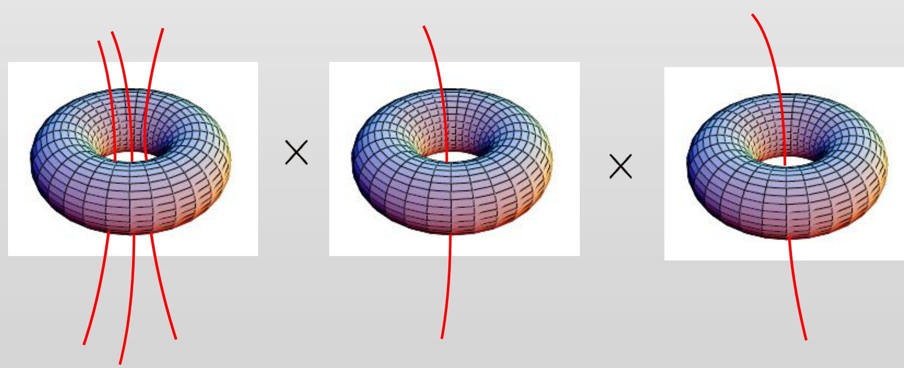
- (The MSSM-like) Model buildings

  - 4D chiral spectra

  - N=1 SUSY (in terms of 4D supercharges)

- Simple toroidal compactifications with magnetic fluxes

$$\begin{array}{c}\text{10D}\\ \text{spacetime}\end{array} = \begin{array}{c}\text{4D}\\ \text{spacetime}\end{array} \times$$



10D U(N) SYM

$$S = \int dx^{10} \sqrt{-G} \left[-\frac{1}{4g^2} \mathrm{tr}\left(F^{MN} F_{MN} \right) + \frac{i}{2g^2} \mathrm{tr}\left(\bar{\lambda} \Gamma^M D_M \lambda \right) \right]$$

- 4D N=1 decomposition $i \ : \ 1, 2, 3$

10D vector field : $A_M = (A_\mu, \ A_i)$

$$A_i \equiv -\frac{1}{\mathrm{Im}\tau_i} (\bar{\tau}_i A_{2+2i} - A_{3+2i})$$

$\rightarrow$ 4D vector field + three complex fields

10D Majorana-Weyl spinor field:

$$\lambda = (\lambda_0, \ \lambda_i) \qquad \begin{array}{ll} \lambda_0 = \lambda_{+++} & \lambda_1 = \lambda_{+--} \\ \lambda_2 = \lambda_{-+-} & \lambda_3 = \lambda_{--+} \end{array}$$

$\rightarrow$ four 4D Wely spinor fields

Single 4D N=1 vector supermultiplet $V = \{A_\mu, \ \lambda_0\}$

Triple 4D N=1 chiral supermultiplets $\phi_i = \{A_i, \ \lambda_i\}$ $i \ : \ 1, 2, 3$

Superfield description on three tori

$$S = \int dx^{10} \sqrt{-G} \left[-\frac{1}{4g^2} \mathrm{tr}\left(F^{MN} F_{MN} \right) + \frac{i}{2g^2} \mathrm{tr}\left(\bar{\lambda} \Gamma^M D_M \lambda \right) \right]$$

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$$= \int d^{10}X\sqrt{-G}\left[\int d^4\theta\,\mathcal{K} + \left\{\int d^2\theta\left(\frac{1}{4g^2}\mathcal{W}^\alpha\mathcal{W}_\alpha + \mathcal{W}\right) + \mathrm{h.c.}\right\}\right]$$

$$\mathcal{K} = \frac{2}{g^2}h^{\bar{i}j}\mathrm{Tr}\left[\left(\sqrt{2}\bar{\partial}_{\bar{i}} + \bar{\phi}_{\bar{i}}\right)e^{-V}\left(-\sqrt{2}\partial_j + \phi_j\right)e^V + \bar{\partial}_{\bar{i}}e^{-V}\partial_j e^V\right] + \mathcal{K}_{\mathrm{WZW}}$$

$$\mathcal{W} = \frac{1}{g^2}\epsilon^{\mathrm{ijk}}e_{\mathrm{i}}{}^i e_{\mathrm{j}}{}^j e_{\mathrm{k}}{}^k\mathrm{Tr}\left[\sqrt{2}\,\phi_i\left(\partial_j\phi_k - \frac{1}{3\sqrt{2}}\left[\phi_j,\phi_k\right]\right)\right]$$

$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$$

$$V \equiv -\theta\sigma^\mu\bar{\theta}A_\mu + i\bar{\theta}\bar{\theta}\theta\lambda_0 - i\theta\theta\bar{\theta}\bar{\lambda}_0 + \tfrac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D$$

$$\phi_i \equiv \tfrac{1}{\sqrt{2}}A_i + \sqrt{2}\theta\lambda_i + \theta\theta F_i$$

Superfield description on three tori

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$$\mathcal{W}_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$$

$$h_{i\bar{j}} = 2\left(2\pi R_i\right)^2\delta_{i\bar{j}} \qquad \partial_i = \frac{\partial}{\partial z^i} \quad z_i \equiv \frac{1}{2}\left(x^{2+2i} + \tau_i x^{3+2i}\right)$$

Non-trivial magnetized background

$$\phi_i \to \langle A_i \rangle + \phi_i \qquad \langle A_i \rangle = \frac{\pi}{\mathrm{Im}\tau_i} M^{(i)} \left(\bar{z}_i + \bar{\zeta}_i \right)$$

$$M^{(i)} = \begin{pmatrix} M_C^{(i)} \mathbf{1}_4 & & \\ & M_L^{(i)} \mathbf{1}_2 & \\ & & M_R^{(i)} \mathbf{1}_2 \end{pmatrix} \quad \zeta^{(i)} = \left(\begin{array}{cc|c|cc} \zeta_C^{(i)} \mathbf{1}_3 & & & & \\ & \zeta_C'^{(i)} & & & \\ \hline & & \zeta_L^{(i)} \mathbf{1}_2 & & \\ \hline & & & \zeta_{R'}^{(i)} & \\ & & & & \zeta_{R''}^{(i)} \end{array} \right)$$

Gauge symmetry breaking : U(8) → U(3) × U(2) × (U(1)) ^{3}

- Dirac's quantization condition

(All the elements of) $M^{(i)} \in \boldsymbol{Z}$

- SUSY condition (to preserve N=1 SUSY)

$$\frac{1}{\mathcal{A}_1} M^{(1)} + \frac{1}{\mathcal{A}_2} M^{(2)} + \frac{1}{\mathcal{A}_3} M^{(3)} = 0$$

Torus area: $\mathcal{A}_i = (2\pi R_i)^2 \mathrm{Im}\tau_i$

Zero-mode equations

$$\bar{\partial}_{\bar{i}}\phi_j + \frac{1}{2}\left[\langle \bar{A}_{\bar{i}}\rangle, \phi_j\right] = 0 \quad \text{for} \quad i = j$$

$$\partial_i\phi_j - \frac{1}{2}\left[\langle A_i\rangle, \phi_j\right] = 0 \quad \text{for} \quad i \neq j$$

The signs of fluxes match with the chirality on the tori

- Well-defined wavefunctions (D. Cremades, L. E. Ibáñez and F. Marchesano)

$$\phi_j^{ab,I} = \mathcal{N} \cdot e^{i\pi M_{ab}^{(i)} z_i \mathrm{Im}\, z_i / \mathrm{Im}\, \tau_i} \cdot \vartheta \begin{bmatrix} I/M_{ab}^{(i)} \\ 0 \end{bmatrix} (M_{ab}^{(i)} z_i, M_{ab}^{(i)} \tau_i) \qquad \vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{l \in \mathbb{Z}} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu + b)}.$$

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$$I : 0,\ 1,\ \cdots,\ M_{ab}^{(i)} - 1$$

- Degenerate zero modes

Magnetic fluxes defined # of the degeneracy

⬇

of the generation

Zero-mode equations

$$\bar{\partial}_{\bar{i}}\phi_j + \frac{1}{2}\left[\langle \bar{A}_{\bar{i}}\rangle, \phi_j\right] = 0 \quad \text{for} \quad i = j$$
$$\partial_i\phi_j - \frac{1}{2}\left[\langle A_i\rangle, \phi_j\right] = 0 \quad \text{for} \quad i \neq j$$

- Opposite charged fields don't appear

  - 4D chiral spectra

Yukawa coupling

Yukawa couplings in 4D EFT

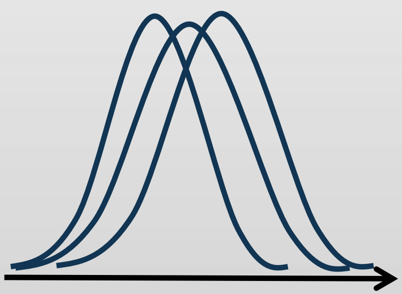
= overlap integral of three (extra dimensional) wavefunctions

$$y_{IJK} = \int \phi_1^{ab,I}(\boldsymbol{z}_i)\phi_2^{bc,J}(\boldsymbol{z}_i)\phi_3^{ca,K}(\boldsymbol{z}_i)d\boldsymbol{z}_i d\bar{\boldsymbol{z}}_i$$

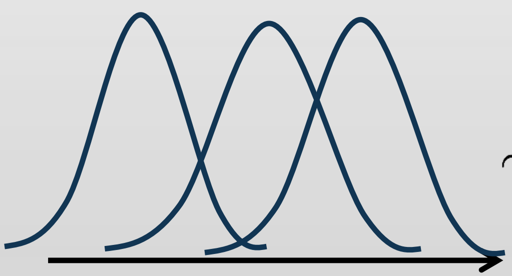
- Zero mode wavefunctions

  - gaussian profiles by magnetic fluxes

  - peak positions are shifted by Wilson lines



$\sim \mathcal{O}(1)$



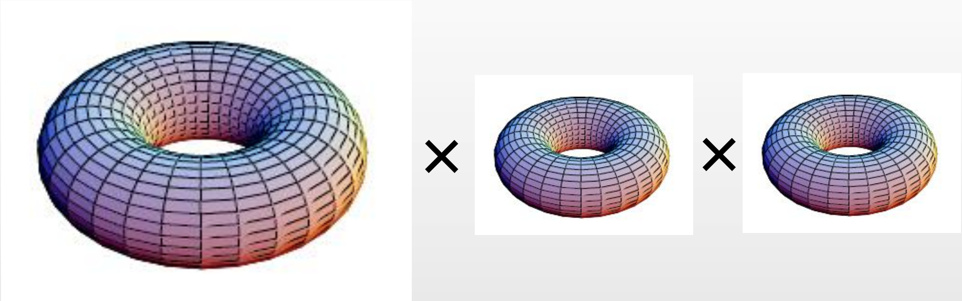
$\sim \mathcal{O}(10^{-5})$

A specific flux configuration

$$\frac{\pi}{\mathcal{A}_1}\begin{pmatrix}0&0&0\\0&3&0\\0&0&-3\end{pmatrix}+\frac{\pi}{\mathcal{A}_2}\begin{pmatrix}0&0&0\\0&-1&0\\0&0&0\end{pmatrix}+\frac{\pi}{\mathcal{A}_3}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}=0 \quad = \quad \frac{1}{\mathcal{A}_1}M^{(1)}+\frac{1}{\mathcal{A}_2}M^{(2)}+\frac{1}{\mathcal{A}_3}M^{(3)}=0$$

1st T^2 2nd T^2 3rd T^2

requires $\mathcal{A}_1/\mathcal{A}_2=\mathcal{A}_1/\mathcal{A}_3=3$



- This gives three generation quark and lepton supermultiplets and six generation higgs supermultiplets
- Wilson-lines on the 1st torus break the Pati-Salam group

Masses and mixings

At the EW scale through the 1-loop RGE

	Sample values	Observed
(m_u, m_c, m_t)	$(2.4 \times 10^{-3}, 0.63, 1.7 \times 10^2)$	$(2.3 \times 10^{-3}, 1.28, 1.74 \times 10^2)$
(m_d, m_s, m_b)	$(4.5 \times 10^{-3}, 2.1 \times 10^{-1}, 10)$	$(4.8 \times 10^{-3}, 0.95 \times 10^{-1}, 4.18)$
(m_e, m_μ, m_τ)	$(8.4 \times 10^{-4}, 7.0 \times 10^{-2}, 5.1)$	$(5.11 \times 10^{-4}, 1.06 \times 10^{-1}, 1.78)$
$\lvert V_{\rm CKM}\rvert$	$\begin{pmatrix} 0.98 & 0.20 & 0.0029 \\ 0.20 & 0.98 & 0.070 \\ 0.017 & 0.068 & 1.0 \end{pmatrix}$	$\begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ 0.23 & 0.97 & 0.041 \\ 0.0087 & 0.040 & 1.0 \end{pmatrix}$

	Sample values	Observed
$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$	$(1.2 \times 10^{-19}, 1.3 \times 10^{-11}, 8.3 \times 10^{-11})$	$< \ 2 \times 10^{-9}$
$\lvert m_{\nu_1}^2 - m_{\nu_2}^2\rvert$	1.7×10^{-22}	7.50×10^{-23}
$\lvert m_{\nu_1}^2 - m_{\nu_3}^2\rvert$	7.0×10^{-21}	2.32×10^{-21}
$\lvert V_{\rm PMNS}\rvert$	$\begin{pmatrix} 0.70 & 0.63 & 0.33 \\ 0.71 & 0.56 & 0.43 \\ 0.091 & 0.54 & 0.84 \end{pmatrix}$	$\begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.51 & 0.58 & 0.64 \\ 0.26 & 0.61 & 0.75 \end{pmatrix}$

$$M^{(N)} = \begin{pmatrix} 1.5 & 2.6 & 0 \\ 2.6 & 0.4 & 6.2 \\ 0 & 6.2 & 4.2 \end{pmatrix} \times 10^{11}\text{GeV}$$

Moduli dependence of 4D effective action

The 4D effective action contains 7 parameters $\{g, \ R_r, \ \tau_r\}$ $\quad r = 1, 2, 3$

Gauge coupling constant	g	⟺	S	: Dilaton
Torus sizes	R_r		T_r	: Kähler moduli
Torus shapes	τ_r		U_r	: Complex structure moduli

$$\langle S \rangle = s + \theta\theta F^S \qquad \langle T_r \rangle = t_r + \theta\theta F^{T_r} \qquad \langle U_r \rangle = u_r + \theta\theta F^{U_r}$$

$$\mathrm{Re}\ s = \frac{1}{g^2}\mathcal{A}_1\mathcal{A}_2\mathcal{A}_3 \qquad \mathrm{Re}\ t_r = \frac{1}{g^2}\mathcal{A}_r \qquad u_r = i\bar{\tau}_r$$

Define the moduli dependence of the 4D effective action

$F^S, \ F^{T_r}, \ F^{U_r}$: Magnitude of ~~SUSY~~ mediated by each modulus

Soft ~~SUSY~~ parameters

- Multi moduli mediation : $\{S,\ T_r,\ U_r\}$
- Anomaly mediation

$$A_{ijk} = -F^m \partial_m \ln\left(\frac{\lambda_{ijk}}{Y_i Y_j Y_k}\right) - \frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2}\frac{F^C}{C_0}$$

$$m_i^2 = -F^m F^{\bar{n}*} \partial_m \partial_{\bar{n}} \ln Y_i - \frac{1}{32\pi^2}\frac{d\gamma_i}{d\ln\mu}\left|\frac{F^C}{C_0}\right|^2 + \frac{1}{16\pi^2}\left(\partial_m \gamma_i F^m (\frac{F^C}{C_0})^* + h.c.\right)$$

$$M_a = F^m \partial_m \ln(\mathrm{Re}(f_a)) + \frac{b_a g_a^2}{8\pi^2}\frac{F^C}{C_0}$$

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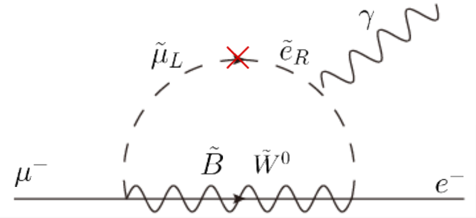
$$M_0 \equiv \sqrt{K_{S\bar{S}}}F^S$$

Ratios of other F-terms

$$R^{T_r} = \frac{\sqrt{K_{T_r\bar{T}_r}}F^{T_r}}{M_{\mathrm{SB}}} \qquad R^{U_r} = \frac{\sqrt{K_{U_r\bar{U}_r}}F^{U_r}}{M_{\mathrm{SB}}} \qquad R^C = \frac{1}{4\pi^2}\frac{F^C/C_0}{M_{\mathrm{SB}}}$$

SUSY flavor violations

- The SUSY flavor structure depends on (almost) only R^{U_1}
- The most stringent bound from $\mu \rightarrow e\gamma$ on $(\delta^E_{LR})_{12,21}$



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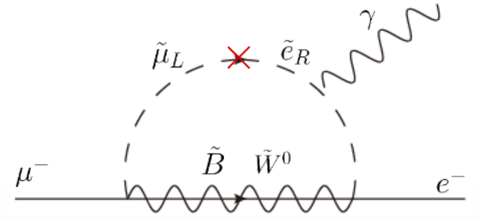
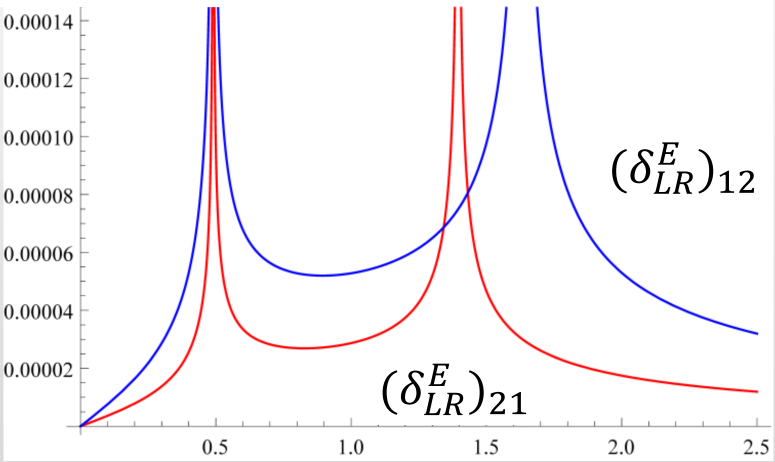
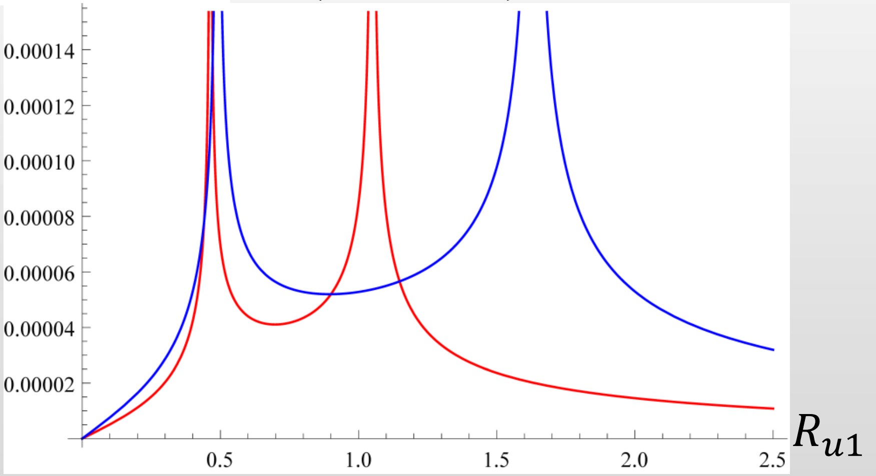


$$m_0 = 2\text{TeV}$$

Mass insertion $(R^C = 0.05)$



Mass insertion $(R^C = 1.7)$



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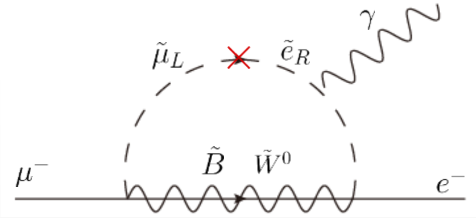

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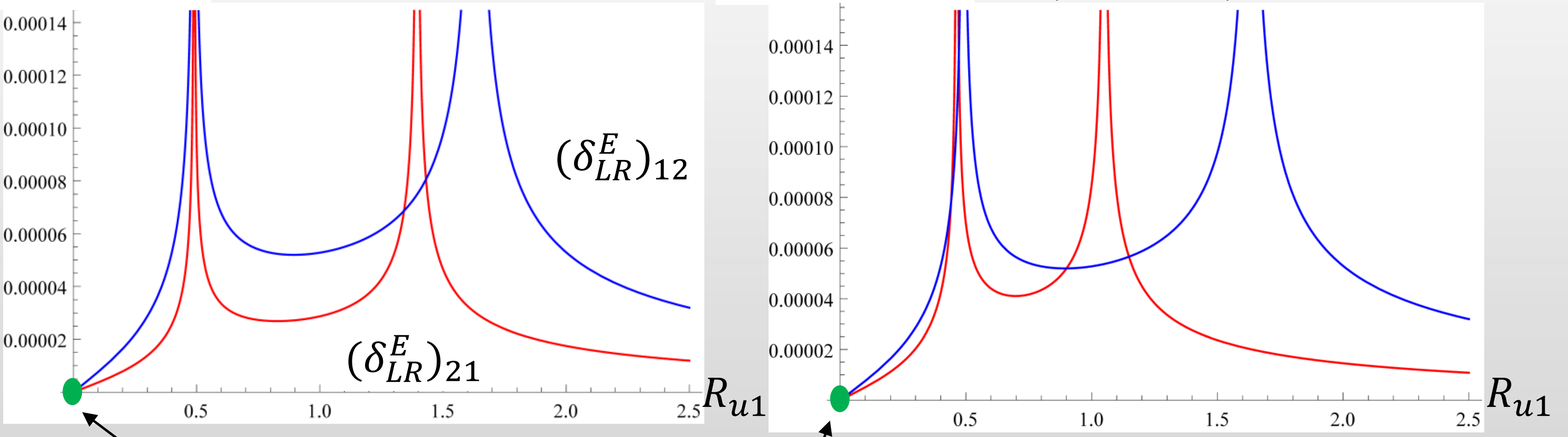
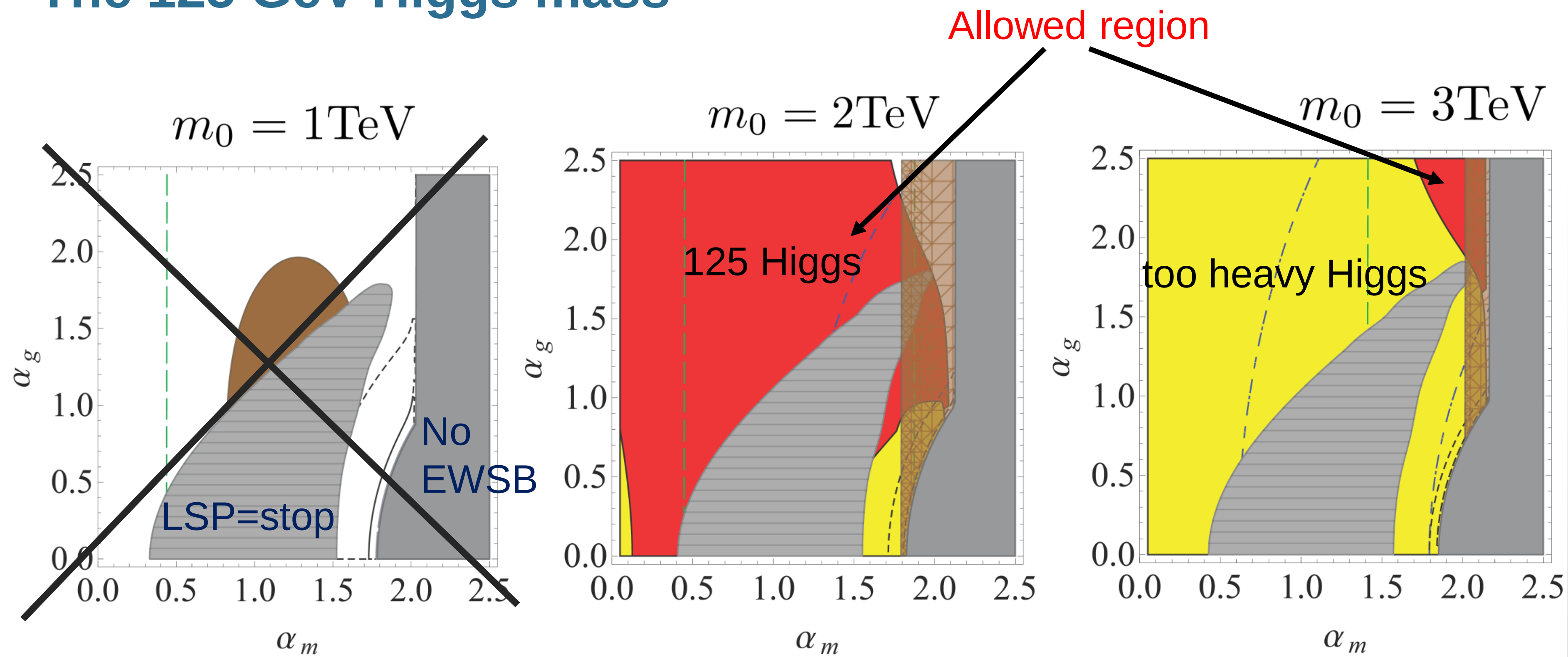


Allowed region $(\delta^E_{LR})_{12,21} < \mathcal{O}(10^{-6})$

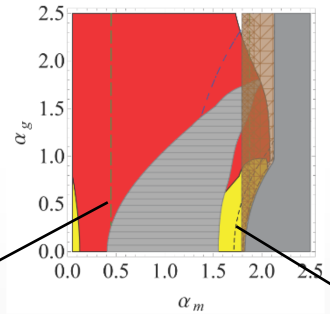
The 125 GeV Higgs mass



$$\alpha_g = R^{T_1} = R^{T_2} = R^{T_3} = R^{T_1} = R^{U_1} = R^{U_2} \quad R^{U_1} = 0$$

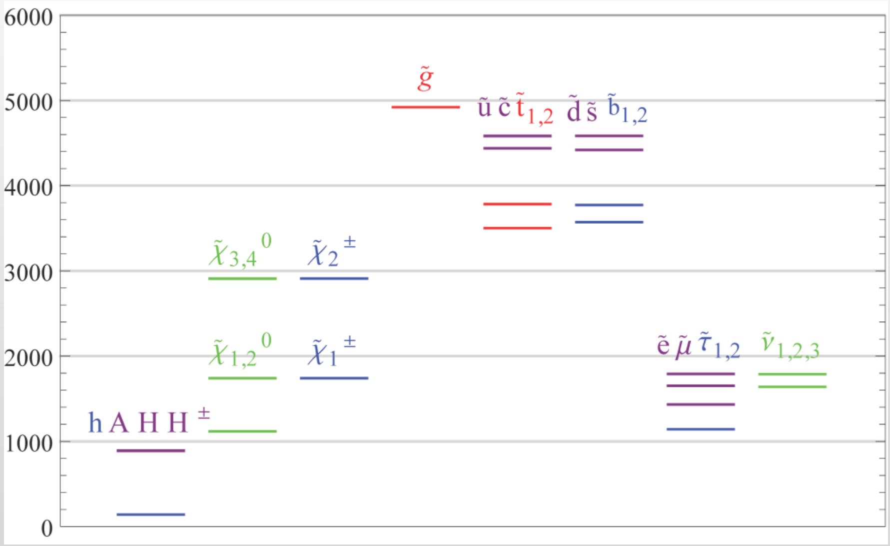
$$\alpha_m = R^C$$

Typical spectra



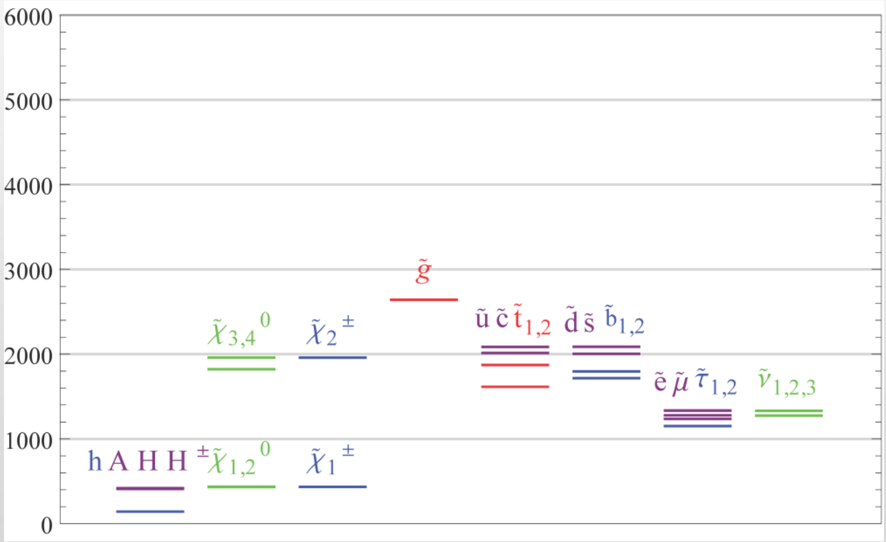
$$m_0 = 2\text{TeV}$$

$$(\alpha_m, \alpha_g) = (0.5, 0.5)$$



$$m_h = 126.2\text{GeV}, \quad \Delta_\mu = 0.053\%$$

$$(\alpha_m, \alpha_g) = (1.8, 0.2)$$



$$m_h = 127.9\text{GeV}, \quad \Delta_\mu = 3.6\%$$

Summary

- 10D SYM theory on magnetized tori

  - magnetic fluxes realize the chiral theory, N=1 SUSY and the flavor structures :

    gauge syms, # of gens, hierarchies･･･

  - No FCNCs while the U_1 moduli is decoupling

  - The 125 GeV Higgs mass

- Typical spectra of various magnetized brane models