Supersymmetrc branes on the conifold with magnetic fluxes

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Ongoing work with
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November 21, 2013@PASCOS 2013,Taipei

Mass hierarchy problem

The Standard Model is very successful model, but

Quark and lepton masses and mixings

	Observed
m_u, m_c, m_t	$(2.3 \times 10^{-3}, 1.28 \times 10^{0}, 1.74 \times 10^{2})$
m_d, m_s, m_b	$(4.8 \times 10^{-3}, 0.95 \times 10^{-1}, 4.18 \times 10^{0})$
m_e, m_μ, m_τ	$(5.11 \times 10^{-4}, 1.06 \times 10^{-1}, 1.78 \times 10^{0})$
$m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$	$< 2 \times 10^{-9}$
$\lvert m_{\nu_1}^2 - m_{\nu_2}^2 \rvert$	7.50×10^{-23}
$\lvert m_{\nu_1}^2 - m_{\nu_3}^2 \rvert$	2.32×10^{-21}
$\lvert V_{CKM} \rvert$	$\begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ 0.23 & 0.97 & 0.041 \\ 0.0087 & 0.040 & 1.0 \end{pmatrix}$
$\lvert V_{PMNS} \rvert$	$\begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.51 & 0.58 & 0.64 \\ 0.26 & 0.61 & 0.75 \end{pmatrix}$

Hierarchical !

Superstring theory

— gravitational interaction

— few free parameters

Extra dimentson
(4+6)=10 dim.

Supersymmetry

Low energy effective theory:
Super Yang-Mills(SYM) theory

If standard model is embedded in SYM

The Standard Model as low energy effective theory of the superstring theory

Type IIB String Theory (10D)

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- **D9-brane**

↓ Low energy limit

- **Super Yang-Mills (SYM) action (10D)**

↓ Compactification

H. Abe, T. Kobayashi, H.Ohki, K. Sumita '12

- **Super Yang-Mills (SYM) effective action (4D)**

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- **MSSM (4D)**

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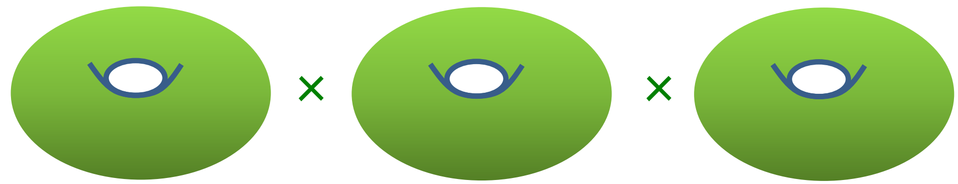
Standard Model (4D)

$$T^2 \times T^2 \times T^2$$



(magnetized)

- **SUGRA effective action (4D)**

Mirage mediation

H. Abe, T. Kobayashi, H. Ohki, A. Oikawa, K.Sumita '13

II. Model Building

Type IIB String Theory (10D)

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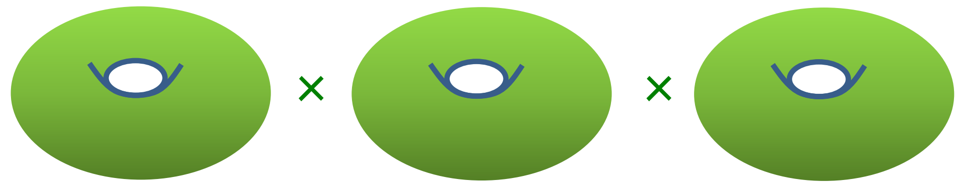
- **D9-brane**

↓ Low energy limit

- **Super Yang-Mills (SYM) action (10D)**

$T^2 \times T^2 \times T^2$



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Torus number $i, j = 1, 2, 3$
$(1), (2), (3)$

Zero-mode wavefunction with magnetic flux M and wilson line ζ

$$\phi(x, z_1, z_2, z_3) = \sum_{I^{(1)}, I^{(2)}, I^{(3)}} \Theta^{I^{(1)}, M^{(1)}}\left(z_1 + \frac{\zeta^{(1)}}{M^{(1)}}\right) \Theta^{I^{(2)}, M^{(2)}}\left(z_2 + \frac{\zeta^{(2)}}{M^{(2)}}\right) \Theta^{I^{(3)}, M^{(3)}}\left(z_3 + \frac{\zeta^{(3)}}{M^{(3)}}\right) \times \phi^{I^{(1)} I^{(2)} I^{(3)}}(x)$$

$$\Theta^{I,M}(z+\eta) = \mathcal{N} e^{i\pi M (z+\eta) \frac{\mathrm{Im}(z+\eta)}{\mathrm{Im}\tau}} \vartheta \begin{bmatrix} I/M \\ 0 \end{bmatrix} (M(z+\eta), M\tau)$$

Jacobi-theta function

$$\Theta^{I,M}(z+\eta) = \mathcal{N} e^{i\pi M(z+\eta)\frac{\mathrm{Im}(z+\eta)}{\mathrm{Im}\tau}} \vartheta \begin{bmatrix} I/M \\ 0 \end{bmatrix} (M(z+\eta), M\tau)$$

$I^{(r)} = 0, 1, \cdots, |M^{(r)}| - 1$

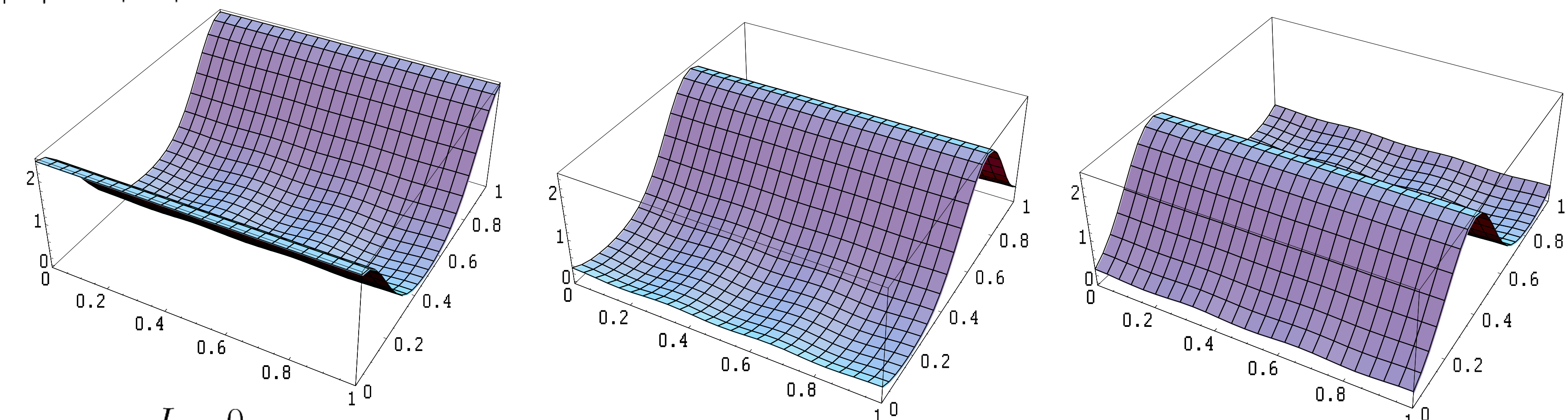
$$\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{l \in \mathbf{Z}} e^{\pi i (a+l)^2 \tau} e^{2\pi i (a+l)(\nu+b)}$$

Generation = # of magnetic flux

D. Cremades, L. E. Ibanez, F. Marchesano, JHEP 0405, 079 (2004)

Wavefunction localization

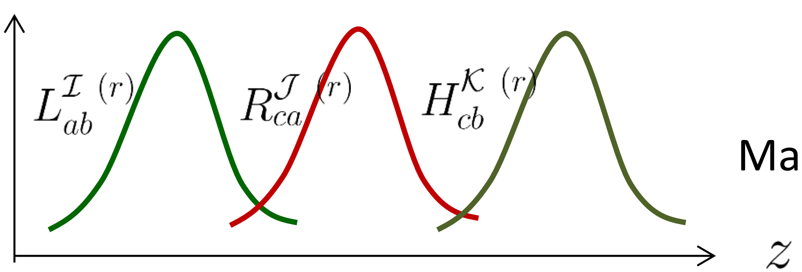
$|\Theta|^2 \quad |M| = 3$



$I = 0$ $\quad$ $I = 1$ $\quad$ $I = 2$

$$\mathcal{L}_{\mathrm{Yukawa}} = Y_{\mathcal{IJK}} L_{ab}^{\mathcal{I}} R_{ca}^{\mathcal{J}} H_{cb}^{\mathcal{K}}$$

$$Y_{\mathcal{IJK}} = \int dz_i d\bar{z}_i$$



Mass hierarchy structure

We choose Higgs VEVs, moduli VEVs, and wilson lines

At the EW scale through 1-loop MSSM RGEs

Quark / charged lepton masses and CKM mixing

	Sample values	Observed
(m_u, m_c, m_t)	$(3.1 \times 10^{-3}, 1.01 \times 10^{0}, 1.70 \times 10^{2})$	$(2.3 \times 10^{-3}, 1.28 \times 10^{0}, 1.74 \times 10^{2})$
(m_d, m_s, m_b)	$(2.8 \times 10^{-3}, 1.48 \times 10^{-1}, 6.46 \times 10^{0})$	$(4.8 \times 10^{-3}, 0.95 \times 10^{-1}, 4.18 \times 10^{0})$
(m_e, m_μ, m_τ)	$(4.68 \times 10^{-4}, 5.76 \times 10^{-2}, 3.31 \times 10^{0})$	$(5.11 \times 10^{-4}, 1.06 \times 10^{-1}, 1.78 \times 10^{0})$
$\lvert V_{CKM}\rvert$	$\begin{pmatrix} 0.98 & 0.21 & 0.0023 \\ 0.21 & 0.98 & 0.041 \\ 0.011 & 0.040 & 1.0 \end{pmatrix}$	$\begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ 0.23 & 0.97 & 0.041 \\ 0.0087 & 0.040 & 1.0 \end{pmatrix}$

Neutrino masses and PMNS matrix

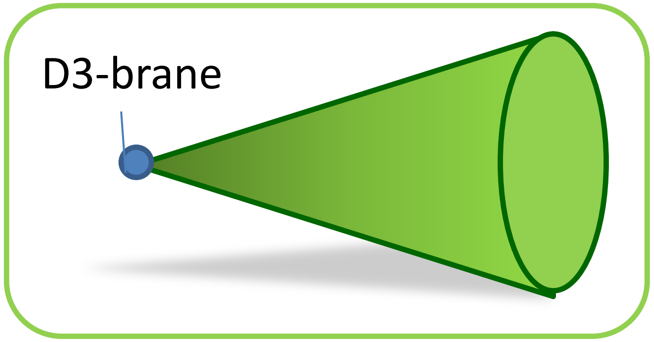
	Sample values	Observed
$(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$	$(3.6 \times 10^{-19}, 8.8 \times 10^{-12}, 2.7 \times 10^{-11})$	$< 2 \times 10^{-9}$
$\lvert m_{\nu_1}^2 - m_{\nu_2}^2\rvert$	7.67×10^{-23}	7.50×10^{-23}
$\lvert m_{\nu_1}^2 - m_{\nu_3}^2\rvert$	7.12×10^{-22}	2.32×10^{-21}
$\lvert V_{PMNS}\rvert$	$\begin{pmatrix} 0.85 & 0.46 & 0.25 \\ 0.50 & 0.59 & 0.63 \\ 0.15 & 0.66 & 0.73 \end{pmatrix}$	$\begin{pmatrix} 0.82 & 0.55 & 0.16 \\ 0.51 & 0.58 & 0.64 \\ 0.26 & 0.61 & 0.75 \end{pmatrix}$

Semi-realistic pattern

Extending the model to Calabi-Yau

-Mass term like majorana neutrino term may be obtained while torus model cannot.

-We want to look for guiding principle for geometry of string theory



Branes at Singularity

-Fractional D3/D7-brane at singularity

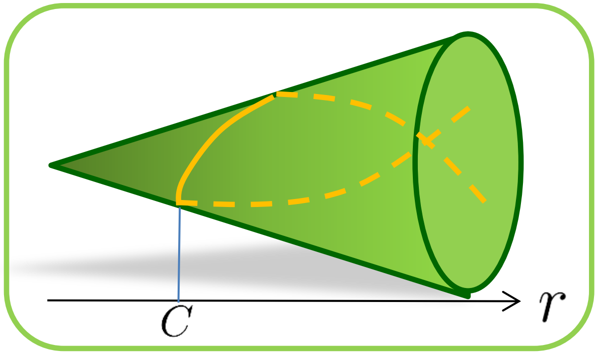
-We can know Gauge groups, Yukawa couplings, and flavor structure as Local Model.

-Moduli stabilization as Global Model.

-There are not models which contains realistic CKM or mass hierarchy structure in Type IIB string framework naively.

Local Model - D7-branes on Conifold

$AdS_5 \times T^{1,1}$



$$ds^2_{T^{1,1}} = \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right) + \frac{1}{9}\left(d\psi + \sum_{i=1}^{2}\cos\theta_i d\phi_i\right)^2$$

J. P. Gauntlett, D. Martelli, J. Sparks, D. Waldram '04

	x_0	x_1	x_2	x_3	θ_1	ϕ_1	θ_2	ϕ_2	ψ	r
dimension	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
D7-brane	O	O	O	O	O	O	O	O		

Kappa symmetry condition (Local SUSY) $\Gamma_\kappa \epsilon = \epsilon$

$$\psi = n_1\phi_1 + n_2\phi_2 + const \qquad n_1, n_2 \in \mathbb{Z}$$

$$r^3 = \frac{C^2}{\left(\sin\frac{\theta_1}{2}\right)^{n_1+1}\left(\cos\frac{\theta_1}{2}\right)^{1-n_1}\left(\sin\frac{\theta_2}{2}\right)^{n_2+1}\left(\cos\frac{\theta_2}{2}\right)^{1-n_2}}$$

Induced metric for D7-brane

D. Arean, E. Crooks, V. Ramallo '04

$$g = \frac{1}{6}\sum_{i=1}^{2}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2) + \frac{1}{9}(C_1(\theta_1)d\theta_1 + C_2(\theta_2)d\theta_2)^2 + \frac{1}{9}(C_1(\theta_1)\sin\theta_1 d\phi_1 + C_2(\theta_2)\sin\theta_2 d\phi_2)^2$$

$$C_i(\theta_i) \equiv \frac{n_i + \cos\theta_i}{\sin\theta_i} \quad (i = 1, 2)$$

Mass hierarchy structure is yielded if zero mode wavefunctions of matter fields (quasi-)localize

When $(n_1, n_2) = (1,1)$, D7-brane has non-zero minimal value of r , $r_* = C$

D. Arean, E. Crooks, V. Ramallo '04

Holomorphic coordinates

Conifold

$$z_1 = r^{3/2} e^{\frac{i}{2}\psi - \phi_1 - \phi_2} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$z_2 = r^{3/2} e^{\frac{i}{2}\psi + \phi_1 + \phi_2} \cos\frac{\theta_1}{2} \cos\frac{\theta_2}{2}$$

$$z_3 = r^{3/2} e^{\frac{i}{2}\psi + \phi_1 - \phi_2} \cos\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$z_4 = r^{3/2} e^{\frac{i}{2}\psi - \phi_1 + \phi_2} \sin\frac{\theta_1}{2} \cos\frac{\theta_2}{2}$$

→

D7-brane

$$z_1 = C$$

$$z_2 = z_3 z_4$$

$$z_3 = C \cot\frac{\theta_1}{2} e^{i\phi_1}$$

$$z_4 = C \cot\frac{\theta_2}{2} e^{i\phi_2}$$

Kahler form

$$J = -\frac{2}{3} Q_1 \Omega_{11} - \frac{2}{3} Q_2 \Omega_{22} - \frac{2}{9} \cot\frac{\theta_1}{2} \cot\frac{\theta_2}{2} (\Omega_{12} + \Omega_{21})$$

$$F = M Q_1 \Omega_{11} - M Q_1 \Omega_{22} \qquad Q_i = \frac{3}{2} + \cot^2\frac{\theta_i}{2}$$

$$J \wedge F = 0 \qquad \Omega_{ij} = d\theta_i \wedge \sin\theta_j d\phi_j$$

Bosonic wavefunctions with magnetic fluxes

Laplacian

$$-g^{\mu\nu}D_\mu D_\nu \Phi = -2g^{z\bar{z}}D_{\bar{z}}D_z\Phi - \underbrace{g^{\bar{z}z}[D_z, D_{\bar{z}}]\Phi}_{=0}$$

$$D_z\Phi = 0$$
$$D_{\bar{z}}\Phi = 0$$

Zero mode function

$$\mathrm{Li}_n(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^n}$$

(PolyLogarithm function)

Wavefunction

$$D_{z_3}\Phi = 0 \rightarrow \Phi_1(z_3, \bar{z}_3) \propto \left(\frac{z_3}{1+z_3\bar{z}_3}\right)^{\frac{M}{2}} f(\bar{z}_3)e^{M\mathrm{Li}_2(-z_3\bar{z}_3)}$$

$$D_{\bar{z}_3}\Phi = 0 \rightarrow \Phi_2(z_3, \bar{z}_3) \propto \left(\frac{\bar{z}_3}{1+z_3\bar{z}_3}\right)^{-\frac{M}{2}} g(z_3)e^{-M\mathrm{Li}_2(-z_3\bar{z}_3)}$$

$$D_{z_4}\Phi = 0 \rightarrow \Phi_3(z_4, \bar{z}_4) \propto \left(\frac{z_4}{1+z_4\bar{z}_4}\right)^{-\frac{M}{2}} h(\bar{z}_4)e^{-M\mathrm{Li}_2(-z_4\bar{z}_4)}$$

$$D_{\bar{z}_4}\Phi = 0 \rightarrow \Phi_4(z_4, \bar{z}_4) \propto \left(\frac{\bar{z}_4}{1+z_4\bar{z}_4}\right)^{\frac{M}{2}} k(z_4)e^{M\mathrm{Li}_2(-z_4\bar{z}_4)}$$

Φ_1	$M > 0$	$M < 0$
Φ_2	$M < 0$	$M > 0$
Φ_3	$M > 0$	$M < 0$
Φ_4	$M < 0$	$M > 0$
Localize at	$z \to 0$	$z \to \infty$

Bosonic wavefunctions with magnetic fluxes

Laplacian

$$-g^{\mu\nu}D_\mu D_\nu \Phi = -2g^{z\bar{z}}D_{\bar{z}}D_z\Phi - \underline{g^{\bar{z}z}[D_z, D_{\bar{z}}]\Phi}$$

$\parallel$
0

$$D_z\Phi = 0$$
$$D_{\bar{z}}\Phi = 0$$

Zero mode function

$$\mathrm{Li}_n(z) \equiv \sum_{k=1}^{\infty} \frac{z^k}{k^n}$$

(PolyLogarithm function)

Wavefunction

$D_{z_3}\Phi = 0 \rightarrow \Phi_1(z_3, \bar{z}_3) \propto \left(\frac{z_3}{1+z_3\bar{z}_3}\right)^{\frac{M}{2}} f(\bar{z}_3)e^{M\mathrm{Li}_2(-z_3\bar{z}_3)}$	$M>0$	$M<0$
$D_{\bar{z}_3}\Phi = 0 \rightarrow \Phi_2(z_3, \bar{z}_3) \propto \left(\frac{\bar{z}_3}{1+z_3\bar{z}_3}\right)^{-\frac{M}{2}} g(z_3)e^{-M\mathrm{Li}_2(-z_3\bar{z}_3)}$	$M<0$	$M>0$
$D_{z_4}\Phi = 0 \rightarrow \Phi_3(z_4, \bar{z}_4) \propto \left(\frac{z_4}{1+z_4\bar{z}_4}\right)^{-\frac{M}{2}} h(\bar{z}_4)e^{-M\mathrm{Li}_2(-z_4\bar{z}_4)}$	$M>0$	$M<0$
$D_{\bar{z}_4}\Phi = 0 \rightarrow \Phi_4(z_4, \bar{z}_4) \propto \left(\frac{\bar{z}_4}{1+z_4\bar{z}_4}\right)^{\frac{M}{2}} k(z_4)e^{M\mathrm{Li}_2(-z_4\bar{z}_4)}$	$M<0$	$M>0$
Localize at	$z \to 0$	$z \to \infty$

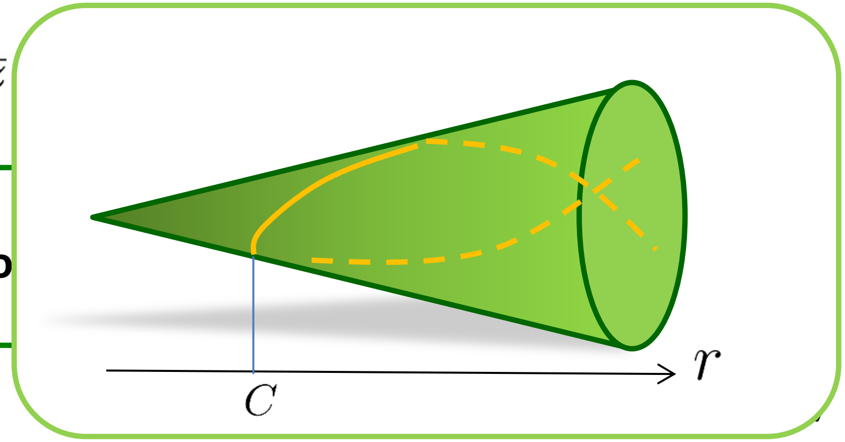
Bosonic wavefunctions with magnetic fluxes

Laplacian

$$-g^{\mu\nu}D_\mu D_\nu \Phi = -2g^{z\bar{z}}$$

$$D_z\Phi = 0$$
$$D_{\bar{z}}\Phi = 0$$

Zero mo



Wavefunction

$$D_{z_3}\Phi = 0 \rightarrow \Phi_1(z_3,\bar{z}_3) \propto \left(\frac{z_3}{1+z_3\bar{z}_3}\right)^{\frac{M}{2}} f(\bar{z}_3)e^{M\mathrm{Li}_2(-z_3\bar{z}_3)}$$

$$D_{\bar{z}_3}\Phi = 0 \rightarrow \Phi_2(z_3,\bar{z}_3) \propto \left(\frac{\bar{z}_3}{1+z_3\bar{z}_3}\right)^{-\frac{M}{2}} g(z_3)e^{-M\mathrm{Li}_2(-z_3\bar{z}_3)}$$

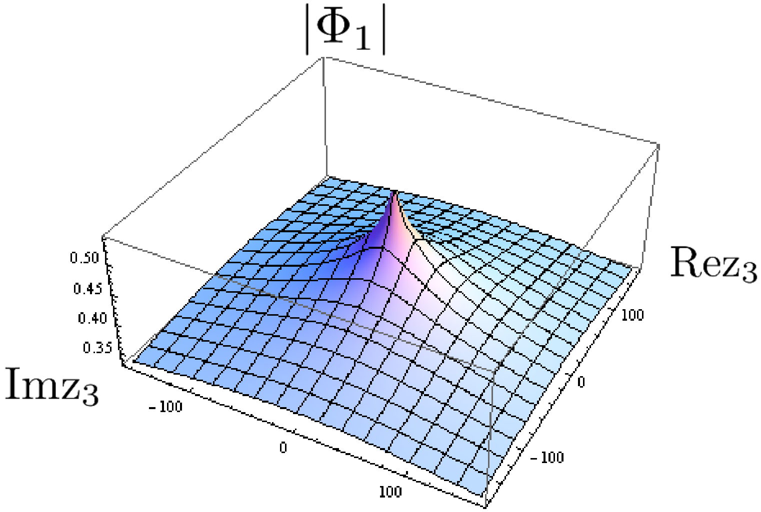
$$D_{z_4}\Phi = 0 \rightarrow \Phi_3(z_4,\bar{z}_4) \propto \left(\frac{z_4}{1+z_4\bar{z}_4}\right)^{-\frac{M}{2}} h(\bar{z}_4)e^{-M\mathrm{Li}_2(-z_4\bar{z}_4)}$$

$$D_{\bar{z}_4}\Phi = 0 \rightarrow \Phi_4(z_4,\bar{z}_4) \propto \left(\frac{\bar{z}_4}{1+z_4\bar{z}_4}\right)^{\frac{M}{2}} k(z_4)e^{M\mathrm{Li}_2(-z_4\bar{z}_4)}$$

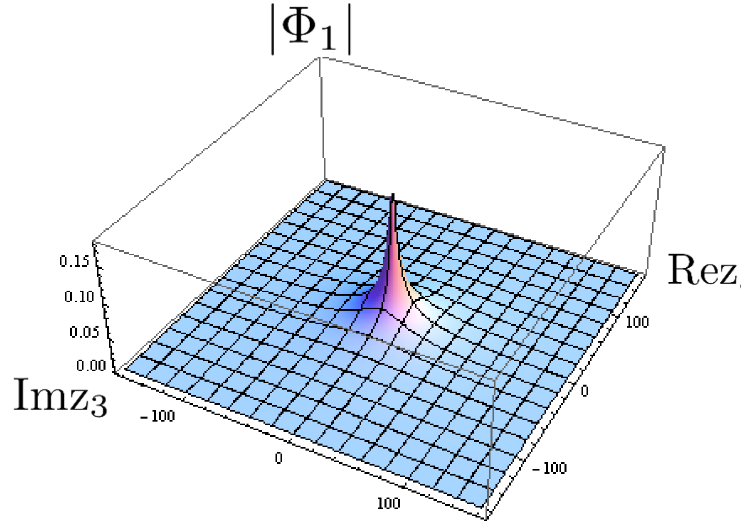
Φ_1	$M>0$	$M<0$
Φ_2	$M<0$	$M>0$
Φ_3	$M>0$	$M<0$
Φ_4	$M<0$	$M>0$
Localize at	$z \rightarrow 0$	$z \rightarrow \infty$
$\longleftrightarrow$	$r \rightarrow C$	$r \rightarrow \infty$

Bosonic wavefunctions with magnetic fluxes

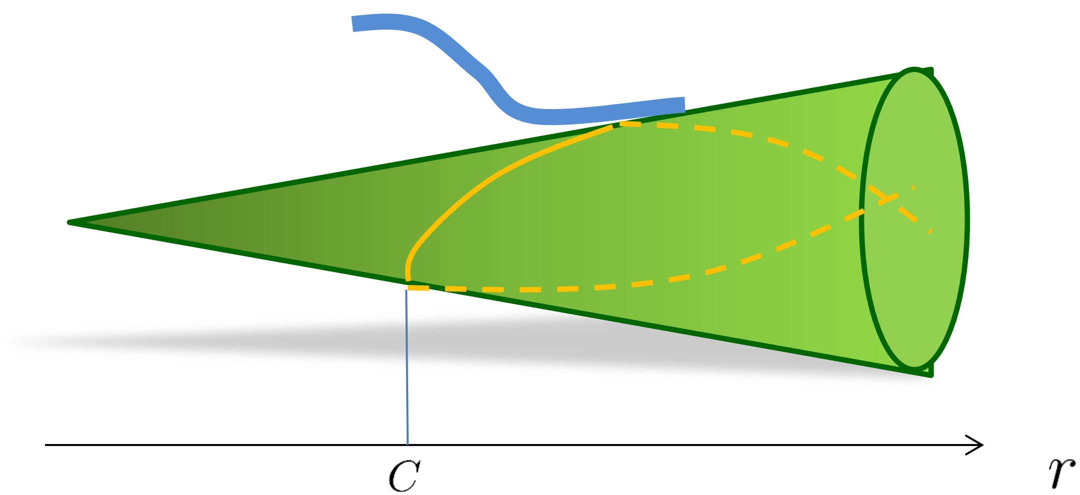
Quasi-localization



$M = 1, \Phi_1(z_3, \bar{z_3})$



$M = 10, \Phi_1(z_3, \bar{z_3})$



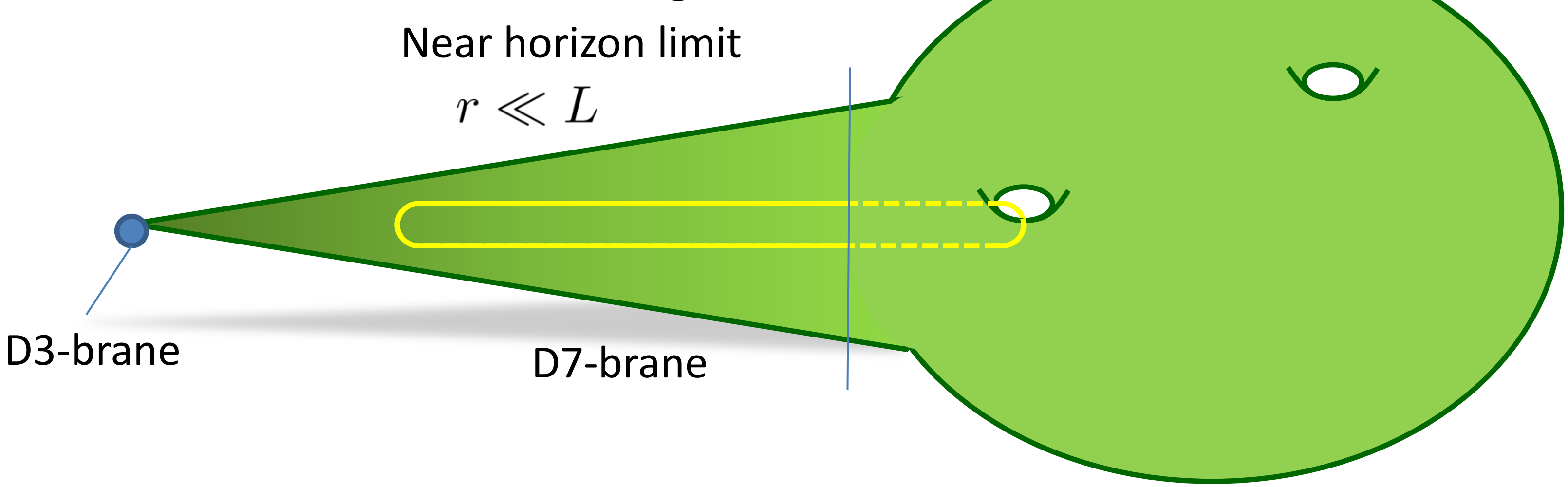
Summary

- We studied 8D SYM theory on supersymmetric D7-brane model in conifold with magnetic flux.
- We obtained analytical solutions of the bosonic zero-mode wavefunction on the D7-brane wrapping conifold
- Quasi-localization of the wavefunctions may derive mass hierarchy structure

Future Work

- Fermionic wavefunction, Yukawa coupling
  - Phenomenological analysis

- Global embedding



Local Model

conifold $z_1 z_2 - z_3 z_4 = 0$

D7-brane $z_1 = C$

Global Model

Compact CY can be constructed by ambient toric space.

- D-branes on Sasaki-Einstein manifolds $Y^{p,q}$