Tree unitarity in Einstein gravity

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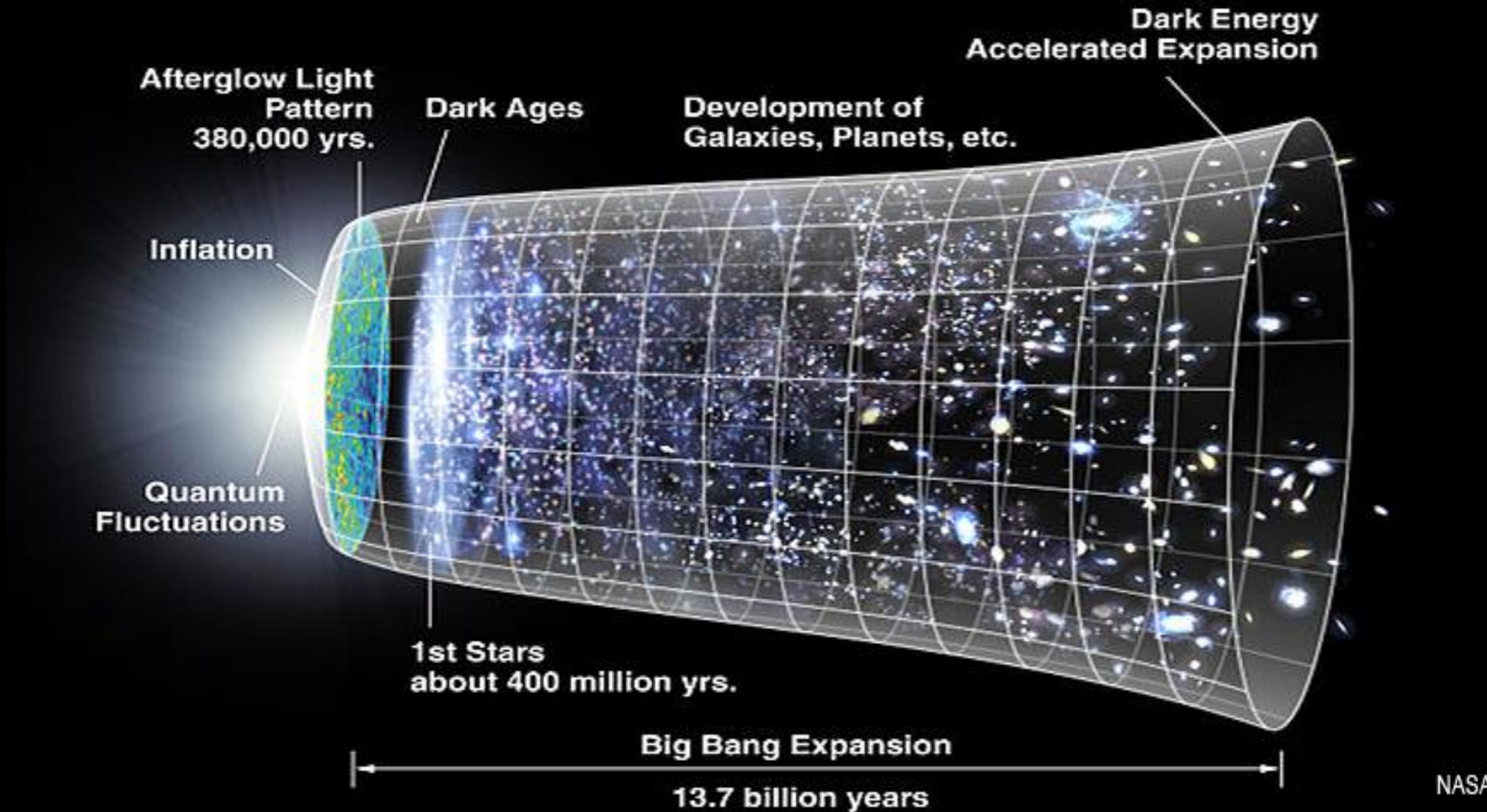
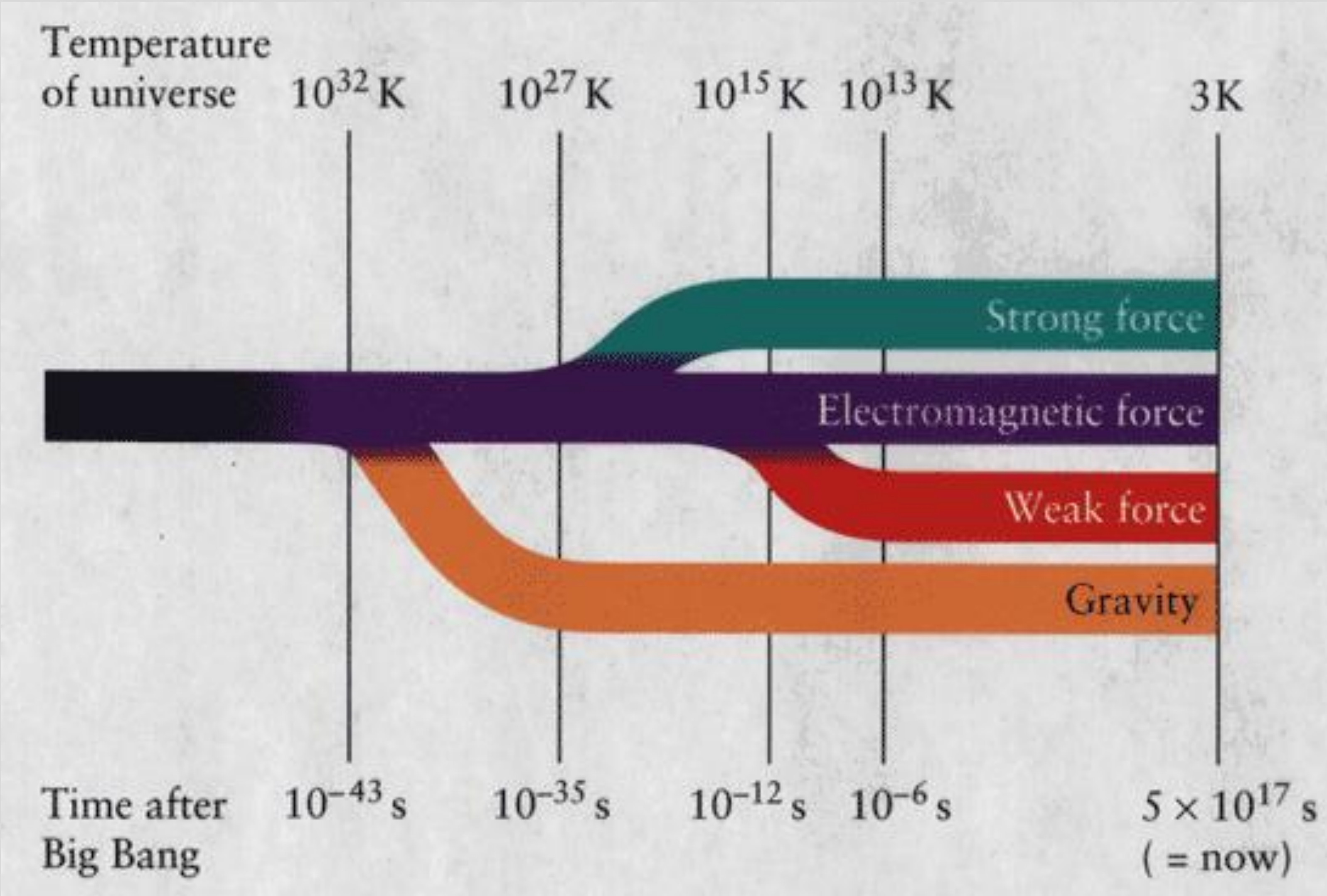
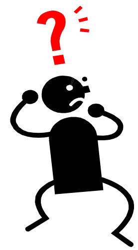
1.Introduction




Temperature of universe
10^32 K
10^27 K
10^15 K
10^13 K
3K
Strong force
Electromagnetic force
Weak force
Gravity
Time after Big Bang
10^−43 s
10^−35 s
10^−12 s
10^−6 s
5 × 10^17 s
(= now)

When considering the early universe and the unified forces, we have to consider a "Quantum Gravity theory".

However, the quantum gravity theory has not yet completed.



《The candidate of quantum gravity》

- Super string theory
- Supergravity theory
- Hořava-Lifshitz gravity etc.

Then, we have to find the theory of gravitation which can be quantized.

We have to find the theory which is renormalizable and does not violate an unitarity.

Although a loop calculation is very hard when we investigate a renormalizability, however using the Llewellyn Smith's suggestion, we can check it without calculating of loop.

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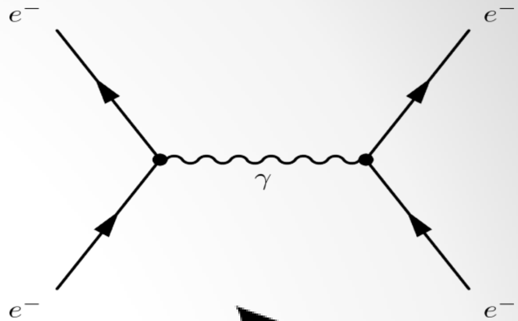
2. Llewellyn Smith's suggestion

Llewellyn Smith (1973)

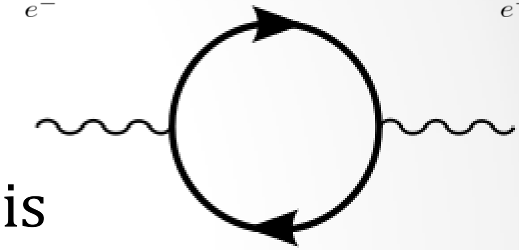
tree unitarity $\cong$ renormalizability

$$\begin{cases} \text{tree unitarity is not violated} \quad \Leftrightarrow \quad \text{reormalizable} \\ \text{tree unitarity is violated} \quad \Leftrightarrow \quad \text{non-reormalizable} \end{cases}$$

◆What is tree unitarity？

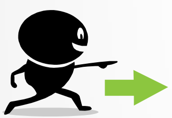


⇒　unitarity at tree level (no loop)



《unitarity in quantum field theory》

The Hilbert space of physical condition is positive definite.



The probability of physical condition has positive value.

in other words ・・・

- Unitarity means probability conservation.
- It means that the condition of unitarity implies a limited scattering cross section.

↓

(We can check tree unitarity by calculating a scattering cross section.)

In fact, it is well-known that the Llewellyn Smith's suggestion holds in Weinberg-Salam model, QED and Yang-Mills theory.

However massive vector theory and 4-Fermi theory doesn't hold the suggestion.

- Weinberg-Salam model
- QED
- Yang-Mills theory

with gauge symmetry

- massive vector theory
- 4-Fermi theory

Then we thought whether the Llewellyn Smith's suggestion also holds in Einstein gravity. Because it is thought that Einstein gravity is also a gauge theory.

Einstein gravity $\approx$ gauge theory

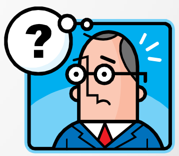
http://arxiv.org/pdf/1306.4035.pdf

If we read the paper which the scattering cross section of Einstein gravity is calculated, the tree unitarity is violated !

Einstein gravity is non-renormalizable and violate tree unitarity.

The Llewellyn Smith's suggestion also holds in Einstein gravity！！

$\Rightarrow$ Is it also true for modified gravity theories ?



We investigate R^2 gravity this time.

Llewellyn Smith's suggestion

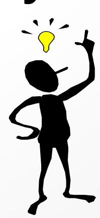
"tree unitarity ≅ renormalizability"

This suggestion holds in gauge theory and Einstein gravity.

Then, we would like to check by investigating tree unitarity whether this suggestion also holds in modified gravity theories.

R^2 gravity

If the above suggestion also holds in modified gravity theories, when a new quantum gravity theory is proposed, it can be a tool to check renormalizability.



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2. R^2 gravity

action

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,(R + \alpha R^2)$$

R: 2nd order derivative

αR^2: 4th order derivative

Einstein-Hilbert action

(α : parameter)

→ This is the theory which add a modification of R^2 to the action of GR.

→ a higher derivative theory

→ renormalizable by power counting

About renormalization

- Einstein gravity

$$S_{EH} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,R$$

$([R] = 2)$

mass dimension of coupling constant：

$[G] = -2 \longrightarrow$ not renormalizable by power counting

- R^2 gravity

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,(R + \alpha R^2)$$

$([R^2]=4)$

$\left[\frac{\alpha}{G}\right] = 0 \longrightarrow$ renormalizable by power counting

The term of R^2 is dominant in the high energy scale.

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3 .How to check the tree unitarity

cross section ： σ
Energy in center of mass system：E

↓ in the high energy limit

$$\sigma \sim E^{\varepsilon} \qquad (E \to \infty)$$

ε > 0 tree unitarity × (cross section is divergent.)

ε ≤ 0 tree unitarity ○ (cross section is not divergent.)

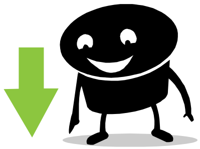
↳ ※The quantity of a scattering cross section has a limit.

We can check the tree unitarity through calculating graviton scattering cross section in the high energy limit.

$$\sigma \sim E^{\varepsilon}$$

Notice this value!

Ex) Einstein gravity



amplitude $\mathcal{M} \sim E^{6}$

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4． Calculation

◆How to calculate the scattering cross section

- expand action to the 4th order of fluctuation

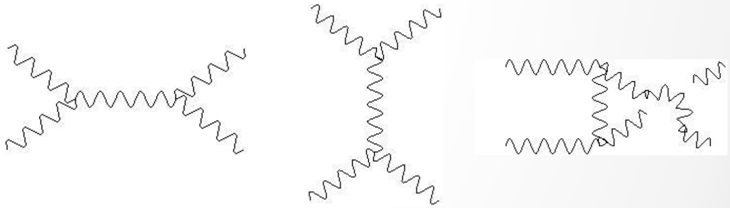
$$\bar{S} = S^{(0)} + S^{(1)} + S^{(2)} + S^{(3)} + S^{(4)}$$

- make Feynman rules

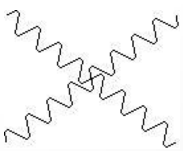
- draw Feynman diagrams

- calculation of scattering cross section

3 point coupling



4 point coupling



◆expansion of the action (Lagrangian)

《expansion of R and $\sqrt{-g}$》

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,(R + \alpha R^2)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad h_{\mu\nu}\text{:fluctuation}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} + h^{\mu\rho}h_{\rho}{}^{\nu} - h^{\mu\rho}h_{\rho\sigma}h^{\sigma\nu} + h^{\mu\rho}h_{\rho\sigma}h^{\sigma\tau}h_{\tau}{}^{\nu}$$

(metric satisfy $g_{\mu\rho}g^{\rho\nu} = \delta_{\mu}{}^{\rho}.$)

$$R = R^{(0)} + R^{(1)} + R^{(2)} + R^{(3)} + R^{(4)}$$

$$\sqrt{-g} = \sqrt{-g}^{(0)} + \sqrt{-g}^{(1)} + \sqrt{-g}^{(2)} + \sqrt{-g}^{(3)} + \sqrt{-g}^{(4)}$$

- (0)～(4) stands for fluctuation from 0th order to 4th.

$$R^{(0)} = 0$$

$$R^{(1)} = \partial_\rho \partial_\mu h^{\mu\rho} - \partial^2 h \qquad ({h_\alpha}^\alpha \equiv h\ )$$

$$R^{(2)} = -\frac{1}{4}\Big[4h^{\rho\nu}\big(\partial_\rho \partial_\mu {h_\nu}^\mu - \partial_\nu \partial_\rho h - \partial^2 h_{\nu\rho} + \partial_\nu \partial_\rho {h_\rho}^\alpha\big) + \big(2\partial_\rho h^{\nu\rho} - \partial^\nu h\big)\big(2\partial^\mu h_{\nu\mu} - \partial_\nu h\big) - \cdots\Big]$$

$$R^{(3)} = \frac{1}{4}\Big[2h^{\mu\alpha}h^{\rho\nu}\big(\partial_\rho \partial_\mu h_{\nu\alpha} - \partial_\rho \partial_\nu h_{\alpha\mu} - \partial_\alpha \partial_\mu h_{\nu\rho} + \partial_\alpha \partial_\nu h_{\rho\mu}\big) + \cdots\cdots\Big]$$

$$R^{(4)} = -\frac{1}{4}\Big[2h^{\rho\lambda}h_{\lambda\sigma}h^{\sigma\nu}\big(\partial_\rho \partial_\mu {h_\nu}^\mu - \partial_\nu \partial_\rho h - \partial^2 h_{\nu\rho} + \partial_\nu \partial_\rho {h_\rho}^\alpha\big) + \cdots\cdots\cdots\Big]$$

$$\sqrt{-g}^{(0)} = 1$$

$$\sqrt{-g}^{(1)} = \frac{1}{2}h$$

$$\sqrt{-g}^{(2)} = -\left(\frac{1}{4}h^{\alpha}{}_{\beta}h^{\beta}{}_{\alpha} - \frac{1}{8}h^2\right)$$

$$\sqrt{-g}^{(3)} = \frac{1}{6}h^{\alpha}{}_{\beta}h^{\beta}{}_{\gamma}h^{\gamma}{}_{\alpha} - \frac{1}{8}hh^{\beta}{}_{\gamma}h^{\gamma}{}_{\beta} + \frac{1}{48}h^3$$

(※Since $R^{(0)} = 0, \sqrt{-g}^{(4)}$ is unnecessary.)

We substitute expanded R and $\sqrt{-g}$ for the action.

$$S = \frac{1}{16\pi G}\int d^4x \sqrt{-g}\left(R + \alpha R^2\right)$$

$$\equiv \frac{1}{16\pi G}\int d^4x \left(\mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)}\right)$$

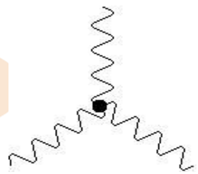
Now we choose "de Donder gauge".

de Donder gauge condition : $\partial_\mu h^{\mu\nu} = \frac{1}{2}\partial^\nu h$

propagator

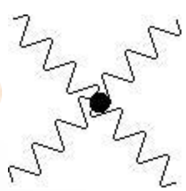
$$\mathcal{L}^{(2)} = \frac{1}{4}\left\{-(\partial_\kappa h^{\rho\nu})^2 + \frac{1}{2}(\partial_\nu h)^2 + \alpha(\partial^2 h)^2\right\}$$

vertex



$$\begin{aligned}\mathcal{L}^{(3)} = &-3h^{\mu\alpha}\partial_\rho h^{\rho\nu}\partial_\mu h_{\nu\alpha} + \tfrac{1}{4}h^{\rho\nu}\partial_\rho h^{\mu\alpha}\partial_\nu h_{\alpha\mu} + h^{\mu\alpha}\partial_\rho h^{\rho\nu}\partial_\nu h_{\alpha\mu}\\ &-2h_\sigma{}^\nu\partial_\rho h^{\rho\sigma}\partial_\mu h_\nu{}^\mu + h_\sigma{}^\nu\partial_\rho h^{\rho\sigma}\partial_\nu h + h^{\rho\sigma}\partial_\rho h_\sigma{}^\nu\,\partial_\nu h\\ &+\tfrac{1}{2}h_\sigma{}^\nu\partial_\kappa h^{\rho\sigma}\partial^\kappa h_{\nu\rho} + \tfrac{1}{2}h^{\lambda\nu}\partial_\rho h_\lambda{}^\rho\partial_\rho h^\alpha{}_\nu\\ &-\tfrac{1}{8}h(\partial_\kappa h^{\rho\nu})^2 + \tfrac{1}{16}h(\partial_\alpha h)^2 - \tfrac{1}{4}h^{\alpha\beta}\partial_\kappa h_{\alpha\beta}\partial^\kappa h\\ &+\alpha\left(\tfrac{1}{8}\partial^2 h(\partial_\kappa h^{\rho\nu})^2 + \tfrac{1}{16}\partial^2 h(\partial_\alpha h)^2 + \tfrac{1}{8}h\partial^2 h\partial^2 h\right)\end{aligned}$$

vertex



$$\mathcal{L}^{(4)} = \cdots\cdots\cdots$$

$$\mathcal{L}^{(2)} = \frac{1}{4}\left\{-(\partial_\kappa h^{\rho\nu})^2 + \frac{1}{2}(\partial_\nu h)^2 + \alpha(\partial^2 h)^2\right\}$$

propagator

contribution of R^2

$$P_{\mu\nu;\lambda\sigma} = \underbrace{\frac{\eta_{\mu\nu}\eta_{\lambda\sigma} - \eta_{\mu\lambda}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\lambda}}{2k^2}}_{\text{propagator of Einstein gravity}} - \frac{\frac{1}{4}\eta_{\mu\nu}\eta_{\lambda\sigma}}{\frac{1}{4\alpha} + k^2}$$

propagator of Einstein gravity

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5. Summary

- We expand the action to the 4th order of fluctuation.

$$\mathcal{L}^{(2)} = \frac{1}{4}\left\{-(\partial_\kappa h^{\rho\nu})^2 + \frac{1}{2}(\partial_\nu h)^2 + \alpha(\partial^2 h)^2\right\},\ \mathcal{L}^{(3)} = \cdots,\ \mathcal{L}^{(4)} = \cdots$$

- make Feynman rules

$$P_{\mu\nu;\lambda\sigma} = \frac{\eta_{\mu\nu}\eta_{\lambda\sigma} - \eta_{\mu\lambda}\eta_{\nu\sigma} - \eta_{\mu\sigma}\eta_{\nu\lambda}}{2k^2} - \frac{\frac{1}{4}\eta_{\mu\nu}\eta_{\lambda\sigma}}{\frac{1}{4\alpha} + k^2}$$

- draw Feynman diagrams

- calculation of scattering cross section

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6． Future work

① We have to finish the calculation.

- We think that the form of the scattering cross section is written by using the Mandelstam variables, because Einstein gravity was so.
- Since the Llewellyn Smith's suggestion holds in Einstein gravity, it is thought that it will be realized also in R^2 gravity.

② If the suggestion holds, it may be meaningful to check tree unitarity of the other theoris.

Example： 5-dimensional Yang-Mills theory

Thank you !