Bayesian statistical approach to dark matter direct detection experiments

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Bolshoi simulation, NASA

Outline

- Bayesian analysis of direct detection data motivated by

    (i) Tension between experiments (4 hints of detection and exclusion bounds)

    (ii) Experimental systematics (e.g. $L_{eff,}$ quenching factors) and backgrounds

    (iii) Astrophysical uncertainties in both the halo parameters and velocity distribution

- Bayesian Evidence for model comparison and compatibility
    - Best scenario that accommodates XENON100 and the hints of detection (DAMA, CoGeNT, CDMS-Si, CRESST)
    - Best particle physics scenario for hints of detection
    - Quantitative measure of incompatibility between XENON100 and hints of detection

- Conclusions

- CA, J.Hamann and Y.Wong, JCAP 1109 (2011)
- CA, J.Hamann, R.Trotta and Y.Wong, JCAP 1203 (2012)
- CA, Phys.Rev.D86 (2012)
- CA, arXiv: 1310.5718, invited review for special issue of PDU

Bayesian Inference framework

X data

$\theta = \{\theta_1, ..., \theta_n, \psi_a, ..., \psi_z\}$

θ_i theoretical model parameters

ψ_k nuisance parameters = astrophysics and systematics

$$\mathcal{P}(\theta|X)\mathrm{d}\theta \propto \mathcal{L}(X|\theta)\cdot\pi(\theta)\mathrm{d}\theta$$

Posterior probability function (PDF)

Likelihood (proper of each EXP)

Prior

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Common prior choices that do not favour any parameter region

$$\pi_{\log}(\log\theta)\ \mathrm{d}\log\theta = \begin{cases} \mathrm{d}\log\theta, & \text{if } \theta_{\min} \leq \theta \leq \theta_{\max}, \\ 0, & \text{otherwise,} \end{cases}$$

$$\pi_{\text{flat}}(\theta)\mathrm{d}\theta \propto \begin{cases} \mathrm{d}\theta, & \text{if } \theta_{\min} \leq \theta \leq \theta_{\max}, \\ 0, & \text{otherwise,} \end{cases}$$

Observable	Prior
WIMP mass (θ_1)	$\log(m_{\rm DM}/\mathrm{GeV}) :\ 0 \to 3$
SI cross-section (θ_2)	$\log(\sigma_n^{\rm SI}/\mathrm{cm}^2) :\ -44(-46) \to -38$

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Posterior sampled with nested sampling techniques (MultiNest) given the likelihood and the prior and marginalized over nuisance parameters

$$\mathcal{P}_{\text{mar}}(\theta_1, ..., \theta_n|X) \propto \int \mathrm{d}\psi_1 ... \mathrm{d}\psi_m\ \mathcal{P}(\theta_1, ..., \theta_n, \psi_1 ..., \psi_m|X)$$

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Profile Likelihood is prior independent (comparison with frequentist approach)

$$\mathcal{L}_{\mathrm{prof}}(X|\theta_1, ..., \theta_n) \propto \max_{\psi_1...\psi_m} \mathcal{L}(X|\theta_1, ..., \theta_n, \psi_1..., \psi_m) \qquad \Delta\chi^2_{\mathrm{eff}}(m_{\mathrm{DM}}, \sigma_n^{\mathrm{SI}}) \equiv -2\ln\mathcal{L}_{\mathrm{prof}}(m_{\mathrm{DM}}, \sigma_n^{\mathrm{SI}})$$

Bayesian Inference framework

Marginalization over all nuisance/new physics parameters

Experiment	Parameter	Prior
DAMA	$q_{\rm Na}$	$0.2 \to 0.4$
DAMA	$q_{\rm I}$	$0.06 \to 0.1$
CoGeNT	C	$0 \to 10$ cpd/kg/keVee
CoGeNT	$\mathcal{E}_0$	$0 \to 30$ keVee
CoGeNT	G_n	$0 \to 10$ cpd/kg/keVee
CRESST	N_α	$5 \to 17$ counts
CRESST	$C_{\rm Pb}$	$1 \to 7$ counts/keV
CRESST	N_n	$3.3 \to 34$ counts
CDMS-Si	N_e	$0 \to 2$
XENON100	$L_{\rm eff}$	$-0.01 \to 0.18$

Background and systematics

Observable	Constraint
Local standard of rest	$v_0^{\rm obs} = 230 \pm 24.4$ km s^{-1}
Escape velocity	$v_{\rm esc}^{\rm obs} = 544 \pm 39$ km s^{-1}
Local DM density	$\rho_\odot^{\rm obs} = 0.4 \pm 0.2$ GeV cm^{-3}
Virial mass	$M_{\rm vir}^{\rm obs} = 2.7 \pm 0.3 \times 10^{12} M_\odot$

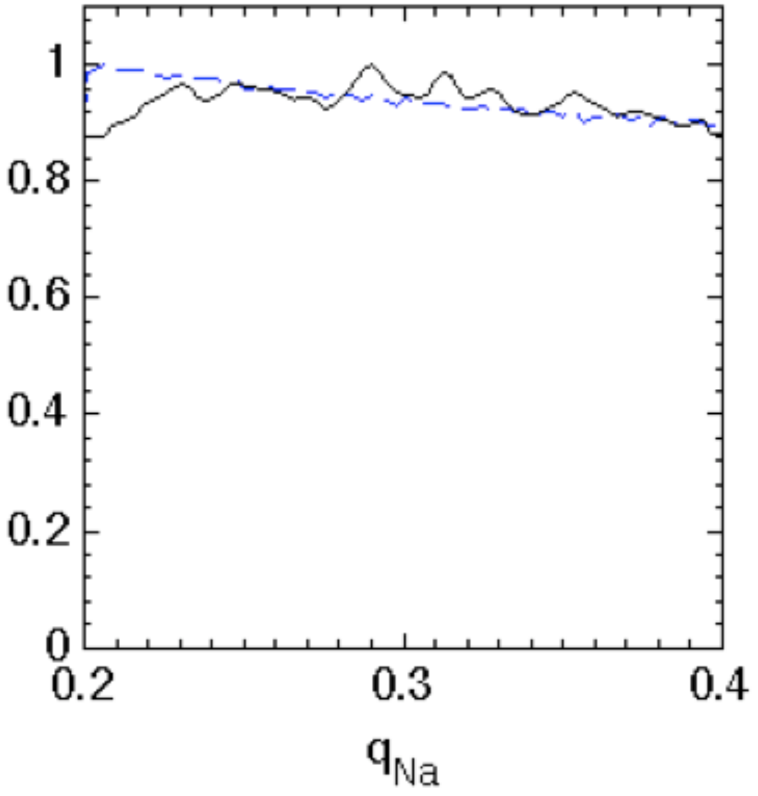
Astrophysical parameters (common to all exp)

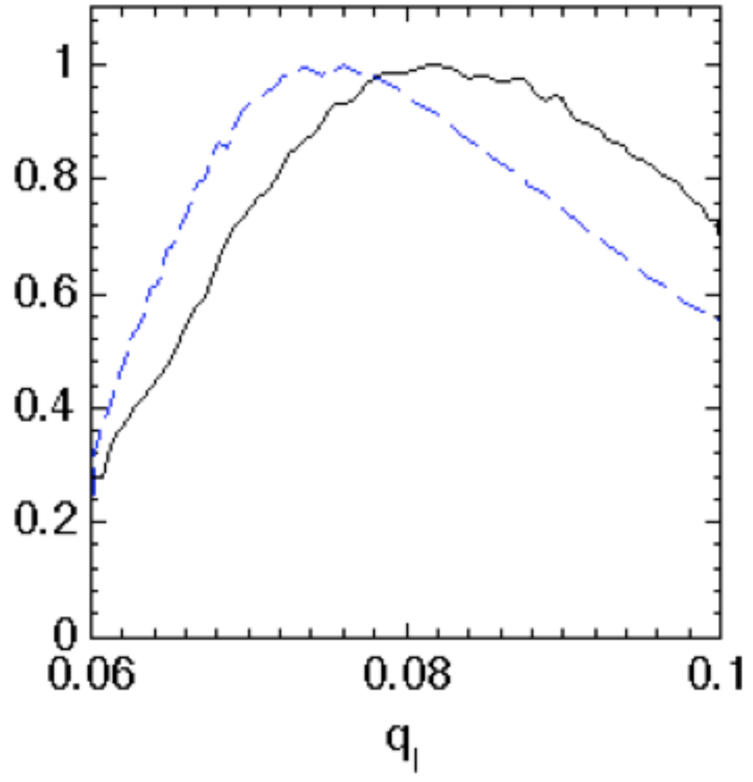
Model	Parameter	Prior
Inelastic	$\delta/({\rm keV})$	$0 \to 300$
Inelastic	$\delta/({\rm keV})$	$-100 \to 0$
Isospin violating	f_n/f_p	$-2 \to 1$

Beyond elastic SI scattering (common to all exp)

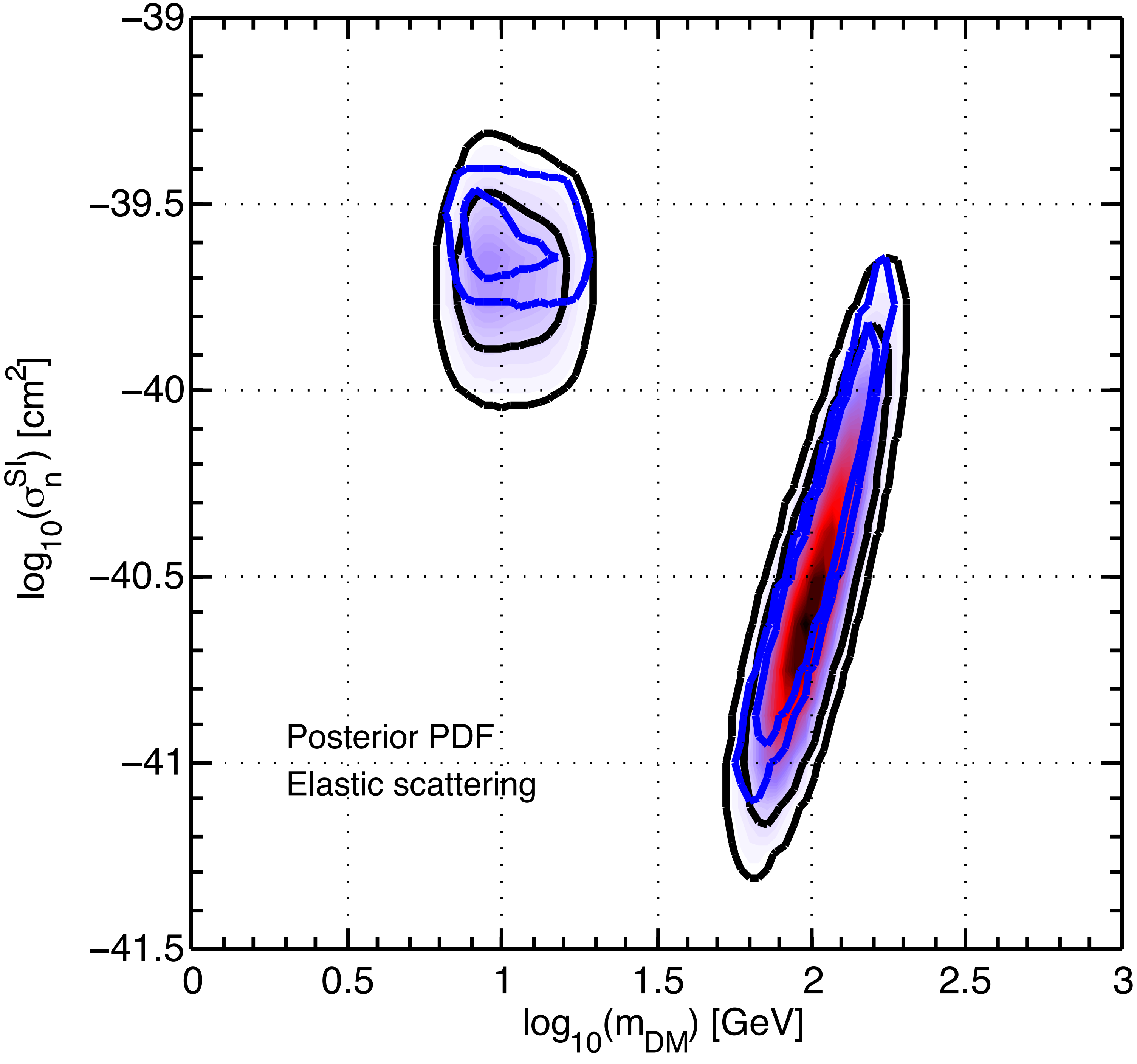
Inference for constraining data , example with DAMA

1D marginalized posterior PDF
quenching factors (nuisance)





2D marginal credible regions at 90 and 99%
(shaded f(v)=NFW, solid blue f(v)= MB)

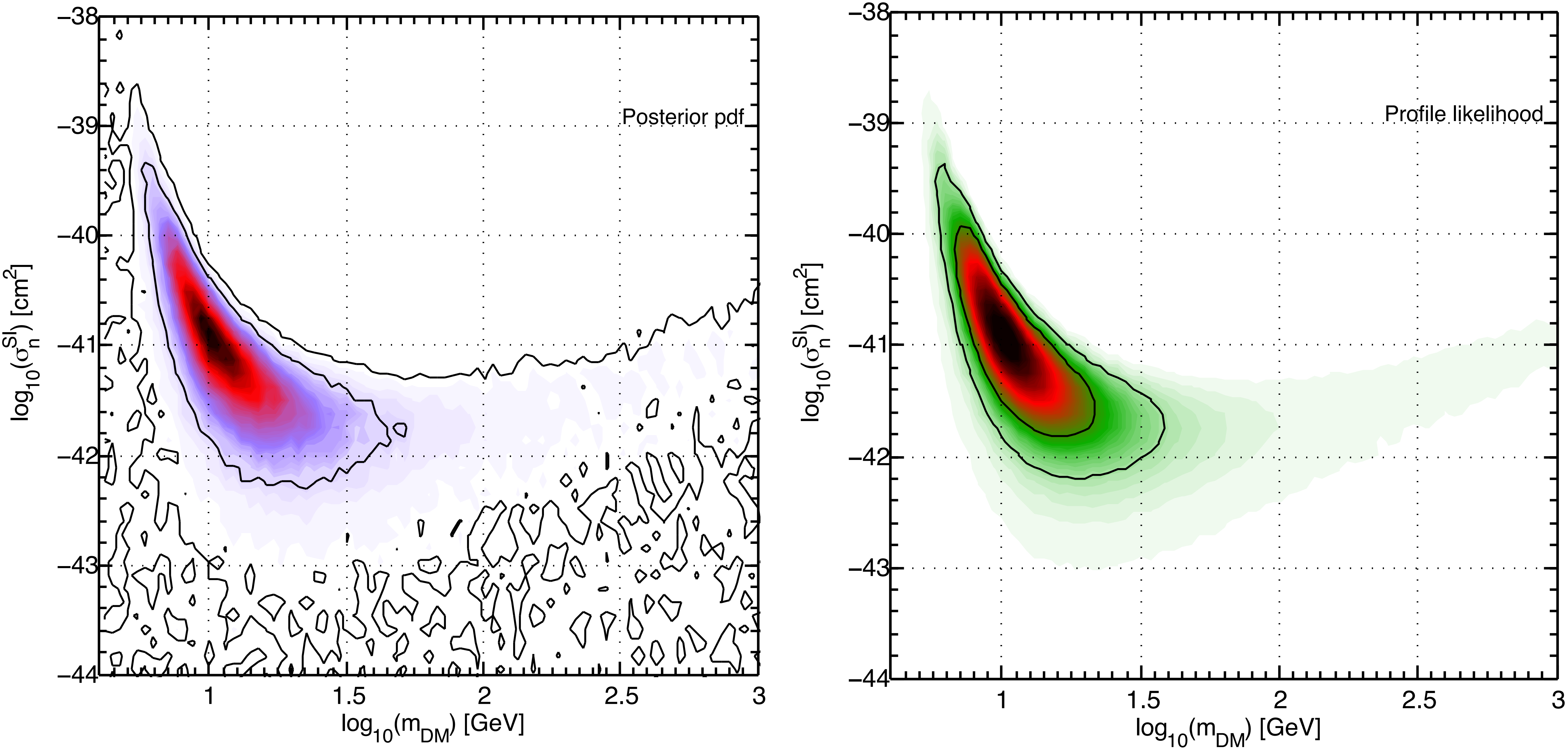


Matches with profile likelihood analysis

Inference for non constraining data: CDMS-Si

data from CDMS-Si collaboration
arXiv:1304.4279

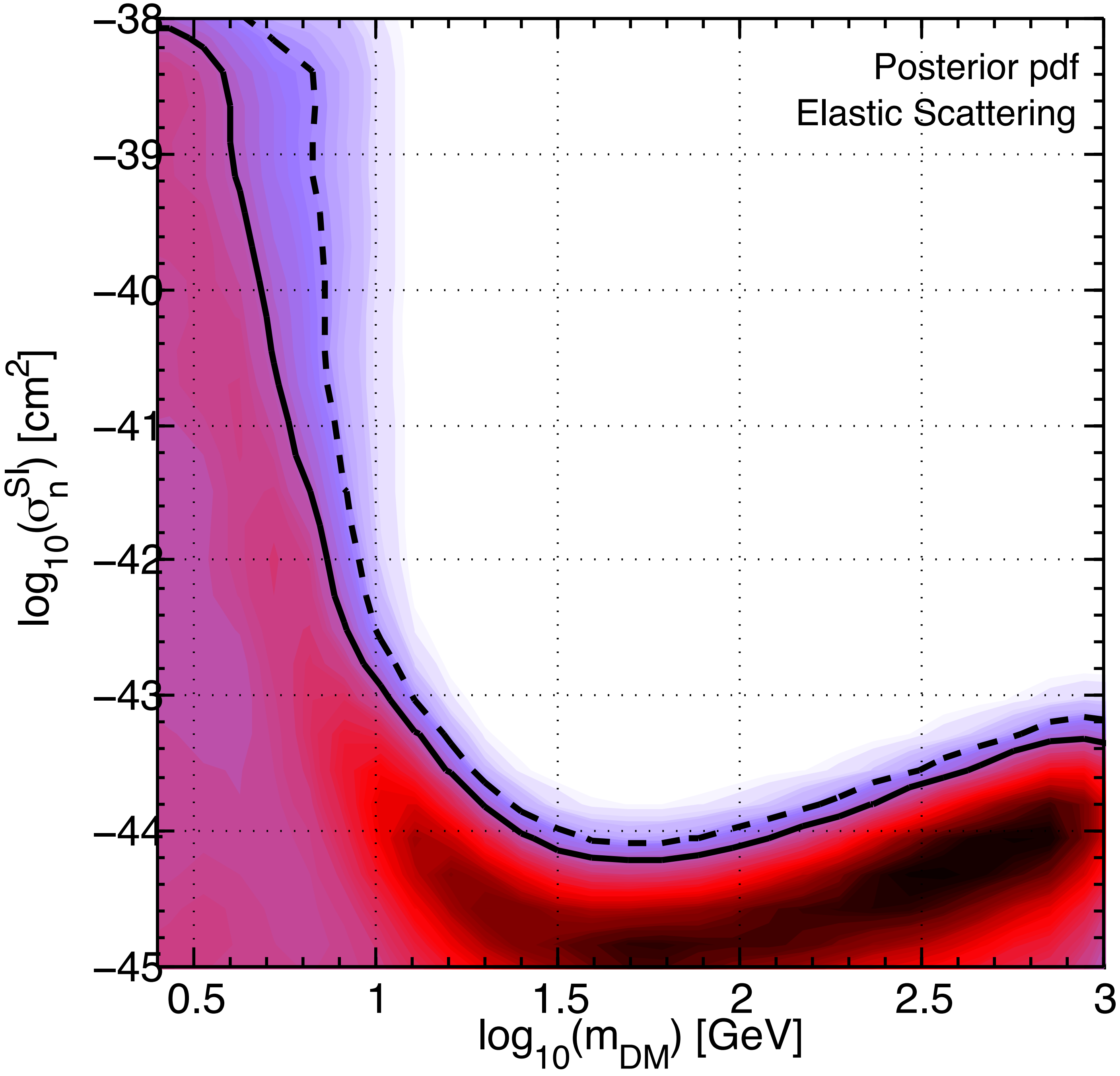
- Likelihood follows a Poisson distribution with spectral information
- 3 events seen with estimated bckg of 0.7: not constraining data



2D marginal credible regions at 68 and 90%for fixed astrophysics

Inference for exclusion bounds: XENON100

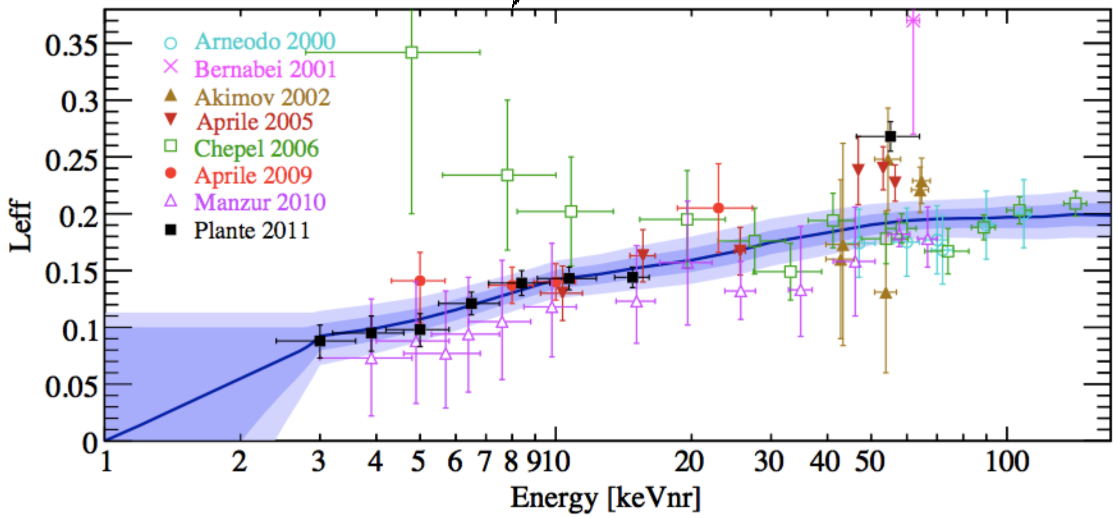
- 2 events seen, likelihood follows a Poisson distribution
- expected background of 1. +- 0.8, analytical marginalization
- considered Poisson fluctuation below threshold



data from XENON100 collaboration, arXiv:1207.5988

$$\ln \mathcal{L}_{\rm Xenon} = \ln \mathcal{L}_{\rm Events} + \ln \mathcal{L}_{\rm L_{eff}}$$

from arXiv:1104.2549



Data are not constraining therefore the upper bound depends on the prior choice:

2D marginal credible regions at 90% +

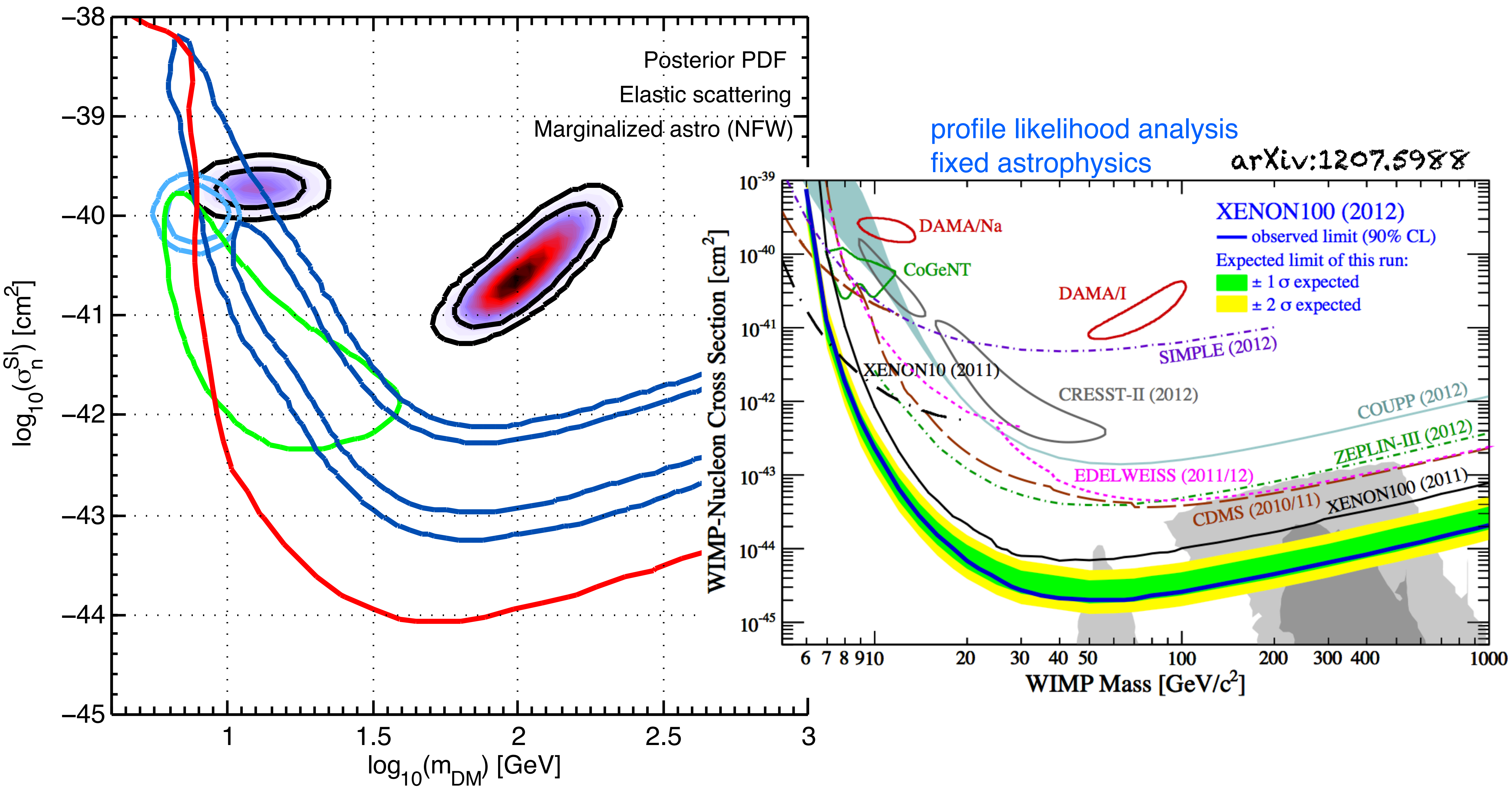
- - - - - - $90_S\%$

Invariant exclusion bound based on the S signal with bayesian interpretation:

$$\Delta\chi^2_{\rm eff} \le 2.7$$

$$\mathcal{P}_{\rm mar}(m_{\rm DM}, \sigma_n^{\rm SI}|X) = \mathcal{P}_{\rm mar}(S_x|X)$$

Inference for elastic SI scattering

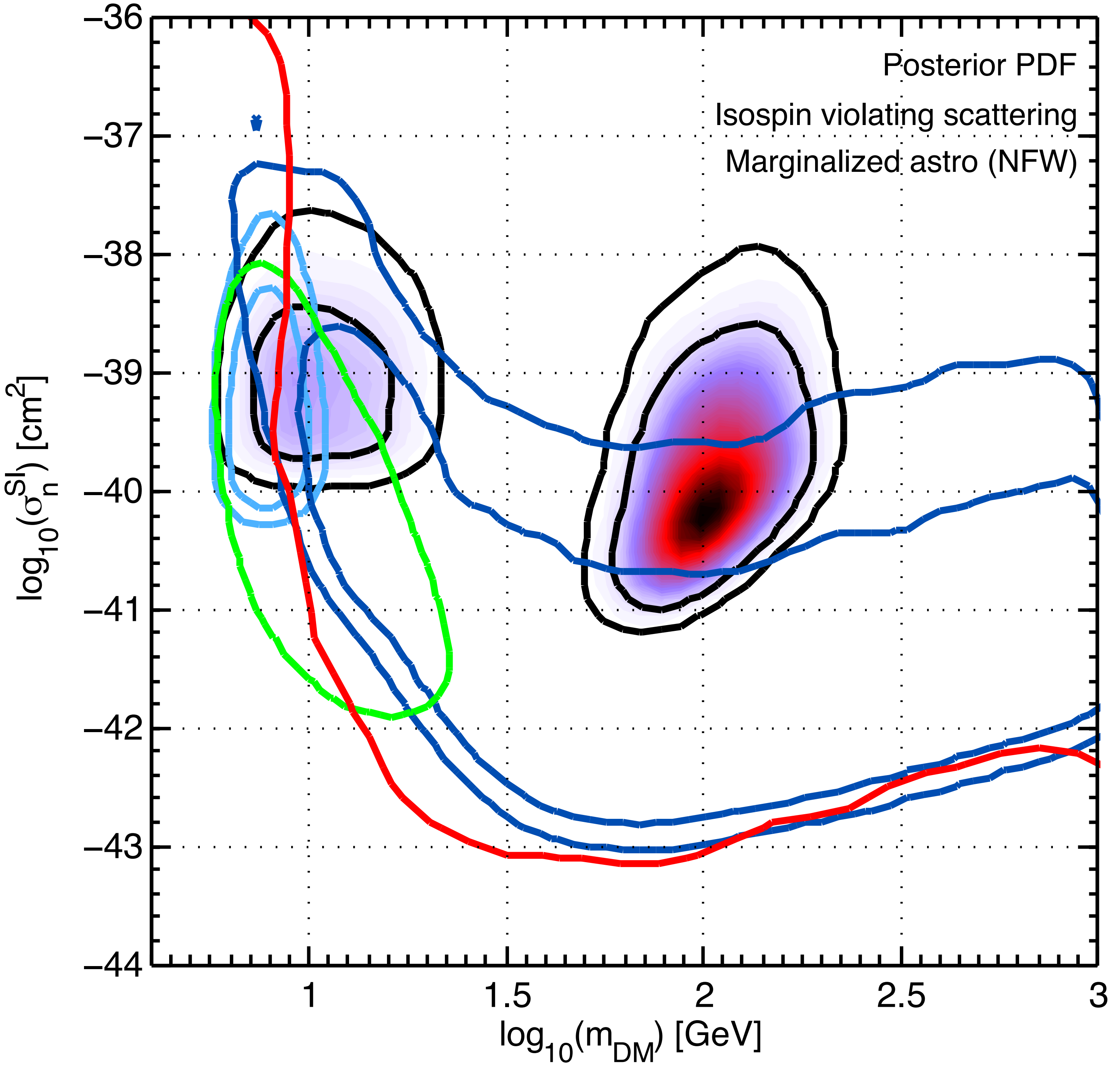


profile likelihood analysis
fixed astrophysics

arXiv:1207.5988

- The marginalization over astrophysics does not improve the compatibility between XENON100 and all detection hints
- The XENON100 bound is less stringent at masses larger than 30 GeV than the one of the collaboration because of the approximate likelihood
- Same analysis can be done with LUX, more difficult to reconcile low mass regions, as its threshold is at 2 PE

Inference for isospin violating scattering



Assumption that interaction of WIMP with proton and neutron is of different strength:

$$f_n \neq f_p$$

$$\frac{d\sigma}{dE} = \frac{M_N \sigma_n^{SI}}{2\mu_n^2 v^2} \frac{\left(f_p Z + (A - Z) f_n\right)^2}{f_n^2} \mathcal{F}^2(E)$$

- The extra parameter is not supported/constrained by current data
- The marginalization over the parameter causes a volume effect: detection regions becomes larger and the exclusion bound moves to the right
- Within the Bayesian approach the hint regions become compatible with the 90% CL of XENON100
- Inelastic and exothermic dark matter have same volume effect, however the agreement between detection regions and exclusion bounds is worst than isospin violating scenario

Bayesian evidence for model comparison

$$\mathcal{P}(\theta \mid X) = \pi(\theta)\,\frac{\mathcal{L}(X|\theta)}{\mathcal{Z}(X)}$$

$$\mathcal{Z} = \int \mathcal{L}(X|\theta)\pi(\theta)d^D\theta$$

Bayesian evidence

1. model averaged likelihood
2. contains notion of Occam's razor principle
3. used for model comparison and statistical test

Posterior pdf for a model:

$$\mathcal{P}(\mathcal{M}|X) \propto \mathcal{Z}\;\pi(\mathcal{M})$$

$$\pi(\mathcal{M}_0) = \pi(\mathcal{M}_1)$$

(non committal prior)

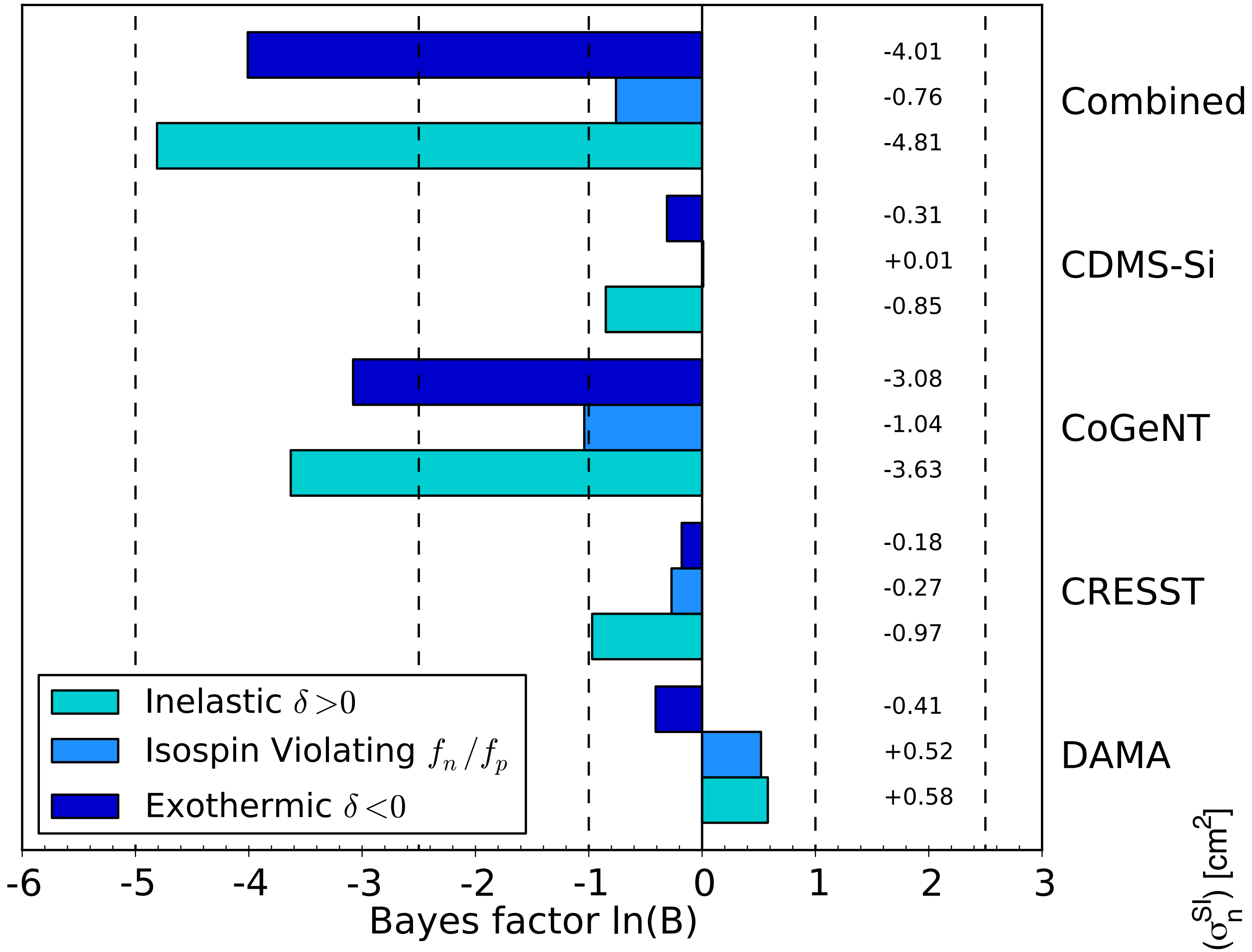
$$\frac{\mathcal{P}(\mathcal{M}_0|X)}{\mathcal{P}(\mathcal{M}_1|X)} = \boxed{B_{01}}\,\frac{\pi(\mathcal{M}_0)}{\pi(\mathcal{M}_1)}$$

Bayes factor: ratio of model's evidences

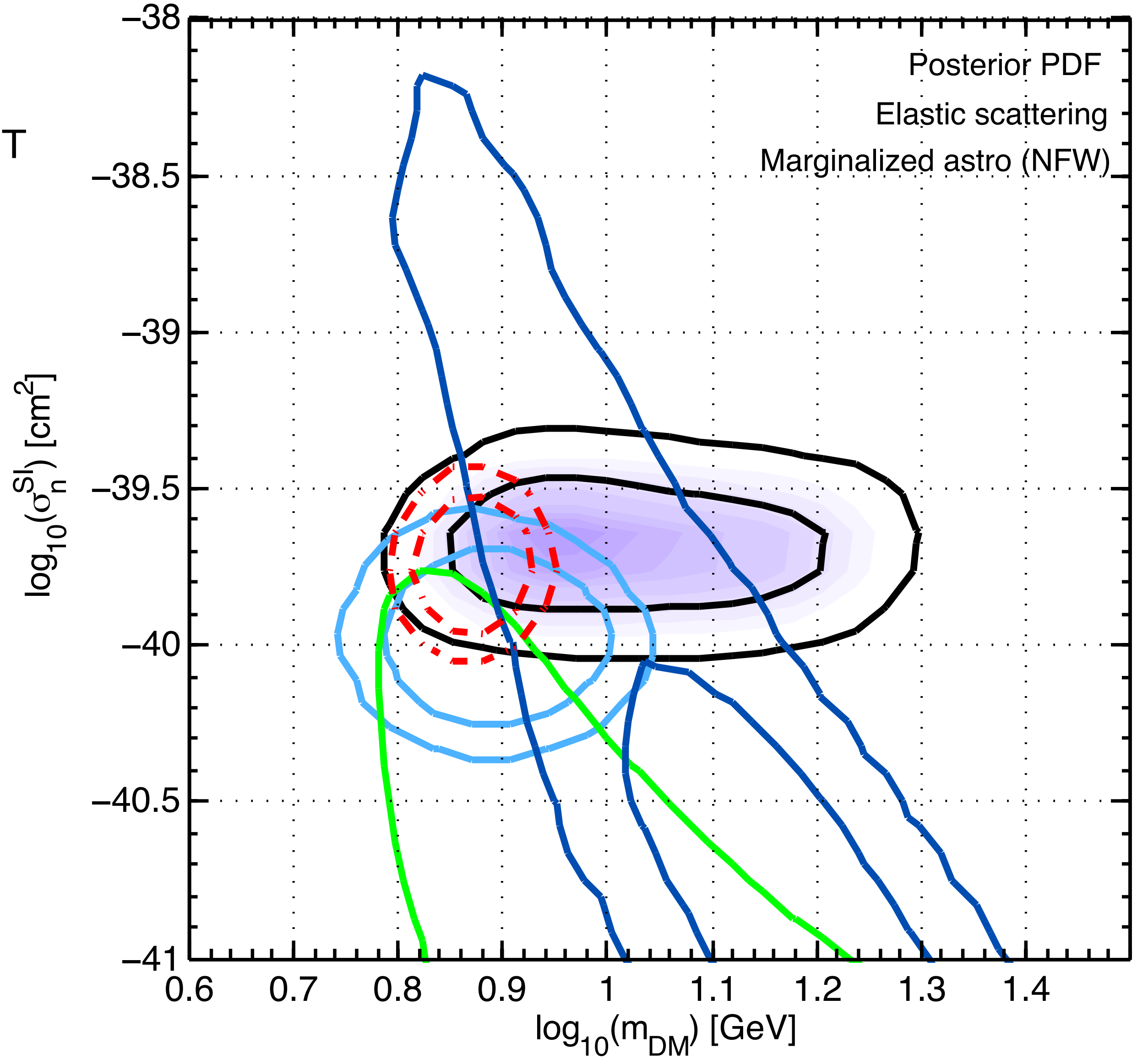
Empirical Jeffreys' scale

$\ln B_{10}$	Odds $\mathcal{M}_1 : \mathcal{M}_0$	Strength of evidence
< -5.0	$< 1 : 150$	Strong evidence for $\mathcal{M}_0$
$-5.0 \rightarrow -2.5$	$1 : 150 \rightarrow 1 : 12$	Moderate evidence for $\mathcal{M}_0$
$-2.5 \rightarrow -1.0$	$1 : 12 \rightarrow 1 : 3$	Weak evidence for $\mathcal{M}_0$
$-1.0 \rightarrow 1.0$	$1 : 3 \rightarrow 3 : 1$	Inconclusive
$1.0 \rightarrow 2.5$	$3 : 1 \rightarrow 12 : 1$	Weak evidence against $\mathcal{M}_0$
$2.5 \rightarrow 5.0$	$12 : 1 \rightarrow 150 : 1$	Moderate evidence against $\mathcal{M}_0$
> 5.0	$> 150 : 1$	Strong evidence against $\mathcal{M}_0$

Combined fit and model comparison



- Evidence between elastic and isospin violating scenarios is inconclusive
- Inelastic and exothermic moderately disfavored with respect to elastic SI



Combined fit pushes the quenching factor and the local standard at rest at corner values (1D pdf not flat anymore but peaked at q_{Na}=0.6)

(In)Compatibility between XENON100 and detection hints ?

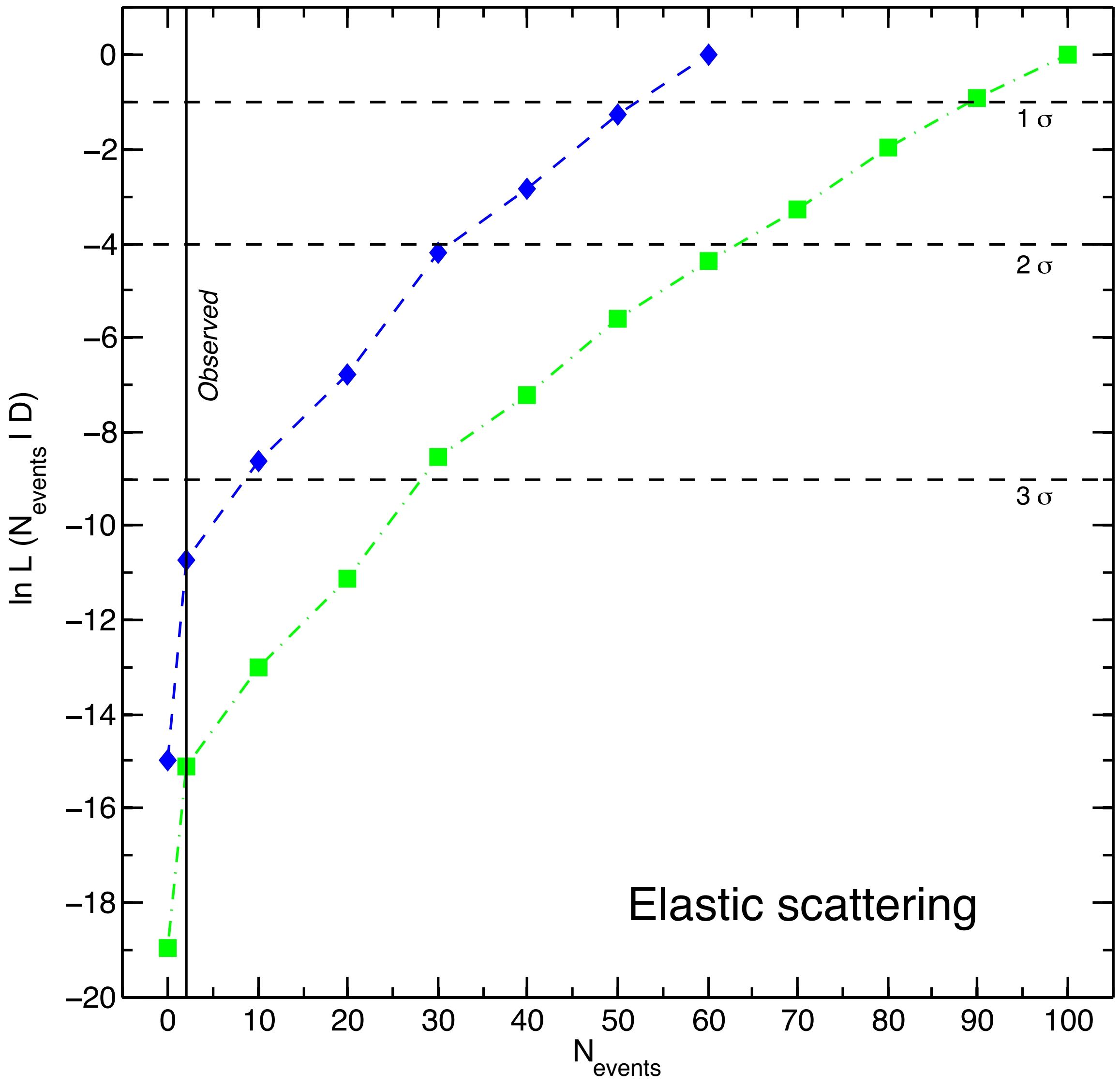
Consider a data set which is given by two subsets: $d = \{\mathscr{D}, D\}$

1. D = this is the data set which is taken as true, hence fixed

2. $\mathscr{D}$ = data set that we want to test, that is we want to quantify if it is compatible with D

$$\mathscr{R}(\mathscr{D}^{\rm obs}) = \frac{p(\mathscr{D}^{\rm obs}, D|\mathcal{H}_0)}{p(\mathscr{D}^{\rm obs}|\mathcal{H}_1)p(D|\mathcal{H}_1)}$$

$\ln \mathscr{R}(N_{\rm obs} = 2)$	Interpretation
-0.32 ± 0.07	Inconclusive evidence against $\mathcal{H}_0$
-0.53 ± 0.07	Inconclusive evidence against $\mathcal{H}_0$
-0.22 ± 0.07	Inconclusive evidence against $\mathcal{H}_0$



$$\mathscr{L}(\mathscr{D}^{\rm obs}|D) = \frac{p(\mathscr{D}^{\rm obs}|D)}{p(\mathscr{D}^{\rm max}|D)} = \frac{p(\mathscr{D}^{\rm obs}, D)}{p(\mathscr{D}^{\rm max}, D)}$$

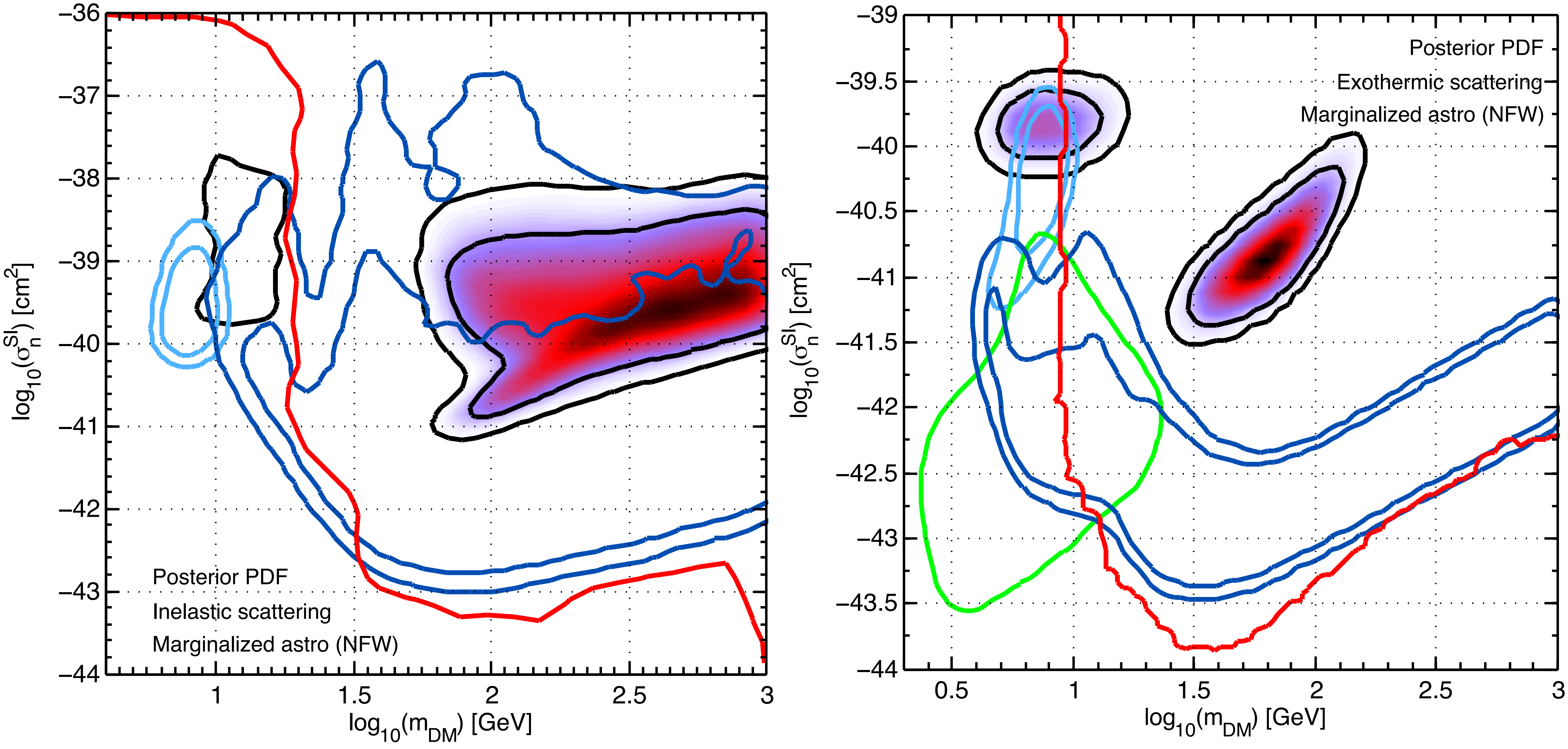
Isospin violating framework: likelihood ratio in data space gives incompatibility at 2σ

Summary

- Bayesian approach for XENON100, DAMA, CoGeNT, CRESST and CDMSI-Si data with marginalization over the systematics and nuisance parameters characteristic of each experiment (can be applied to LUX, similar to XENON100 procedure)
- Inclusion of velocity distributions arising from DM density profile and marginalization over astrophysical variables (NFW)
- Difficult to reconcile at 90% CL all detection hints and XENON100
- Going beyond the elastic SI scattering (isospin violating, inelastic and exothermic scattering) ameliorates the compatibility between experiments: the additional physics parameter is not constrained by the current data
- Astrophysical uncertainties can not be yet constrained by direct detection experiment alone (however combined fit can constrain astrophysics)
- Combined fit implies large value of the quenching factor on Sodium for DAMA and small local standard of rest velocity
- For hints of detection the elastic and isospin violating scenarios have the strongest support form the data; isospin violating framework ameliorate the compatibility between hints of detection and exclusion bounds

Back up slides

Inference for inelastic/exothermic SI scattering



Changing the WIMP physics interaction

Isospin violating interaction

Feng et al. '11

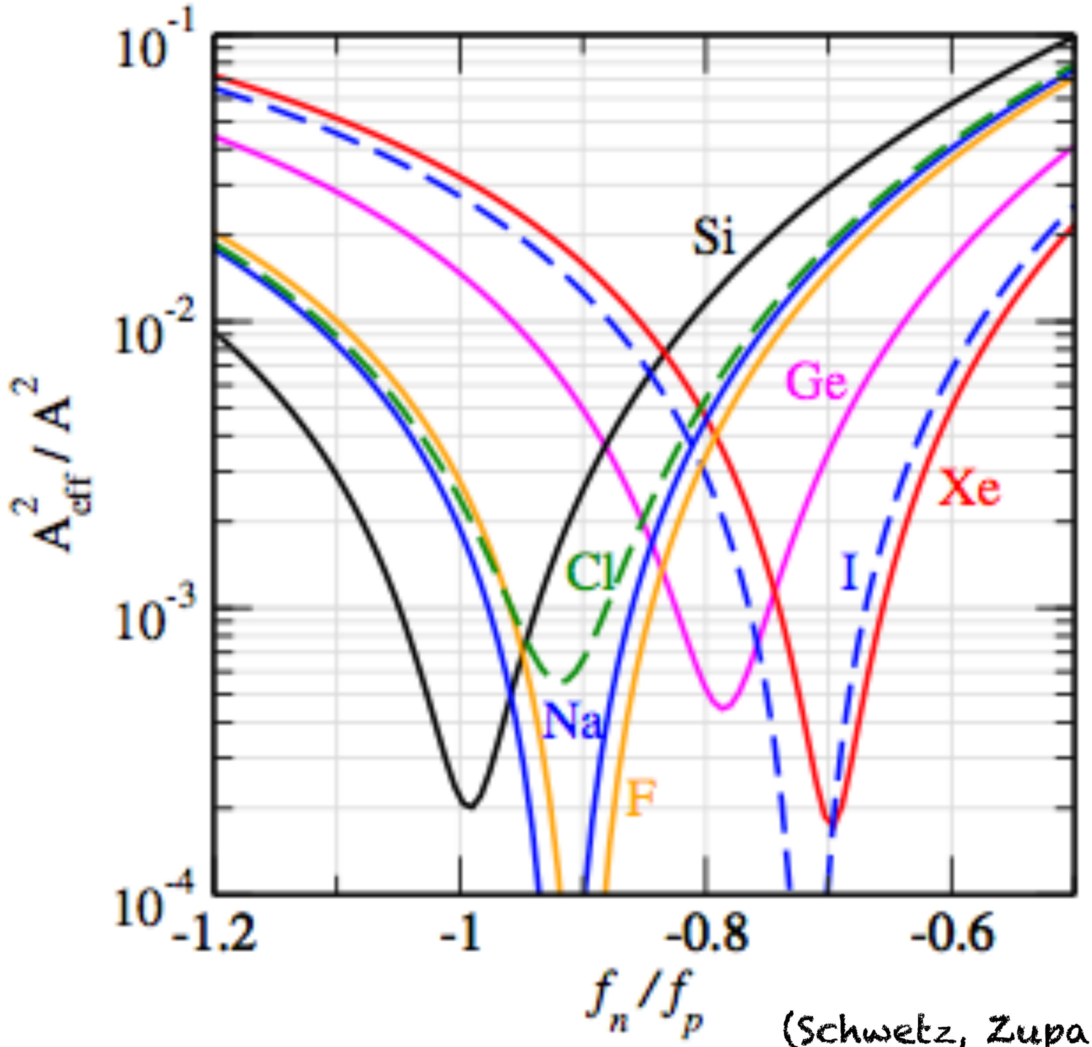
- Assumption that interaction of WIMP with proton and neutron is of different strength:

$$f_n \neq f_p$$

- Defined a mean SI cross-section with an effective couplings to nuclei:

$$\sigma^{\rm SI} = \frac{\sigma_n^{\rm SI} + \sigma_p^{\rm SI}}{2}$$

$$A^2_{\rm eff} = \sum_{i=isotopes} 2r_i\,[Zf_p + (A_i - Z)f_n]^2$$



(Schwetz, Zupan '11)

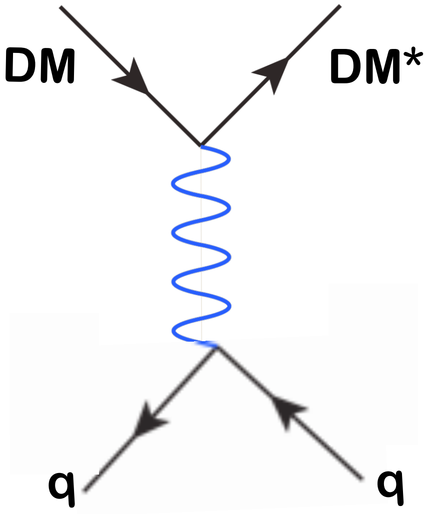
- Example of realization in WIMPs model: The couplings neutralino-squark-quark violate isospin, however in the most common scenarios they are not the dominant contributions to elastic scattering

- Other possibilities: long range interactions, inelastic scattering, spin-dependent interaction

Inelastic scenario

Tucker-Smith, Weiner '01

$$\mathrm{DM\,q} \to \mathrm{DM}^\star\,\mathrm{q}$$



$$v'_{\min} = \sqrt{\frac{1}{2M_{\mathcal{N}}E_R}}\left(\frac{M_{\mathcal{N}}E_R}{\mu_n} + \delta\right)$$

$$\Delta m \equiv (m_{\mathrm{DM}^\star} - m_{\mathrm{DM}}) = \delta$$

- If the splitting is positive the DM scatters into an heavier state: kinematic condition implies that the scatter occurs most probably with heavy nuclei (hence more sensitive to heavy WIMPs)

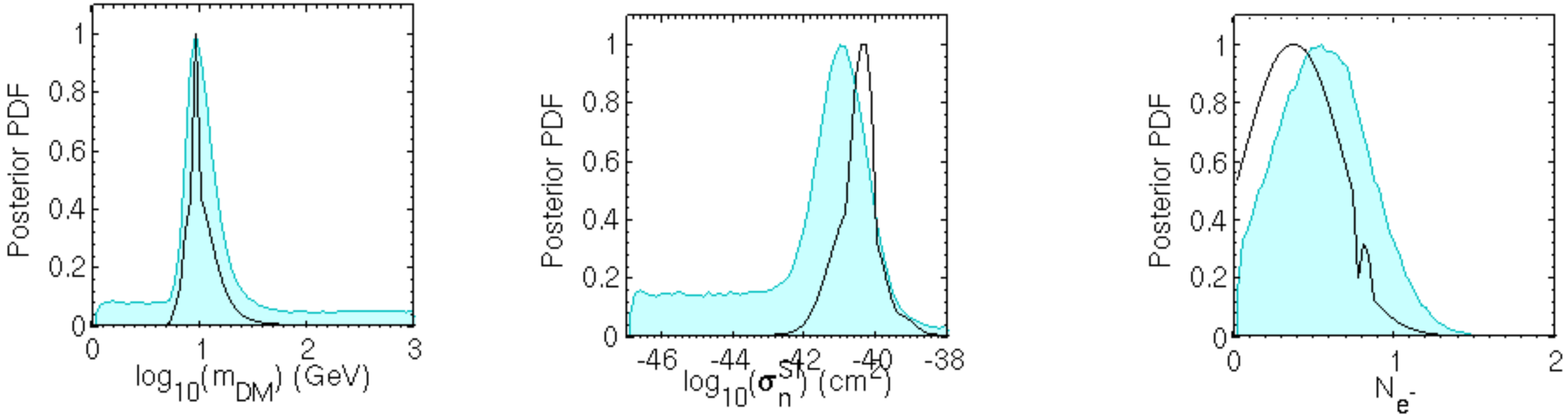
- If the splitting is negative exothermic Dark Matter, it decays into a lighter states and light target are favoured

SI elastic scattering scenario CDSM-Si

data from CDMS-Si collaboration
arXiv:1304.4279

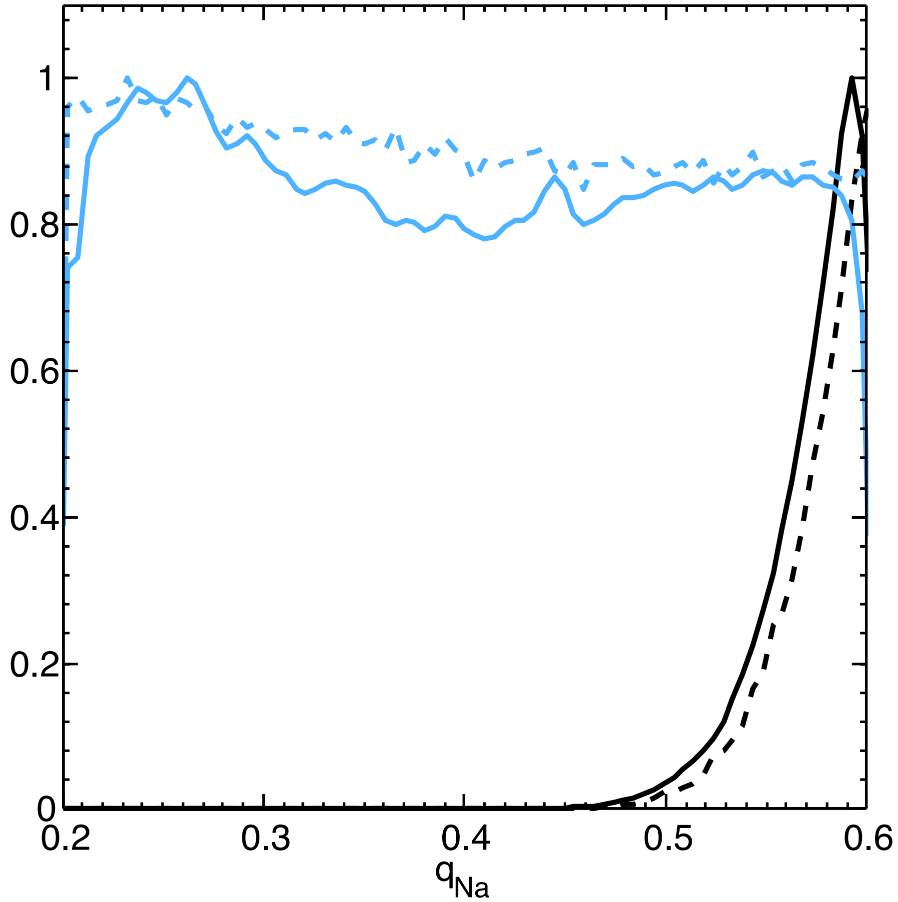
$$\ln \mathcal{L}_{CDMSSi} = \left[\sum_{i=1}^{3} P_1(S+B)\right] + \left[\sum_{j} P_0(S+B)\right] + \ln \mathcal{L}_{bck}$$

1D marginalized posterior PDF for all parameters:



Combined fit more details

combined fit pushes the quenching factor and the local standard at rest at corner values (1D pdf not flat anymore but peaked at qNa=0.6)



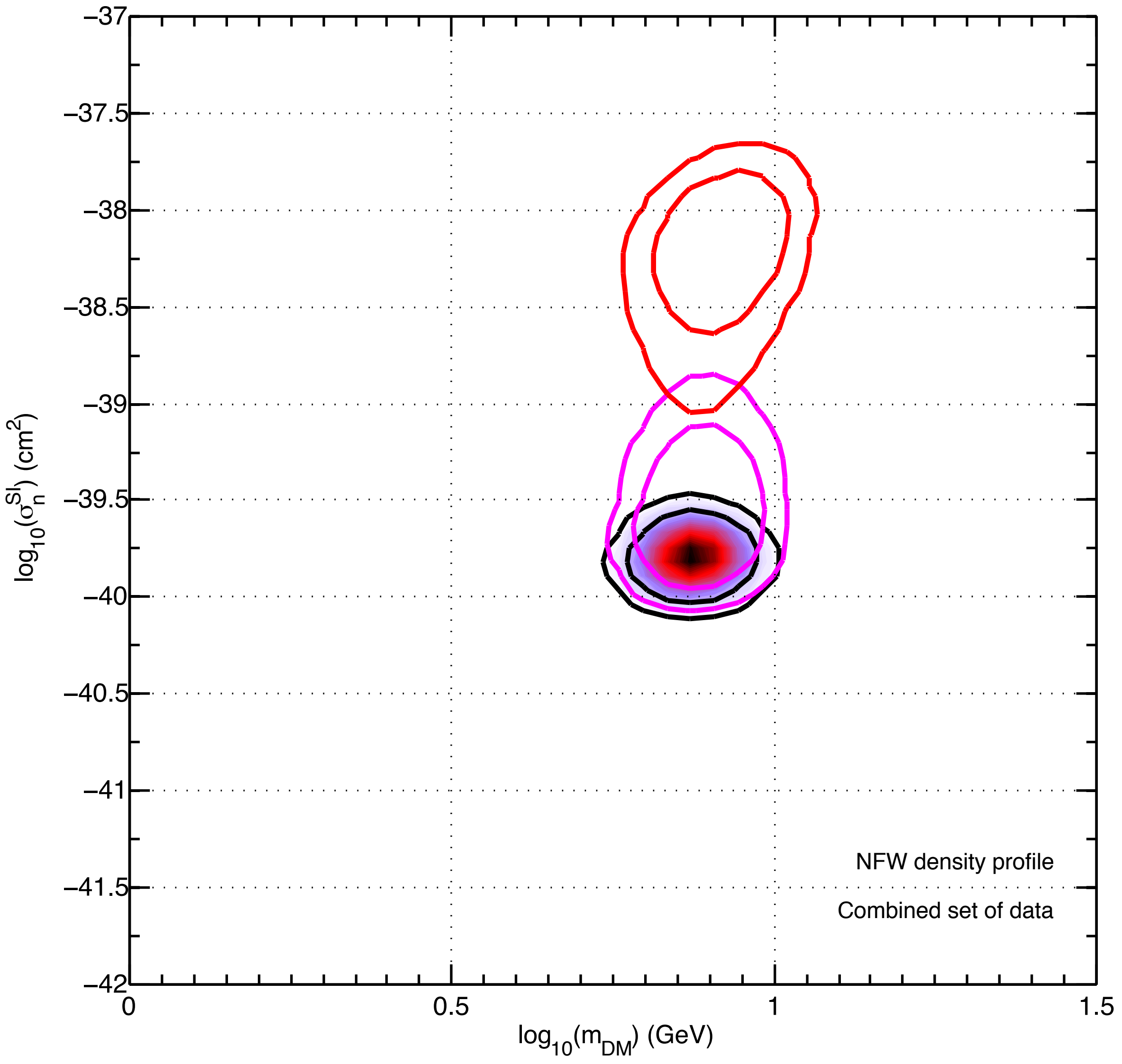
Combined (elastic SI)

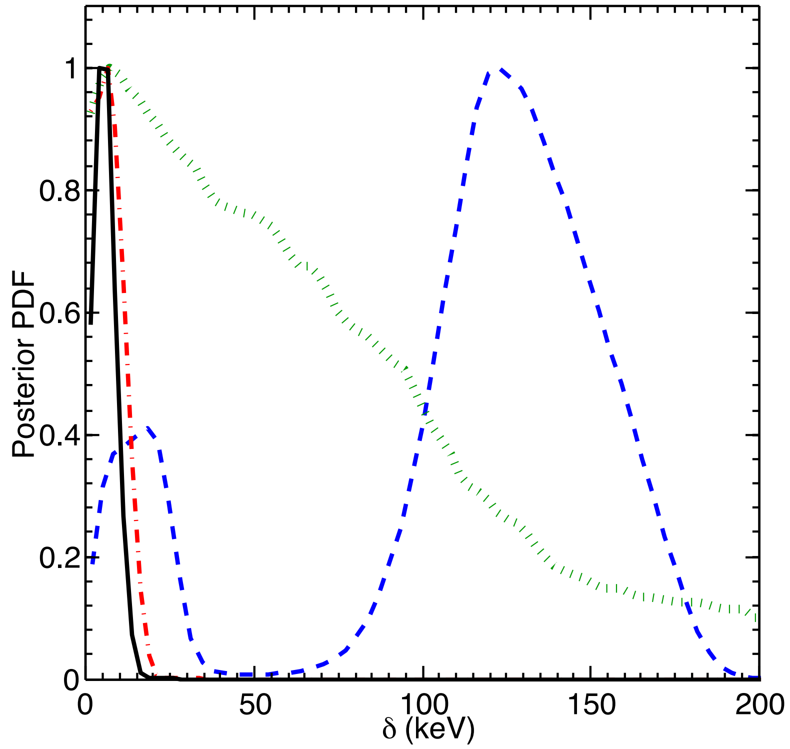
$$v_0 = 214^{+32}_{-21}\ (\mathrm{km\ s^{-1}})$$
$$v_{\rm esc} = 556^{+14}_{-15}\ (\mathrm{km\ s^{-1}})$$
$$\rho_\odot = 0.35^{+0.14}_{-0.09}\ (\mathrm{GeV\ cm^{-3}})$$

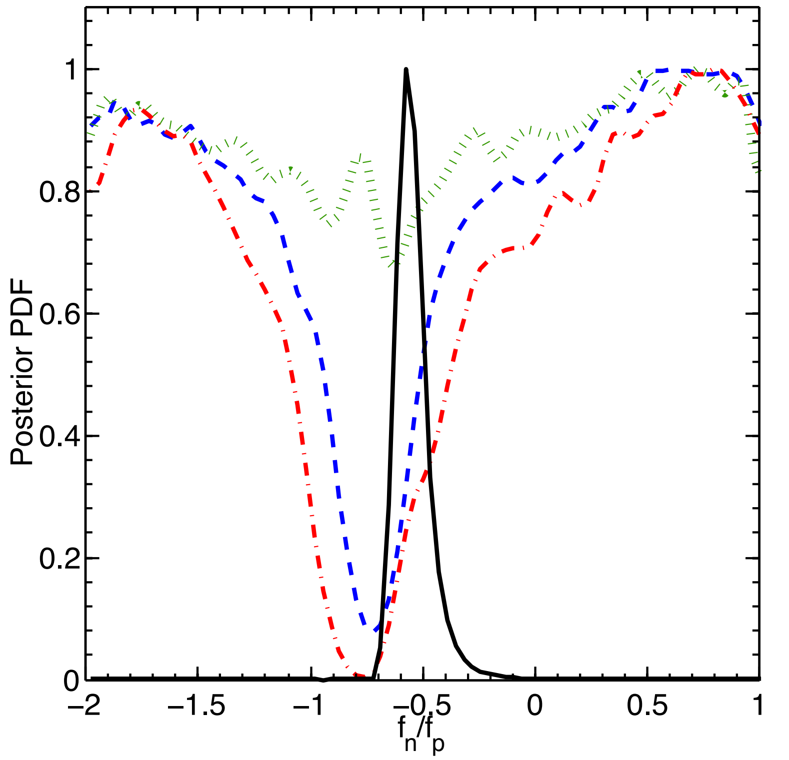
Single experiments

$$v_0^{\rm obs} = 219^{+39}_{-22}\ \mathrm{km\ s^{-1}}$$
$$v_{\rm esc}^{\rm obs} = 557^{+18}_{-15}\ \mathrm{km\ s^{-1}}$$
$$\rho_\odot^{\rm obs} = 0.37^{-0.1}_{+0.15}\ \mathrm{GeV\ cm^{-3}}$$

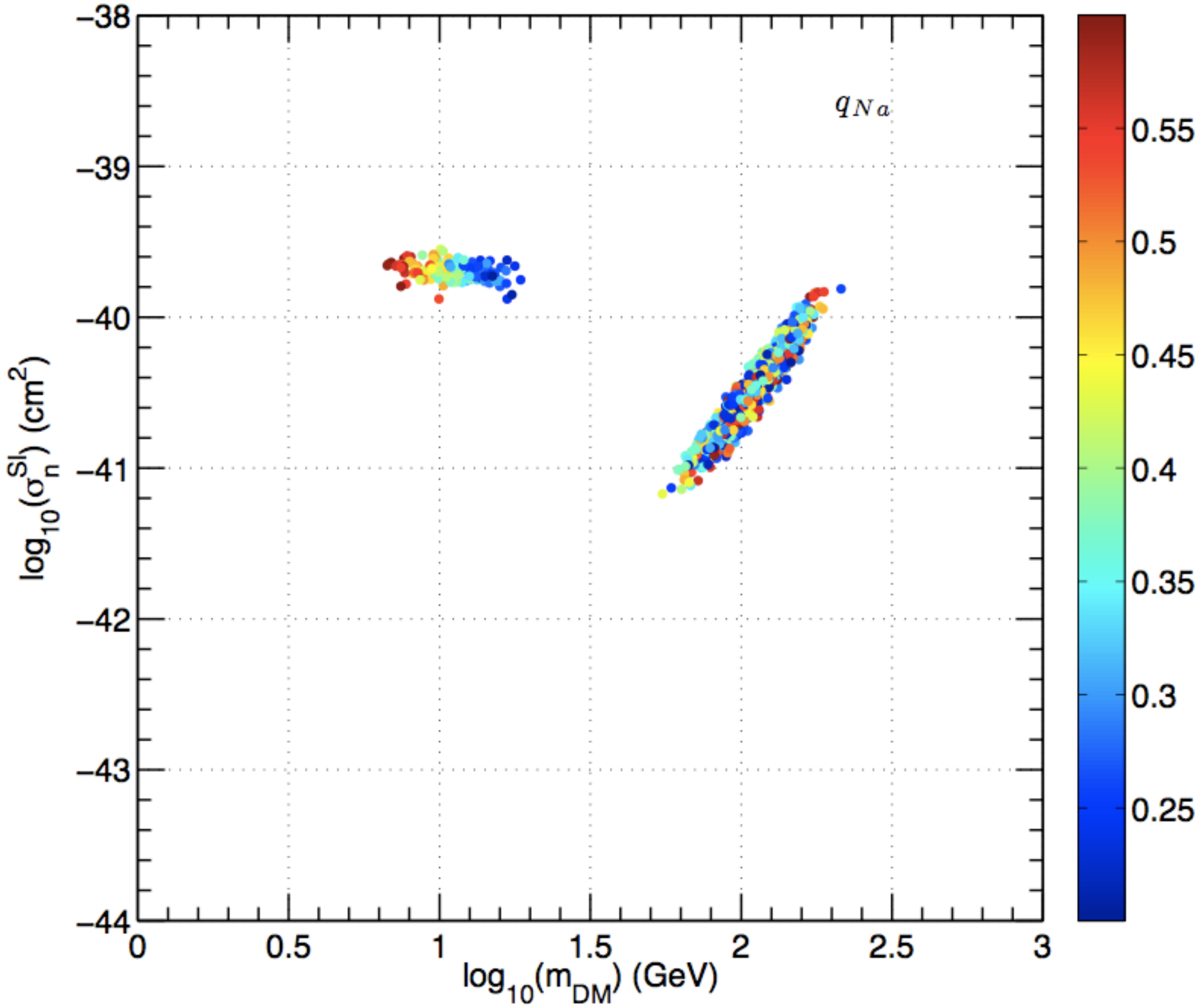
Combined fit more details



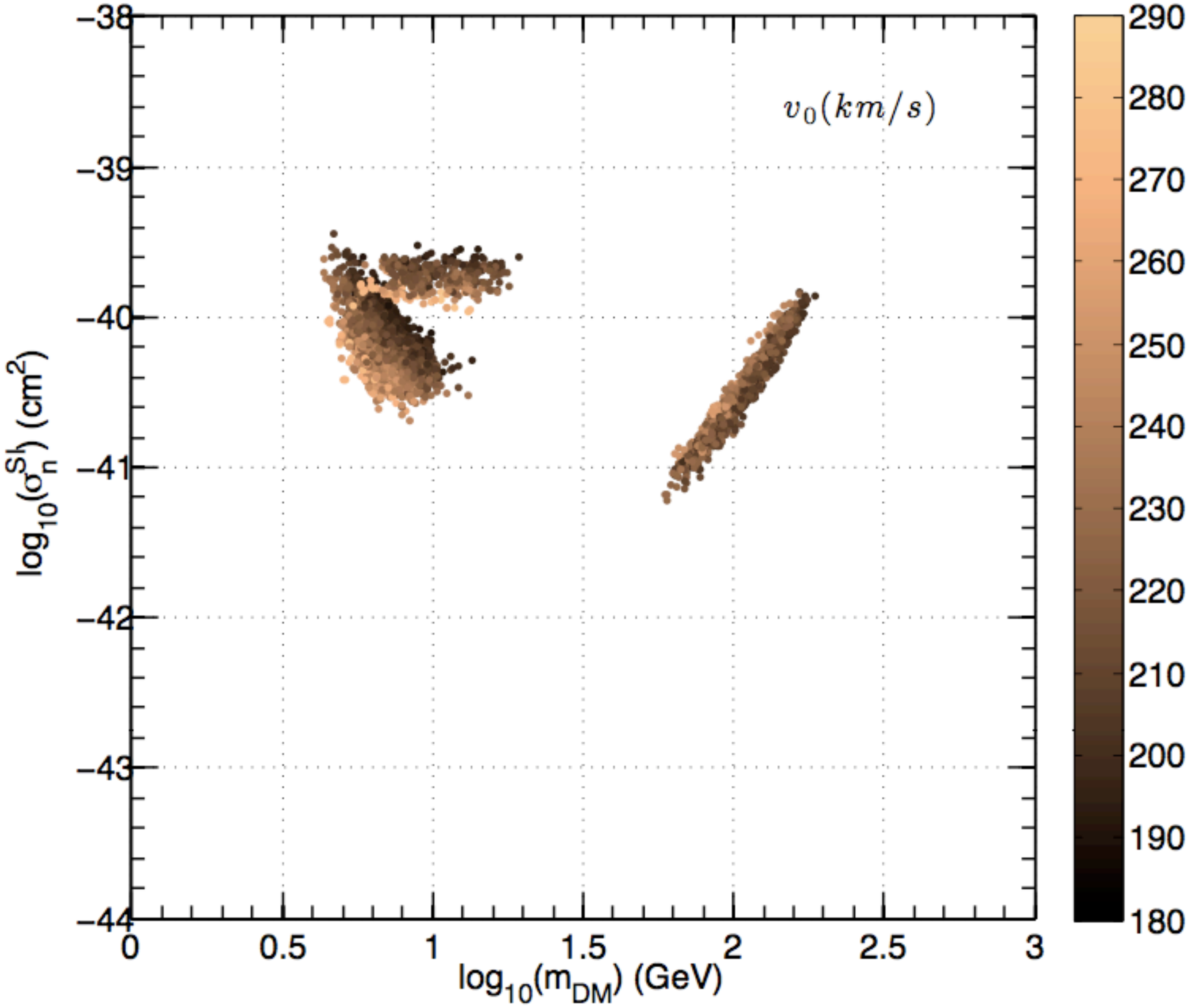




DAMA and CoGeNT, combined fit: hidden directions behavior



- the larger q_{Na} the smaller the WIMP mass
- low mass region is independent on q_I



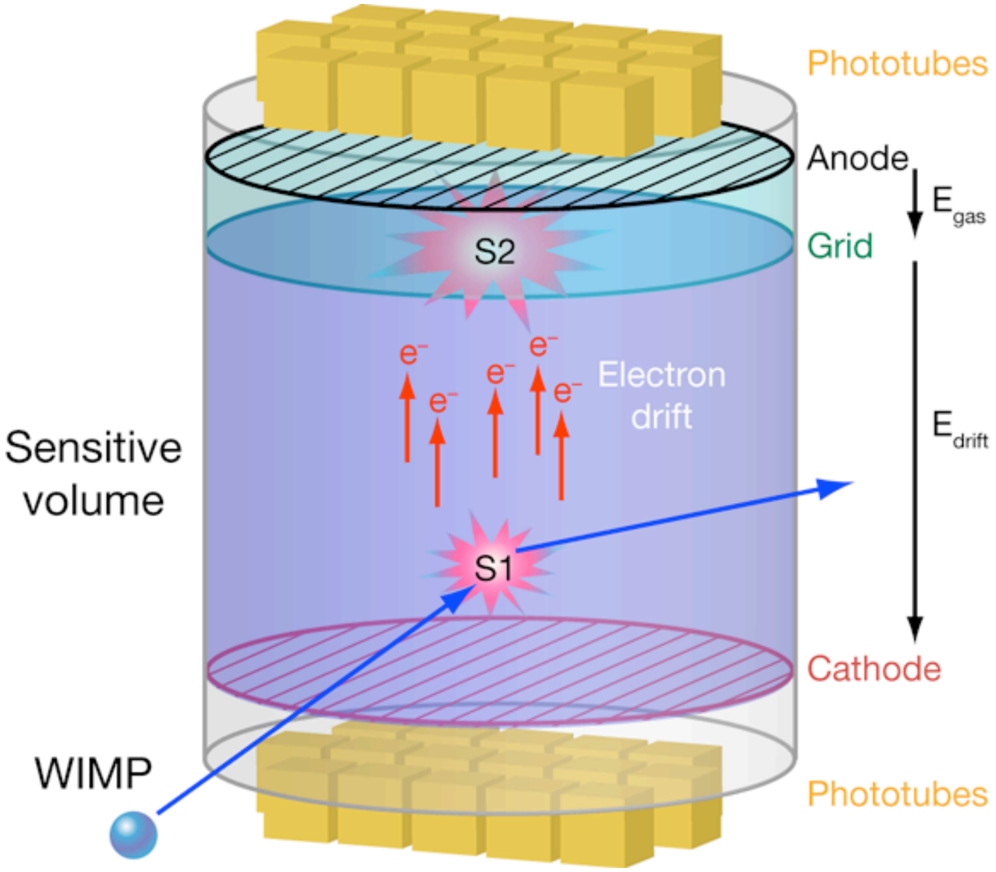
- similar behavior for the DM density at the sun position
- less sensitive to the escape velocity value

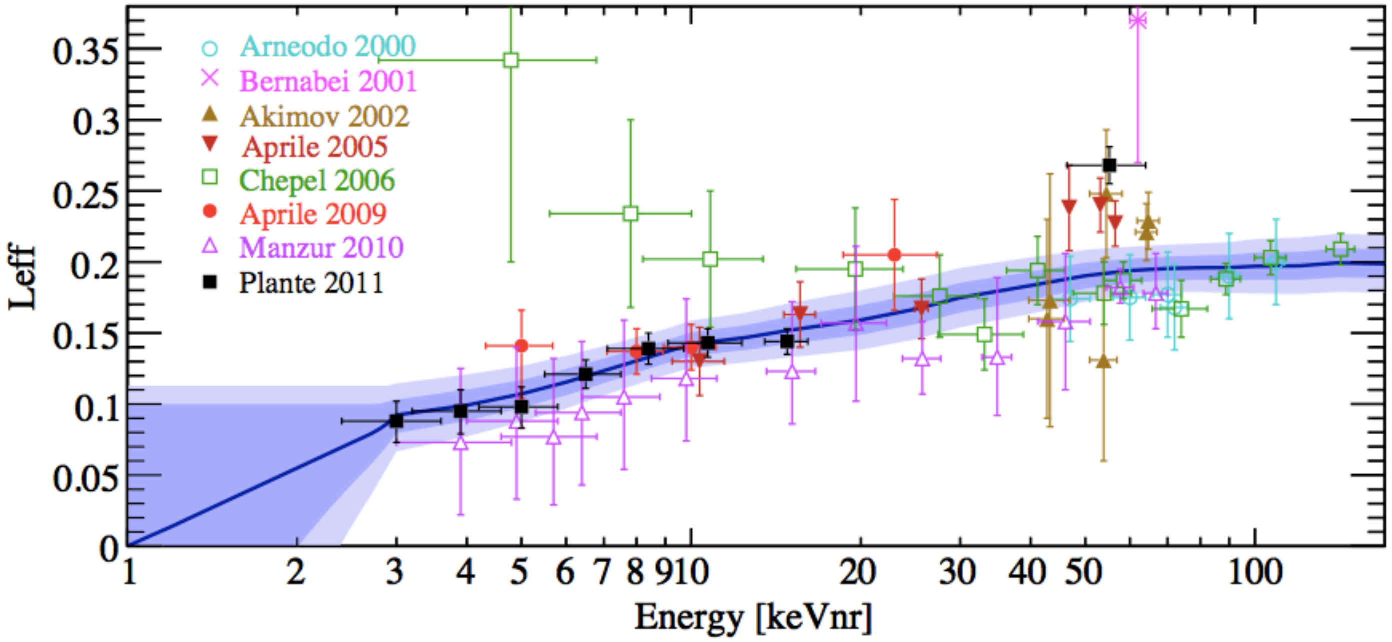
XENON100

XENON100 collaboration, arXiv:1207.5988
Aprile et al. arXiv:1104.2549

- S = 2 (seen events), likelihood follows a Poisson distribution
- B = 1. +- 0.8
- Total exposure 2323.7 kg days

$$\ln \mathcal{L}_{\mathrm{Xenon}} = \ln \mathcal{L}_{\mathrm{Events}} + \ln \mathcal{L}_{\mathrm{L_{eff}}}$$





$$S_1(E) = \mathrm{L_{eff}}(E)\ \mathrm{L}_y\ E\ \frac{S_{\mathrm{nr}}}{S_{\mathrm{ee}}}$$

- Scintillation efficiency is a systematic of the experimental set-up
- treated as nuisance parameter with truncated gaussian prior and marginalized over

XENON100

$$\ln \mathcal{L}_{\rm events}(N_{\rm obs}|S,B) = -S - \bar{B} + \tfrac{\sigma_B^2}{2} + 2 + \ln\left[\tfrac{\sigma_B^2+(S+\bar{B}-\sigma_B^2)^2}{4}\right] ;$$

$$\ln \mathcal{L}_{\rm L_{eff}} = -\frac{(m-\bar{m})^2}{2\sigma_m^2}$$

$$\mathrm{L_{eff}}(E) = \begin{cases} \bar{\mathrm{L}}_{\rm eff}(E), & E \geq 3 \text{ keVnr}, \\ \max\{m[\ln(E/\text{keVnr}) - \ln 3] + 0.09,\ 0\}, & 1 < E/\text{keVnr} < 3 \end{cases}$$

$$S_1(E) = \mathrm{L_{eff}}(E)\ \mathrm{L}_y\ E\ \frac{S_{\rm nr}}{S_{\rm ee}}$$

conversion between keVnr and PE

$$S = M_{\rm det} T \sum_{n=\mathrm{PE_{min}}}^{\mathrm{PE_{max}}} \frac{\mathrm{d}R}{\mathrm{d}S_1}$$

$$\mathrm{d}N_B/\mathrm{d}S_1 = 0.069/(1481 \text{ kg days})$$

$$\frac{\mathrm{d}R}{\mathrm{d}S_1} = \int_0^\infty \mathrm{d}E\, \frac{\mathrm{d}R}{\mathrm{d}E} \times P(S_1|\bar{S}_1(E))$$

All the likelihoods are normalized such that $\ln \mathcal{L} = 0$ if the background matches exactly the number of observed events

CoGeNT 2011

Germanium cryogenic detector
detector mass 0.33 kg
live time 442 days
total exposure 145.86 kg days

- Data analysis and binning follow arXiv:1106.0650 [astro-ph.CO]
- Radioactive peaks subtracted as prescribed by the collaboration
- Analysis of the total rate with a background (27 bins)
- Analysis of the modulated rate without background in 3 energy bins
- All data are corrected by the efficiency factor, ranging from 0.7 to 0.82

$$\ln \mathcal{L}_{\rm TR} = -\frac{\chi^2}{2} = -\sum_{i=1}^{27} \frac{((S_i + b_i) - C_i)^2}{2\sigma_i^2}$$

$$\ln \mathcal{L}_{MR} = -\frac{\chi^2}{2} = -\sum_{j=1}^{3} \frac{(S^i_{\rm theo} - S^i_{\rm m})^2}{2\sigma_i^2}$$

Total rate : 27 bins of width 0.1 keVee
energy range 0.5- 3.2 keVee

Modulated rate:

ΔE_i (keVee)	S_m (cpd/kg/keVee)
$0.5 - 0.9$	1.10 ± 0.39
$0.9 - 3.0$	0.60 ± 0.12
$3.0 - 4.5$	0.07 ± 0.9

3 nuisance parameters for the non modulating background

$$b_i = \frac{1}{\Delta_b} \int_{\mathcal{E}_i}^{\mathcal{E}_{i+1}} \frac{dB}{d\mathcal{E}} d\mathcal{E}$$

$$\frac{dB}{d\mathcal{E}} = C + A \exp(-\mathcal{E}/\mathcal{E}_0)$$

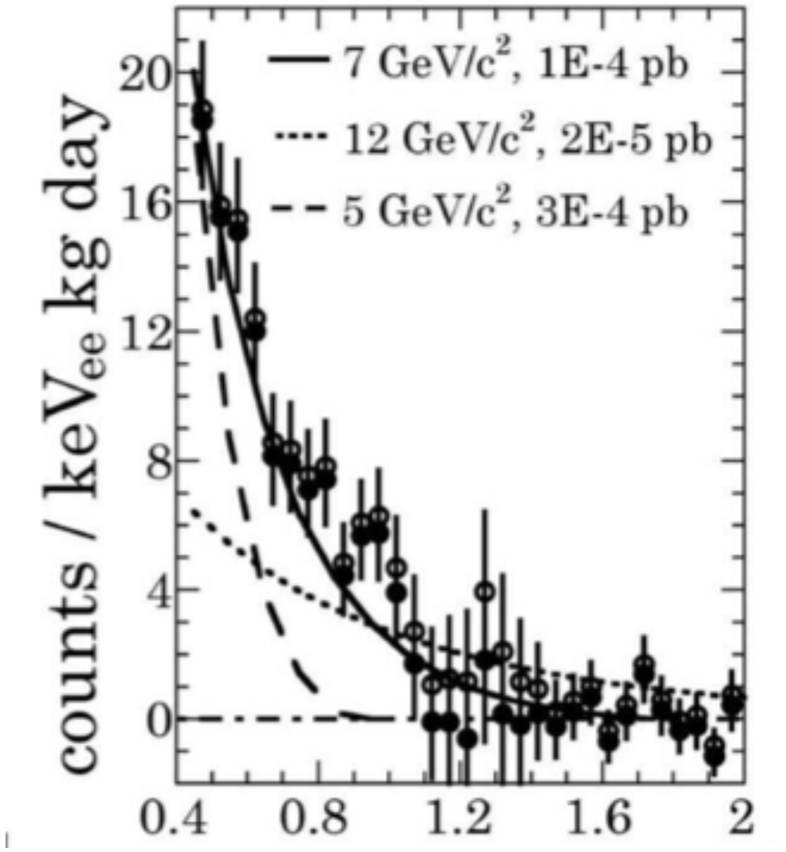
Experiment	Parameter	Prior
CoGeNT	C	$0 \rightarrow 10$ cpd/kg/keVee
CoGeNT	$\mathcal{E}_0$	$0 \rightarrow 30$ keVee
CoGeNT	A	$0 \rightarrow 10$ cpd/kg/keVee

quenching factor: $\mathcal{E}(\text{keVee}) = 0.19935 \times E^{1.1204}(\text{keVnr})$

CoGeNT

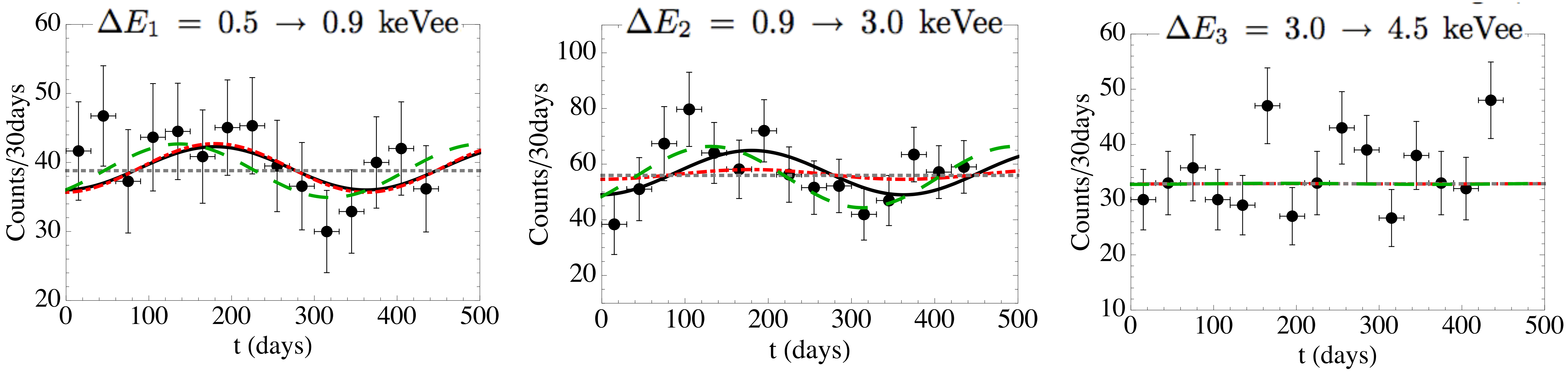
Aalseth et al. arXiv:1106.0650

Ge detector, 146 kg days
Very low threshold:
0.4 keVee = 2.7 keV



Gaussian likelihood $\ln \mathcal{L}_{\rm CoGeNT} = \ln \mathcal{L}_{\rm TR} + \ln \mathcal{L}_{\rm MR}$

- Background
  1. does not modulate, included only for the total rate
  2. constant + exponential background (mimic surface events)
  3. 3 nuisance parameters
- Radioactive peaks subtracted



Modulation: from 2.3σ **to** 1.6σ

CA, J.Hamann, R.Trotta & Y.Wong arXiv:1111.3238 [hep-ph];

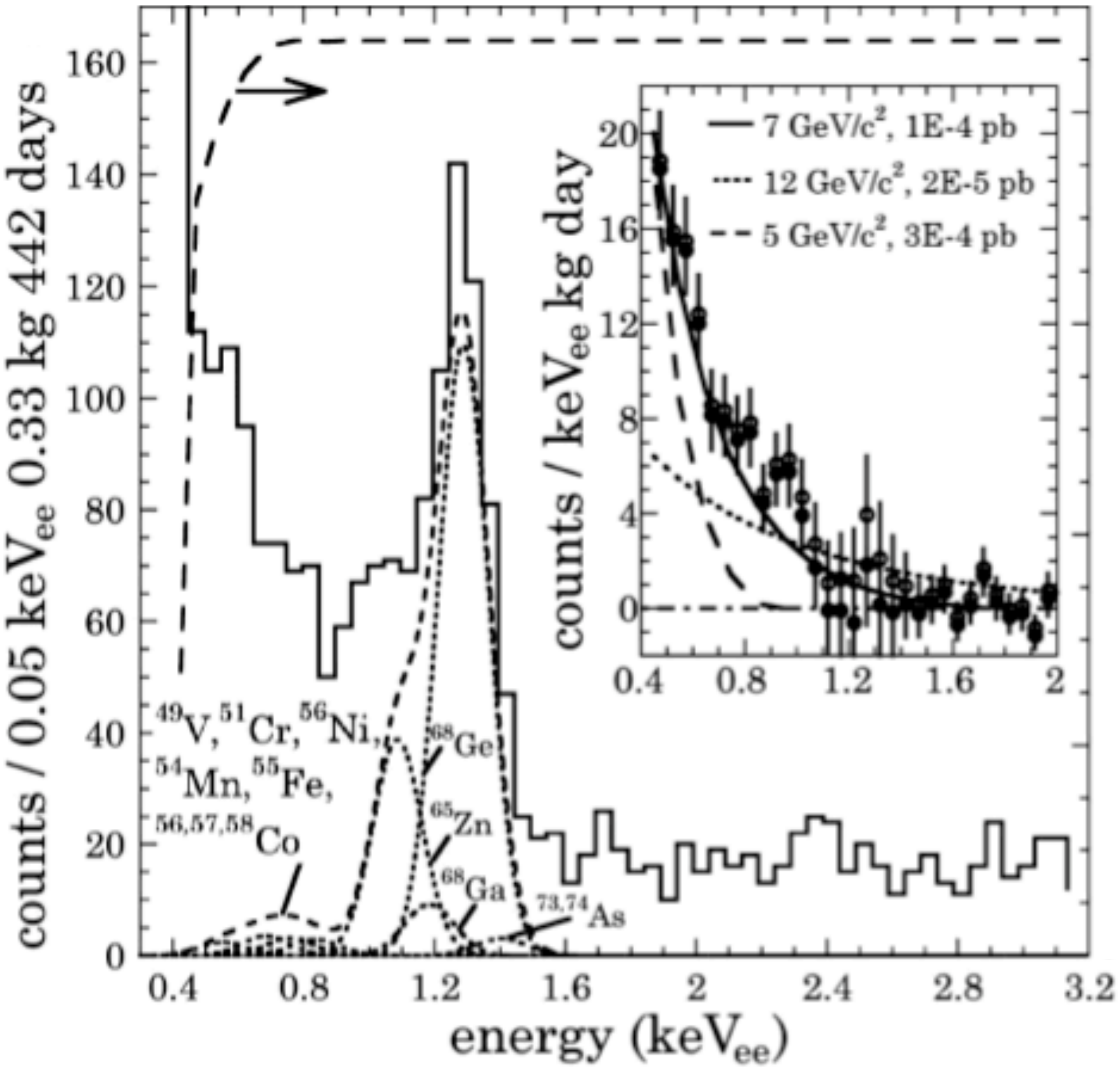
CoGeNT 2011

Data analysis

Radioactive peaks

Element	$\mathcal{E}_p$ (keVee)	σ_p (keVee)	$\tau_{1/2}$ (days)	N_0
^{73}As	1.414	0.077	80.	12.7
^{68}Ge	1.298	0.077	271.	638.9
^{68}Ga	1.194	0.076	271.	52.8
^{65}Zn	1.096	0.075	244.	211.2
^{56}Ni	0.926	0.075	5.9	1.53
^{56,58}Co	0.846	0.074	71.	9.44
^{57}Co	0.846	0.074	271.	2.59
^{55}Fe	0.769	0.074	996.	44.9
^{55}Mn	0.695	0.073	312.	21.1
^{51}Cr	0.628	0.073	28.	2.93
^{49}V	0.564	0.073	330.	14.9

arXiv:1106.0650



$$P^A_{\rm rad}(\mathcal{E}_{\rm min},\mathcal{E}_{\rm max}) = \int_{\mathcal{E}_{\rm min}}^{\mathcal{E}_{\rm max}} {\rm Gaussian}(\mathcal{E},\mathcal{E}_p,\sigma_p){\rm d}\mathcal{E}$$

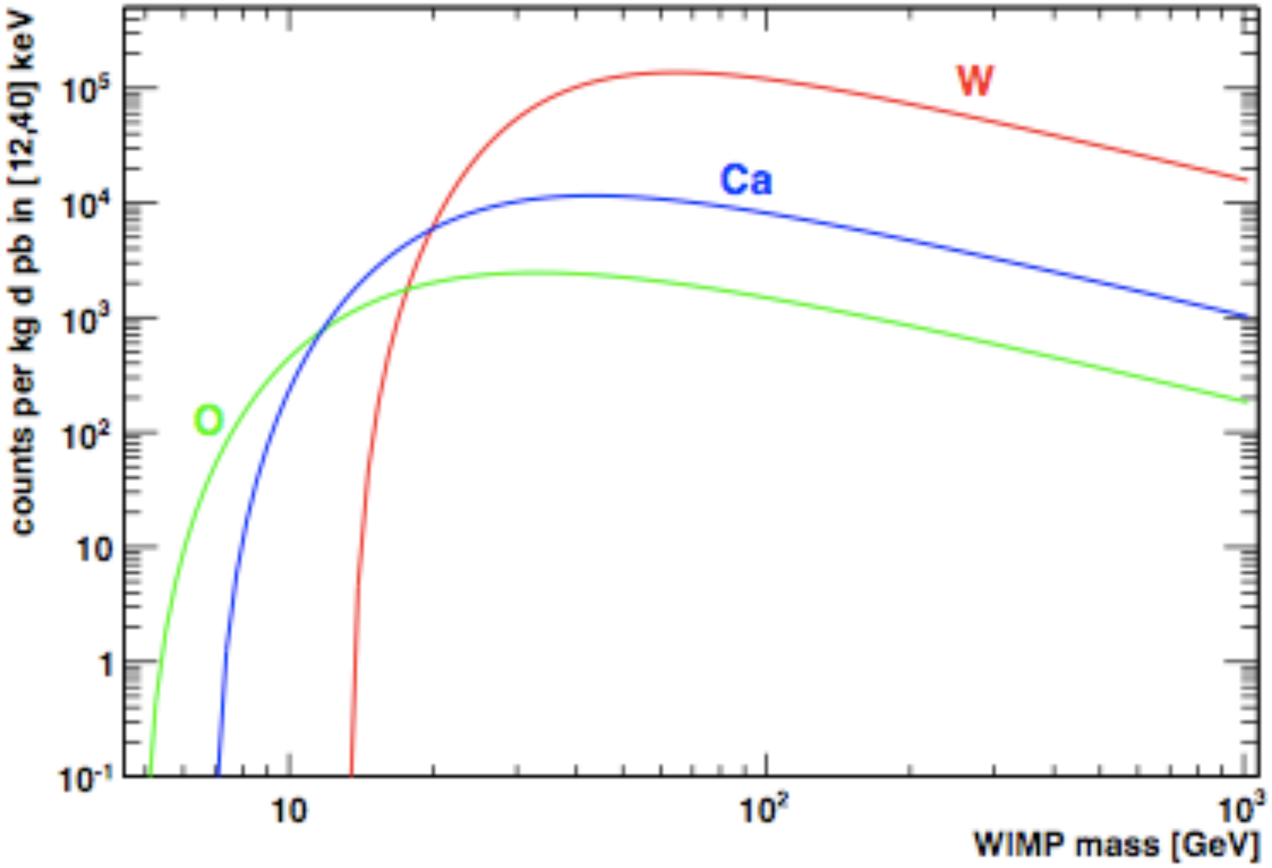
$$D^A(t_1,t_2) = \left(\exp(-\frac{\ln 2}{\tau_{1/2}}t_1) - \exp(-\frac{\ln 2}{\tau_{1/2}}t_2) \right)$$

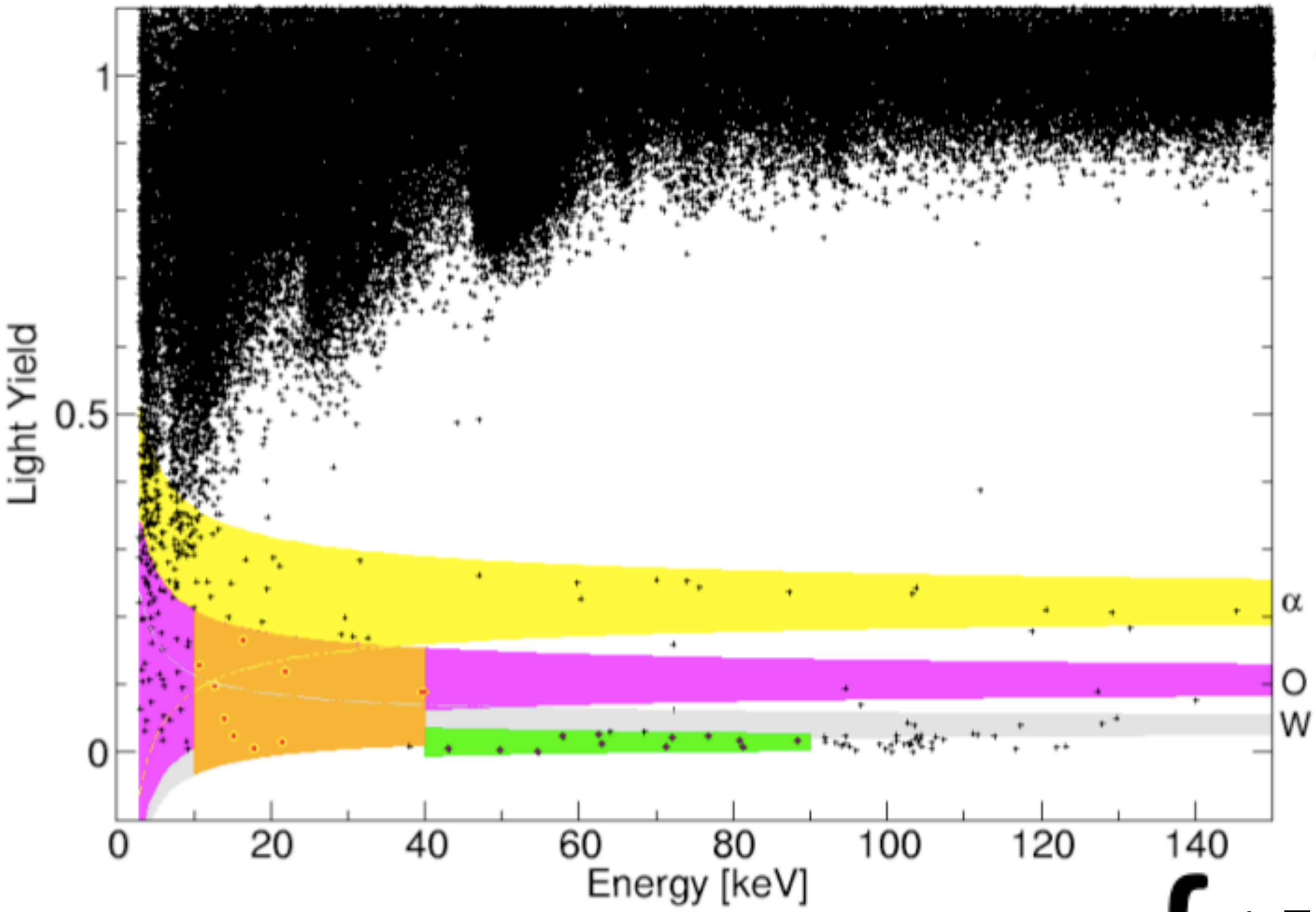
$$N^A_{\rm tot}(\mathcal{E}_{\rm min},\mathcal{E}_{\rm max},t_1,t_2) = N_0 P^A_{\rm rad}(\mathcal{E}_{\rm min},\mathcal{E}_{\rm max}) D^A(t_1,t_2)$$

CRESST-II

Angloher et al., arXiv:1109.0702
evidence at 4 sigma

- 8 detector modules made by $CaWO_4$ crystals (multi-target detectors)
- scintillation + ionization to disentangle background (e, n, alpha, decays of Pb isotopes)
- exposure of 730 kg days
- S = 67 events (background can account only for 65% of S)





light elements are more sensitive to light particles and viceversa!

Likelihood → Poisson distribution

1. Total counts in each module
2. Global spectral information
3. Background as nuisance parameters: 3 nuisance parameters

CRESST-II

Angloher et al., arXiv:1109.0702

- 8 detector module made by CaWO4 crystals (multi-target detectors)
- scintillation + ionization to disentangle background (e, n, alpha, decays of Pb isotopes)
- exposure of 730 kg days
- S = 67 events (background can account only for 65% of N)

$$\ln \mathcal{L}_{\rm CRESST}(N_{\rm tot}|S,B) = \ln \mathcal{L}_{\rm module} + \ln \mathcal{L}_{\rm Spectral} + \ln \mathcal{L}_B$$

$$\ln \mathcal{L}_{\rm module} = \sum_{i=1}^{8} \ln \mathcal{L}_i(n^i_{\rm obs}|S_i, \sum_j B_{ij})$$

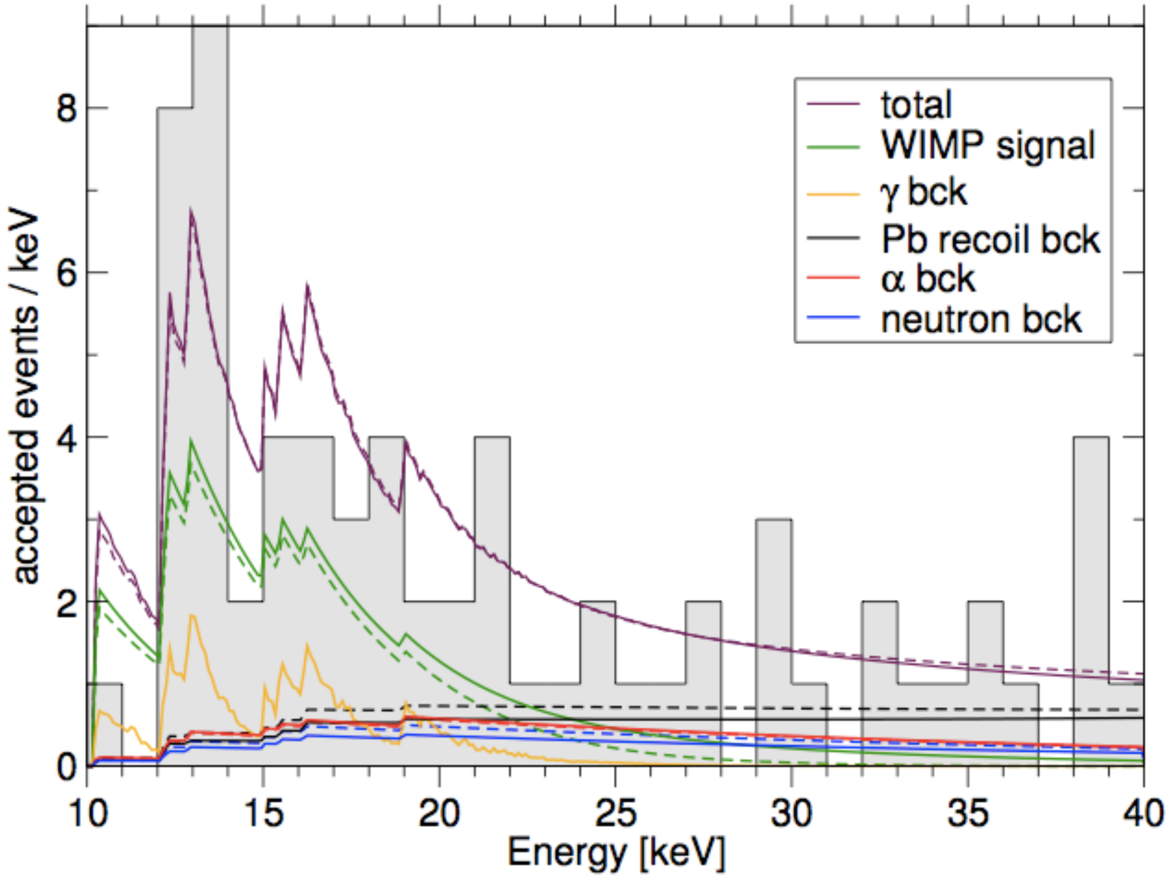
$$\ln \mathcal{L}_i(n^i_{\rm obs}|S_i, \sum_j B_{ij}) =$$

$$\ln \left[\frac{(S_i + \sum_j B_{ij})^{n^i_{\rm obs}} \exp(-S_i - \sum_j B_{ij})}{n^i_{\rm obs}!} \right]$$

$$B_i = B_{i\alpha} + B_{i\,e/\gamma} + B_{i\,n} + B_{i\,\rm Pb}$$

$$\frac{{\rm d}B_{\rm Pb}}{{\rm d}E} = C_{\rm Pb}\left[0.13 + \exp\left(\frac{E-90}{13.72}\right)\right]$$

$$B_n = N_n \left[\exp\left(-\frac{E_{\rm min}}{23.54}\right) - \exp\left(-\frac{E_{\rm max}}{23.54}\right)\right]$$

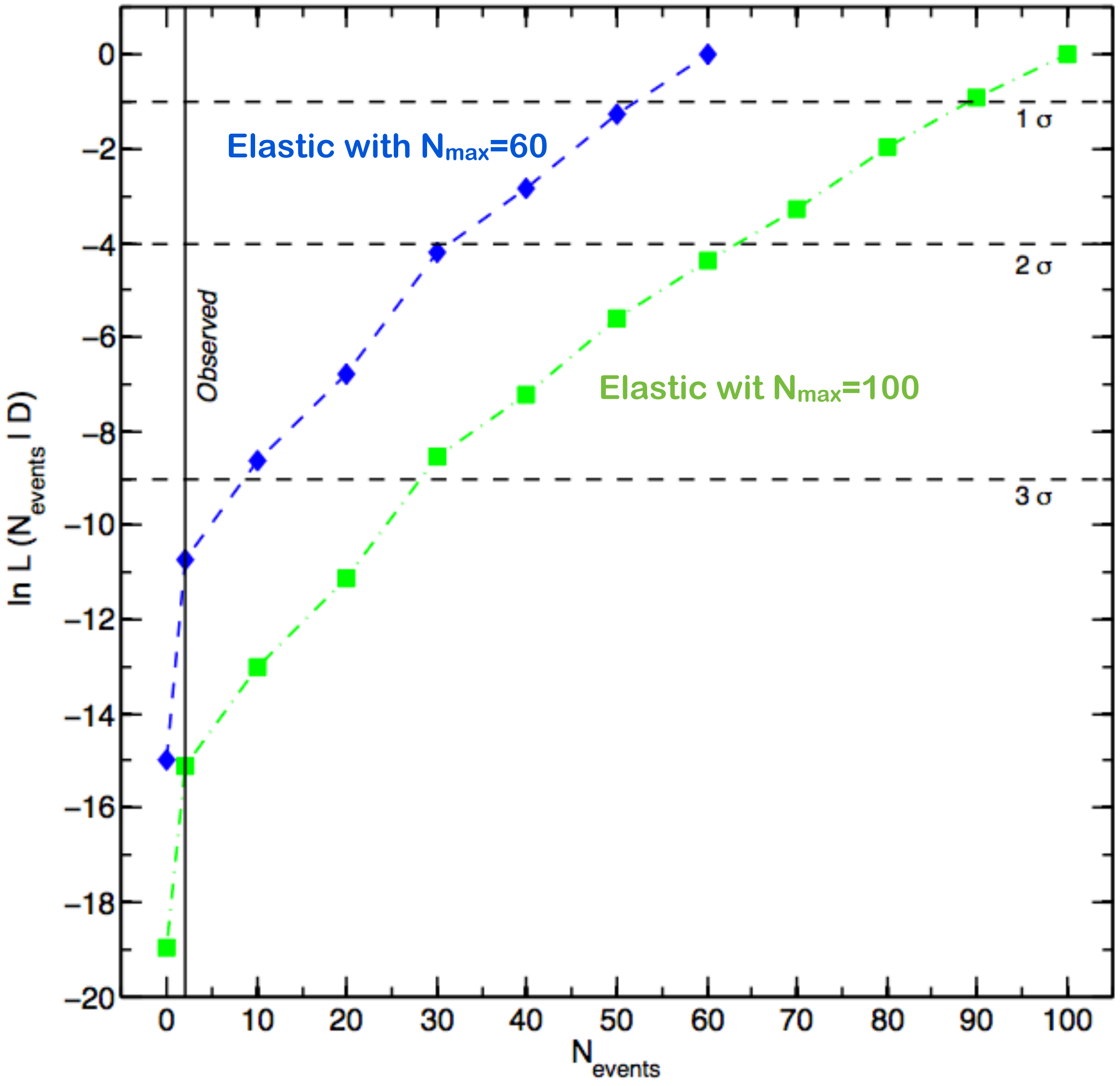


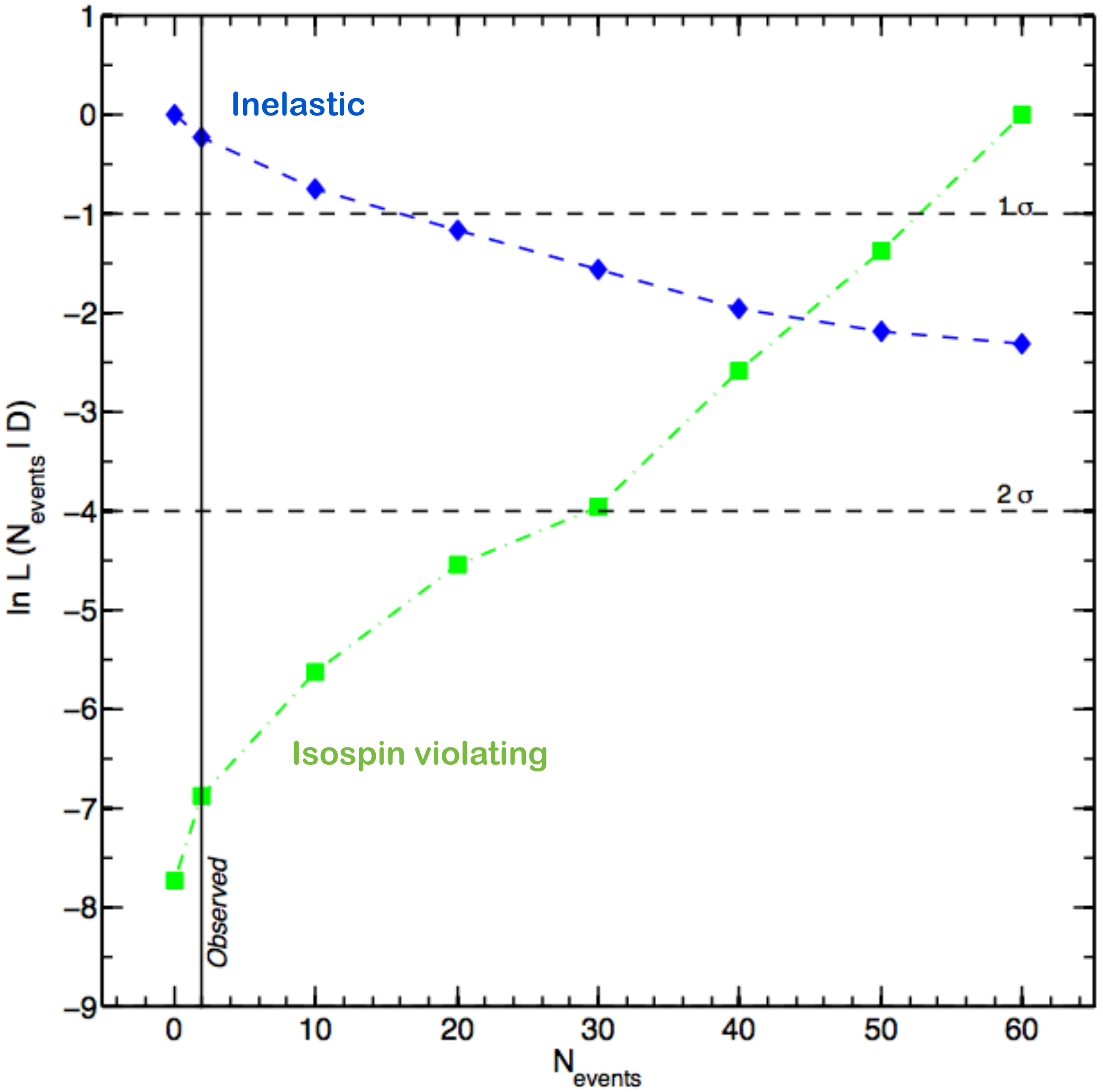
$$\begin{aligned} \bar{B}_\alpha \pm \sigma_\alpha &= 9.2 \pm 2.3, \\ \bar{B}_n \pm \sigma_n &= 9.7 \pm 5.1 \\ \bar{B}_{\rm Pb} \pm \sigma_{\rm Pb} &= 19 \pm 5 \end{aligned}$$

(In)Compatibility test

- We assume that:
  1. The fixed set is: $D = \{\mathrm{DAMA}, \mathrm{CoGeNT}, \mathrm{CRESST}\}$
  2. The result to be tested is the number of events seen in the XENON100 detector

$$\mathscr{D} \equiv N_{\mathrm{events}}$$



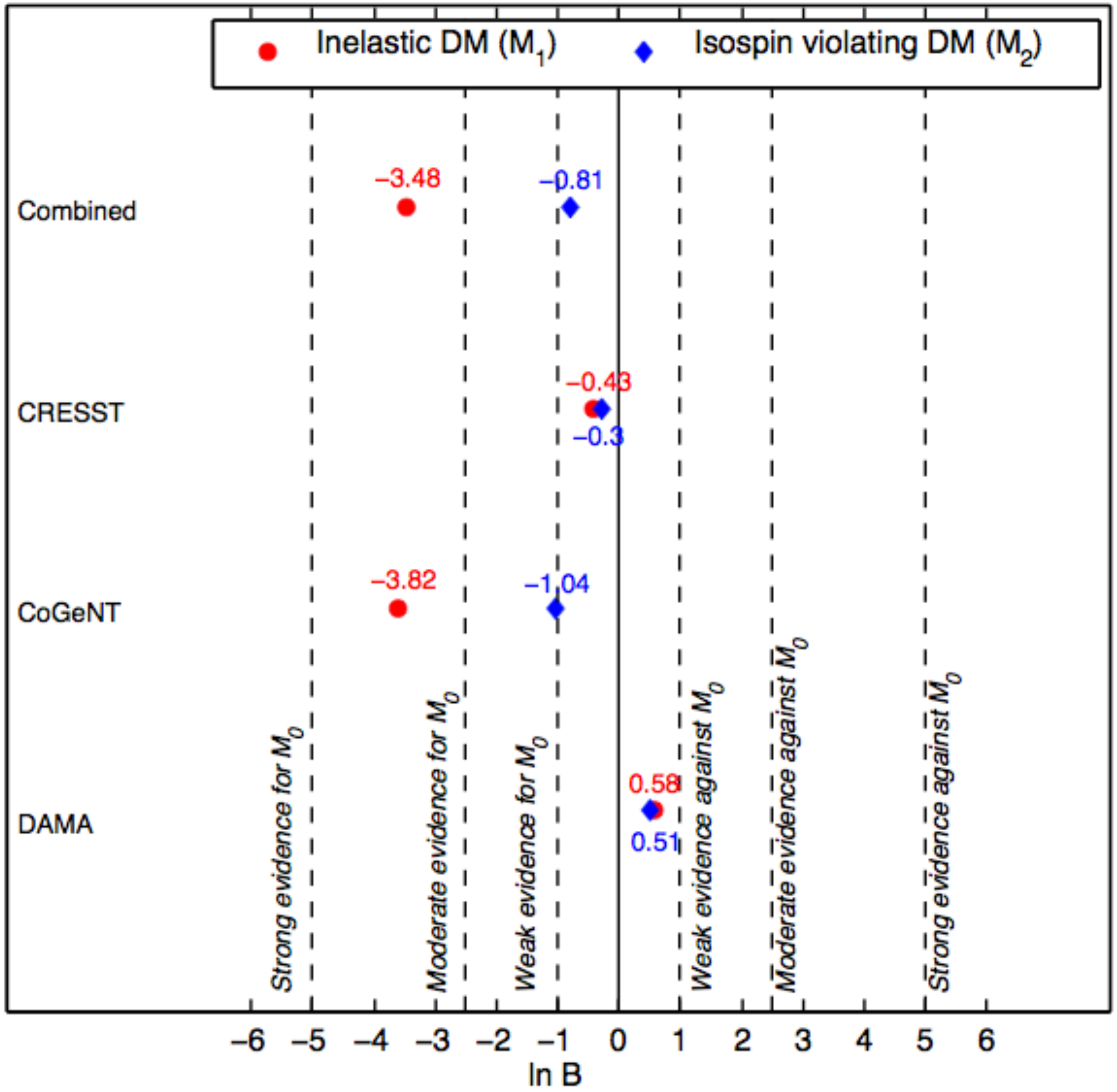


We find that elastic and isospin violating models are incompatible at 2 sigma level, while inelastic scattering is within 1 sigma

Which is the best model that accounts for the excessat low WIMP mass?

Elastic, inelastic and isospin violating models are nested models:

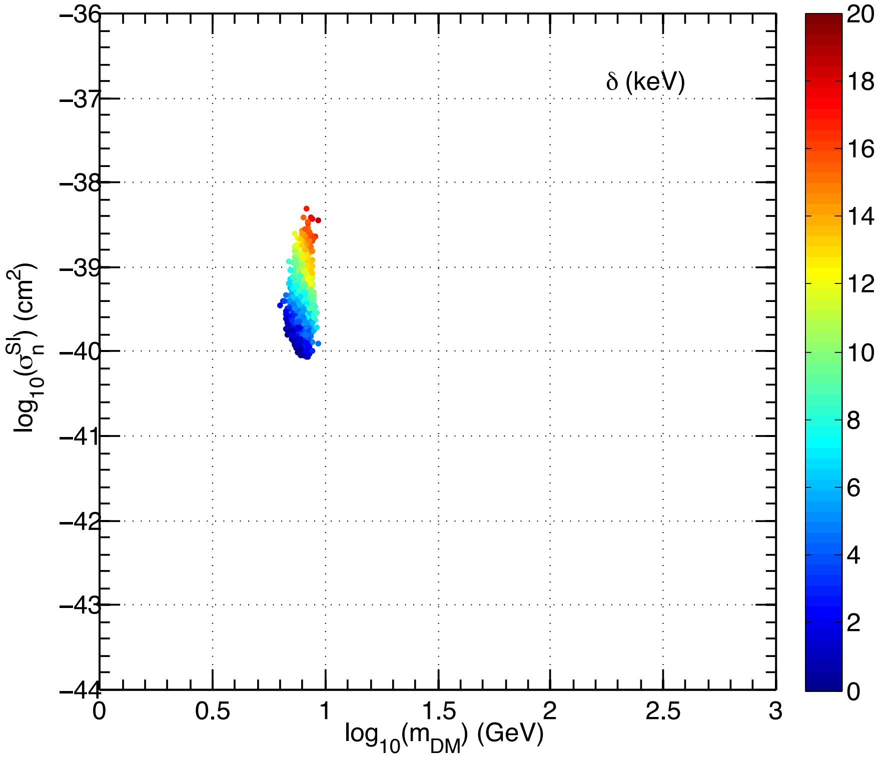
Inelastic reduces to elastic for $\delta = 0$
Isospin violating reduces to elastic for $f_n/f_p = 1$

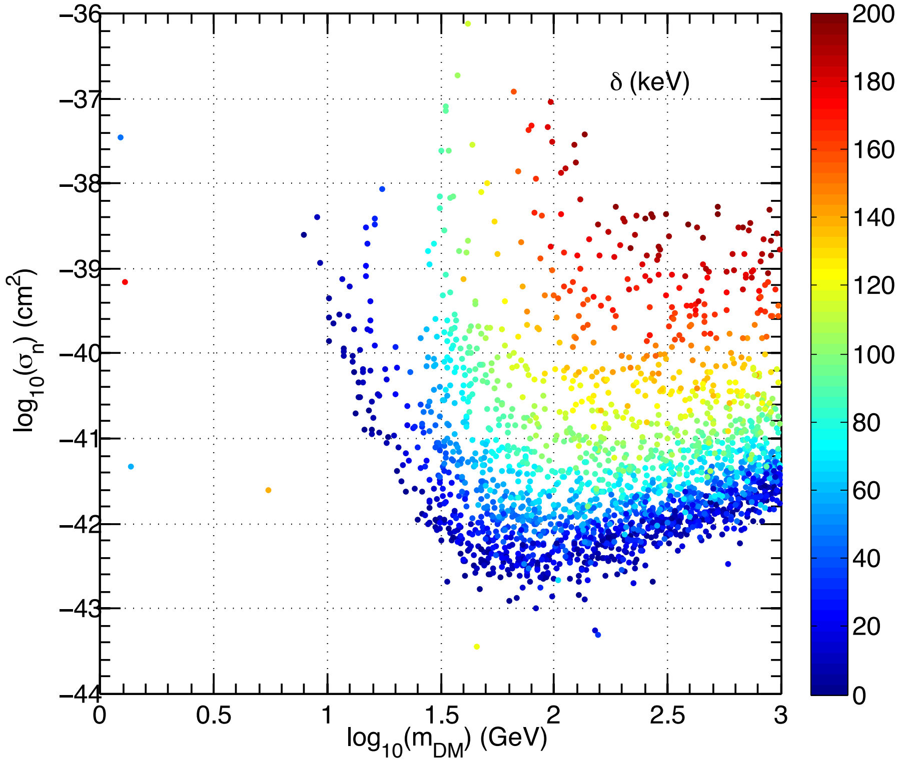


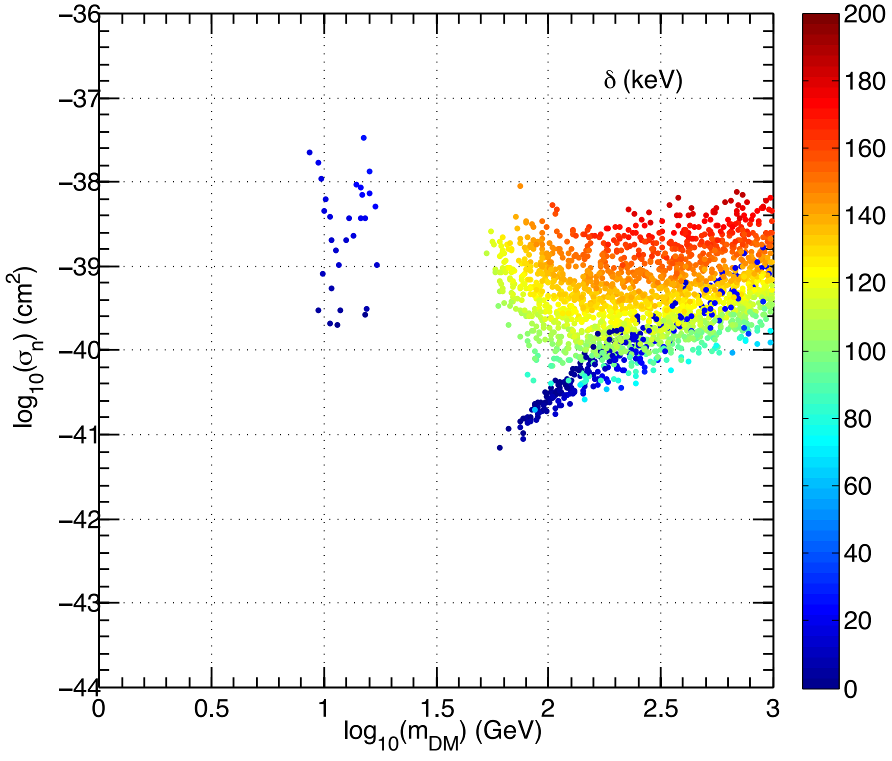
	$\mathcal{M}_i : \mathcal{M}_0$		
$\mathcal{M}_1$ Inelastic DM	odds	$\Delta\chi^2_{\rm eff}$	p-values
DAMA	2 : 1	1.95	0.08
CoGeNT	1 : 37	0.87	0.18
CRESST	1 : 2	0.04	0.42
Combined	1 : 32	0.71	0.20
$\mathcal{M}_2$ Isospin violating DM			
DAMA	2 : 1	1.88	0.09
CoGeNT	1 : 3	0.12	0.36
CRESST	1 : 1	0.03	0.43
Combined	1 : 2	8.56	0.002

CoGeNT and combined fit disfavour inelastic scattering because the excess is in the low energy region and it prefers light WIMP masses

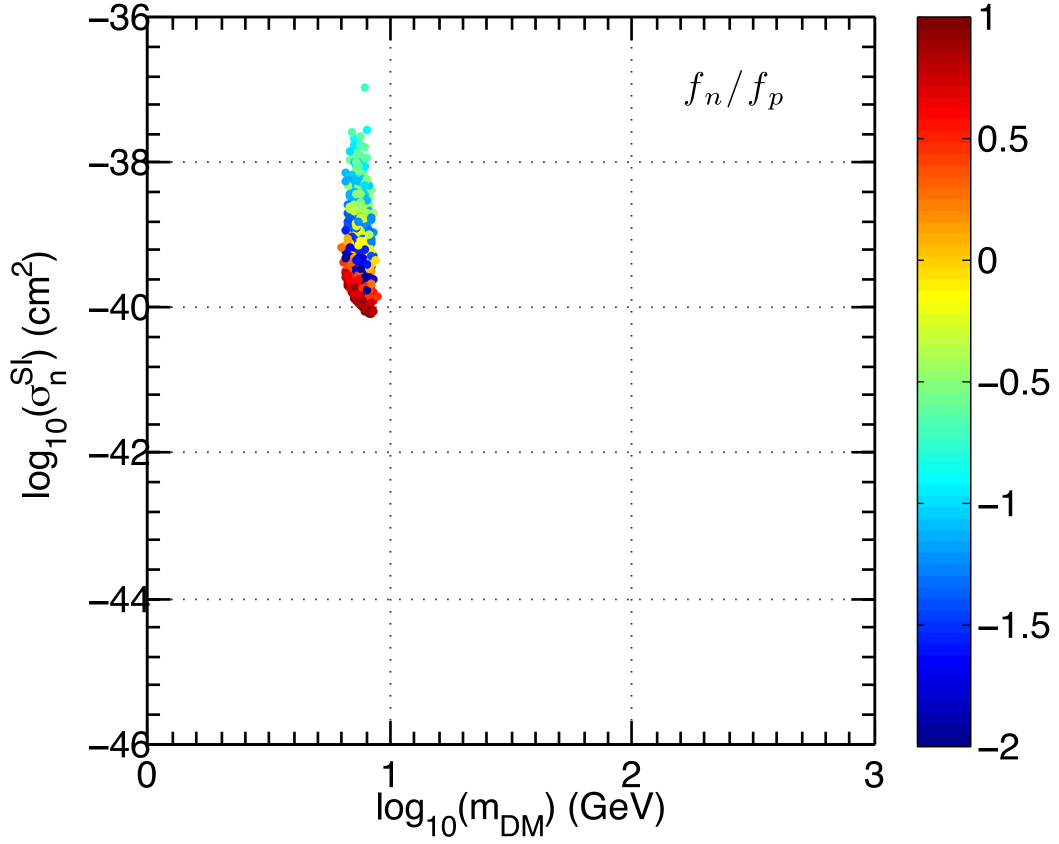
Parameter inference 3D: inelastic case

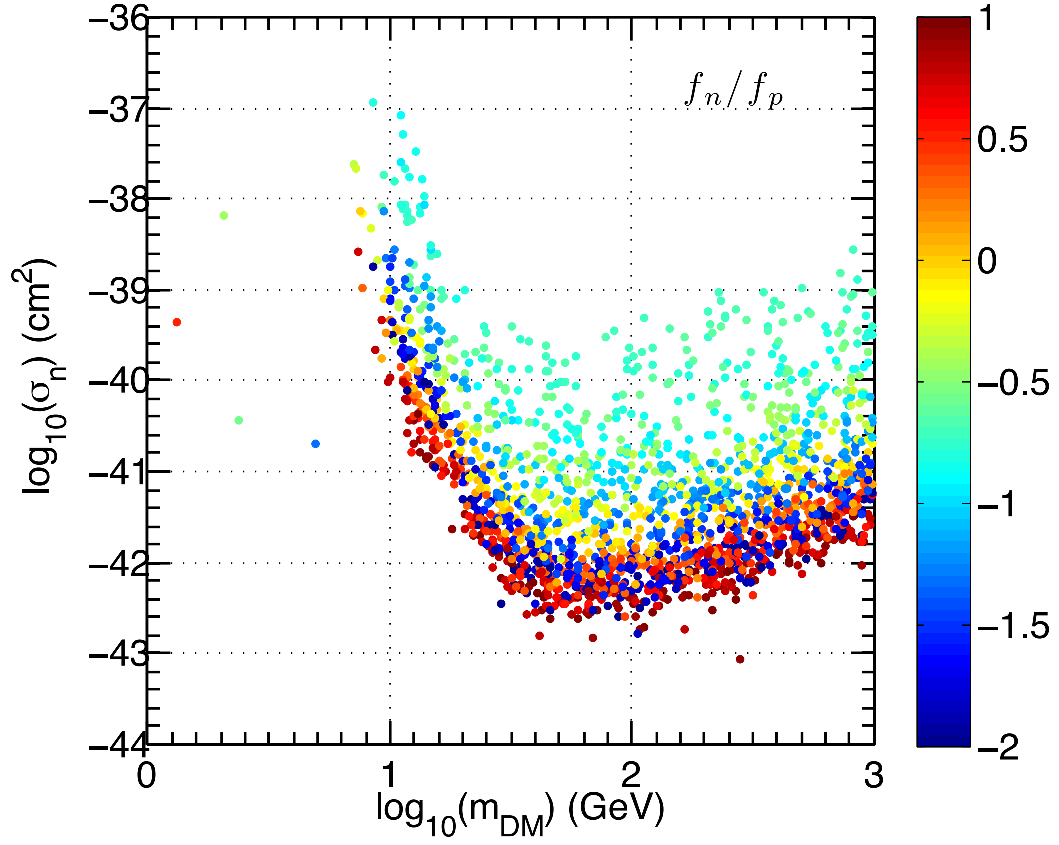


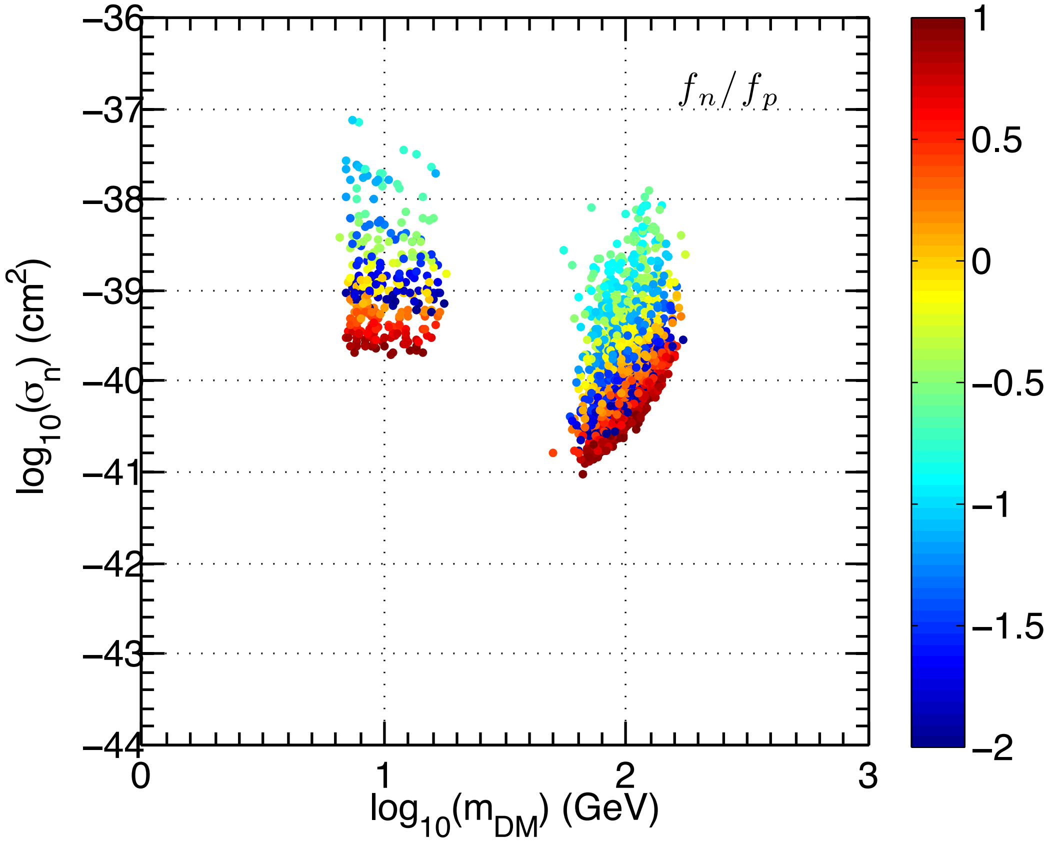




Parameter inference 3D: isospin violating case







Construction of DM velocity distribution (1)

$$\int_{v'>v'_{\rm min}} \mathrm{d}^3 v' \frac{f(\vec{v}'(t))}{v'} \longrightarrow \begin{array}{l} f(\vec{v}'(t)) \equiv F(\vec{v},\vec{R}_\odot)/\rho_\odot \\ \rho_\odot \equiv \rho_{\rm DM}(R_\odot) \end{array}$$

DD depends on the distribution function (DF) at the sun position arising from the WIMPs phase-space distribution $F(\vec{r},\vec{v})\ d^3r\ d^3v$

$$\rho_{\rm DM}(\vec{r}) = \int \mathrm{d}^3 v\ F(\vec{v},\vec{r})$$

- DF obtained inverting the above equation
- Symmetries assumed: density profile spherically symmetric and f(v) isotropic -> DF only function of the energy

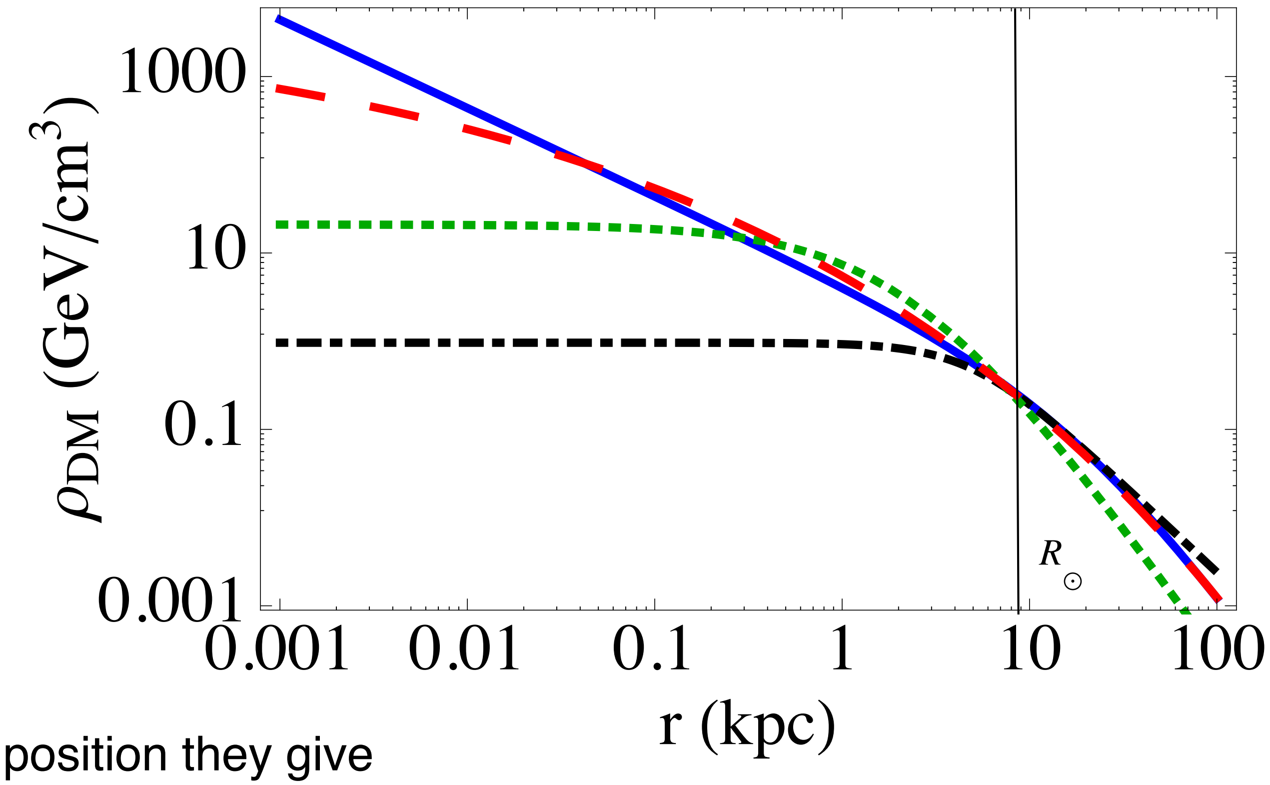
$$F(\varepsilon) = \frac{1}{\sqrt{8}\pi^2}\left[\int_0^\varepsilon \frac{\mathrm{d}^2\rho_{\rm DM}}{\mathrm{d}\Psi^2}\frac{\mathrm{d}\Psi}{\sqrt{\varepsilon-\Psi}} + \frac{1}{\sqrt{\varepsilon}}\left(\frac{\mathrm{d}\rho_{\rm DM}}{\mathrm{d}\Psi}\right)\Bigg|_{\Psi=0}\right]$$

- f(v) is a function of the gravitational potential (including baryon contribution)
- f(v) is a function of the DM density profile

Construction of DM velocity distribution (2)

Spherically symmetric DM density profiles $\rho_{\rm DM} = \rho_{\rm DM}(c_{\rm vir}, M_{\rm vir})$:

- NFW
- Einasto
- Cored Isothermal
- Burkert



The profiles mostly differ near the galactic center, at the sun position they give similar behavior for f(v)
In what follow only shown comparison between NFW and SMH

Likelihood for astrophysical observables (nuisance parameters for ALL EXP)

$$\ln \mathcal{L}_{\rm Astro} = -\frac{(v_0 - \bar{v}_0^{\rm obs})^2}{2\sigma_{v_0}^2} - \frac{(v_{\rm esc} - \bar{v}_{\rm esc}^{\rm obs})^2}{2\sigma_{v_{\rm esc}}^2} - \frac{(\rho_\odot - \bar{\rho}_\odot^{\rm obs})^2}{2\sigma_{\rho_\odot}^2} - \frac{(M_{\rm vir} - \bar{M}_{\rm vir}^{\rm obs})^2}{2\sigma_{M_{\rm vir}}^2}$$

Observable/Parameter	Constraint/Prior
Local standard of rest	$v_0^{\rm obs} = 230 \pm 24.4$ km s^{-1}
Escape velocity	$v_{\rm esc}^{\rm obs} = 544 \pm 39$ km s^{-1}
Local DM density	$\rho_\odot^{\rm obs} = 0.4 \pm 0.2$ GeV cm^{-3}
Virial mass	$M_{\rm vir}^{\rm obs} = 2.7 \pm 0.3 \times 10^{12} M_\odot$
Concentration parameter (NFW, Einasto)	$c_{\rm vir}$: $5 \rightarrow 20$
Concentration parameter (ISO, Burkert)	$c_{\rm vir}$: $50 \rightarrow 200$

$$v_{\rm esc} = \sqrt{2\Psi}\Big|_{r=R_\odot}$$

$$v_0 \equiv \sqrt{-r\frac{d\Psi}{dr}}\Bigg|_{r=R_\odot}$$

$$\rho_\odot \equiv \rho_{\rm DM}(R_\odot)$$

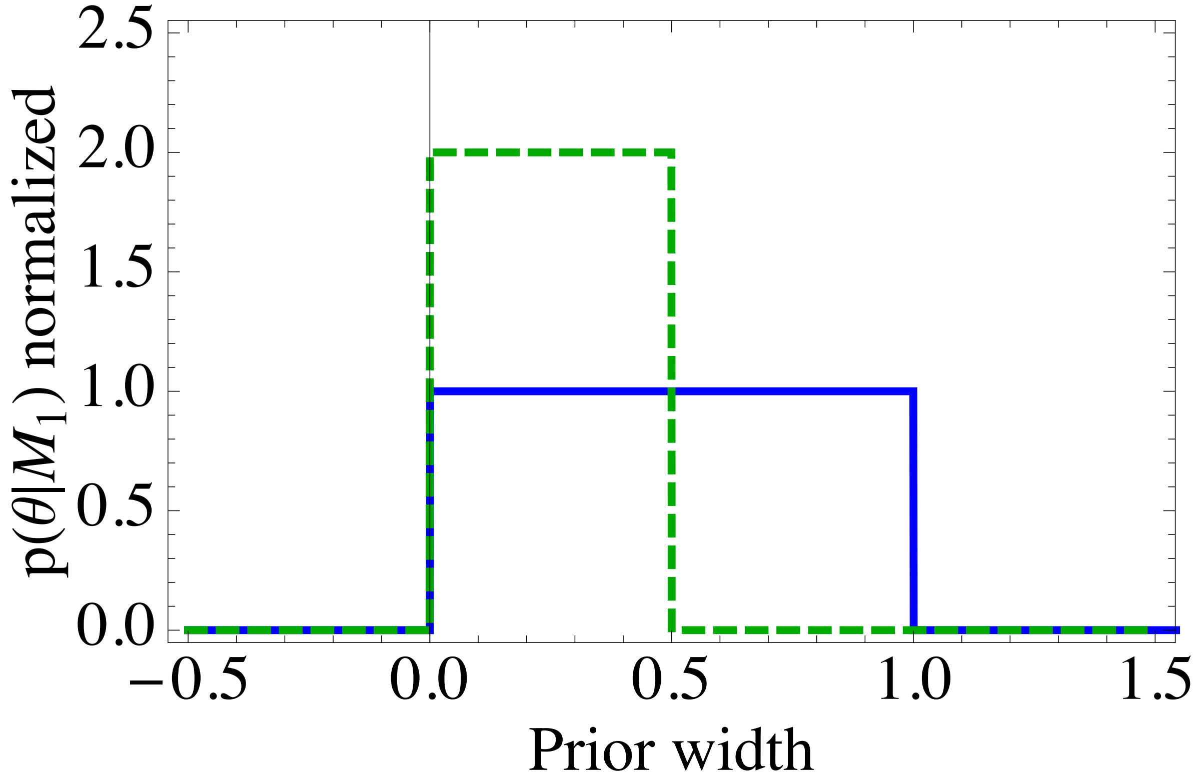
Sensitivity analysis

For nested models with parameter priors separable the Savage Dickey density ratio (SDDR) gives an analytical estimate of the effect on lnB changing the width of the prior

marginal normalized prior density computed at fixed value of ϑ

$$B_{10} = \frac{p(\vartheta^\star|\mathcal{M}_1)}{p(\vartheta^\star|d,\mathcal{M}_1)}$$

marginal posterior pdfs, computed at fixed value of the parameters



EXAMPLE

$$S^i_{\rm m} = 0 \rightarrow 0.5$$
$$\ln 2^3 \simeq 2.1$$
$$\ln B_{2a} = -1.06$$

- lnB of 1a:2a is now 3.11 instead of 5.21, still moderate evidence

- Results are robust from a Bayesian point of view!

Velocity distribution from DM density profile

Assuming equilibrium between gravitational force and pressure:

$$F(\varepsilon) = \frac{1}{\sqrt{8}\pi^2}\left[\int_0^{\varepsilon} \frac{\mathrm{d}^2\rho_{\mathrm{DM}}}{\mathrm{d}\Psi^2}\frac{\mathrm{d}\Psi}{\sqrt{\varepsilon-\Psi}} + \frac{1}{\sqrt{\varepsilon}}\left(\frac{\mathrm{d}\rho_{\mathrm{DM}}}{\mathrm{d}\Psi}\right)\bigg|_{\Psi=0}\right]$$

Eddigton formula for spherically symmetric DM density profiles that lead to isotropic f(v)

Poisson equation for the gravitational potential including contribution from the bulge and disk:

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\Psi}{\mathrm{d}r} = -4\pi G[\rho_{\mathrm{DM}} + \rho_{\mathrm{disk}} + \rho_{\mathrm{bulge}}]$$

$$\rho_{\mathrm{DM}}(r) = \rho_s\left(\frac{r}{r_s}\right)^{-1}\left(1+\left(\frac{r}{r_s}\right)\right)^{-2}$$

$$\rho_{\mathrm{disk}}(r) = \frac{M_{\mathrm{disk}}}{4\pi r_{\mathrm{disk}}^2}\frac{e^{-r/r_{\mathrm{disk}}}}{r}$$

$$\rho_{\mathrm{bulge}}(r) = M_{\mathrm{bulge}}\delta_D^{(3)}(\vec{r})$$

The velocity distribution is translated to the reference frame of the Earth:

$$\int_{v'>v'_{\mathrm{min}}} \mathrm{d}^3v' \frac{f(\vec{v'}(t))}{v'} \to 2\pi\rho_{\odot}^{-1}\int_{v'>v'_{\mathrm{min}}} \mathrm{d}v'\, v' \int_{-1}^{1} \mathrm{d}\alpha\, F\Big(\Psi_{\odot} - \frac{1}{2}v^2\Big)$$

$$v_0 \equiv \sqrt{-r\frac{\mathrm{d}\Psi}{\mathrm{d}r}}\bigg|_{r=R_{\odot}}$$

$$v^2 = |\vec{v'} + \vec{v}_{\oplus}|^2 = v'^2 + v_{\oplus}^2 + 2v'v_{\oplus}\alpha\,,$$

$$v_{\oplus} = |\vec{v}_{\odot} + \vec{v''}_{\oplus,\mathrm{rot}}| = v_{\odot} + v''_{\oplus,\mathrm{rot}}\cos\gamma\cos[2\pi(t-t_0)/T]$$

$$v_{\mathrm{esc}} = \sqrt{2\Psi}\Big|_{r=R_{\odot}}$$

DM density profiles

$$r_s(M_{\rm vir}, c_{\rm vir}) = \frac{r_{\rm vir}(M_{\rm vir})}{c_{\rm vir}} \qquad M_{\rm vir} = 4\pi \int_0^{r_{\rm vir}} {\rm d}r\, r^2 \rho_{\rm DM}(r) = \frac{4}{3}\pi r_{\rm vir}^3 \delta_c \rho_{\rm crit}$$

Cored isothermal	$\rho_{\rm DM}(r) = \rho_s \left[1 + \left(\frac{r}{r_s}\right)^2\right]^{-1}$ $\rho_s(c_{\rm vir}) = \frac{\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3}{c_{\rm vir} - \tan^{-1}(c_{\rm vir})}$
Navarro–Frenk–White (NFW)	$\rho_{\rm DM}(r) = \rho_s \left(\frac{r}{r_s}\right)^{-1} \left(1 + \left(\frac{r}{r_s}\right)\right)^{-2}$ $\rho_s(c_{\rm vir}) = \frac{\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3}{\ln(1 + c_{\rm vir}) - c_{\rm vir}/(1 + c_{\rm vir})}$
Einasto	$\rho_{\rm DM}(r) = \rho_s \exp\left(-\frac{2}{a}\left[\left(\frac{r}{r_s}\right)^a - 1\right]\right)$ $\rho_s(c_{\rm vir}) = \frac{\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3 [2^{-\frac{3}{\alpha}} \exp(\frac{2}{\alpha}) \alpha^{\frac{3}{\alpha}-1}]^{-1}}{\Gamma\left(\frac{3}{\alpha}\right) - \Gamma\left(\frac{3}{\alpha}, \frac{2c_{\rm vir}^\alpha}{\alpha}\right)}$
Burkert	$\rho_{\rm DM}(r) = \rho_s \left(1 + \frac{r}{r_s}\right)^{-1} \left(1 + \frac{r}{r_s}\right)^{-2}$ $\rho_s(c_{\rm vir}) = \frac{4\delta_c \rho_{\rm crit}}{3} \frac{c_{\rm vir}^3}{2\ln(1 + c_{\rm vir}) + \ln(1 + c_{\rm vir}^2) - 2\tan^{-1}(c_{\rm vir})}$

Theoretical predictions for elastic spin-independent scattering off nucleus

Differential rate

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\rho_\odot}{m_{\mathrm{DM}}} \int_{v'>v'_{\mathrm{min}}} \mathrm{d}^3 v' \, \frac{\mathrm{d}\sigma}{\mathrm{d}E} \, v' \, f(\vec{v'}(t))$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}E} = \frac{M_{\mathcal{N}} \sigma_n^{\mathrm{SI}}}{2\mu_n^2 v'^2} \frac{\left(f_p Z + (A-Z) f_n\right)^2}{f_n^2} \mathcal{F}^2(E)$$

$$\mathcal{E} = qE$$

$$S(t) = M_{\mathrm{det}} T \int_{\mathcal{E}_1/q}^{\mathcal{E}_2/q} \mathrm{d}E \, \epsilon(qE) \, \frac{\mathrm{d}R}{\mathrm{d}E}$$

Modulated rate

$$s = \frac{1}{\mathcal{E}_2 - \mathcal{E}_1} \sum_{X=\mathrm{Na,I}} w_X \int_{\mathcal{E}_1/q_X}^{\mathcal{E}_2/q_X} \mathrm{d}E \, \frac{1}{2} \left[\frac{\mathrm{d}R_X}{\mathrm{d}E}(\mathrm{June}\ 2) - \frac{\mathrm{d}R_X}{\mathrm{d}E}(\mathrm{Dec}\ 2) \right]$$

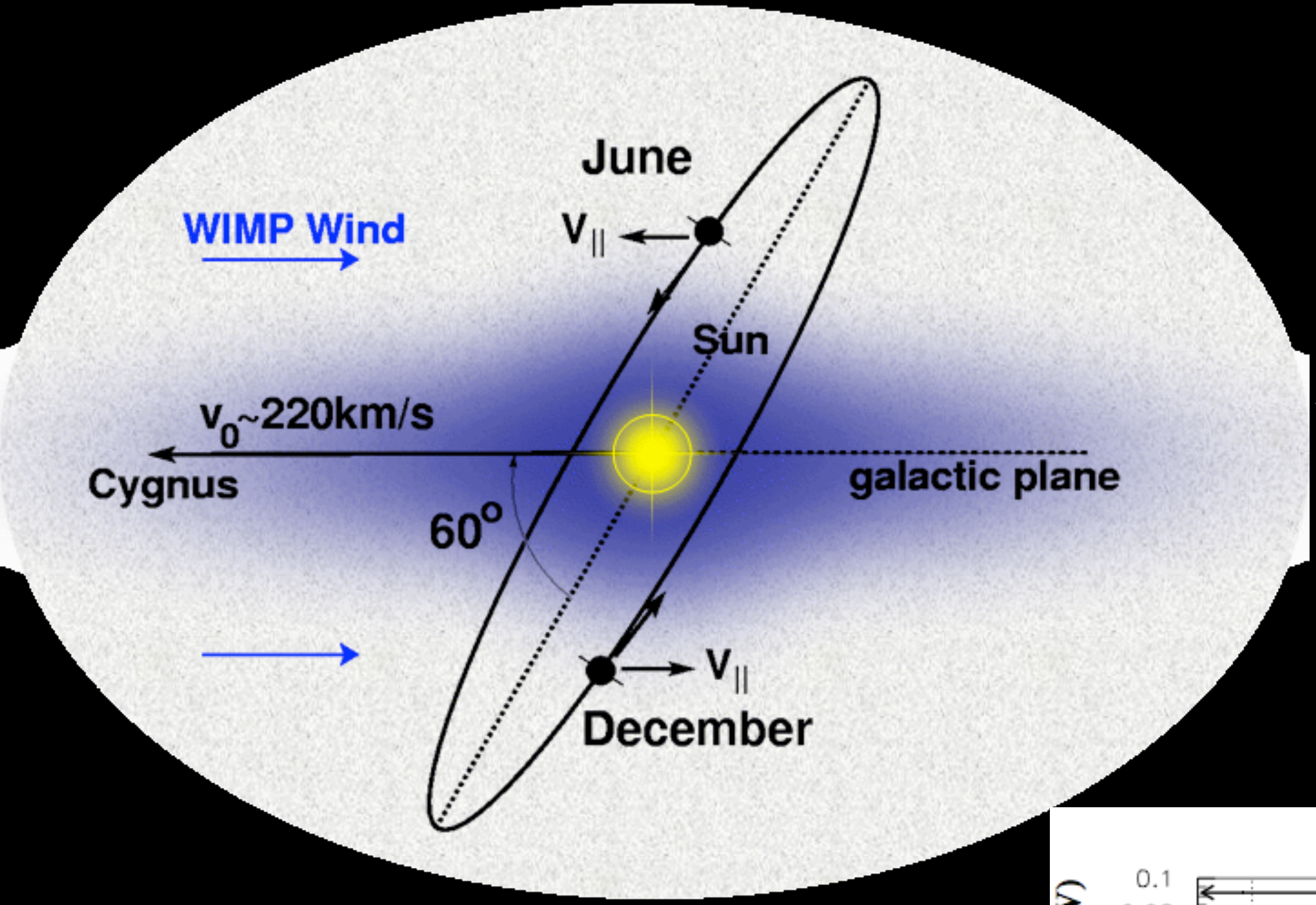
$$S_{\mathrm{m\%}} = \frac{R(\mathrm{June2}) - R(\mathrm{Dec2})}{R(\mathrm{June2}) + R(\mathrm{Dec2})}$$

Annual Modulation

Drukier, Freese and Spergel '86,
Freese, Frieman and Gould '88

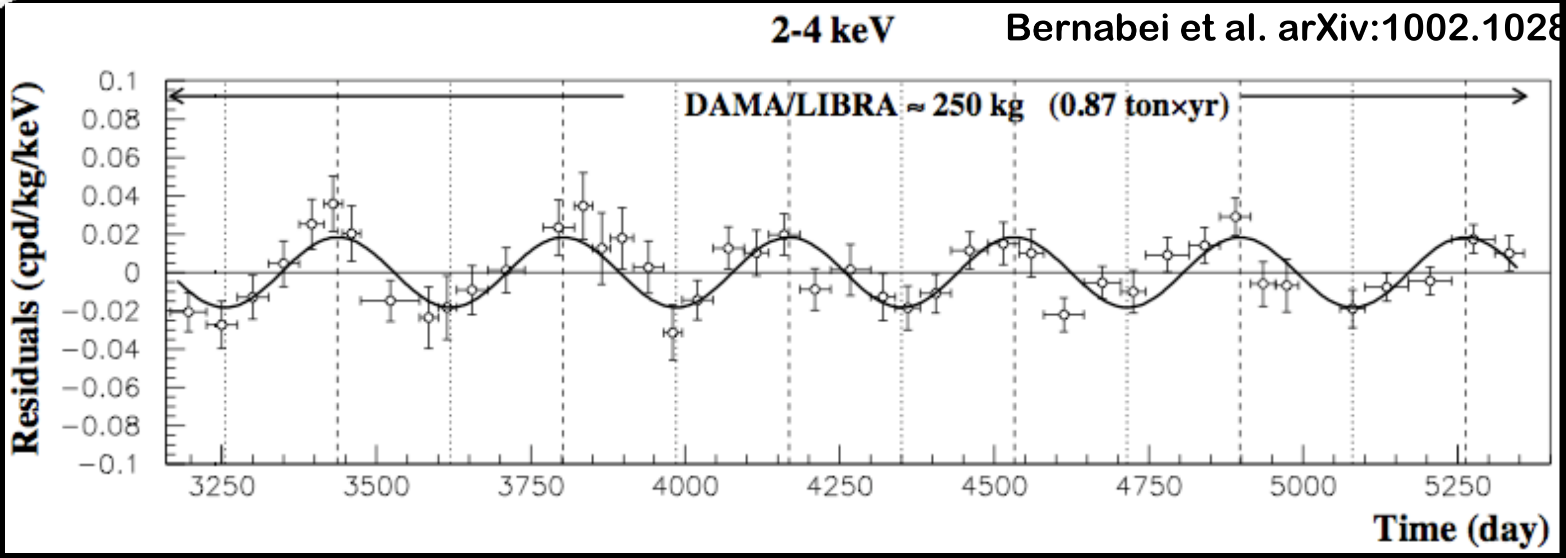
Signature of WIMP recoil in the detector

In the Earth's rest frame the DM velocity distribution acquires a time dependence, which follows a sinusoidal behavior



$$v^2 = |\vec{v'} + \vec{v}_\oplus|^2$$

$$v_\oplus = |\vec{v}_\odot + \vec{v''}_{\oplus,\mathrm{rot}}|$$
$$= v_\odot + v''_{\oplus,\mathrm{rot}} \cos\gamma \cos[2\pi(t-t_0)/T]$$



Bernabei et al. arXiv:1002.102

$$\gamma = 60^\circ$$

effect of O(10%)

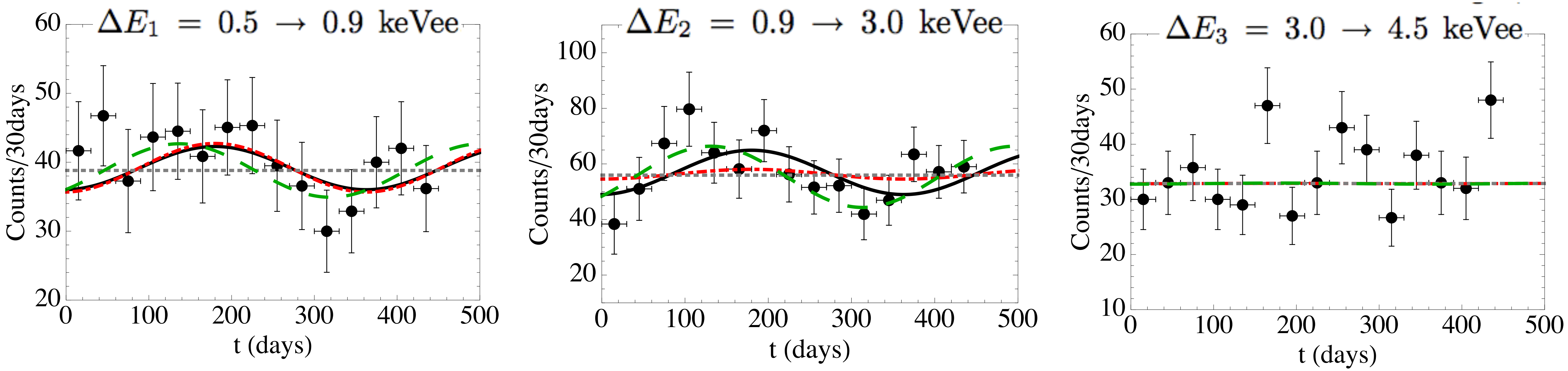
CoGeNT modulation

Is there evidence for DM modulation in CoGeNT?

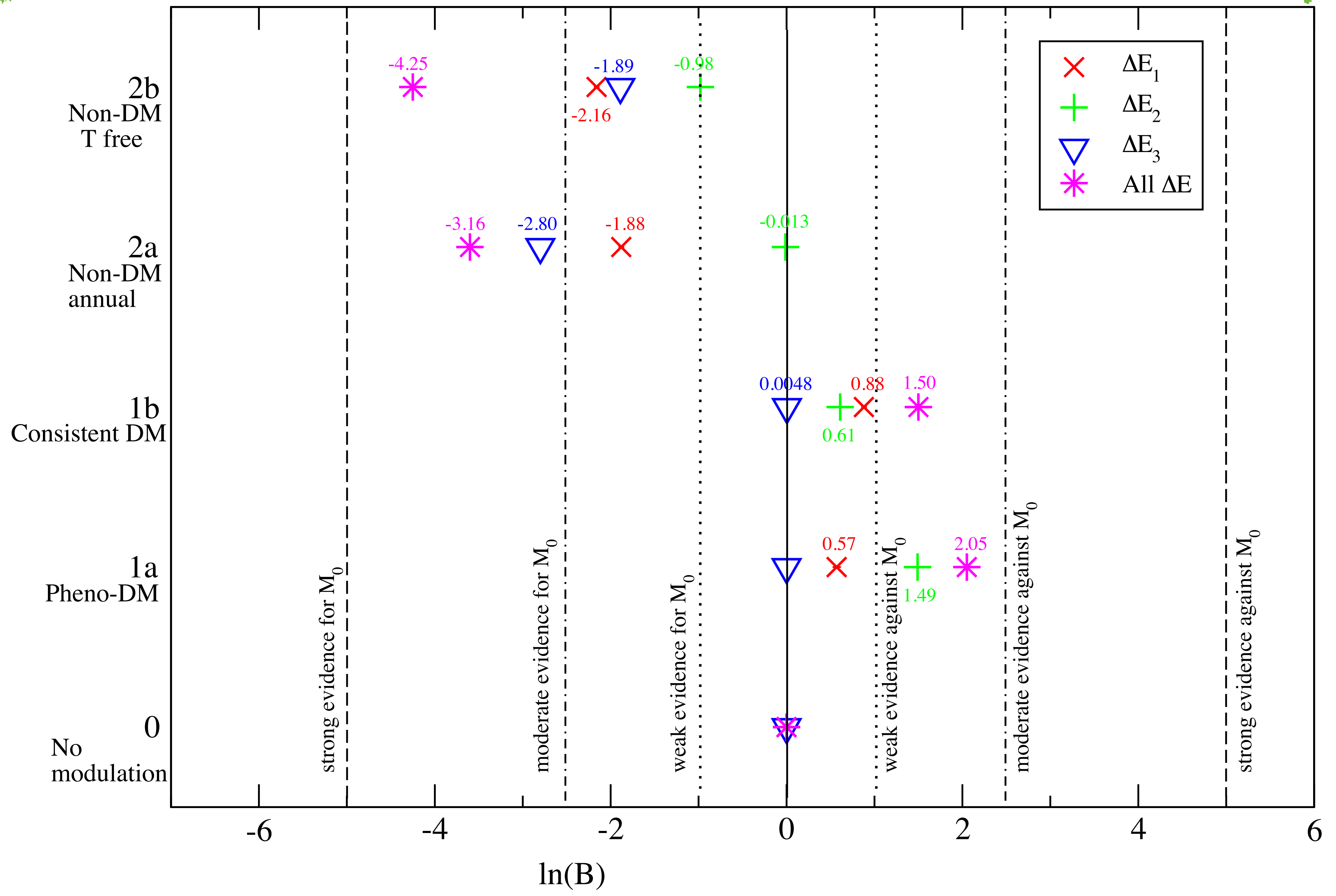
Comparison between 5 phenomenological models that describe a sinusoidal modulation:

$$R_i(t) = U^i_{\mathrm{m}} \left(1 + S^i_{\mathrm{m}} \cos[2\pi(t - t_{\max} - 28)/T]\right)$$

Model	Description	Fractional modulation S^i_{m}	Phase $t_{\max}$ (days)	Period T (days)	Extra params
0	No modulation	0	—	—	$\nu = 0, 0$
1a	Pheno-DM	$S^{1,2}_{\mathrm{m}} = 0 \to 0.2$ $S^3_{\mathrm{m}} = 0$	152	365	$\nu = 1, 2$
1b	Consistent DM	Gaussian, clipped at 0 $(S^i_{\mathrm{m}} \geq 0)$ $S^1_{\mathrm{m}} = 0.098 \pm 0.021$ $S^2_{\mathrm{m}} = 0.026 \pm 0.011$ $S^3_{\mathrm{m}} = (0.37 \pm 36) \times 10^{-4}$	152	365	$\nu = 1, 3$
2a	Non-DM, annual	$0 \to 1$	$0 \to 365$	365	$\nu = 2, 4$
2b	Non-DM, free period	$0 \to 1$	$0 \to 365$	$1 \to 365$	$\nu = 3, 5$



Bayes factor: results for model comparison



	$\mathcal{M}_i : \mathcal{M}_0$			
Model $\mathcal{M}_i$	Bin 1	Bin 2	Bin 3	All 3 bins
1a	2 : 1	4 : 1	1 : 1	8 : 1
1b	2 : 1	2 : 1	1 : 1	5 : 1
2a	1 : 7	1 : 1	1 : 16	1 : 37
2b	1 : 9	1 : 3	1 : 6	1 : 70

$\mathcal{M}_i : \mathcal{M}_j$	Bin 1	Bin 2	Bin 3	All 3 bins
1a:2a	12 : 1	5 : 1	16 : 1	183 : 1
1a:2b	15 : 1	12 : 1	7 : 1	545 : 1
1b:2a	16 : 1	2 : 1	17 : 1	107 : 1
1b:2b	21 : 1	5 : 1	7 : 1	314 : 1

Classical p-values

$$\wp \equiv \int_{t_{\rm obs}}^{\infty} p(t|H_0)$$

probability of obtaining more extreme data than observed assuming the null hypothesis is correct and NOT probability for hypothesis

$$\Delta\chi^2_{\rm eff} \equiv -2\ln\left[\frac{\mathcal{L}(\vartheta^{\star},\hat{\psi})}{\mathcal{L}(\hat{\hat{\vartheta}},\hat{\hat{\psi}})}\right]$$

test statistics for nested models if

1. additional dof distributed as a gaussian
2. unbounded likelihood
3. all additional dof identifiable under the null

	$\Delta\chi^2_{\rm eff}$ relative to model 0			
Model	Bin 1	Bin 2	Bin 3	All 3 bins
1a	2.04	4.23	–	6.26
	$\wp = 0.08$	$\wp = 0.02$	–	$\wp = 0.02$
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Classical p-values

$$\wp \equiv \int_{t_{\rm obs}}^{\infty} p(t|H_0)$$

probability of obtaining more extreme data than observed assuming the null hypothesis is correct and NOT probability for hypothesis

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test statistics for nested models if

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2. unbounded likelihood
3. all additional dof identifiable under the null

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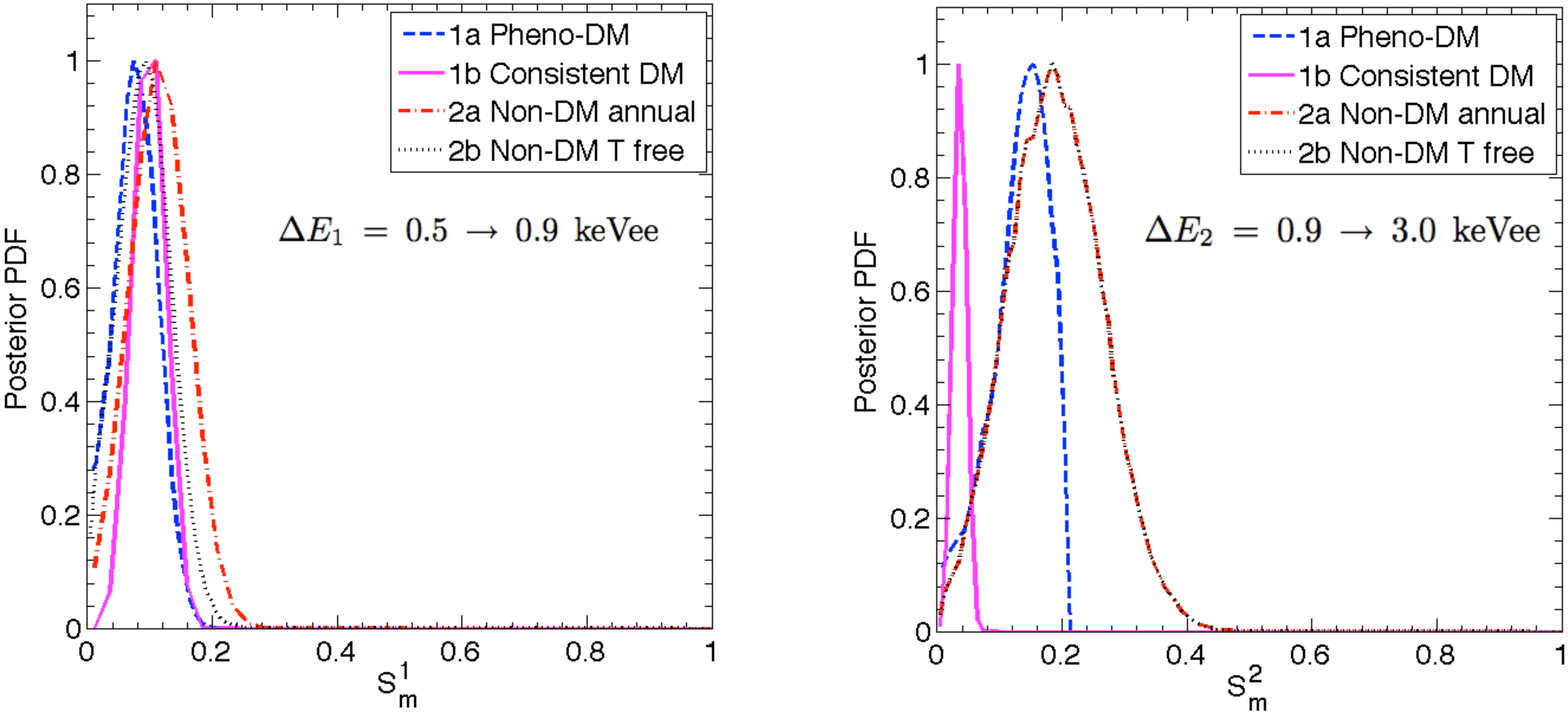
Rely on Monte Carlo simulation for mapping the t statistic into p-values

Parameter inference: amplitude of



Similar behavior for the All bin case: the inference is driven by bin 2

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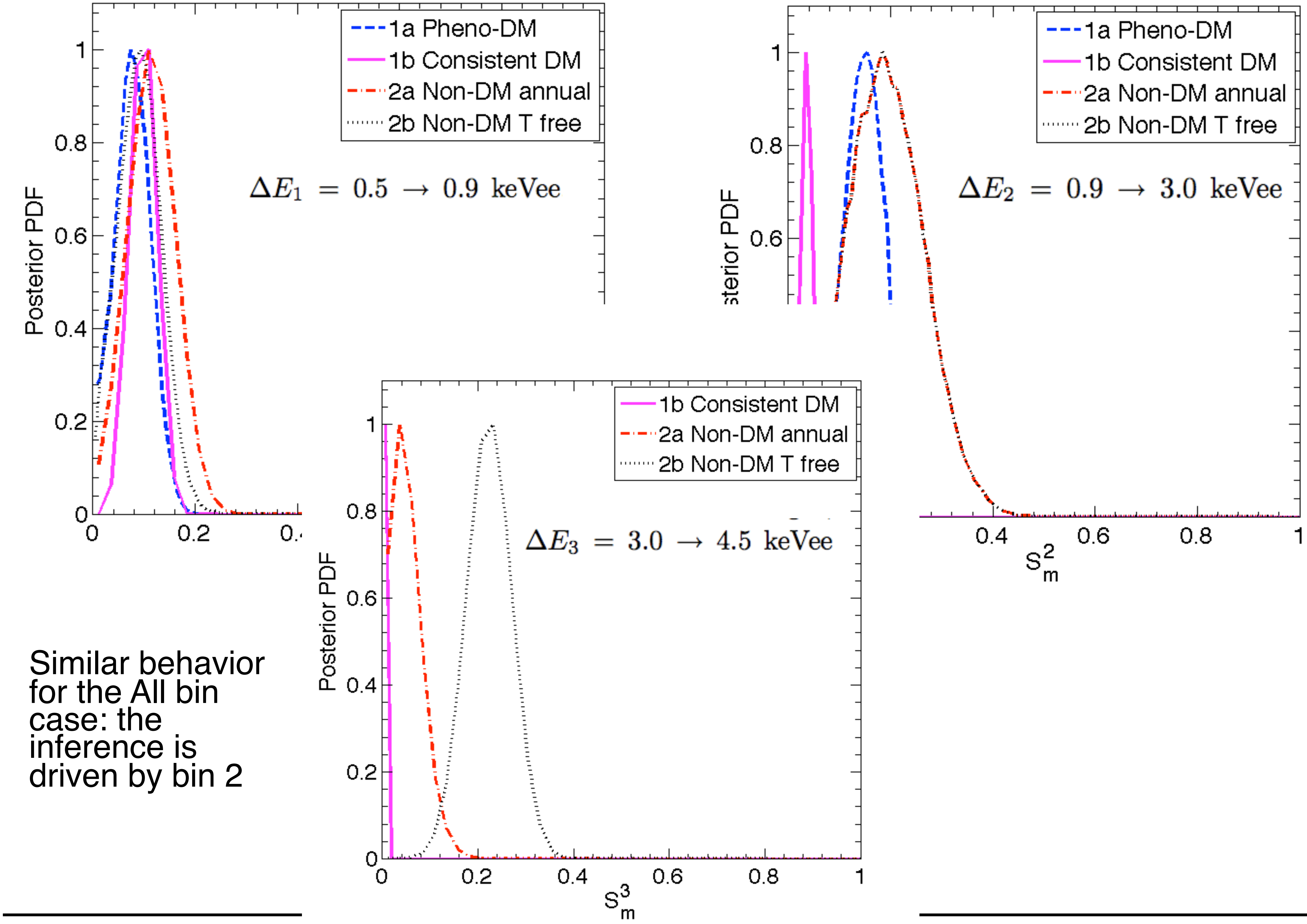
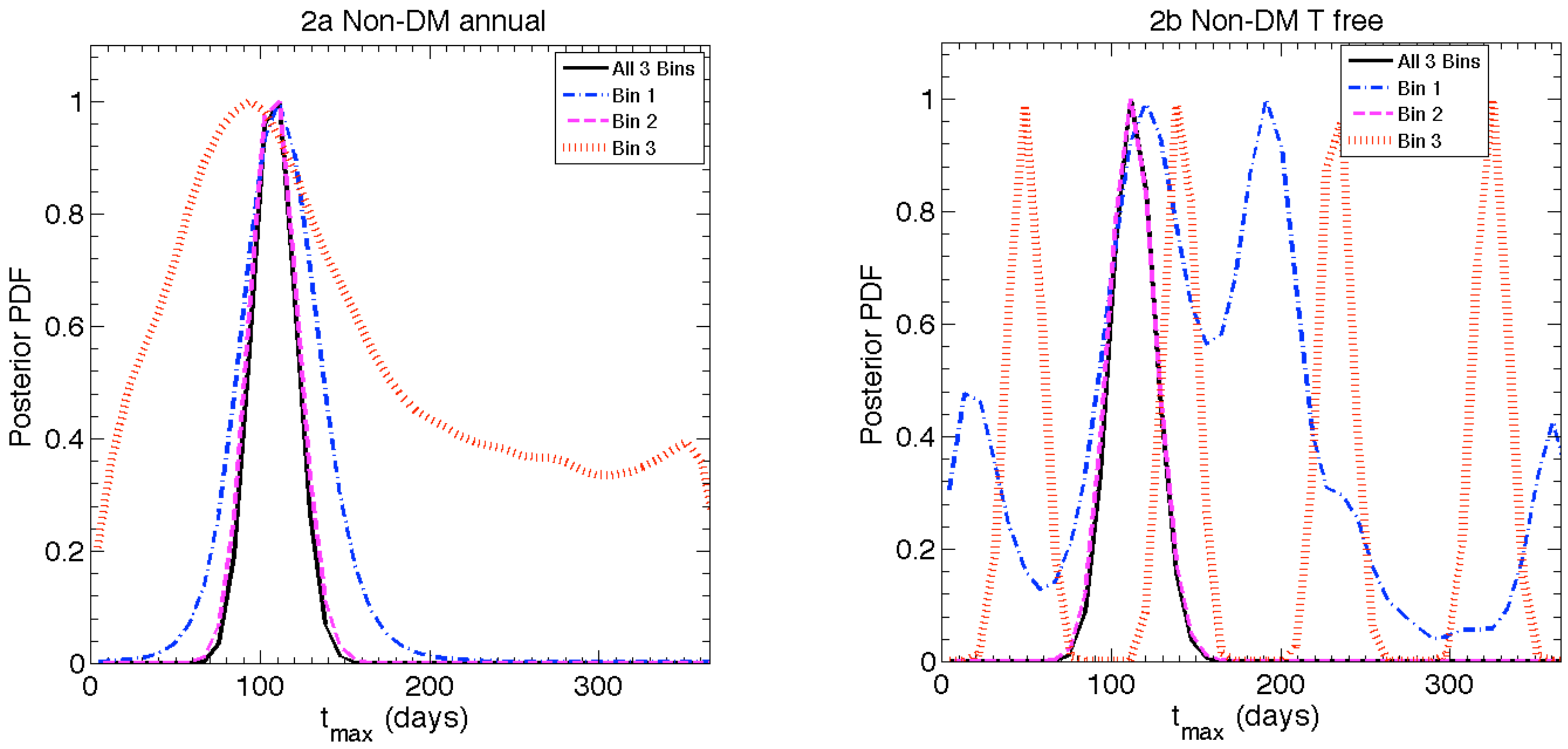
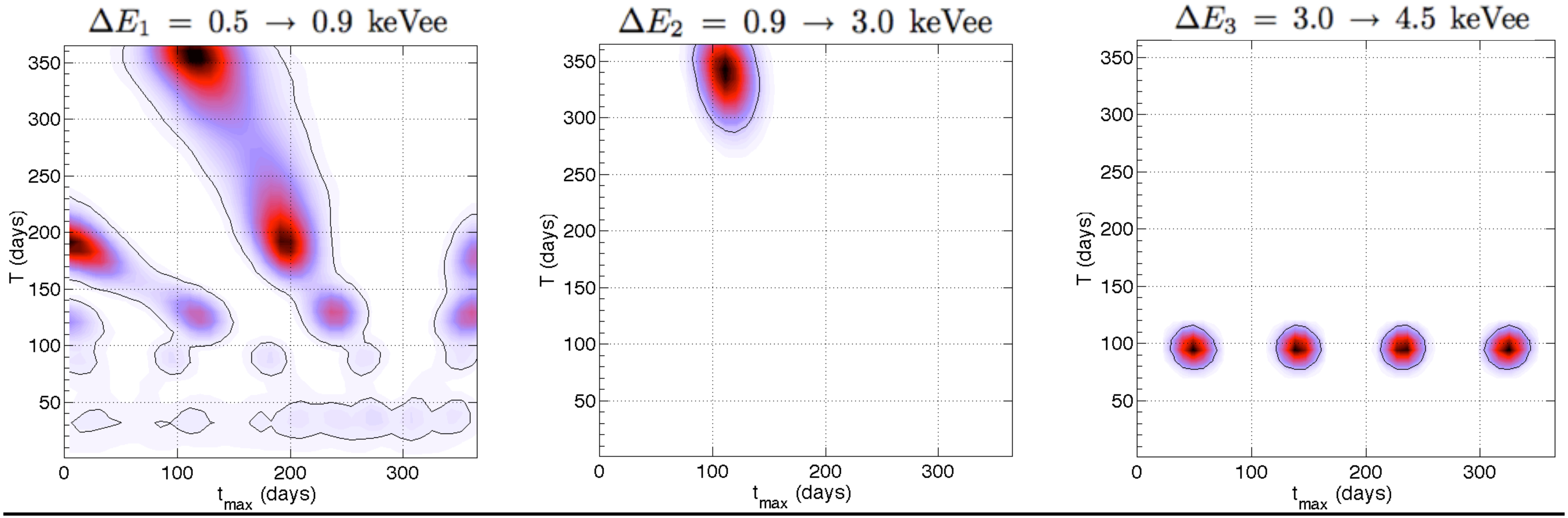


Similar behavior for the All bin case: the inference is driven by bin 2

Parameter inference: phase and period


2a Non-DM annual

2b Non-DM T free


$\Delta E_1 = 0.5 \rightarrow 0.9$ keVee

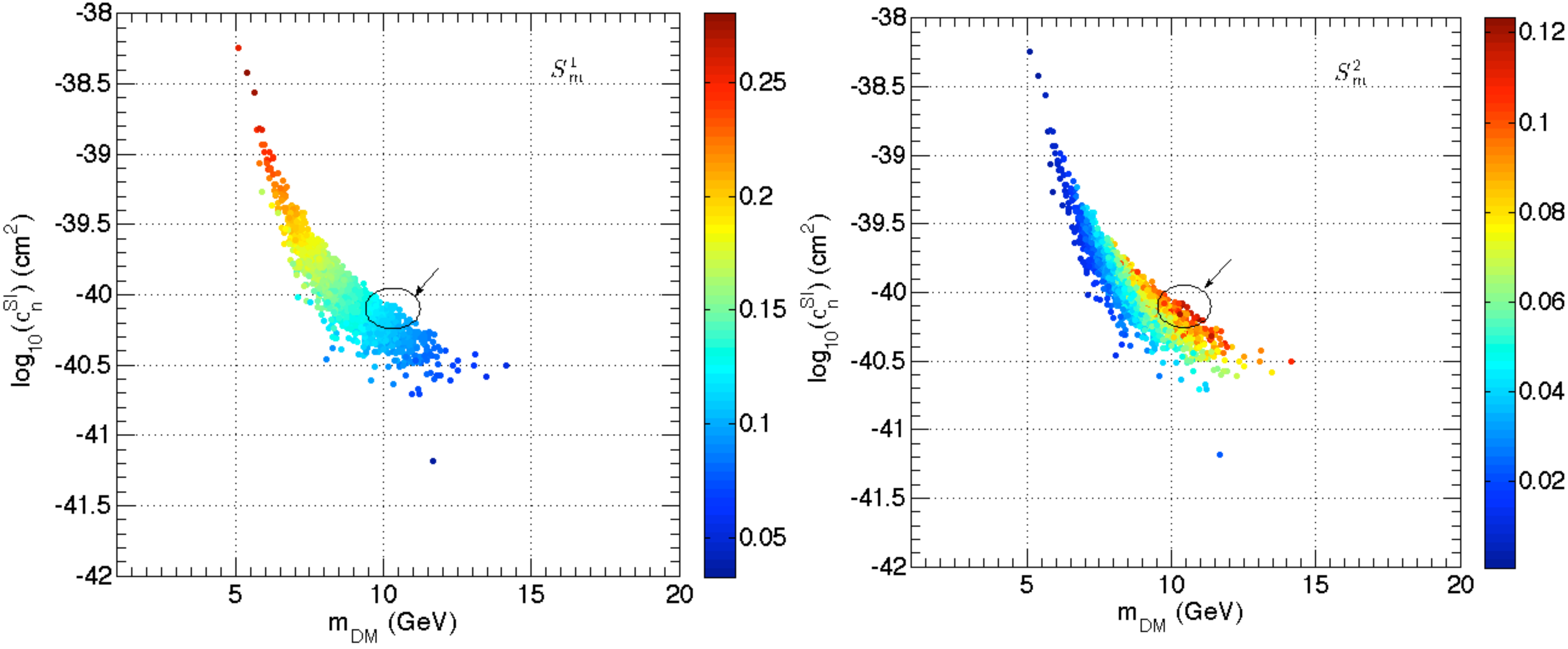
$\Delta E_2 = 0.9 \rightarrow 3.0$ keVee

$\Delta E_3 = 3.0 \rightarrow 4.5$ keVee

Locally anisotropic DM velocity

Ellipsoidal, triaxial DM halo model gives rise to a triaxial gaussian velocity distribution:

$$f(\vec{v'}(t)) = \frac{1}{(2\pi)^{3/2}\sigma_R\sigma_\phi\sigma_z} \exp\left[-\frac{v_R'^2}{2\sigma_R^2} - \frac{(v_\phi' + v_\oplus)^2}{2\sigma_\phi^2} - \frac{v_z'^2}{2\sigma_z^2}\right]$$



Background CoGeNT

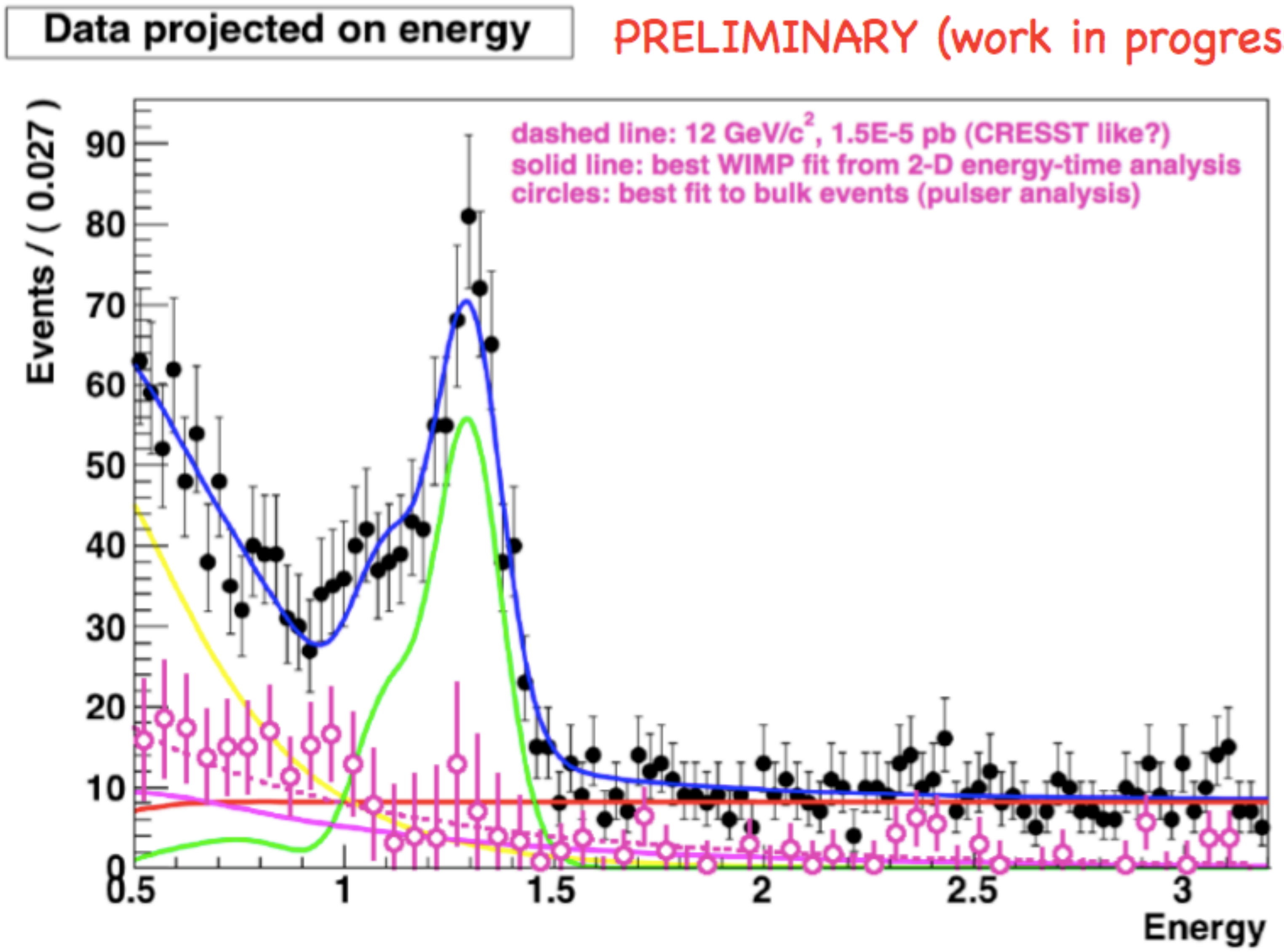
Priors on the fractional modulated amplitude predicted from configurations of DM mass and sigma that account for the CoGeNT total rate

$$R(t) = S(t) + B$$

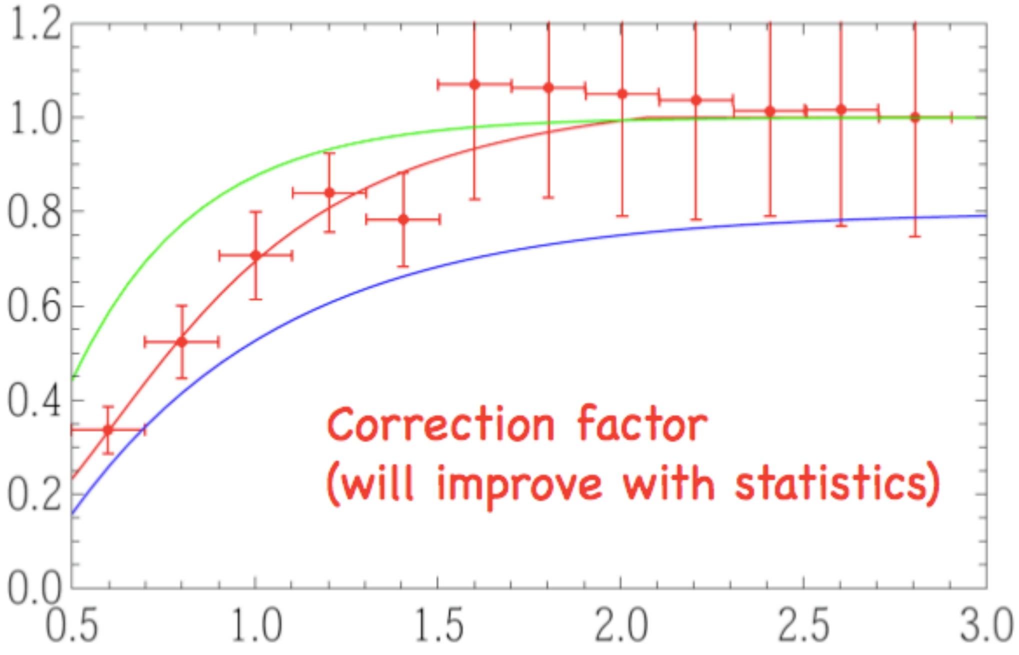
$$\frac{\mathrm{d}B}{\mathrm{d}\mathcal{E}} = \mathcal{C} + \mathcal{A}\exp(-\mathcal{E}/\mathcal{E}_0)$$

Background:

1. does not modulate, included only for the total rate
2. constant + exponential background (mimic surface events)



PRELIMINARY (work in progress)



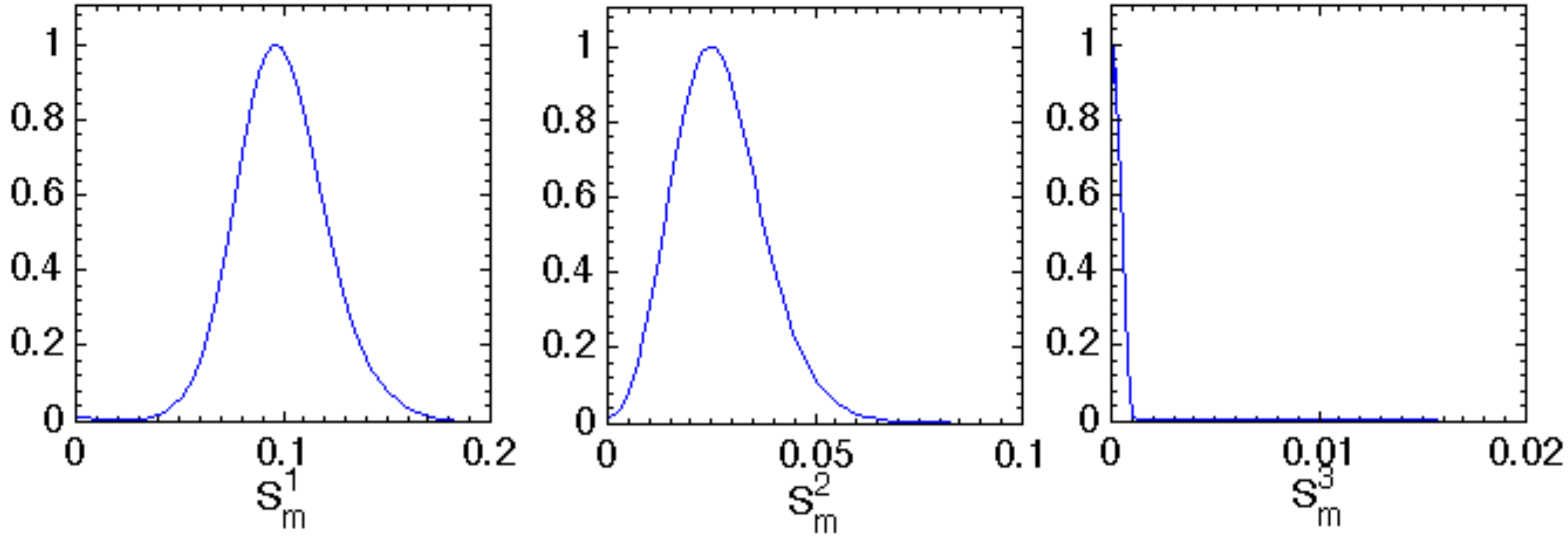
Correction factor (will improve with statistics)

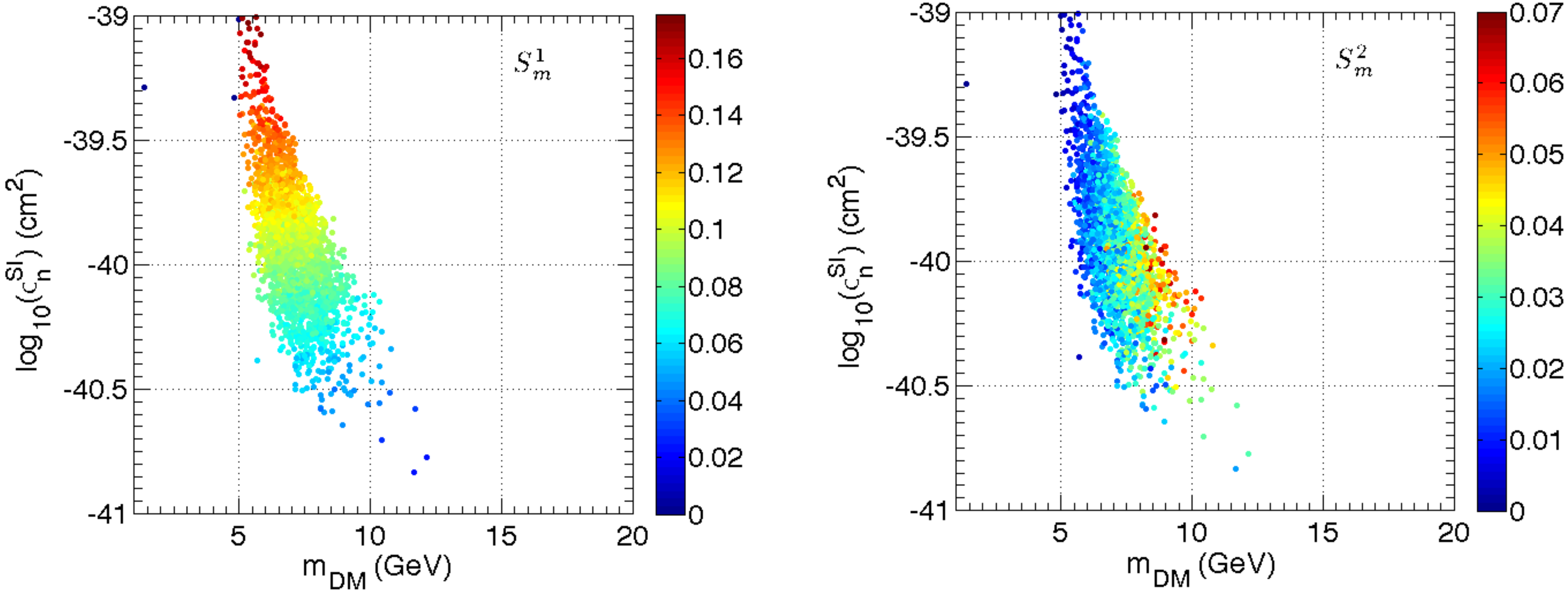
J. Collar talk @ TAUP 2011.

Model 1b: consistent DM

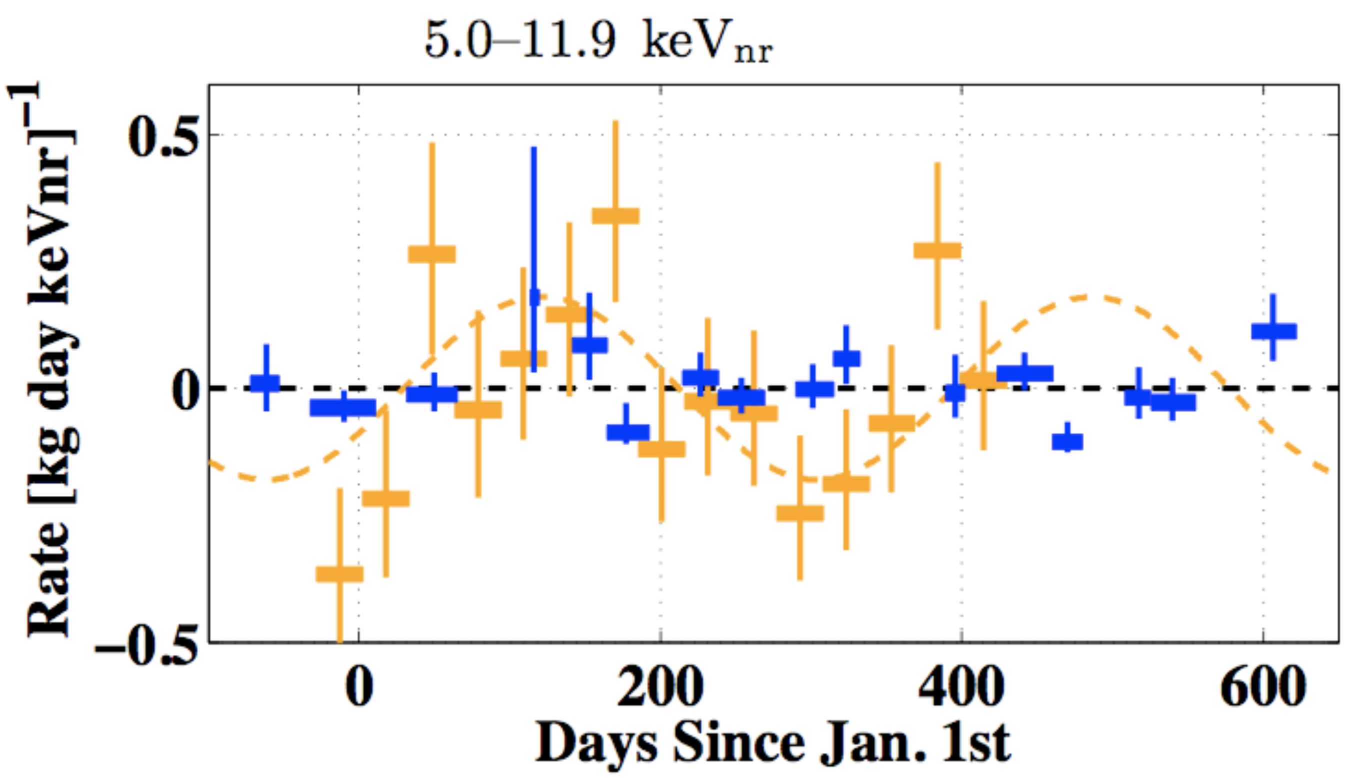
Priors on the fractional modulated amplitude predicted from configurations of DM mass and sigma that account for the CoGeNT total rate $R(t) = S(t) + B$

$$S_{\rm m} = \frac{R(t_{\rm max}) - R(t_{\rm min})}{R(t_{\rm max}) + R(t_{\rm min})}$$





Annual modulation in CoGeNT and in



CDMS collaboration, Z. Amhed et al., arXiv:1203.1309 [astro-ph.CO]

- 214 kg days exposure
- requirement that event scatter only once in the detector
- 5 keVnr as threshold
- No evidence for annual modulation

Amplitudes larger than $0.06\ [\mathrm{keV_{nr}\ kg\ day}]^{-1}$ are excluded at 99% C.L.