A Holographic Model of the Kondo Effect

Jackson Wu

National Center for Theoretical Sciences

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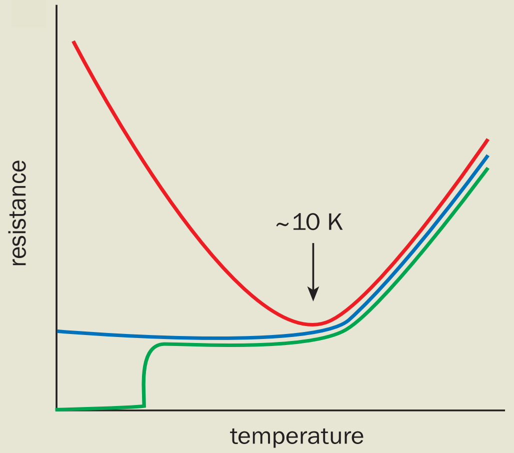
Based on the joint work 1310.3271 with

J. Erdmenger (Max-Planck-Institut für Physik, München)

C. Hoyos (Tel-Aviv University)

A. O'Bannon (DAMPT Cambridge University)

The Kondo Effect



Progress of Theoretical Physics, Vol. 32, No. 1, July 1964

Resistance Minimum in Dilute Magnetic Alloys

Jun KONDO

The Kondo Effect

Kondo Hamiltonian:

$$H_K = \sum_{k,\sigma} \epsilon(k) \psi^\dagger_{k\sigma} \psi_{k\sigma} + \hat{\lambda}_K \, \delta(\vec{x}) \, \vec{S} \cdot \sum_{k\sigma k'\sigma'} \psi^\dagger_{k\sigma} \frac{1}{2} \vec{\tau}_{\sigma\sigma'} \, \psi_{k'\sigma'}$$

- $\psi_{k\sigma}$, $\psi^\dagger_{k\sigma}$ conduction electrons
- $\sigma = \uparrow, \downarrow$ **spin** $SU(2)$, $\vec{\tau}$ Pauli matrices
- $\psi_{k\sigma} \to e^{i\alpha} \psi_{k\sigma}$ **charge** $U(1)$
- $\epsilon(k) = \frac{k^2}{2m} - \epsilon_F$ **dispersion relation**
- $\vec{S}$ **impurity spin**
- $\hat{\lambda}_K$ **Kondo coupling**
  - $\hat{\lambda}_K < 0$ ferromagnetic
  - $\hat{\lambda}_K > 0$ anti-ferromagnetic

Logarithmic behaviour at low temperatures

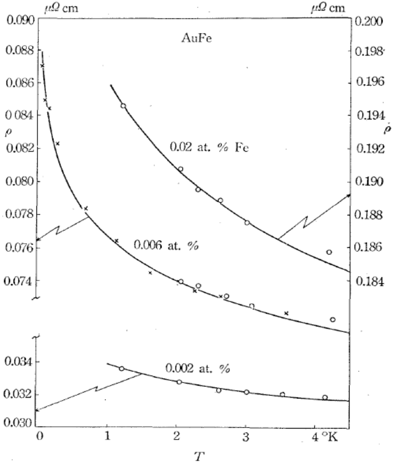


Fig. 1. Comparison of experimental and theoretical ρ–T curves for dilute AuFe alloys.

Jun Kondo:

- Progress of Theoretical Physics
- Volume 32, Issue 1
- Pp. 37-49

Resistivity:

$$\rho(T) = \rho_0 + aT^2 + bT^5 + c\hat{\lambda}_K^2 - \tilde{c}\hat{\lambda}_K^3 \log \frac{T}{\Lambda}$$

- c, $\tilde{c} \propto$ impurity concentrations, Λ cutoff scale
- $\rho(T) \uparrow$ as $T \downarrow$ if $\hat{\lambda}_K > 0$ (anti-ferromagnetic)

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Kondo temperature: $T_K \approx \Lambda \exp\left(-\frac{c}{\tilde{c}}\frac{1}{\hat{\lambda}_K}\right)$

- Perturbation theory breaks down when $\mathcal{O}(\hat{\lambda}_K^2) \sim \mathcal{O}(\hat{\lambda}_K^3)$
- T_K dynamically generated scale

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Asymptotic freedom: "$T_K \sim \Lambda_{QCD}$"

- $\beta(\hat{\lambda}_K) \sim -\hat{\lambda}_K^2 + \mathcal{O}(\hat{\lambda}_K^3)$
- Coupling diverges at low energy

The Kondo problem

What is the ground state of H_K at low temperature?

Solution known for single-impurity problem:

UV: Fermi liquid + decoupled spin

IR: Fermi liquid + non-magnetic impurity + special BC

- Magnetic impurity screened by the formation of the Kondo resonance (heuristically: $e^- +$ impurity $\to SU(2)$ singlet)
- Electron wavefunction vanish at impurity

Generalisations

Entend the spin group: $SU(2) \rightarrow SU(N)$

Enlarge the impurity spin representation: $s_{imp} = 1/2 \rightarrow R_{imp}$

Multiple channels ("flaovurs"): $\psi \rightarrow \psi_\alpha$, $\alpha = 1, \ldots, k$

Total symmetry: $SU(N) \times SU(k) \times U(1)$

Kondo model specified by N, k, and R_{imp}

CFT Description [Affleck & Ludwig 90s]

EFT: Chiral fermions in 1D interacting with impurity at origin

$$H = \frac{v_F}{2\pi}\,\psi_L^\dagger i\partial_x \psi_L + v_F\,\lambda_K\,\delta(x)\,\vec{S}\cdot\psi_L^\dagger\,\frac{1}{2}\vec{\tau}\,\psi_L,$$

- $\lambda_K = \frac{m^2}{2\pi^2} v_F \hat{\lambda}_K$ classically marginal

UV: $\lambda_K \to 0$, free $(1+1)$-d CFT

- Kac-Moody algebra: $SU(N)_k \otimes SU(k)_N \times U(1)$
  - $SU(N)$ **spin**: $\vec{J} = \psi_L^\dagger \vec{\tau} \psi_L$
  - $SU(k)$ **channel**: $J^A = \psi_L^\dagger t^A \psi_L$
  - $U(1)$ **charge**: $J = \psi_L^\dagger \psi_L$
  - $J^a(z) = \sum_{n\in\mathbb{Z}} z^{-n-1} J_n^a$, $[J_n^a, J_m^b] = i\, f^{abc} J_{n+m}^c + \eta\,\frac{n}{2}\,\delta^{ab}\,\delta_{n,-m}$
- Spectrum determined by current algebra and BC's.

IR Fixed Point

Sugawara form:

$$H = \frac{1}{2\pi(N+k)} J^a J^a + \frac{1}{2\pi(k+N)} J^A J^A + \frac{1}{4\pi Nk} J^2 + \lambda_K \, \delta(x) \, \vec{S} \cdot \vec{J}$$

$$\to \frac{1}{2\pi(N+k)} \mathcal{J}^a \mathcal{J}^a + \frac{1}{2\pi(k+N)} J^A J^A + \frac{1}{4\pi Nk} J^2$$

$$\mathcal{J}^a \equiv J^a + \pi(N+k)\lambda_K \delta(x) S^a$$

- Impurity spin "absorbed" by the conduction electrons
- Same $SU(N)_k \times SU(k)_N \times U(1) \Leftrightarrow \lambda_K = \frac{2}{N+k}$

Kondo problem: How reps. rearrange between UV and IR

- Fusion rules: $R_{spin}^{IR} = R_{spin}^{UV} \times R_{imp}$
- IR CFT = UV CFT + shifted spectrum

Large N Approach

$SU(N)$ spin $\Rightarrow$ standard large N limit

Kondo effect appear as (0+1)-d superconductivity

Slave fermions: $S^a = \chi^\dagger T^a \chi \,, a = 1, \ldots, N^2 - 1.$

- Impurity in totally antisymmetric $SU(N)$ rep. Q
- Extra $U(1)$ symmetry $\Rightarrow$ **constraint** $\chi^\dagger \chi = Q$.

$\mathcal{O} \equiv \psi_L^\dagger \chi$ $SU(N)$ singlet, $SU(k) \times U(N_f)$ bi-fundamental

$$\lambda_K \, \delta(x) \, J^a S^a = \lambda_K \, \delta(x) \, \left(\psi_L^\dagger T^a \psi_L\right) \, \left(\chi^\dagger T^a \chi\right)$$
$$= \frac{1}{2} \lambda_K \, \delta(x) \mathcal{O} \mathcal{O}^\dagger + \mathcal{O}(1/N)$$

- $\mathcal{O}\mathcal{O}^\dagger$ classically marginal "double trace" deformation.
- $\langle \mathcal{O} \rangle \neq 0$ when $T \leq T_c \leftrightarrow$ formation of **Kondo singlet**.

Essential Ingredients

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
N_c D3	•	•	•	•	–	–	–	–	–	–
N_7 D7	•	•	–	–	•	•	•	•	•	•
N_5 D5	•	–	–	–	•	•	•	•	•	–

(1+1)-d chiral fermions $\psi_L \leftrightarrow$ **probe D7 along** $AdS_3 \times S^5$

FT $U(N_7)$ current J_μ obeying Kac-Moody algebra (N, k)

Dual Chern-Simons gauge field A_μ

Impurity slave fermion $\chi \leftrightarrow$ **probe D5 along** $AdS_2 \times S^5$

FT $U(N_5)$ current j_μ with $\chi^\dagger \chi = Q$ (R_{imp})

Dual Yang-Mills gauge field a_μ with flux Q

Kondo interaction:

FT Bilinear scalar operator $\mathcal{O} = \psi_L^\dagger \chi$

Dual Bi-fundamental complex scalar Φ (tachyon)

Holography: Bottom-Up Model

Action:

$$S = S_{CS} + S_{AdS_2}$$

$$S_{CS} = -\frac{N}{4\pi}\int \mathrm{tr}\left(A \wedge dA + \frac{2}{3}A \wedge A \wedge A\right)$$

$$S_{AdS_2} = -N\int d^3x\, \delta(x)\sqrt{-g}\left[\frac{1}{4}\mathrm{tr} f^2 + |D\Phi|^2 + V(\Phi^\dagger\Phi)\right]$$

$$D\Phi = \partial\Phi + i\, A\, \Phi - i\, a\, \Phi$$

Bottom-up: Choose $V(\Phi^\dagger\Phi) = M^2\Phi^\dagger\Phi$

Finite temperature: BTZ black hole

$$ds^2 = \frac{1}{z^2}\left(\frac{dz^2}{h(z)} - h(z)\, dt^2 + dx^2\right), \quad h(z) = 1 - \frac{z^2}{z_H^2}$$

The Kondo Coupling

Near the boundary $z \to 0$:

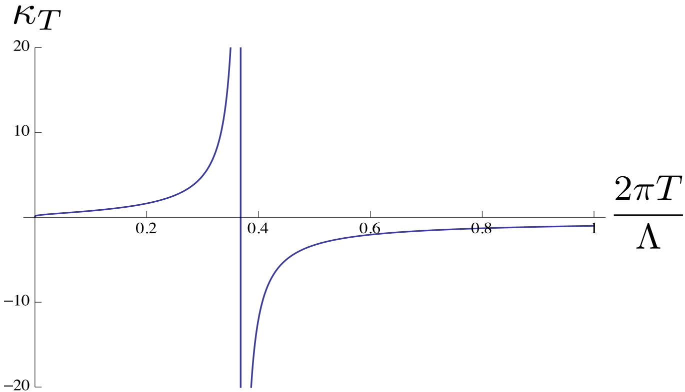
$$a_t(z) \sim \frac{Q}{z} + \mu\,, \quad \phi(z) \sim \sqrt{z}\,(\alpha \log(\Lambda z) + \beta)\ , \quad A_x(z) \to 0$$

- **BC**: Boundary flux $\sqrt{-g} f^{tz}|_{z=0} = -Q$
- Double trace coupling: $\alpha = \kappa\beta \propto \langle \mathcal{O} \rangle$ [Witten 01]

Running of coupling: $\phi(z)$ independent of scaling

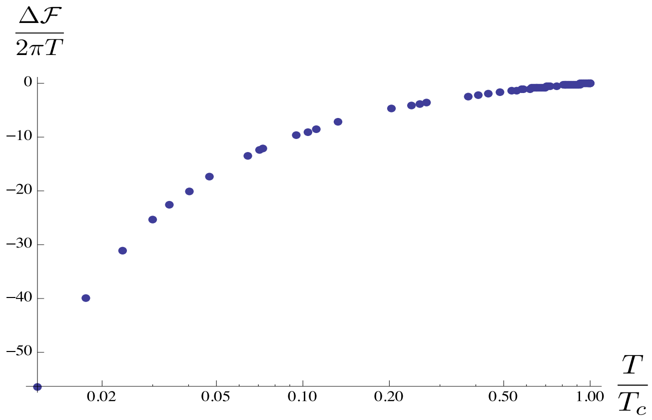
$$\kappa_T \beta_T = \frac{\kappa\beta}{2\pi T}\,, \quad \kappa_T = \frac{\kappa}{1 + \kappa \log \frac{\Lambda}{2\pi T}}$$

Dynamical Scale Generation



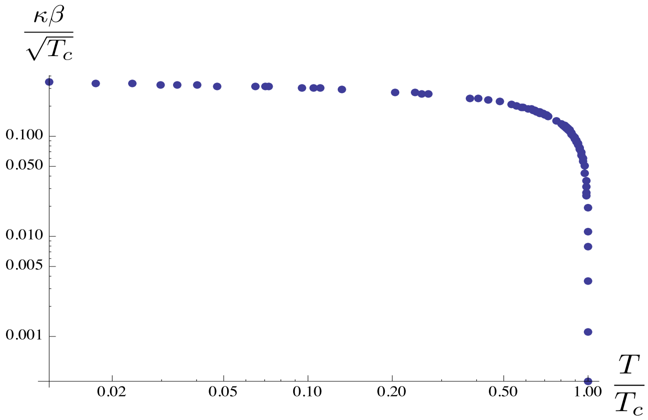
Divergence of κ_T determines $T_K = \frac{1}{2\pi}\Lambda e^{1/\kappa}$

Phase Transition



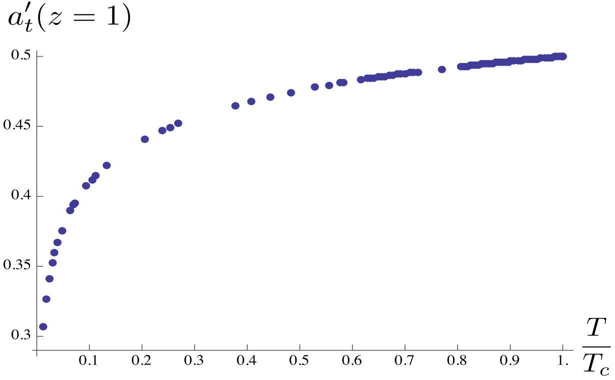
$\Delta\mathcal{F} = \mathcal{F}_{\phi(z)\neq 0} - \mathcal{F}_{\phi(z)=0}$, $T_c/T_K \approx 0.90$

The Condensate



Mean-field transition: $\langle \mathcal{O} \rangle \propto \left(1 - \frac{T}{T_c}\right)^{1/2}$, $T \lesssim T_c$

Screening of Impurity



Flux at horizon: $\sqrt{-g} f^{tz}|_{z=1} = a_t'(z=1)$

Non-trivial $\phi(z)$ draws charge away from $a_t(z)$, reducing flux at horizon $\Rightarrow R_{imp}^{IR} < R_{imp}^{UV} = Q$, i.e. **impurity screened!**

Summary

A simple and realistic holographic model that describes the Kondo effect along the entire RG flow; useful for further model building.

Holographic dual of Kondo effect at large N:

Holographic superconductor in AdS_2 with the "double trace" boundary condition imposed on the scalar field coupled as a defect to the CS gauge field in AdS_3.

Open problems: Multi-impurities, Kondo lattice, quantum quenches, entanglement entropy, ...