Calculation of absolute neutrino masses in the seesaw extension

D. Jurčiukonis, T. Gajdosik and A. Juodagalvis

Vilnius University, Institute of Theoretical Physics and Astronomy

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Seesaw Mechanism

• The original SM has massless neutrinos, but the observation of neutrino oscillations requires that neutrinos are massive.

• The seesaw mechanism than suggests an explanation for the observed smallness of neutrino masses.

• The simple extesion of adding gauge singlet fermions to the particle spectrum allows normal Yukawa mass terms for neutrinos:

$$\mathscr{L}_{\mathrm{Y}} = -\sum_{k=1}^{2} \left(\Phi_k^\dagger \bar{\ell}_R \Gamma_k + \tilde{\Phi}_k^\dagger \bar{\nu}_R \Delta_k \right) D_L + \mathrm{H.c.}$$

left-handed lepton doublet	$D_L = (\nu_L \ \ell_L)^T$
right-handed leptons	$\ell_R,\ \nu_R$
Yukawa coupling matrices	Γ_k are $n_L \times n_L$ Δ_k are $n_R \times n_L$

• The mass terms for the neutrino are:

$$-\bar{\nu}_R M_D \nu_L - \frac{1}{2} \bar{\nu}_R C M_R \bar{\nu}_R^T + \mathrm{H.c.}$$

Seesaw Mechanism

• **In this model (with 2 Higgs doublets), spontaneous symmetry breaking of the SM gauge group is achieved by the vacuum expectation values**

$$\langle \Phi_k \rangle_{\text{vac}} = \begin{pmatrix} 0 \\ v_k/\sqrt{2} \end{pmatrix}$$

Which satisfy the condition:

$$v = \sqrt{|v_1|^2 + |v_2^2|} \simeq 246 \text{ GeV}$$

• **Without loss of generality we are free to choose certain bases for the various fields. By a unitary rotation of the Higgs doublets, we can achieve**

$$\langle \Phi_1^0 \rangle_{\text{vac}} = v/\sqrt{2} > 0 \qquad \langle \Phi_2^0 \rangle_{\text{vac}} = 0$$

• **The charged-lepton mass matrix**

$$M_\ell = \frac{v}{\sqrt{2}} \Gamma_1$$

• **The Dirac neutrino mass matrix**

$$M_D = \frac{v}{\sqrt{2}} \Delta_1$$

with assumption that: $M_\ell = \text{diag}\left(m_e, m_\mu, m_\tau\right)$

Seesaw Mechanism

- **Neutral fermion mass matrix is given by:**

$M_D = vY_\nu$ Dirac mass term for SM neutrinos

M_L Majorana mass term for SM neutrinos

- it does not exist at tree level $M_L|_{\text{tree}} = 0$
- it can be generated at the loop level $\delta M_L|_{\text{loop}} \neq 0$

$$M_\nu = \begin{pmatrix} M_L & M_D^T \\ M_D & M_R \end{pmatrix} \quad (n_L + n_R)$$

$$(n_L + n_R)$$

- **Diagonalization with unitary matrix *U* gives rise to a split spectrum consisting of heavy and light states of neutrino masses**

$$U^T M_\nu U = \hat{m} = \mathrm{diag}\left(m_1, m_2, \ldots, m_{n_L+n_R}\right)$$

- **In order to implement see-saw mechanism we assume:**

M_D —order→ m_D

M_R —order→ m_R

$$m_D \ll m_R$$

m_i ⇒

$i = 1, 2, \ldots, n_L$ —order→ m_D^2/m_R

$i = n_L + 1, \ldots, n_L + n_R$ —order→ m_R

Seesaw Mechanism

- **It is useful to decompose the unitary matrix U to two submatrices U_L and U_R**

$$U = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix}$$

- **where the submatrices have dimensions:**

$$U_L \Longrightarrow n_L \times (n_L + n_R)$$
$$U_R \Longrightarrow n_R \times (n_L + n_R)$$

- **and fulfill unitarity relations:**

$$U_L U_L^\dagger = \mathbb{1}_{n_L} \qquad U_L^* \hat{m} U_L^\dagger = 0$$
$$U_R U_R^\dagger = \mathbb{1}_{n_R} \qquad U_R \hat{m} U_R^T = M_R$$
$$U_L^\dagger U_L + U_R^T U_R^* = \mathbb{1}_{n_L+n_R} \qquad U_L U_R^T = 0_{n_L \times n_R} \qquad U_R \hat{m} U_L^\dagger = M_D$$

- **with these submatrices, the left- and right-handed neutrinos are written as linear superpositions of the physical Majorana neutrino fields χ_i**

$$\nu_R = U_R P_R \chi$$
$$\nu_L = U_L P_L \chi$$

One-loop corrections

- **The one-loop corrections to the tree mass matrix are determined by the neutrino intereactions with the Z bozon , the neutral Goldstone boson (G^0), and the Higgs boson (H^0)**

- **The interaction of the *Z* bozon with the neutrinos is given by:**

$$\mathcal{L}_{\rm nc}^{(\nu)} = \frac{g}{4c_w} Z_\mu \bar{\chi} \gamma^\mu \left[P_L \left(U_L^\dagger U_L \right) - P_R \left(U_L^T U_L^* \right) \right] \chi$$

where *g* is the SU(2) gauge coupling constant and c_w is the cosine of Weinberg angle

- **The Yukawa coupling of the Higgs boson and the neutrinos is given by:**

$$\mathcal{L}_{\rm Y}^{(\nu)}\left(H^0\right) = -\frac{1}{2\sqrt{2}} \sum_b H_b^0 \, \bar{\chi} \Big[\left(U_R^\dagger \Delta_b U_L + U_L^T \Delta_b^T U_R^* \right) P_L + \left(U_L^\dagger \Delta_b^\dagger U_R + U_R^T \Delta_b^* U_L^* \right) P_R \Big] \chi$$

- **The Yukawa coupling of the Goldstone boson and the neutrinos is given by:**

$$\mathcal{L}_{\rm Y}^{(\nu)}\left(G^0\right) = -\frac{1}{2\sqrt{2}} G^0 \, \bar{\chi} \Big[\left(U_R^\dagger \Delta_{b_Z} U_L + U_L^T \Delta_{b_Z}^T U_R^* \right) P_L + \left(U_L^\dagger \Delta_{b_Z}^\dagger U_R + U_R^T \Delta_{b_Z}^* U_L^* \right) P_R \Big] \chi$$

One-loop corrections

- **In 2 Higgs doublets model neutral scalars are characterized by 4 complex unit vectors b_k, k = 1,2,3 and b_Z**

$$\Delta_b = \sum_k b_k \Delta_k$$

The b_k vectors characterize states of neutral Higgses

For the chosen basis b_Z vector receive precise form

$$(b_Z)_k = i\frac{v_k}{v}$$

The b_Z vector characterize Goldstone boson

$$b_Z = \begin{pmatrix} i \\ 0 \end{pmatrix}$$

- **Vectors have to fulfill the orthogonality conditions:**

$$\sum_j \mathrm{Re}\,(b_j (b_Z)_j) = 0 \qquad \sum_b b_j b_k + (b_Z)_j (b_Z)_k = 0 \qquad \sum_b \mathrm{Re}\, b_k \,\mathrm{Re}\, b_{k'} = \sum_b \mathrm{Im}\, b_k \,\mathrm{Im}\, b_{k'} = \delta_{kk'}$$

$$\sum_k (\mathrm{Re}\, b_k \,\mathrm{Re}\, b'_k + \mathrm{Im}\, b_k \,\mathrm{Im}\, b'_k) = \mathrm{Re}\,(b^\dagger b') = \delta_{bb'} \qquad \sum_b \mathrm{Re}\, b_k \,\mathrm{Im}\, b_{k'} = 0$$

- **Once one-loop corrections are taken into account the neutral fermion mass matrix is given by:**

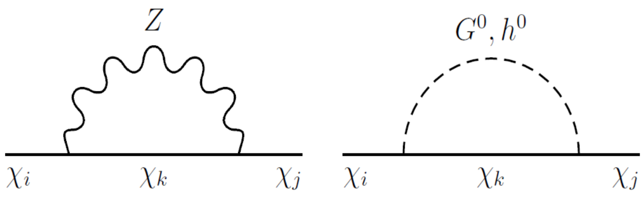
$$M_\nu^{(1)} = \begin{pmatrix} \delta M_L & M_D^T + \delta M_D^T \\ M_D + \delta M_D & \hat{M}_R + \delta M_R \end{pmatrix} \approx \begin{pmatrix} \delta M_L & M_D^T \\ M_D & \hat{M}_R \end{pmatrix}$$

where the 0_{3x3} submatrix appearing in tree level is replaced by the contribution δM_L

One-loop corrections

- The self energy functions arise from the self-energy Feynman diagrams (evaluated at zero external momentum) involving the Z, the neutral Goldstone (G^0), and the Higgs (h^0) bosons

$$-i\Sigma_L^S(0) = -i\left[\Sigma_L^{S(Z)}(0) + \Sigma_L^{S(G^0)}(0) + \Sigma_L^{S(h^0)}(0)\right]$$



- The Z and Goldstone (G^0) bosons contributes as:

$$\delta M_L\,(Z) = i\,\frac{g^2}{4c_w^2}\int\frac{d^Dk}{(2\pi)^D}\left[\frac{D}{k^2-m_Z^2} + \frac{1}{m_Z^2}\left(\frac{k^2}{k^2-\xi_Z m_Z^2} - \frac{k^2}{k^2-m_Z^2}\right)\right]U_L^*\frac{\hat{m}}{k^2-\hat{m}^2}U_L^\dagger$$

$$\delta M_L\left(G^0\right) = -\frac{ig^2}{4m_W^2}\int\frac{d^Dk}{(2\pi)^D}\frac{1}{k^2-\xi_Z m_Z^2}U_L^*\frac{\hat{m}^3}{k^2-\hat{m}^2}U_L^\dagger$$

- Finally the finite contributions from the Z and Higgs self-energy functions reads:

$$\delta M_L = \sum_b \frac{1}{32\pi^2}\Delta_b^T U_R^* \hat{m}\left(\frac{\hat{m}^2}{m_{H_b^0}^2} - \mathbb{1}\right)^{-1}\ln\left(\frac{\hat{m}^2}{m_{H_b^0}^2}\right)U_R^\dagger \Delta_b + \frac{3g^2}{64\pi^2 m_W^2}M_D^T U_R^* \hat{m}\left(\frac{\hat{m}^2}{m_Z^2} - \mathbb{1}\right)^{-1}\ln\left(\frac{\hat{m}^2}{m_Z^2}\right)U_R^\dagger M_D$$

Infinities parts and the contribution from the neutral Goldstone boson cancels after summation of all terms

Case $n_R = 1$

• **For this case (toy model) we consider the minimal extension of standard model adding only one right-handed ν_R field to the three left-handed fields contained in ν_L**

we use parametrization of $\Delta_1 = \frac{\sqrt{2}m_D}{v}\vec{a}_1^{\,T}$ $\Delta_2 = \frac{\sqrt{2}m_D}{v}\vec{a}_2^{\,T}$

$$U^T M_\nu U = U^T \begin{pmatrix} 0 & 0 & 0 & m_D a_1 \\ 0 & 0 & 0 & m_D a_2 \\ 0 & 0 & 0 & m_D a_3 \\ m_D a_1 & m_D a_2 & m_D a_3 & m_R \end{pmatrix} U = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_l & 0 \\ 0 & 0 & 0 & m_h \end{pmatrix}$$

• **after diagonalization we receive one light and one heavy states of neutrino masses**

$$M_l = \mathrm{diag}(0, 0, m_l) \qquad M_h = m_h$$

the non zero masses are determined analytically by finding eigenvalues of the hermitian matrix $M_\nu M_\nu^\dagger$ which are the squares of the masses of the neutrinos

these analytical solutions correspond to the seesaw mechanism:

$$m_D^2 = m_h m_l$$
$$m_R^2 = (m_h - m_l)^2 \sim m_h^2$$

Case $n_R = 1$

- **We can construct the diagonalization matrix U for the tree level from two diagonal matrices of phases and three rotation matrices**

$$U_{\rm tree} \;=\; U_\phi(\phi_i)U_{12}(\alpha_1)U_{23}(\alpha_2)U_{34}(\beta)U_i$$

- **detailed expressions of matrices:**

$$U_{12} = \begin{pmatrix} \cos(\alpha_1) & \sin(\alpha_1) & 0 & 0 \\ -\sin(\alpha_1) & \cos(\alpha_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad U_{23} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_2) & \sin(\alpha_2) & 0 \\ 0 & -\sin(\alpha_2) & \cos(\alpha_2) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad U_{34} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\beta) & \sin(\beta) \\ 0 & 0 & -\sin(\beta) & \cos(\beta) \end{pmatrix}$$

$$U_\phi = \begin{pmatrix} \arg(a_1) & 0 & 0 & 0 \\ 0 & \arg(a_2) & 0 & 0 \\ 0 & 0 & \arg(a_3) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

the values of α_i and φ_i can be chosen to cover variations in M_D

$$\alpha_1 = \arctan\left(\frac{a_1}{a_2}\right)$$

$$\alpha_2 = \arctan\left(\frac{\sqrt{a_1^2 + a_2^2}}{a_3}\right)$$

$$U_i = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

the angle β is determined by the masses of light and heavy neutrinos

$$\beta = \operatorname{arccot}\left(\sqrt{\frac{m_h}{m_l}}\right)$$

Case $n_R = 1$

- **The final expression for *U* is as follows:**

$$U_{\text{tree}} = \begin{pmatrix} e^{i\phi_1}\cos(\alpha_1) & e^{i\phi_1}\sin(\alpha_1)\cos(\alpha_2) & ie^{i\phi_1}\sin(\alpha_1)\sin(\alpha_2)\cos(\beta) & e^{i\phi_1}\sin(\alpha_1)\sin(\alpha_2)\sin(\beta) \\ -e^{i\phi_2}\sin(\alpha_1) & e^{i\phi_2}\cos(\alpha_1)\cos(\alpha_2) & ie^{i\phi_2}\cos(\alpha_1)\sin(\alpha_2)\cos(\beta) & e^{i\phi_2}\cos(\alpha_1)\sin(\alpha_2)\sin(\beta) \\ 0 & -e^{i\phi_3}\sin(\alpha_2) & ie^{i\phi_3}\cos(\alpha_2)\cos(\beta) & e^{i\phi_3}\cos(\alpha_2)\sin(\beta) \\ 0 & 0 & -i\sin(\beta) & \cos(\beta) \end{pmatrix}$$

- **neutral fermion mass matrix expressed by angles:**

$$M_\nu = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2}e^{i\phi_1}m_R\sin(\alpha_1)\sin(\alpha_2)\tan(2\beta) \\ 0 & 0 & 0 & \frac{1}{2}e^{i\phi_2}m_R\cos(\alpha_1)\sin(\alpha_2)\tan(2\beta) \\ 0 & 0 & 0 & \frac{1}{2}e^{i\phi_3}m_R\cos(\alpha_2)\tan(2\beta) \\ \frac{1}{2}e^{i\phi_1}m_R\sin(\alpha_1)\sin(\alpha_2)\tan(2\beta) & \frac{1}{2}e^{i\phi_2}m_R\cos(\alpha_1)\sin(\alpha_2)\tan(2\beta) & \frac{1}{2}e^{i\phi_3}m_R\cos(\alpha_2)\tan(2\beta) & m_R \end{pmatrix}$$

- **diagonalization of the mass matrix after calculation of one-loop corrections is performed with unitary matrix U_{loop}**

$$U_{\text{loop}} = U_{\text{egv}}U_\varphi(\varphi_1, \varphi_2, \varphi_3)$$

We use singular value decomposition (SVD) for calculation of U_{loop}

where U_{egv} is an eigenmatrix of $M_\nu^{(1)}M_\nu^{(1)\dagger}$ and U_φ is a phase matrix

Case $n_R = 1$

in general case we use parametrization:

$$\Delta_1 = \frac{\sqrt{2} m_D}{v} \vec{a}_1^{\,T}$$

$$\Delta_2 = \frac{\sqrt{2} m_D}{v} \vec{a}_2^{\,T}$$

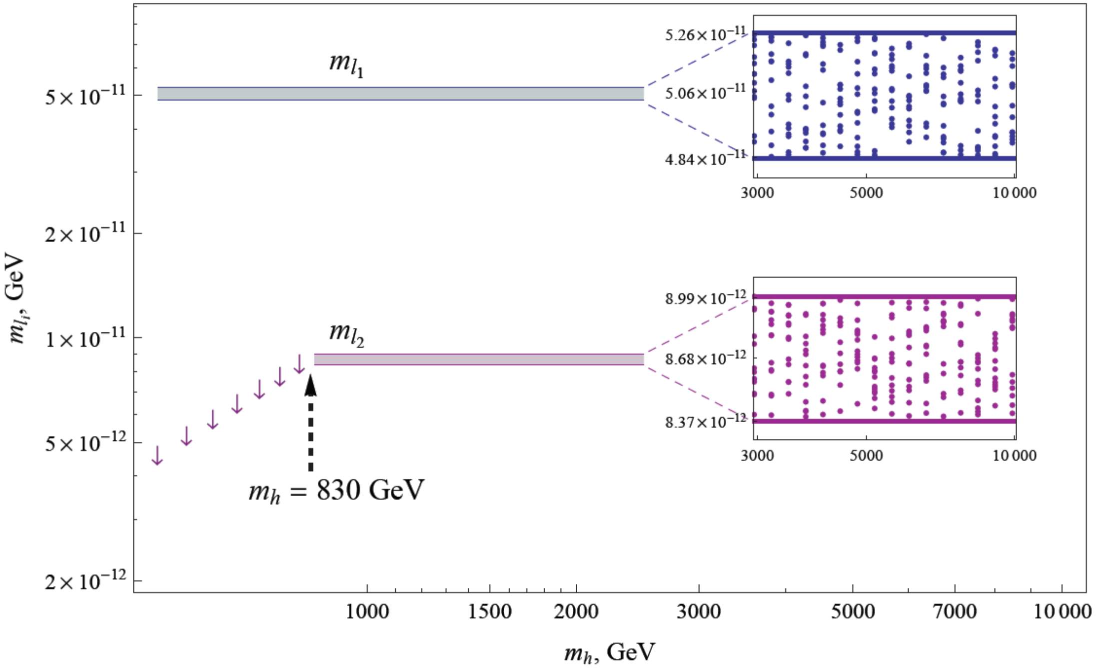
We vary: $\{m_h, m_{H_2}, m_{H_3}, \vec{a}_1, \vec{a}_2\}$

We fix: $\{m_{l_1}^{\text{tree}} = 5 \times 10^{-11} \text{ GeV}, \; m_{H_1} = 125 \text{ GeV}\}$

Set of *b* vectors used for numerical calculations

$$b_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad b_2 = \begin{pmatrix} 0 \\ i \end{pmatrix} \qquad b_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **masses of two light neutrinos as a function of the heavy neutrino mass m_h**



- Masses of the light neutrino scattered in their 3σ-band
- Here we do not sort values according 3σ-band of oscillation angles
- For this case min(m_h) = 830 GeV

Case $n_R = 1$ (reduced)

A basis transformation on ν_l can be performed in such a way that: $\Delta_i = \left(\sqrt{2}\, m_D/v\right) \vec{a}_i^T$

$$\vec{a}_1^T = (0,0,1) \qquad \vec{a}_2^T = (0,n,n') \qquad m_D, n > 0, \quad n' \in \mathbb{C}$$

We use orthogonal vectors

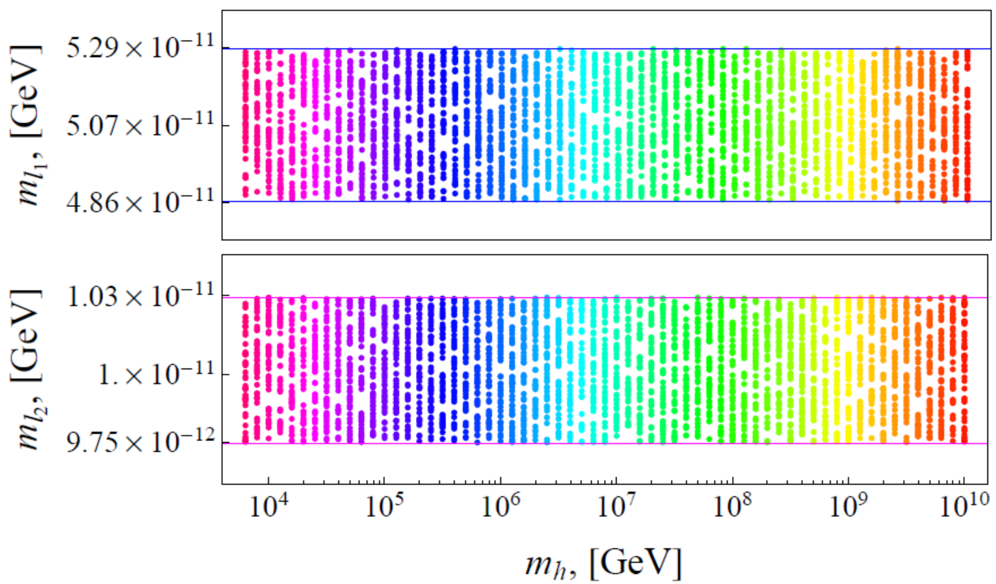
$$\vec{a}_1^T = (0,0,1) \qquad \vec{a}_2^T = \left(0, n, e^{i\phi}\sqrt{1-n^2}\right)$$

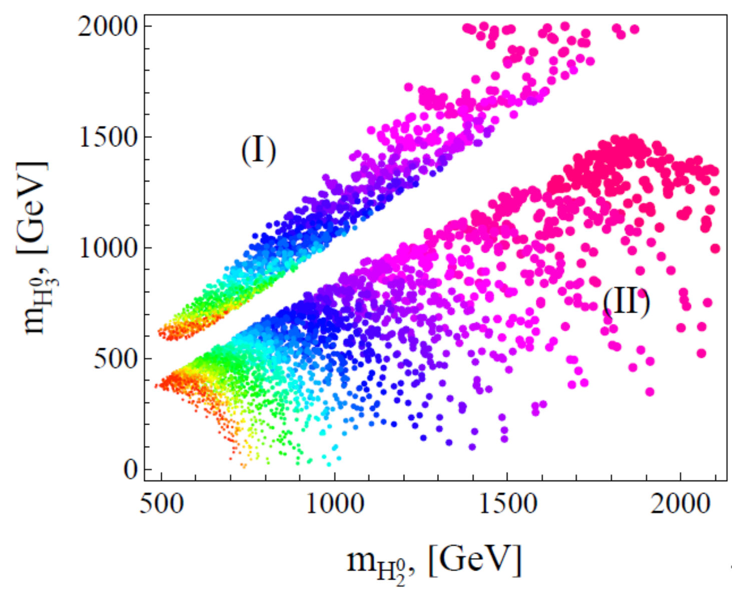
Set of b vectors used for numerical calculations:

$$b_Z = \begin{pmatrix} i \\ 0 \end{pmatrix}, \qquad b_1 = \begin{pmatrix} 0 \\ i \end{pmatrix}$$

$$b_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad b_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

- **masses of two light neutrinos and Higgses as a function of the heavy neutrino mass m_h**

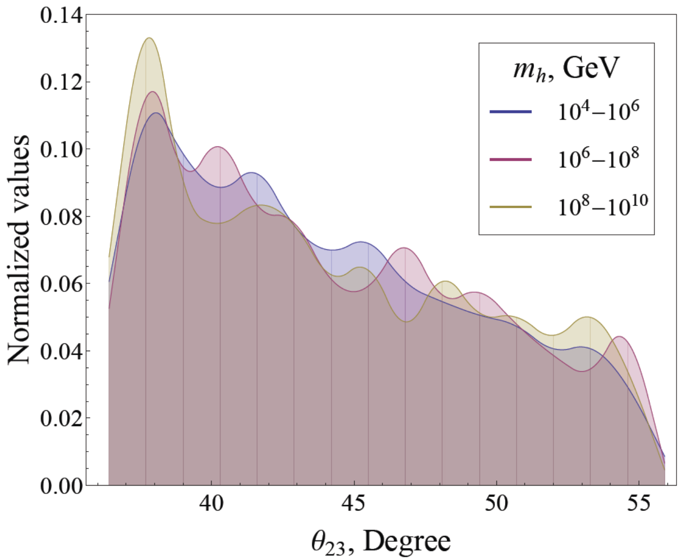


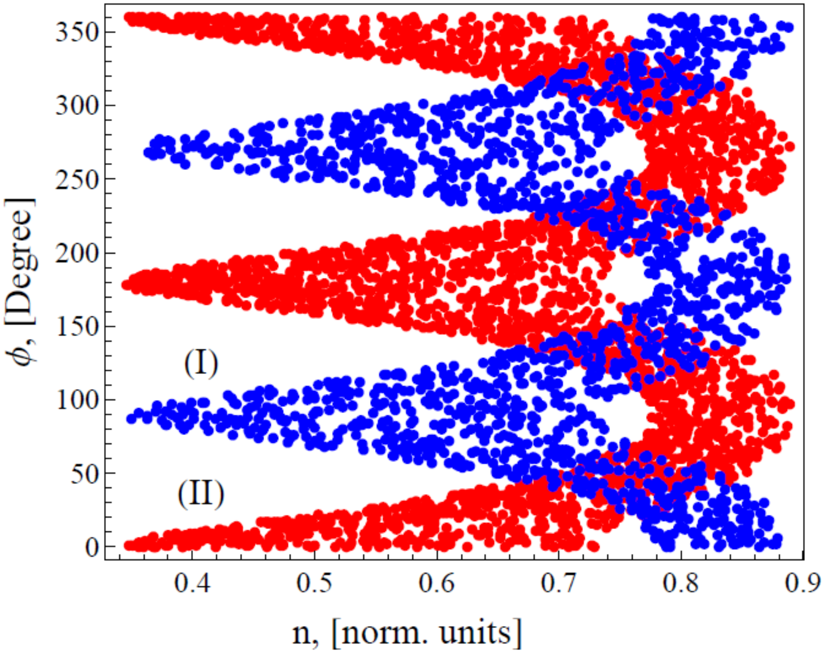


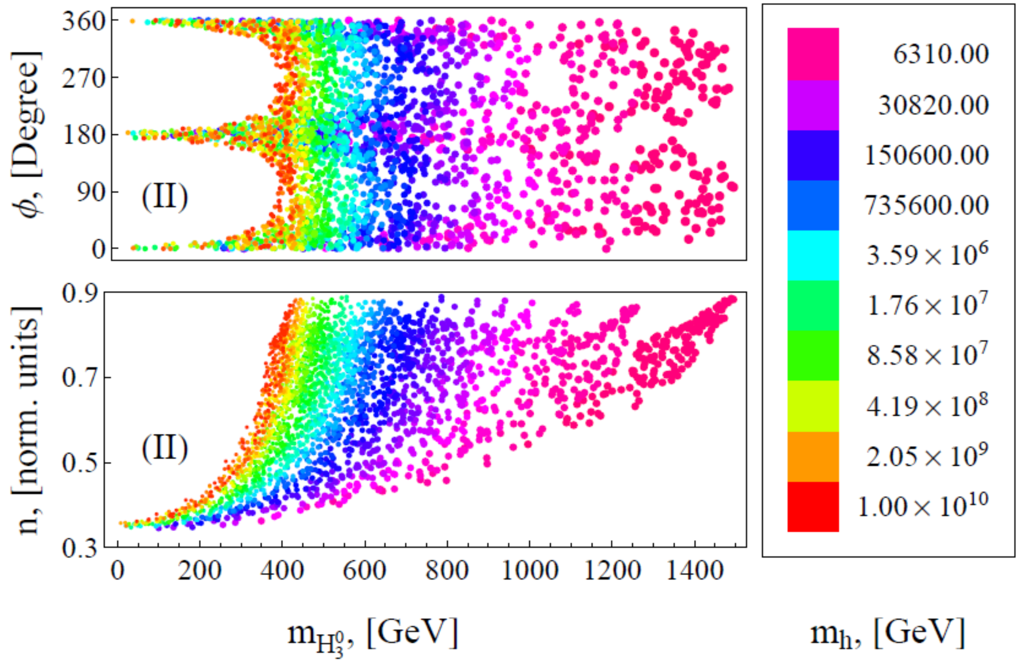
Case $n_R = 1$ (reduced)

- We select values which fulfill Δm^2_{atm}, Δm^2_{sol} and θ_{23} experimental 3σ-band

- Model parameters *n* and φ forms two sets of Higgs masses but they are independent from m_h







Case $n_R = 2$

- **For this case we consider the minimal extension of standard model adding two right-handed ν_R fields to the three left-handed fields contained in ν_L**

in general case we use parametrization:

$$\Delta_1 = \frac{\sqrt{2}}{v}\begin{pmatrix} m_{D_2}\vec{a}_1^{\,T} \\ m_{D_1}\vec{b}_1^{\,T} \end{pmatrix}$$

$$\Delta_2 = \frac{\sqrt{2}}{v}\begin{pmatrix} m_{D_2}\vec{a}_2^{\,T} \\ m_{D_1}\vec{b}_2^{\,T} \end{pmatrix}$$

orthogonality of vectors reduce the number of the parameters

$$|\vec{a}| = 1$$

$$|\vec{b}| = 1$$

$$U^T M_\nu U = U^T \begin{pmatrix} 0 & 0 & 0 & m_{D_1}a_1 & m_{D_2}b_1 \\ 0 & 0 & 0 & m_{D_1}a_2 & m_{D_2}b_2 \\ 0 & 0 & 0 & m_{D_1}a_3 & m_{D_2}b_3 \\ m_{D_1}a_1 & m_{D_1}a_2 & m_{D_1}a_3 & M_{R_1} & 0 \\ m_{D_2}b_1 & m_{D_2}b_2 & m_{D_2}b_3 & 0 & M_{R_2} \end{pmatrix} U = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & M_{l_1} & 0 & 0 & 0 \\ 0 & 0 & M_{l_2} & 0 & 0 \\ 0 & 0 & 0 & M_{h_1} & 0 \\ 0 & 0 & 0 & 0 & M_{h_2} \end{pmatrix}$$

- **after diagonalization we receive two light and two heavy states of neutrino masses at tree level**

the diagonalization matrix for the tree level:

$$U_{\text{tree}} = U_{12}(\alpha_1, \alpha_2)U_{\text{egv}}(\beta_i)U_\phi(\phi_i)$$

where U_{egv} is an eigenmatrix of $M_\nu^{(1)} M_\nu^{(1)\dagger}$

the non zero masses are determined by the seesaw mechanism

$$m_{D_i}^2 \approx m_{h_i} m_{l_i}$$

$$m_{R_i}^2 \approx m_{h_i}^2$$

Case $n_R = 2$

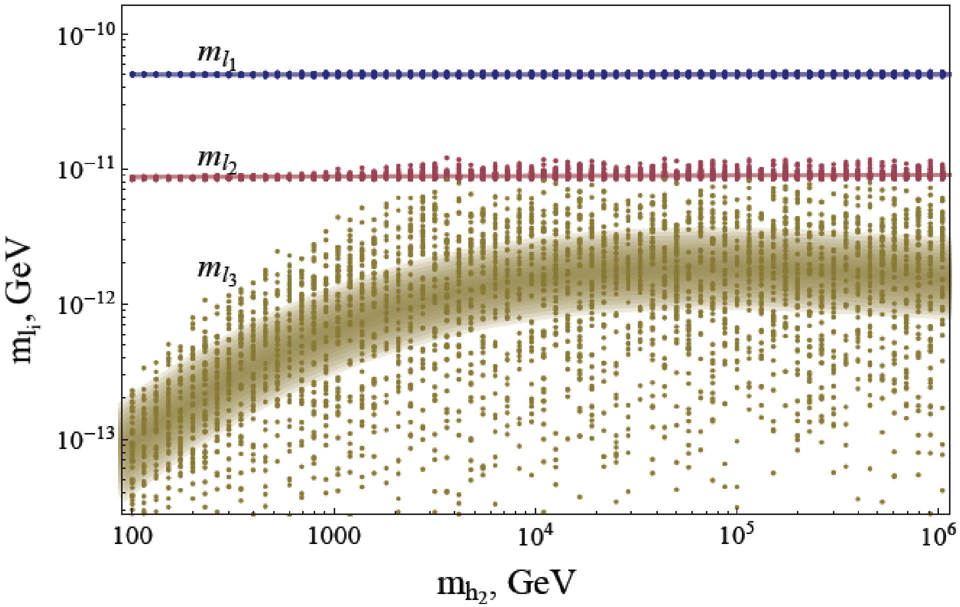
Set of *b* vectors used for numerical calculations

In numerical analysis we fix m_{H1} = 125 GeV, but m_{H2} and m_{H3} we generate randomly in the range 1 to 2000 GeV

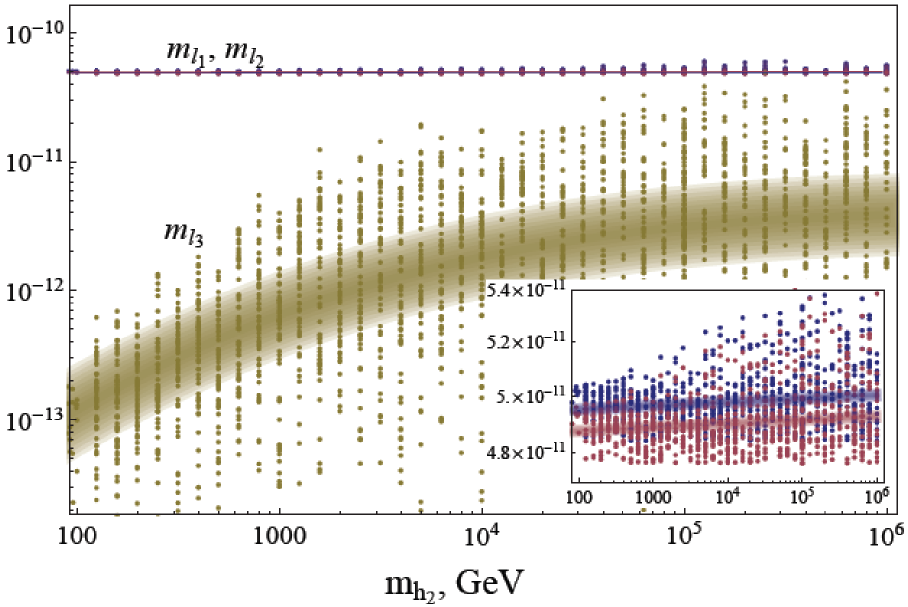
$$b_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad b_2 = \begin{pmatrix} 0 \\ i \end{pmatrix} \quad b_3 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- **The masses of the light neutrinos as functions of the heaviest right-handed neutrino mass.**

Normal hierarchy



Inverted hierarchy



wide solid line indicate the place of the most frequent values of the scatter data

one of heavy neutrino is fixed: $m_{h_1} = 100$ GeV

Case $n_R = 2$ (with Z_4)

- **The idea is substituting the lepton-number symmetry $L_e - L_\mu - L_\tau$ by a discrete symmetry Z_4**

Parametrization with the Z_4 symmetry (complex parameters):

$$\Delta_1 = \frac{\sqrt{2}m_D}{v}\begin{pmatrix} a_1 & 0 \\ 0 & a_2 \\ 0 & a_3 \end{pmatrix}^T \quad \Delta_2 = \frac{\sqrt{2}m_D}{v}\begin{pmatrix} 0 & b_1 \\ b_2 & 0 \\ b_3 & 0 \end{pmatrix}^T$$

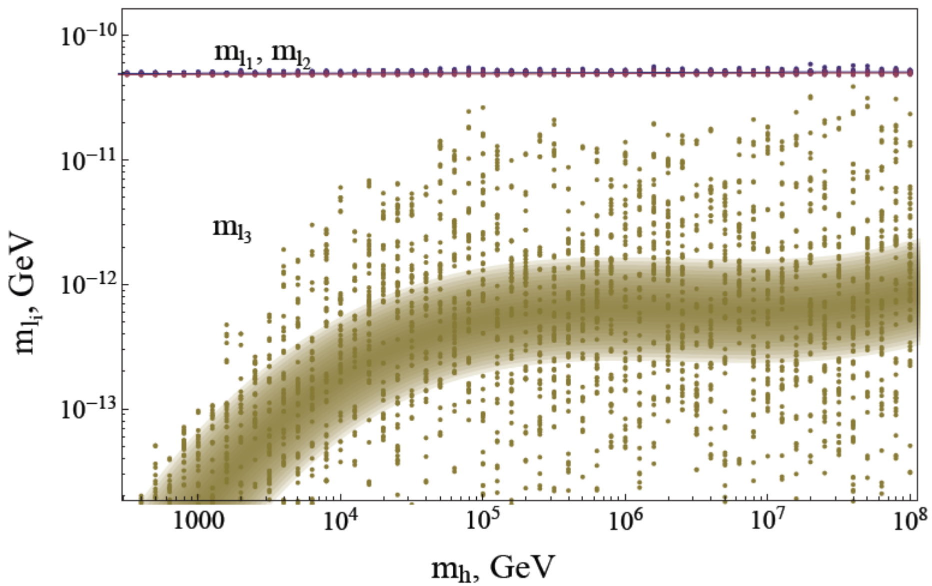
$$M_R = \begin{pmatrix} 0 & m_r \\ m_r & 0 \end{pmatrix}$$

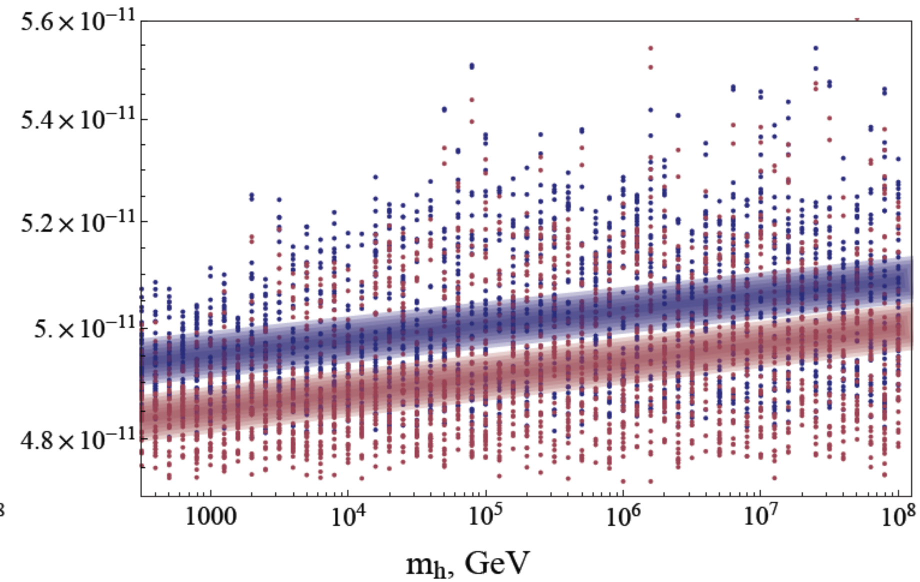
Set of b vectors used for numerical calculations

$$b_1 = \begin{pmatrix} 0 \\ i \end{pmatrix} \quad b_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \quad b_3 = \begin{pmatrix} -\frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$$

For this case is possible to find analytical solutions:

$$m_D^2 = m_h m_l$$
$$m_R^2 = (m_h - m_l)^2 \sim m_h^2$$

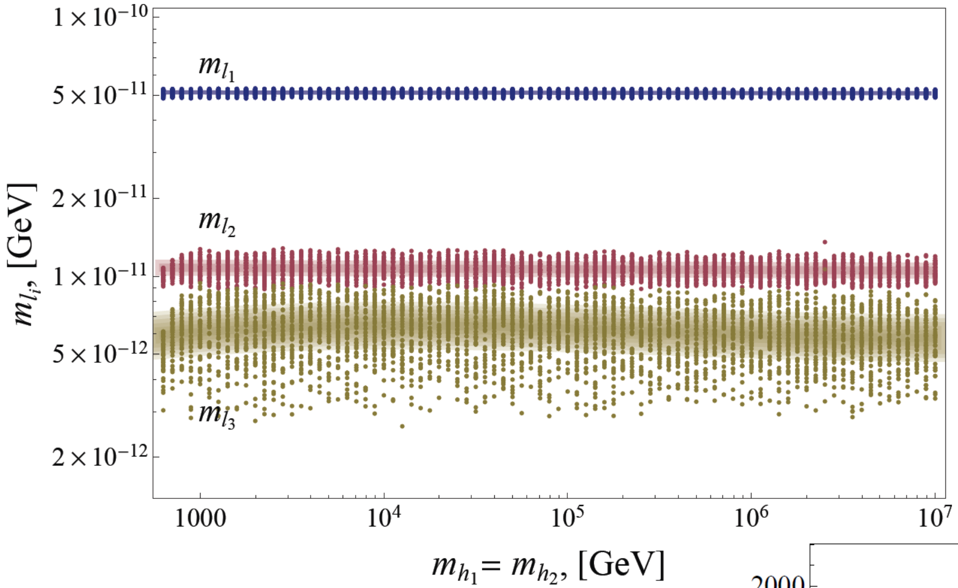




Only inverted hierarchy is possible for this case

Case $n_R = 2$ (reduced)

- **The case with minimal number of free parameters which fulfill experimental boundaries**



reduced parametrization with real parameters:

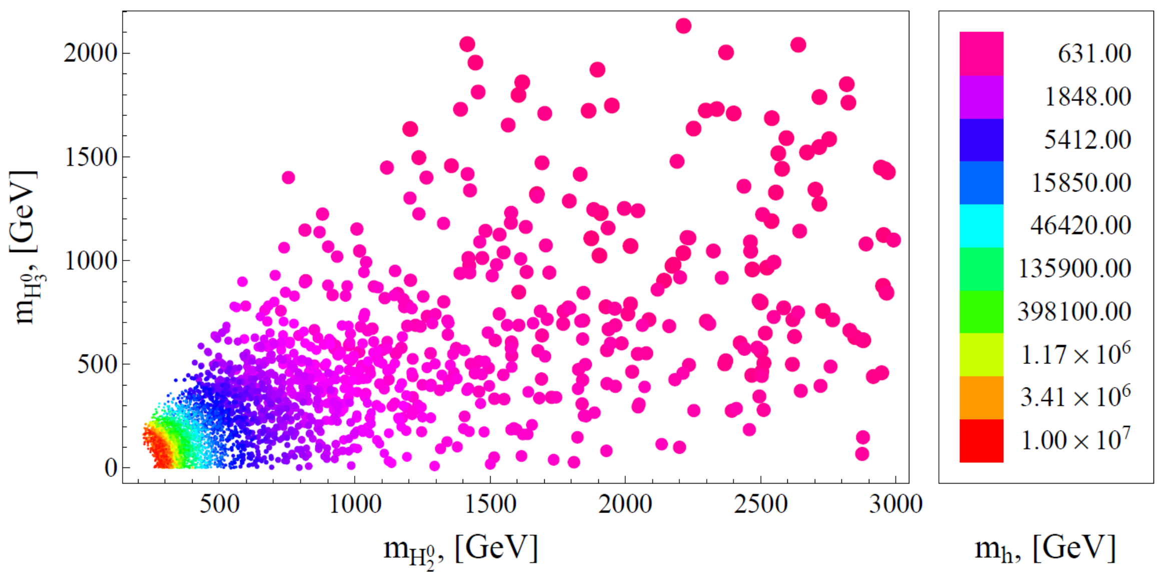
$$\Delta_1 = \frac{\sqrt{2} m_{D_1}}{v} \begin{pmatrix} a_1 & \sqrt{1-a_1^2} & 0 \\ 0 & b_1 & \sqrt{1-b_1^2} \end{pmatrix}$$

$$\Delta_2 = \frac{\sqrt{2} m_{D_2}}{v} \begin{pmatrix} 0 & a_2 & \sqrt{1-a_2^2} \\ b_2 & \sqrt{1-b_2^2} & 0 \end{pmatrix}$$

Set of b vectors used for numerical calculations

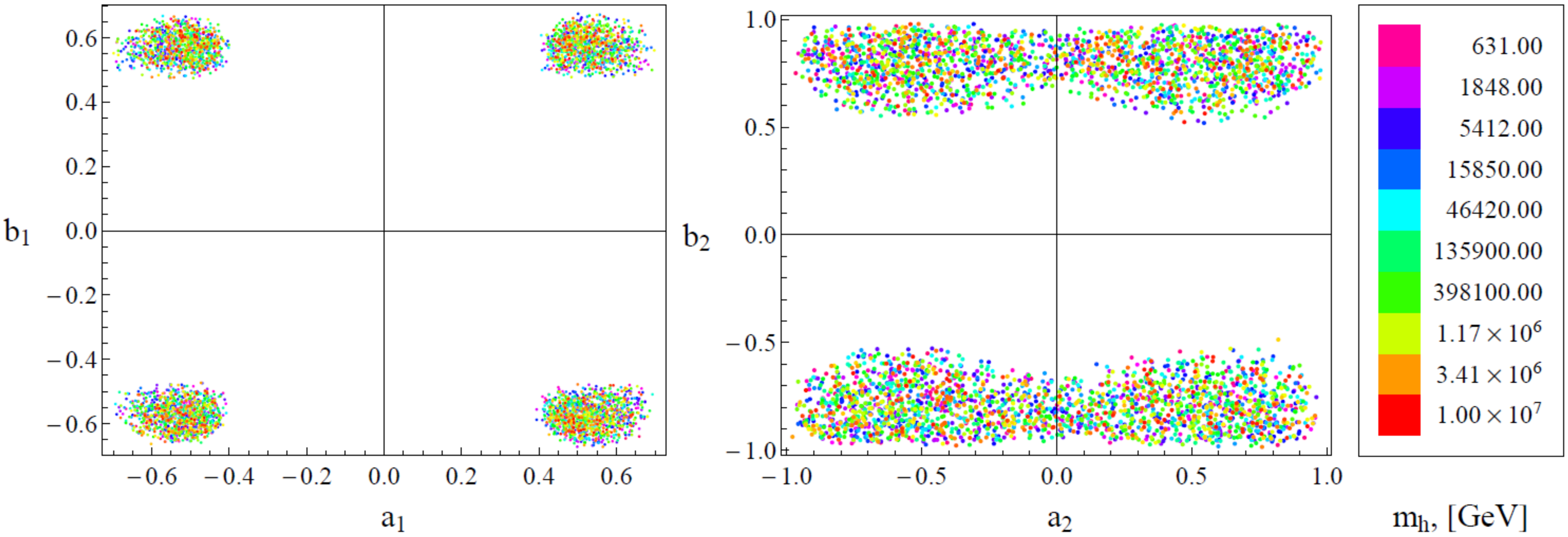
$$b_Z = \begin{pmatrix} i \\ 0 \end{pmatrix}, \qquad b_1 = \begin{pmatrix} 0 \\ i \end{pmatrix}$$

$$b_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad b_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$



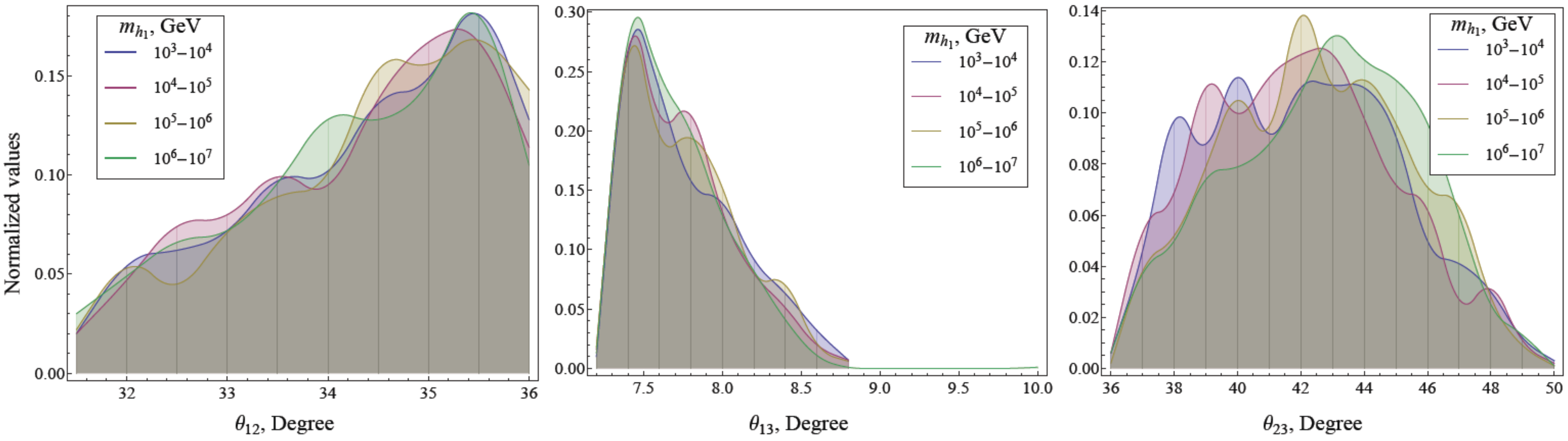
Case $n_R = 2$ (reduced)

- **The plots of free parameters**
  - parameters are real and orthogonal
  - they are selected according to 3σ-band of experimental data
  - they have weak dependence from m_h



Case $n_R = 2$ (reduced)

- **The plots of oscillation angles**
  - the angles are calculated by parametrizing diagonalization matrix U_{loop} with parameters from U_{PMNS}
  - they are selected according 3σ-band of experimental data
  - the histograms shows that they have weak dependence from m_{h}



Case $n_R = 2$ (μ-τ symmetry)

- **We introduce symmetry to the mass matrix which reduce number of the free parameters**
  - at tree level μ-τ symmetry disagrees with experiment but radiative corrections break μ-τ symmetry and we receive correct values
  - for this case we receive light neutrino masses and oscilation angles which fulfill 3σ-band of experimental data
  - by choosing real and orthogonal parameters we have only 4 free parameters
  - work in progress…

Texture which agrees with experimental data:

$$M_R = \begin{pmatrix} m_{R_1} & m_{R_2} \\ m_{R_2} & m_{R_1} \end{pmatrix}$$

$$\Delta_1 = \frac{\sqrt{2} m_{D_1}}{v} \begin{pmatrix} a_1 & a_2 & a_3 \\ a_1 & a_3 & a_2 \end{pmatrix}$$

$$\Delta_2 = \frac{\sqrt{2} m_{D_2}}{v} \begin{pmatrix} b_1 & b_2 & b_3 \\ -b_1 & -b_3 & -b_2 \end{pmatrix}$$

Set of *b* vectors used for numerical calculations:

$$b_Z = \begin{pmatrix} i \\ 0 \end{pmatrix}, \qquad b_1 = \begin{pmatrix} 0 \\ i \end{pmatrix}$$

$$b_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad b_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Conclusions

- **For the case $n_R = 1$ we can receive two states for light neutrino but the third neutrino remains massless. Only normal ordering of neutrino masses is possible.**

- **In the case $n_R = 2$ we obtain three non vanishing masses of light neutrinos for normal and inverted hierarchies.**

- **The radiative corrections generate the lightest neutrino mass and have a big impact on the second lightest neutrino mass.**

- **By introducing special symmetries and reducing the number of free parameters it is possible to obtain some dependence between Higgses and parameters.**

- **The case with μ-τ symmetry is interesting and promising…**

Thank You…