

Detector electronics

Part 2

Applications



Rev 220314

Fourier transforms

- Any periodic function $f(t)$ can be represented as **Fourier series**

$$f(t) = a_0 + a_1 \cos \omega_1 t + b_1 \sin \omega_1 t + a_2 \cos \omega_2 t + b_2 \sin \omega_2 t + \dots$$

i.e. as a superposition of sinusoidal functions of increasing frequency



$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$
one can **compose any periodic function**

- Fourier Analysis** is the inverse operation:
Decomposition of a given time function into its discrete periodic components with coefficients a_i and b_i
- It can be shown * that the Fourier coefficients can be calculated by averaging over all time periods T

$$a_0 = \frac{1}{T} \int_0^T f(t) dt$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(n\omega t) dt$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(n\omega t) dt$$

- A very powerful mathematical simplification is the addition of an **imaginary part** to the coefficients

$$f_n = [a_n \cos n\omega t + b_n \sin n\omega t] + i [a_n \sin n\omega t - b_n \cos n\omega t]$$

$$f_n = a_n - i b_n = |f_n| e^{i\omega t}$$

Note: the addition of the imaginary part does not change the real part of $f(t)$ as long as real and imaginary part are only linearly connected and do not interfere

- Re-writing the discrete sum $f(t)$ as **continuous Integral**, the Fourier function $F(t)$ of a periodic function $f(t)$ is

$$F(t) = \text{Real} \left[\frac{1}{\pi} \int_0^{\infty} f(\omega) e^{i\omega t} d\omega \right]$$

$$= \text{Real} \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} f(\omega) e^{i\omega t} d\omega \right]$$

and inversely the continuous function of coefficients (**spectral function**) is obtained* as

$$a(\omega) = \int_{-\infty}^{\infty} F(t) e^{-i\omega t} dt$$

- The two above equations represent a **transformation** of the a time function into a spectral function $f(\omega)$ and vice versa:

$$F(t) \Leftrightarrow a(\omega)$$

- The calculation with **spectral functions** significantly reduces the problem of solving differential equations in the $F(t)$ space by much simpler algebraic equations in the $f(\omega)$ space.

Laplace transforms

- **Laplace transformations** are generalized Fourier transformation which cover also non-periodic functions by changing the Fourier variable $i\omega$ to $s = \sigma + i\omega$, i.e. for $\sigma \rightarrow 0$ Laplace and Fourier transforms are the same.
- Some functions can only be represented as Laplace transforms, therefore they are more popular

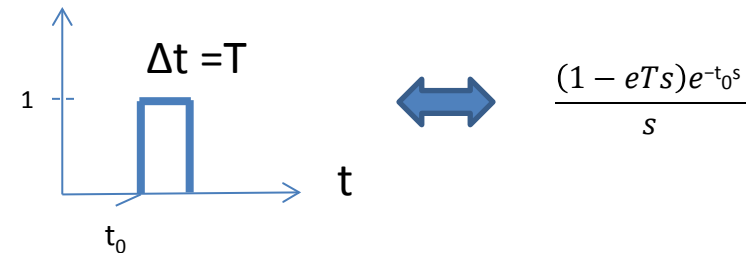
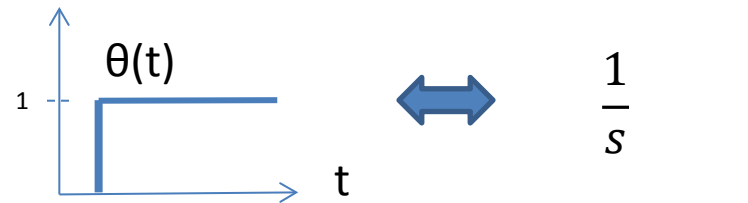
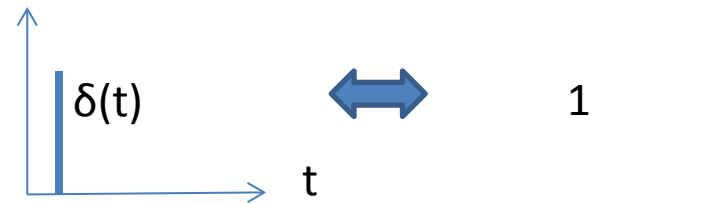
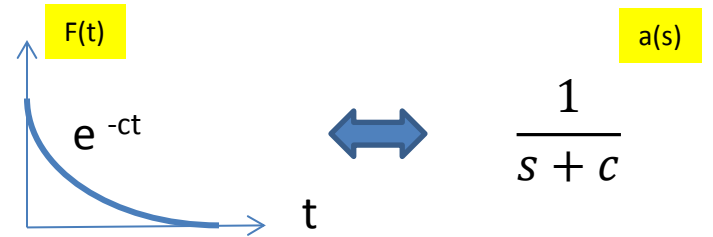
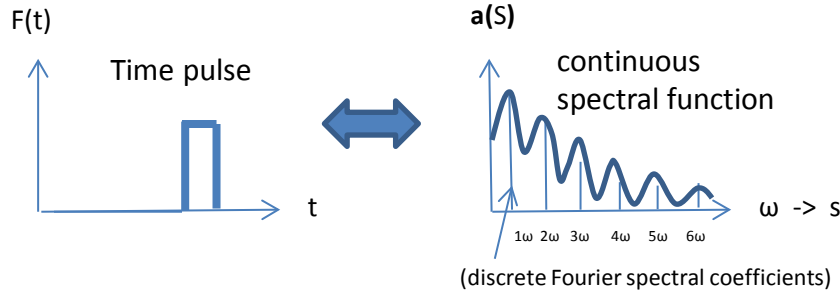
$$L(s) = \int_{-\infty}^{\infty} F(t) \exp(-st) dt$$

$$F(t) = \frac{1}{2\pi i} \int_{\sigma - \infty}^{\sigma + \infty} L(s) \exp(st) ds$$

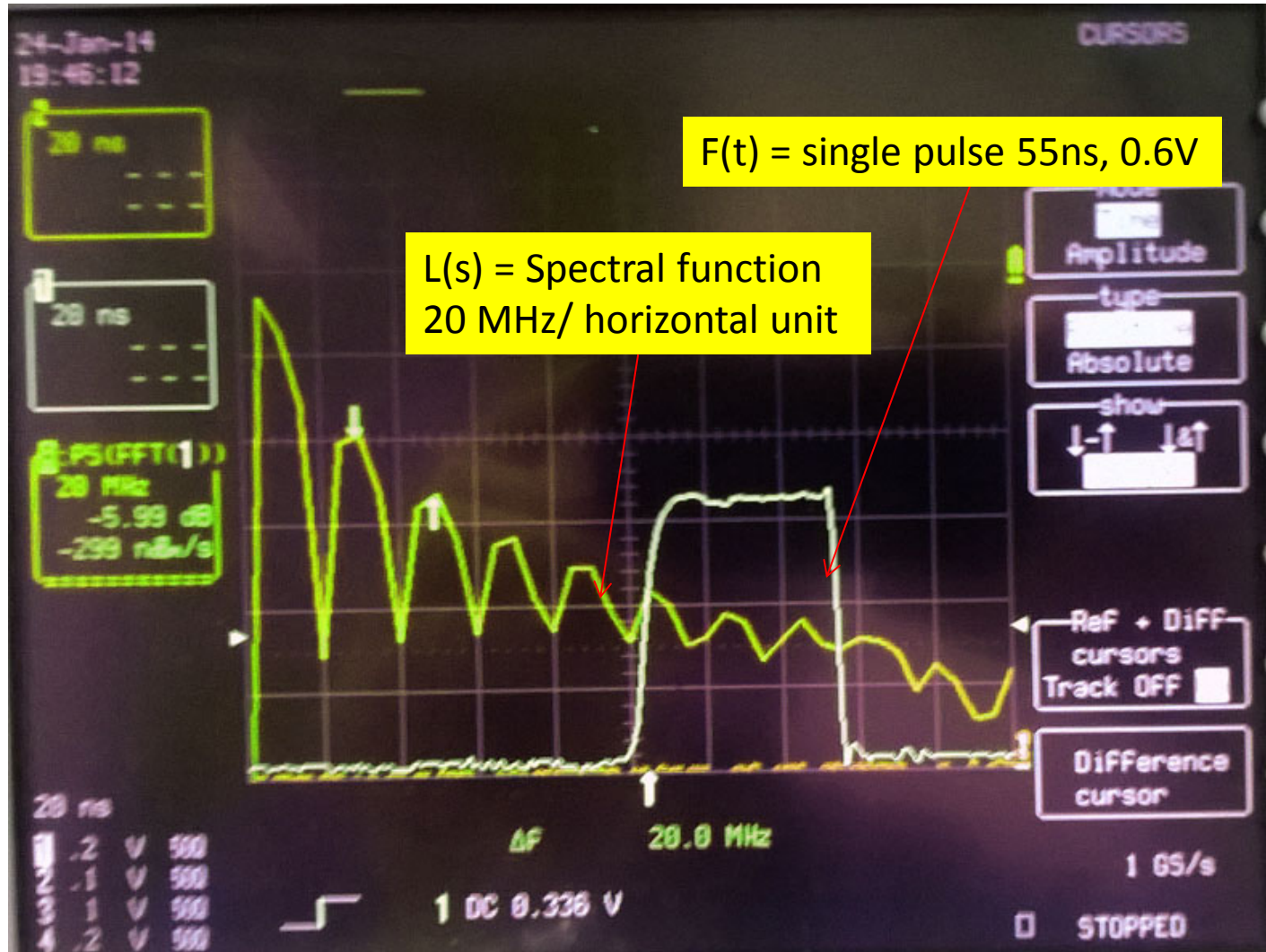
these functions have some important properties :

$$F'(t) = s L(s) - f(0)$$

$$t^n F(t) = (-1)^n L^{(n)}(s)$$

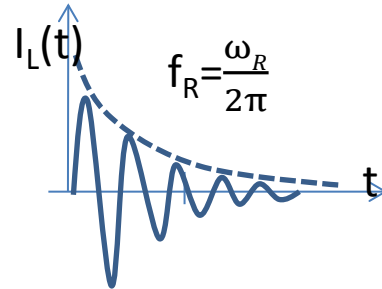
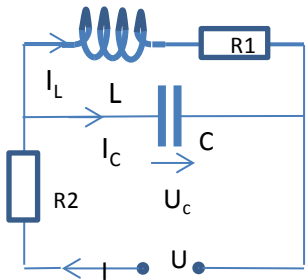


Fourier Analysis via Oscilloscope



Example: behaviour of
a resonance network:

Resonance circuit



- $$F(t): \quad I \cdot R_2 + U_c = U \quad I_L(0) = 0$$

$$I_c = C \frac{dU_c}{dt} \quad U_c(0) = 0$$

$$L \frac{dI_L}{dt} + R_1 I_L = U_c$$

⇔ transform

- $$L(s): \quad I \cdot R_2 + U_c = U$$

$$I_c = C \cdot s \cdot U_c$$

$$L \cdot s \cdot I_L + R_1 \cdot I_L = U_c$$

⇒
$$I_L = \frac{U}{[R_2 L C s^2 + (L + R_2 C)s + R_2 + R_1]}$$
 from table equivalence:

$$\frac{1}{s^2 + 2as + b^2}$$

⇔
$$I_L(t) = \frac{1}{\omega} e^{-at} \sin \omega t$$
 (a damped oscillation)

$$\omega_R = \sqrt{\frac{R_2 + R_1}{R_2 L C} - \frac{1}{4} \left(\frac{R}{L} + \frac{1}{R_2 C} \right)^2}$$

(ω_R resonance frequency)

Function name	Original function $f(t) = \mathcal{L}^{-1}\{F(s)\}$	Laplace spectral function $F(s) = \mathcal{L}\{f(t)\}$
Deltafunktion	$\delta(t)$	1
Theta function	$\Theta(t)$	$\frac{1}{s}$
Exponential funktion	$e^{-at} \quad e^{at}$	$\frac{1}{s+a} \quad \frac{1}{s-a}$
Exponential distribution	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
time	t	$\frac{1}{s^2}$
nth order time	t^n	$\frac{n!}{s^{n+1}}$
damped oscillation	$\frac{1}{\omega} e^{-iat} \sin \omega t$ $\omega = \sqrt{b^2 - a^2}$	$\frac{1}{s^2 + 2as + b^2}$
damped potential funktion	$t e^{-at}$	$\frac{1}{(s+a)^2}$
nth Root	$\sqrt[n]{t}$	$s^{-(n+1)/n} \cdot \Gamma\left(1 + \frac{1}{n}\right)$
Sinus	$\sin(at)$	$\frac{a}{s^2 + a^2}$
Cosinus	$\cos(at)$	$\frac{s}{s^2 + a^2}$
Logarithmus naturalis	$\ln(at)$	$-\frac{1}{s} \left(\ln\left(\frac{s}{a}\right) + \gamma \right)$

Dezibel and Bode plots

- The Bel scale is originally defined as **ratio of intensity**

$$\text{Bel} = \log_{10} (W_{\text{out}}/W_{\text{in}})$$

- The **dezi Bel** (10 dezi Bel = 1 Bel) is a finer measure, generalized for any ratios of equal observables that have linear relation with Intensity.

$$P_{\text{dB}} = \left\{ 10 \log \frac{W_{\text{out}}}{W_{\text{in}}} \right\} [\text{dB}] \quad \text{for ratios of power}$$

- Intensity is related via the square with Voltage or Current and therefore in the logarithmic world this become a factor 2

$$P_{\text{dB}} = \left\{ 20 \log \frac{I, U_{\text{out}}}{I, U_{\text{in}}} \right\} [\text{dB}] \quad \text{for ratios of Voltages, Currents}$$

- Ratios are in general are a measure of **Transfer Functions like the RC or LC networks** with a frequency dependent input and output.
- A very frequent use in electronics is the **bode plot** as characteristic transfer function of the **magnitude of a spectral function H(s)**
- Asymtotes** describe the essential features of Bode plots. The **magnitude** of the transfer function $H(s) = s + 1$ as function of frequency $\omega = 2\pi f$ is

$$|H(s)| = \sqrt{\omega^2 + 1}$$

remember : $s = \sigma + i\omega$, i.e. for $\sigma \rightarrow 0$ $s \rightarrow i\omega$

- Two simple Bode plot with poles and zeros of their corresponding polynomials are shown *:

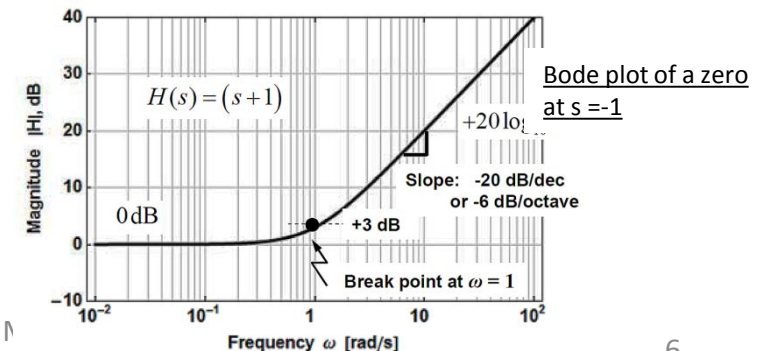
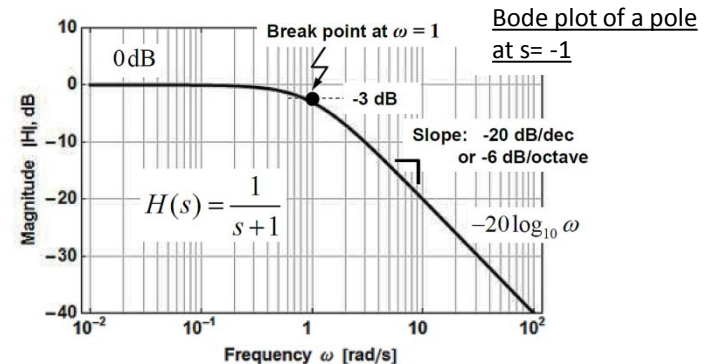
$$H(s) = \frac{1}{s+1} \quad \text{has a pole at } s = -1 = -i\omega$$

with $|H(s)| = 1/\sqrt{1+1}$ this is the half power point (-3dB)

$H(s) = s+1$ has a **zero** cross at $s = -1$ with +3 dB break point

ratios of transfer functions

H(s) transfer ratio	H(s) dB
1	$20 \log(1) = 0\text{dB}$
$\sqrt{2}$	$20 \log \sqrt{2} = 3\text{dB}$
2	$20 \log(2) = 6\text{dB}$
4	$20 \log(4) = 12\text{dB}$
5	$20 \log(5) = 14\text{dB}$
10	$20 \log(10) = 20\text{dB}$



* <http://my.ece.ucsb.edu/York/Bobsclass/2B/Frequency%20Response.pdf>

Bode poles and zeros

- The spectral function of the resonance circuit

$$I_L = \frac{U}{[R_2LC s^2 + (L + R_2C)s + R_2 + R_1]}$$

is a polynomial of second order

- The spectral functions $L(s)$ are in general **polynomials** of order N . In general these polynomials have the form

$$H(s) = K \frac{(s-z_1)(s-z_2)\dots(s-z_n)}{(s-p_0)(s-p_1)\dots(s-p_m)}$$

where the zero crossings are Z_i

and p_i are **poles**

Poles and zeros can be written as $(s + a)$ if $a = -p$

- In a logarithmic world the magnitude of $H(s) = A \frac{s+z}{s+p}$ becomes

$$|H(s)|_{dB} = 20\log(A) + 20\log|s+z| + 20\log\frac{1}{|s+p|}$$

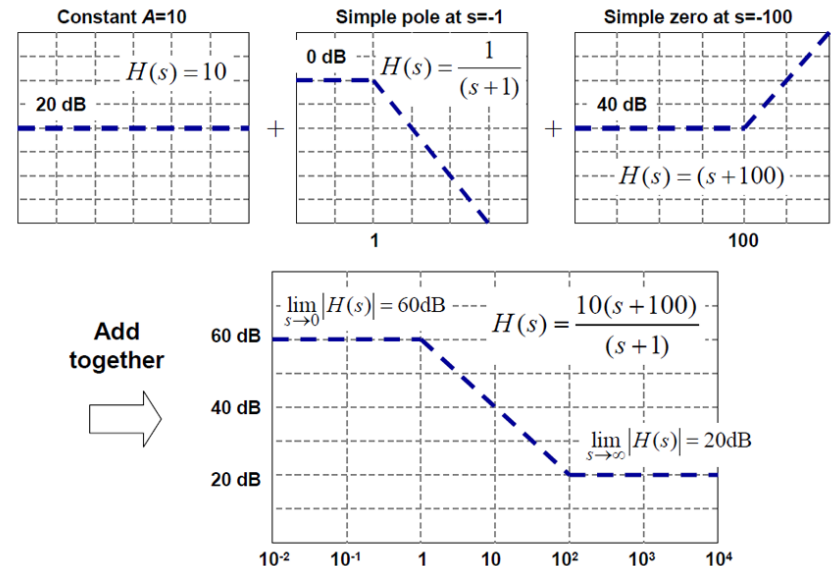
One can **compose bode plots** from the **sums of the poles and zeros** of a Laplace polynomial (*)

* "frequency and bode plots", Bob York
<http://my.ece.ucsb.edu/York/Bobsclass/2B/Frequency%20Response.pdf>

Example of transfer function

$$H(s) = \frac{10(s+100)}{s+1}$$

Illustration of how the composite Bode plot of the above transfer function with $A=10$ is a superposition of the individual terms



Waves

- Cables (or PCB traces) can be de-composed in a series of infinitesimal small LC lumped elements along a line x. With this equivalent circuit

$$dU = -L\delta x \frac{\partial I}{\partial t}$$

(partial differentiation $\frac{\partial}{\partial t}$ since I depends also on position)

- The negative sign of dU on the capacitor C gives

$$dQ = -C \delta x dU$$

$$I = \frac{\partial Q}{\partial t} = -C \delta x \frac{\partial U}{\partial t}$$

by differentiation to x and t
 $\frac{\partial U}{\partial x} = -L \frac{\partial I}{\partial x}$ and $\frac{\partial I}{\partial x} = -C \frac{\partial U}{\partial t}$
 by further differentiation and putting equals to equals one obtains the

WAVE EQUATION

$$\frac{\partial^2 U}{\partial x^2} = LC \frac{\partial^2 U}{\partial t^2} \quad \frac{\partial^2 I}{\partial x^2} = LC \frac{\partial^2 I}{\partial t^2} \quad \text{in general} \quad \frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2}$$

- Solutions of the general wave equation are **any functions** $f(x - ct)$

c is the **propagation speed**

therefore, for U and I, the propagation speed is $c = \frac{1}{\sqrt{LC}}$

sinusoidal voltage and current waves:

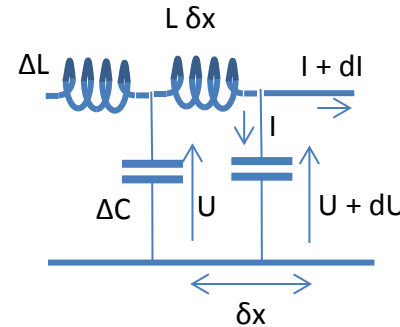
$$U = U_0 \cos\left(x - \frac{1}{\sqrt{LC}} t\right)$$

$$I = I_0 \cos\left(x - \frac{1}{\sqrt{LC}} t\right)$$

- Applying as above $\frac{\partial U}{\partial x} = -L \frac{\partial I}{\partial x}$ one obtains the impedance of the cable

$$\frac{U}{I} = \frac{U_0}{I_0} = \sqrt{\frac{L}{C}} = Z_0$$

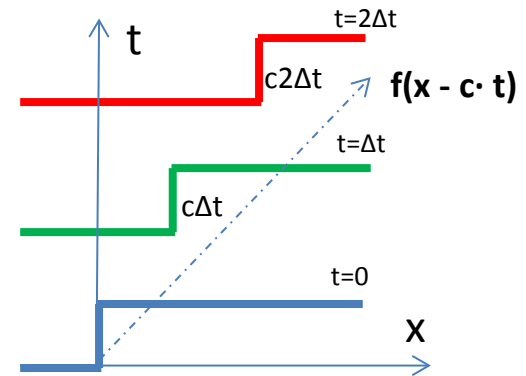
The **impedance of the transmission line** is a **real number** (ideal case)



$$\Delta L = L \delta x$$

$$\Delta C = C \delta x$$

travelling wave in x and t



waves are shapes along x that propagate in time t

Signal transmission

- In the real world the wave equation is extended to include ohmic, magnetic and dielectric losses. This is described by the **telegraph equation**:

$$\frac{\partial^2 U}{\partial t^2} = 1/LC \frac{\partial^2 U}{\partial x^2} + \frac{LA-RC}{2LC} U$$

where **A** represents dielectric energy loss per length

- The solution for $R \sim 0$ is a combination of waves that travel in time-space forward and backwards

$$U(x,t) = e^{-\mu t} [f(x-ct) + g(x+ct)]$$

losing their energy exponentially in time

- A very popular cable for wave transmission is the **coaxial cable** because the full shield provides high noise immunity and a well defined cable impedance
- The Inductance per length coaxial cable is

$$L = 2 \ln\left(\frac{D}{d}\right) \left[\frac{nH}{cm} \right]$$

- The capacitance per length with ϵ as relative dielectric constant of the insulator material is

$$C = \frac{\epsilon}{2 \ln\left(\frac{D}{d}\right)} \left[\frac{pF}{cm} \right]$$

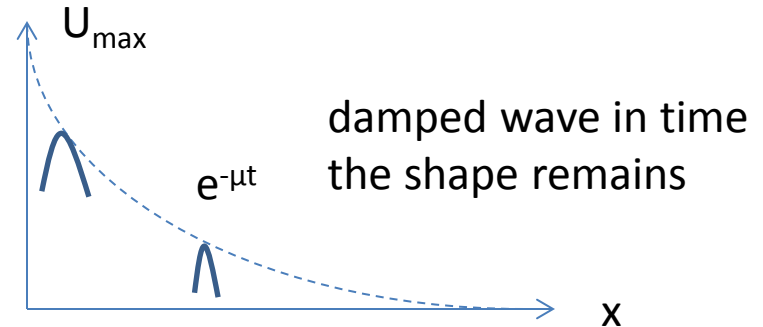
The impedance per length of the coax cable is the real number

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{60}{\sqrt{\epsilon}} \ln\left(\frac{D}{d}\right) \text{ [OHM]}$$

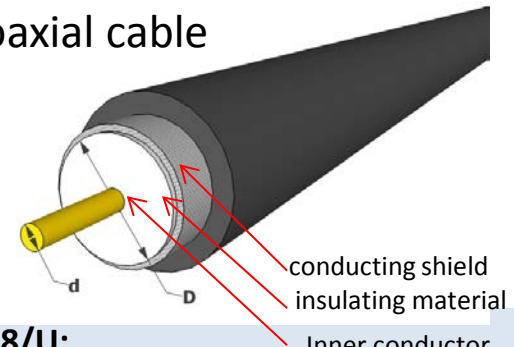
Standard values are 50 or 75 OHM

The lowest damping occurs at $\frac{D}{d} = 3.64$

- RG-58/U** is a coaxial cable which is often used for low-power signal and RF connections



Coaxial cable



RG58/U:

thickness 5 mm, 300 V max

50 OHM, $C = 82 \text{ pF/m}$, inner conductor solid

prop. speed $\frac{1}{\sqrt{LC}} = 2.44 \cdot 10^8 \text{ m/s} \Rightarrow \mathbf{4.1 \text{ ns/m}}$

RG-58A/CU *:

inner conductor 7 or 19 strand

Attenuation dB/100 feet:

1 MHz - 0.4 dB, 100 MHz 4.9 dB, 1 GHz 21.5 dB

Reflected waves

- The wave in an open cable without loss consists of an outgoing and incoming part

$$U(x,t) = [f(x-ct) + g(x+ct)]$$

- By a termination of the cable with some resistive impedance, energy is transferred to the termination and the reflected wave is attenuated.
- Defining $c = \omega/k$ as **velocity** at which the wave travels in time, and k as the **wavenumber** which counts the number of shapes of the wave per unit distance (or radian)

$$x-ct \Rightarrow \omega t - kx = -k(x - \frac{\omega}{k} t)$$

- In complex notation the wave in one direction is

$$U_{in}(x,t) = U_0 e^{i(\omega t - kx)}$$

- The reflected wave is

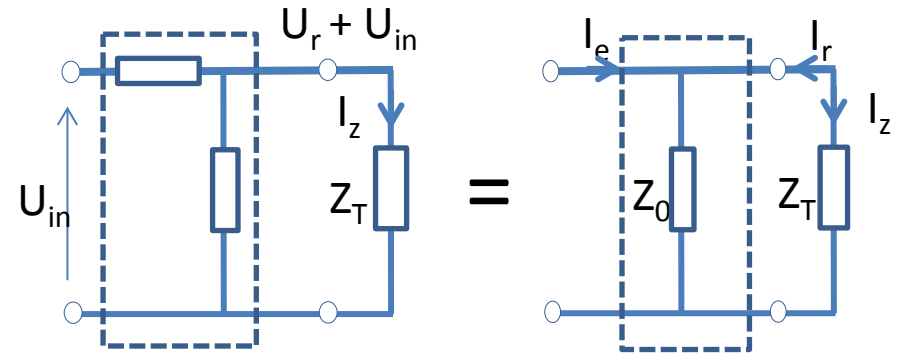
$$U_r(x,t) = r \cdot U_0 e^{i(\omega t + kx)}$$

where $1 > r > 0$ is the reflection coefficient

- The current through the termination resistor Z_T

$$I_z = I_e - I_r = \frac{U_{in} - r \cdot U_{in}}{Z_0} = \frac{U_{in} + r \cdot U_{in}}{Z_T}$$

$$1/Z_0 - r/Z_0 = \frac{1+r}{Z_T}$$



- The reflection coefficient for the wave in the cable is defined by the impedance Z_0 of the cable and the termination impedance Z_T

$$r = \frac{Z_T - Z_0}{Z_T + Z_0}$$

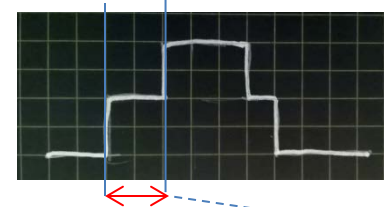
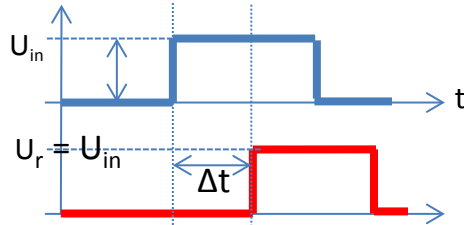
- If the termination is a pure resistor

$$Z_T = R \Rightarrow r = 0$$

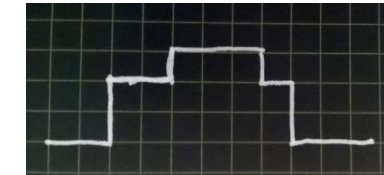
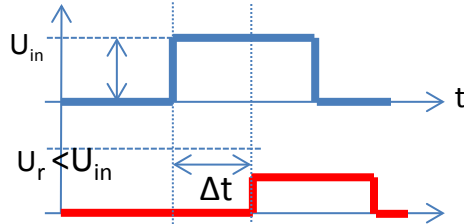
for $r = 0$ there is no reflected wave and the energy transfer from the cable to the termination resistor is at maximum.

Terminations

$Z = \infty$



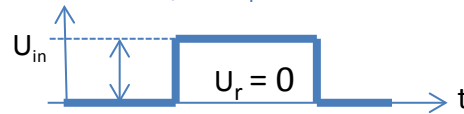
$\infty > Z > Z_0$



$\Delta t = 2l \sqrt{LC}$

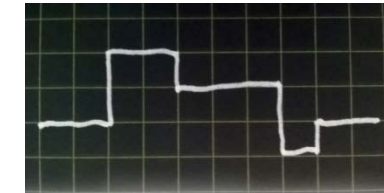
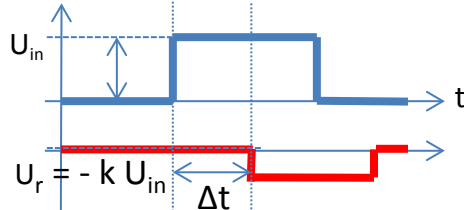
propagation speed
= $\frac{1}{\sqrt{LC}}$

$Z = R$

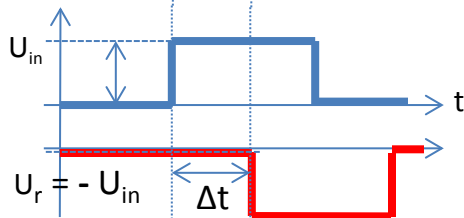


proper termination
= no reflection

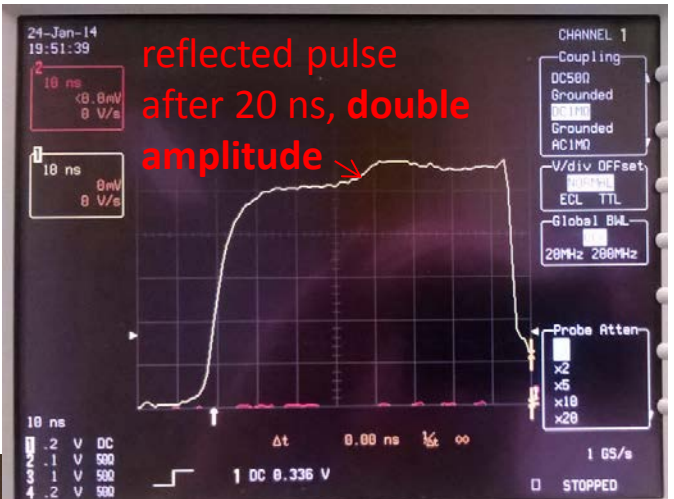
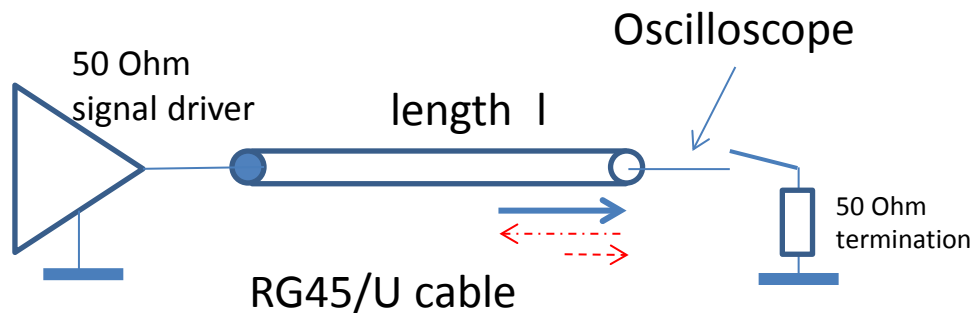
$0 < Z < Z_0$



$Z = 0$



Coaxial Cable termination



The reflection delay Δt is a measure of the cable length

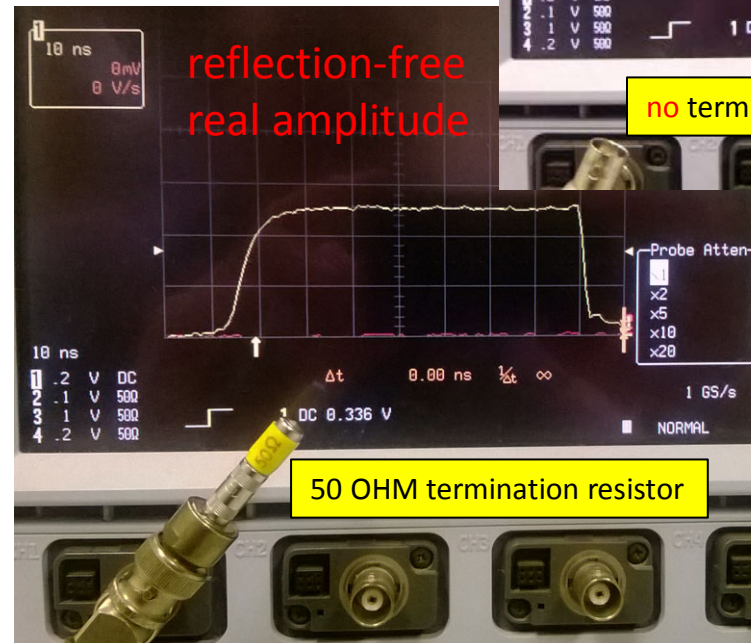
Example :

$$\Delta t = 20 \text{ ns}$$

$$\text{RG58/U cable} : \frac{1}{\sqrt{LC}} = 2.44 \cdot 10^8 \text{ m/s}$$

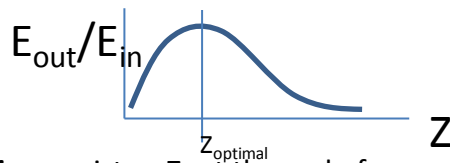
$$l = \frac{1}{2} \Delta t \frac{1}{\sqrt{LC}} =$$

$\rightarrow 2.44 \text{ m cable}$

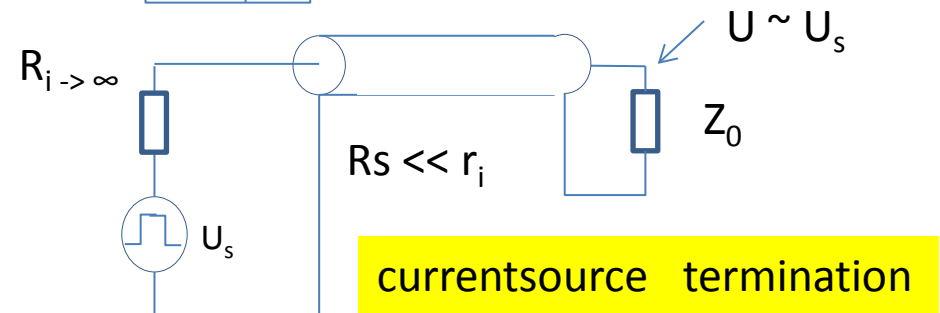
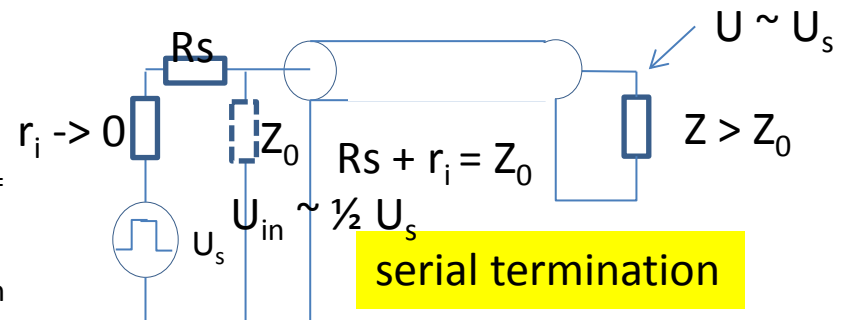
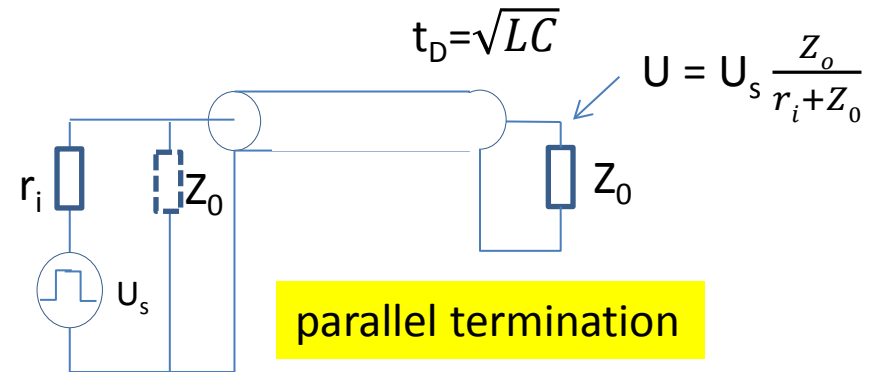


Impedance matching

- The transfer of energy from the signal source over a transmission line to a termination resistance depends on the termination scheme



- A **parallel termination** resistor Z_0 at the end of a transmission line of impedance Z_0 results in a reflection-free transfer of energy with an amplitude that depends on the internal resistance of the signal source
- A **serial termination** at the signal source is used when the impedance on the end of the transmission line is high. $U \sim U_s \frac{Z_0}{r_i + R + Z_0}$. The series resistor is dimensioned to be $R_s + r_i = Z_0$ such that for small r_i the input voltage is $\frac{1}{2} U_s$. Due to the high termination at the output the transmitted pulse is reflected such that in total $U \sim U_s$. Due to the full reflection no energy is transmitted however if the input is a driver with high ohmic input an undistorted signal is amplified.
- The **serial current termination** is a special case of the serial termination. Since R_i of a current source is high, one can remove R to get $U = U_s \frac{Z_0}{r_i + Z_0}$ at the output. Any chosen termination resistance will allow to transfer a maximum of energy.



Oscilloscope probes

- Direct X1 probes have a limited bandwidth due to the low-pass made by $R_s || R_{in}$ and $C_{in} + C_{tip}$

$$f_{-3dB} = \frac{1}{2\pi(R_s || R_{in})(C_{in} + C_{tip})}$$

- X10 probes have a resistor and capacitor $R1 || C1$ inserted. If $R_1 C_1 = R_2 C_2$, the effect of both capacitors cancel. In practice, this condition may not be met exactly but can be approximated:

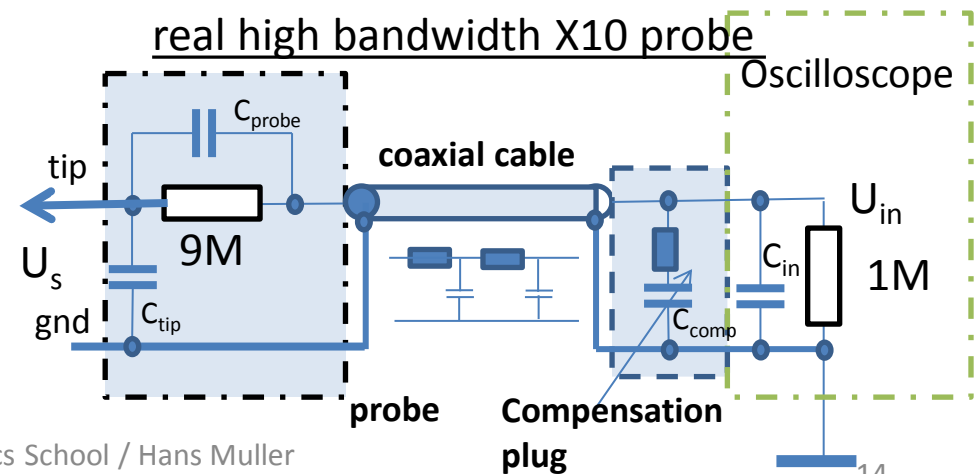
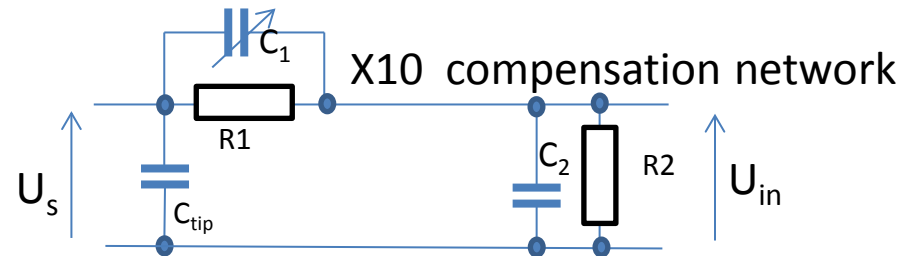
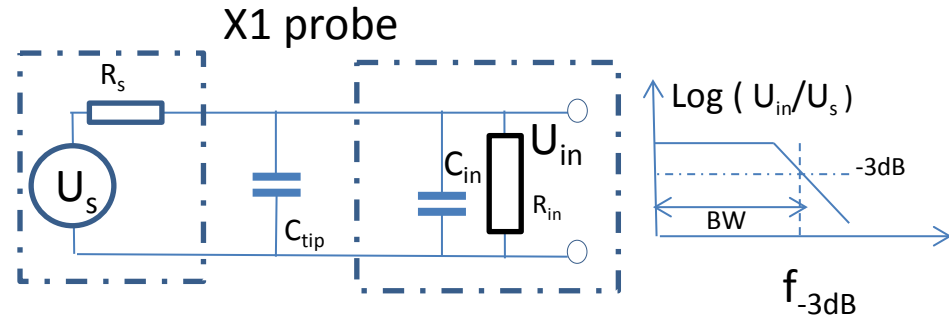
$$U_{in} = U_s \left(\frac{R2}{R1 + R2} \right)$$

- The net effect is that the low-pass bandwidth limitation is practically eliminated at the expense of a smaller signal at the oscilloscope. By choosing $R1 = 9 \times R2$

$$U_{in} = [1/10] U_s$$

with a standard 1M input impedance, the load on the test voltage is 10 M

- In a real X10 probe, the compensation capacitor is placed in the oscilloscope plug and a specially tuned coaxial cable makes the connection between probe and oscilloscope. In high BW probes a specially tuned, lossy high Z coaxial cable and an extra series resistor extend the flat bandwidth response even to higher frequencies.

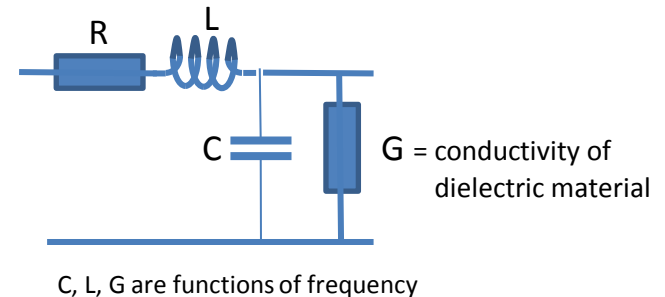


Dispersion

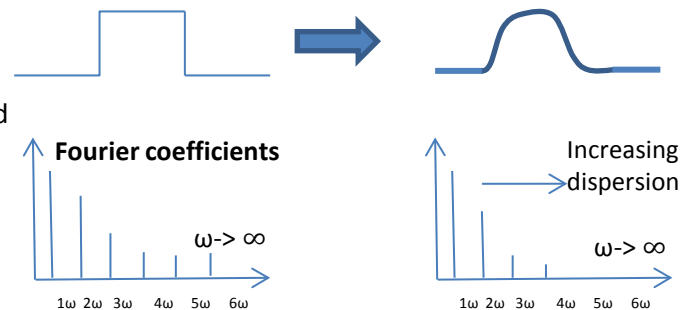
- With the wave equation, a travelling wave $g(\mathbf{x} - \mathbf{ct})$ in the vacuum propagates at the speed of light c
- A **monochromatic periodic wave** of frequency $\omega = 2\pi f$ has a wave function $g(\mathbf{x} - \frac{\omega}{k}t)$ that is travelling at the **phase velocity**

$$v = \frac{\omega}{k}$$
- In vacuum the phase velocity is equal to the speed of light
- In a **dispersive material**, the frequency of a wave does not change, however the wavelength changes and with it the phase velocity
- A pulse-shape is a superposition of monochromatic waves of different frequencies (Fourier Analysis). In dispersive materials, the **phase velocity is less for components with increasing frequencies**. The reasons for dispersion are dielectric losses, skin effect and resistivity. Therefore the Ansatz for an equivalent lumped element model adds resistivity and conductivity elements to the simple LC model.
- The Fourier analysis shows that a **rectangular pulse** shape is equivalent to an infinite series of mono-chromatic frequency components, of which the higher ones make the sharp edges of the rectangular shape.
- The **dispersion in a cable like RG48/U** makes that a sharp rectangular shaped input pulse is transmitted as a smoothed output pulse shape.
- The magnitude of the dispersion depends on the electric constant of the material and the skin effect.
- Printed circuits made from **FR4** with a dielectric constant of 4.2 @ 5 GHz) show significant dispersion above 100 MHz (-2dB/m @100 MHz, -7dB/m @ 1 GHz, -45 dB/m at 10 GHz). **High frequency materials** like NELCO 4000-13 has a lower dielectric constant 3.7 @ 2.5 GHz (-1dB/m @100 MHz, -4dB/m @ 1 GHz, -20dB/m at 10 GHz)

Lumped element equivalent of a **lossy transmission line**

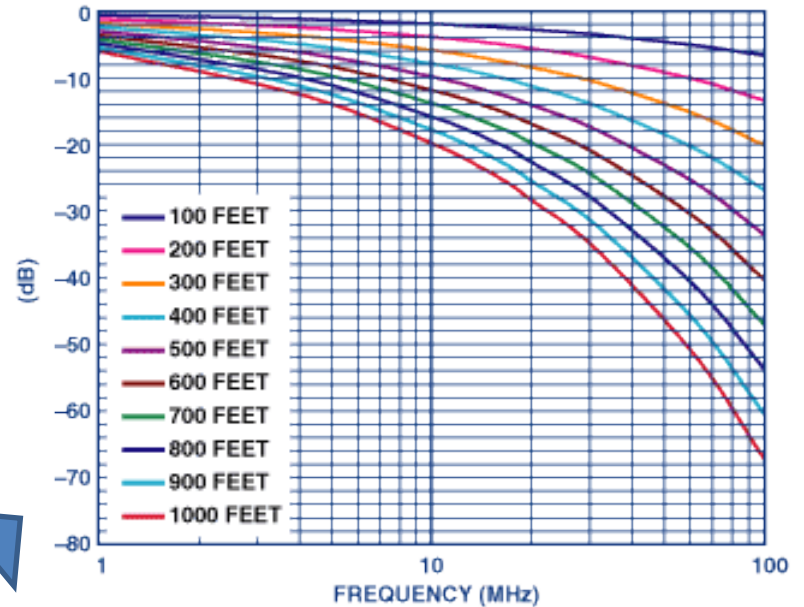
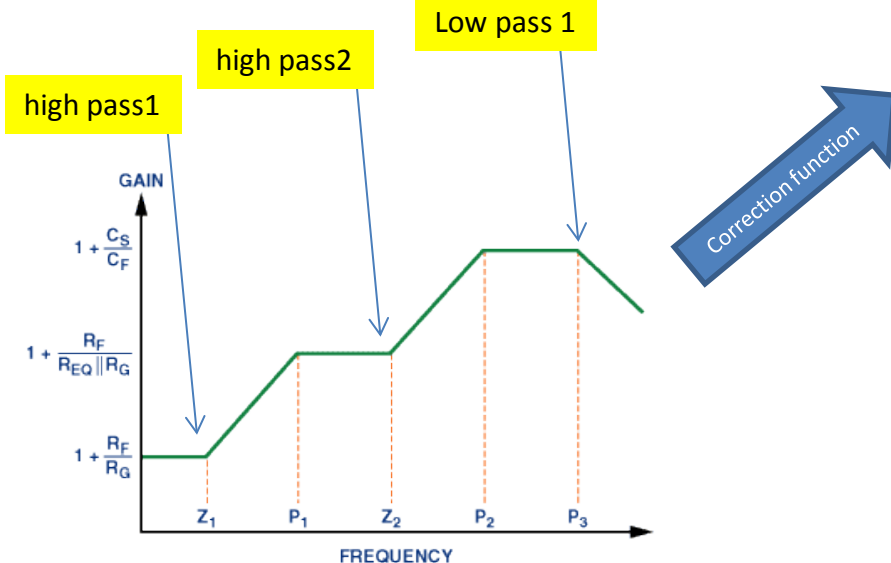


dispersive transmission line

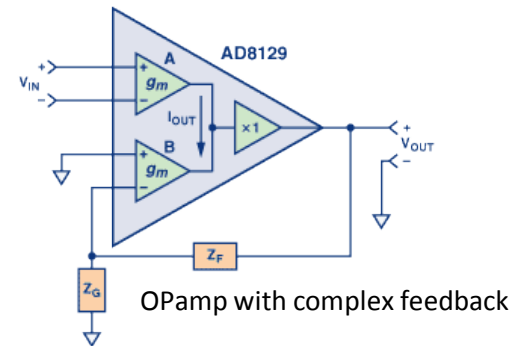


Equalizers

- In order to **compensate dispersion losses, equalizer circuits** can be designed to **enhance the Fourier components** which get attenuated in known cable loss characteristics.
- A correction circuit uses an operational amplifier with differential input for the cable and a **feedback circuit consisting of complex impedances** which is calculated to provide a frequency dependent reciprocal correction of the cable loss.
- The design of the correcting transfer function $H(s)$, a **bode plot can be** composed as addition of sections with high and low pass transfers (poles and zeros). These represent the corrections to the different frequency loss sections*



Frequency responses for various lengths of Cat-5 (un-shielded UTP twisted pair) cable*.



* <http://www.analog.com/library/analogDialogue/archives/38-07/equalizer.html>

Diode basics

- The current through a diode is highly non-linear and defined by the Schottky equation

$$I = I_0 \left[\exp\left(\frac{e \Delta U}{n k T}\right) - 1 \right]$$

ring at cathode



- the bandgap $\Delta E = e \Delta U$ for Silizium is 1.1 eV.
- the diode-dependent reverse current I_0 is $10^{-6} \dots 10^{-12}$ A
- The emission coefficient is $n = 1..2$
- the kT/e is a temperature-dependent Voltage

$$U_T = 25 \text{ mV @ } 25 \text{ }^\circ\text{C.}$$

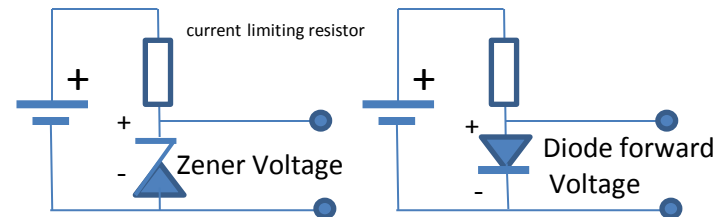
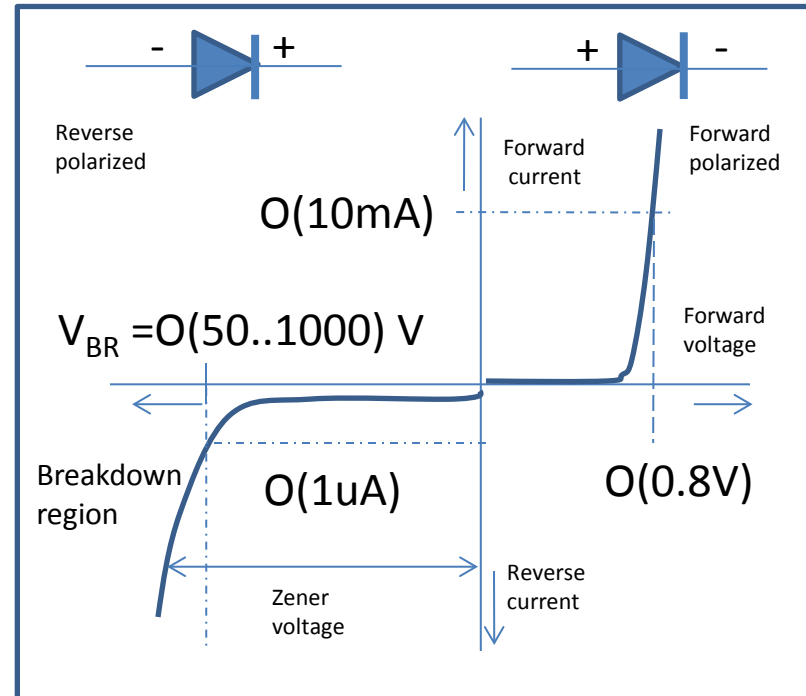
With $n = 2$ and $I_0 = 10^{-8}$ the current at $\Delta U = +0.8 \text{ V}$

$$I_{\text{forward}} = I_0 \left[\exp\left(\frac{\Delta U}{n 25 \text{ mV}}\right) - 1 \right] = 8.8 \text{ mA}$$

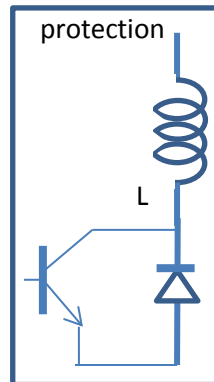
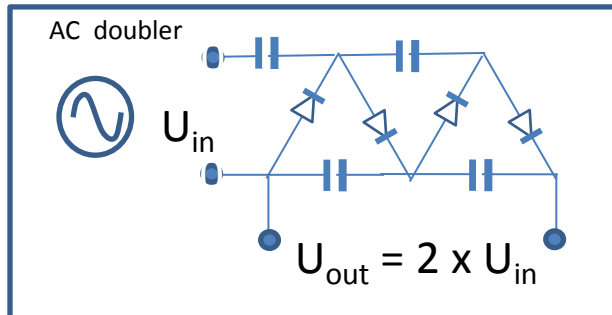
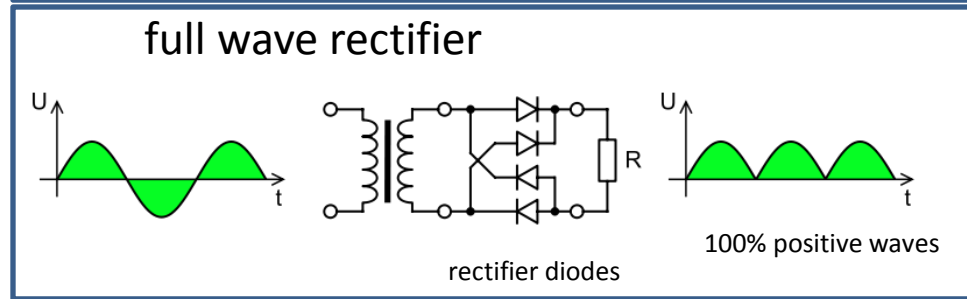
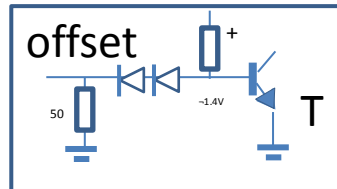
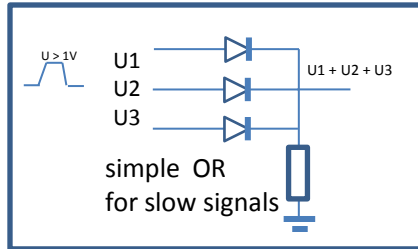
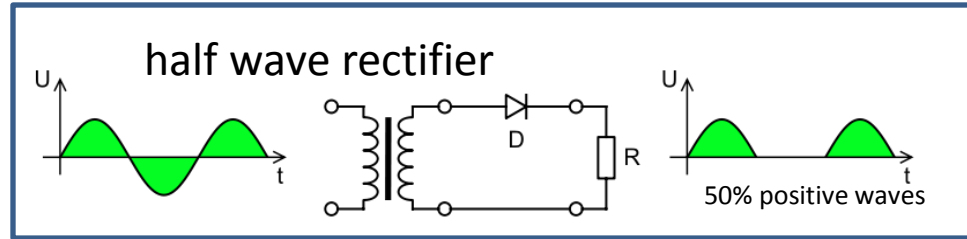
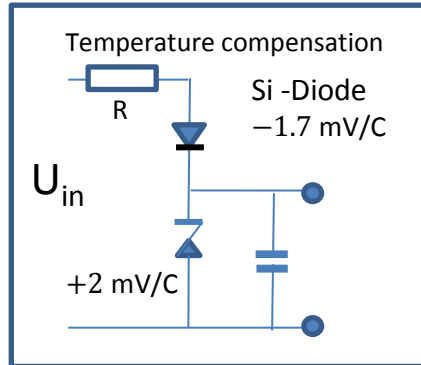
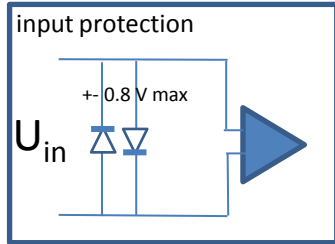
at $\Delta U = -0.8 \text{ V}$

$$I_{\text{reverse}} = I_0 \left[\exp\left(\frac{\Delta U}{n 25 \text{ mV}}\right) - 1 \right] = I_0$$

- The temperature coefficient of Si diodes is $\frac{\partial U}{\partial T}$ is -1.7 mV/K
- Depending on the diode the **reverse current** rises fast at a **breakdown voltage** V_{BR} . This effect is normally not desired and may destroy the diode due to the high **avalanche current**, however **Zener diodes** are used in breakdown mode for the creation of almost constant Voltage differences. Zener diodes are available in a large range of **Zener voltages**.
- The **forward voltage drop** of Si diodes is equally used as voltage difference of the order 0.8 V. For example the **pulse levels of NIM electronics** correspond to the forward diode voltage.

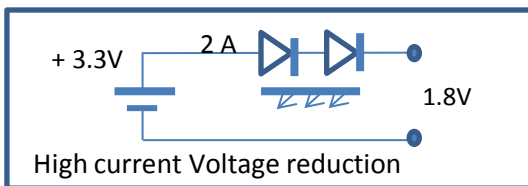
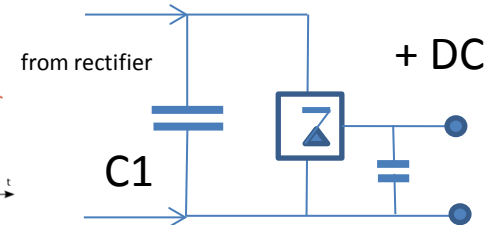
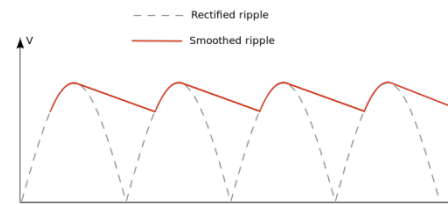


Diode applications



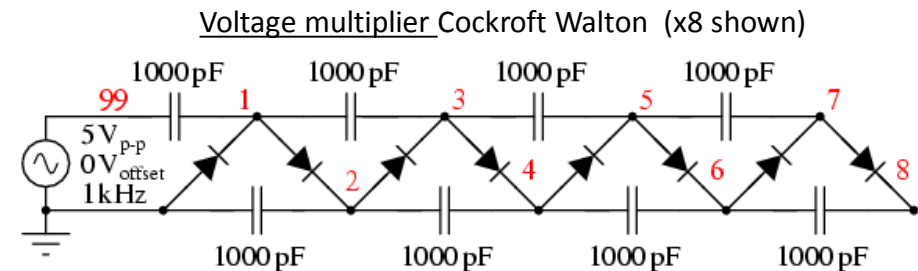
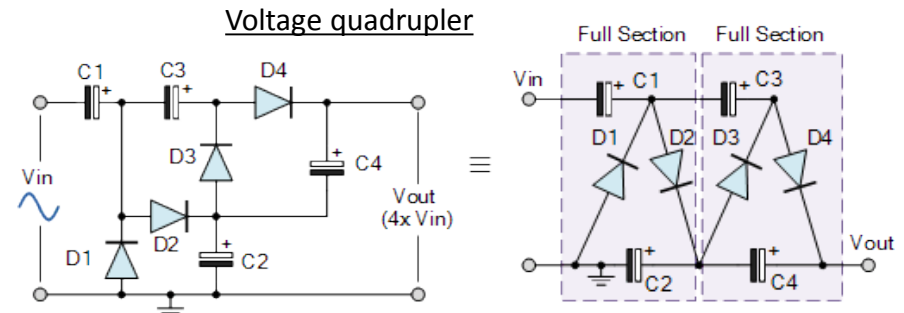
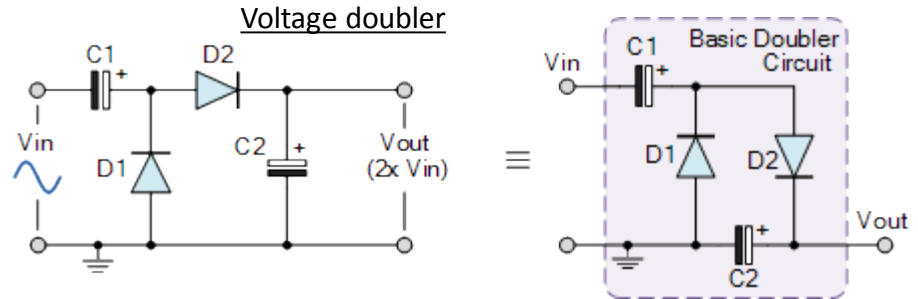
A large capacitor C1 in parallel the bridge output smooths out a DC level with some remaining ripple

DC + ripple A voltage regulator with low impedance output and a small capacitor C2 practically eliminates the ripple



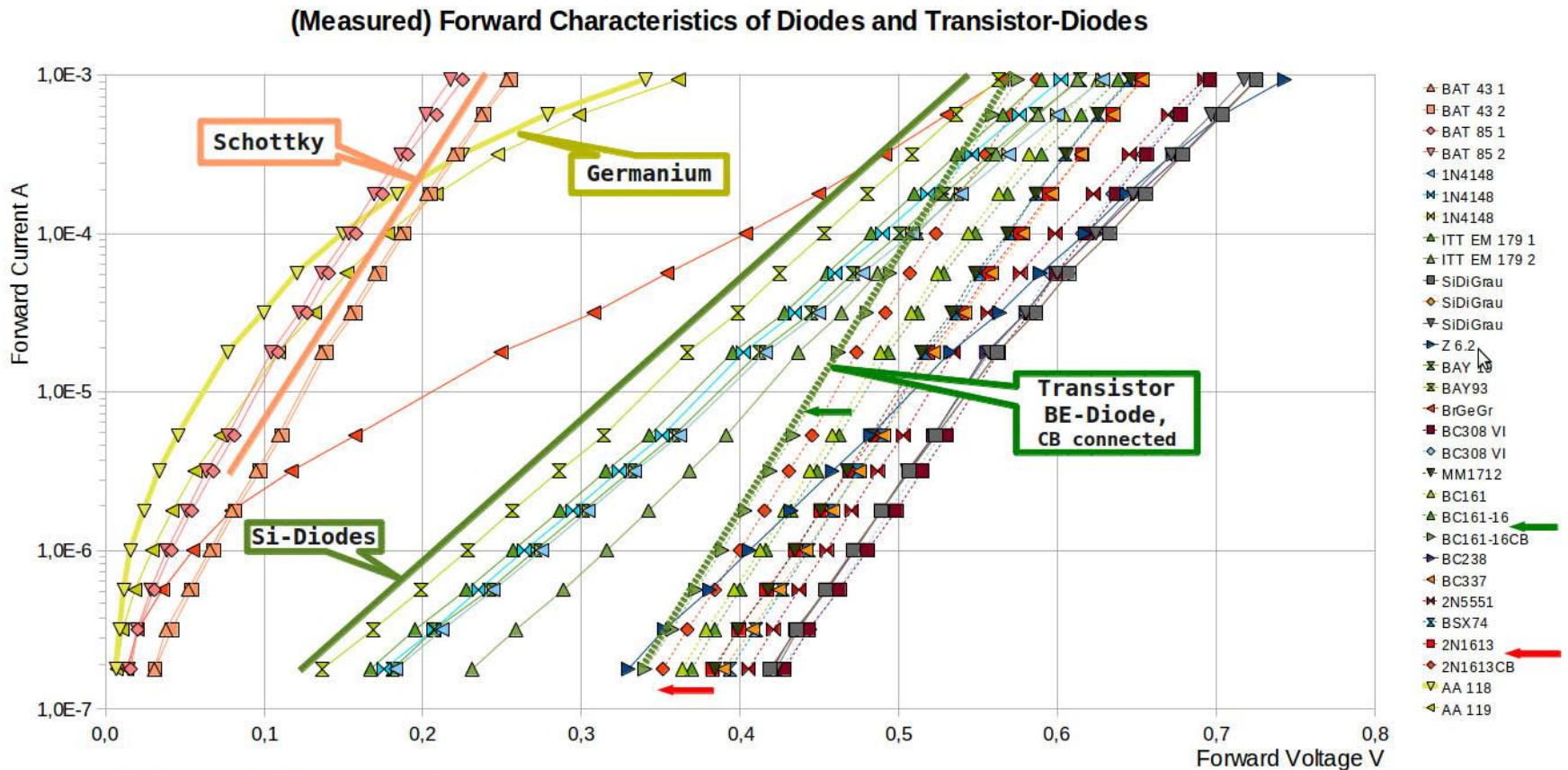
Voltage multipliers

- Voltage multipliers* are generalizations of diode rectifiers which generate DC output voltages that are integer times the AC peak input, for example, 2, 3, or 4 times the AC peak input.
- Top picture: the voltage doubler charges in the first negative half-wave of V_{in} the C1 capacitor negative through D1 whilst D2 is not conducting. In the following positive half-wave, D1 is not conducting, so the negative V_{in} half-wave and the negative pre-charged C2 add up to twice the peak voltage which is passed through D2 which is now conducting. With this, C2 gets charged to twice the peak voltage of the input voltage.
- Middle picture: the Voltage quadrupler consist of 2 successive stages of a voltage doubler.
- http://www.youtube.com/watch?v=ep3D_LC2UzU
- Bottom picture: the N x voltage multiplier (Cockroft Walton) is a simple succession of voltage doublers. Note that it only doubles in each stage the input voltage and not the previous voltage. With a 5V p-p AC input an 8-fold voltage doubler generates theoretically $5 \times 2 + 5 \times 2 + 5 \times 2 + 5 \times 2 + 5 \times 2 + 5 \times 2 + 5 \times 2 + 5 \times 2 = 80$ Volt. In real, the diode forward voltages have to be deduced, the doubling factor of each successive stage is reduced due to the impedance of the RC made by a diode and a capacitor. When current is flowing at the output, frequency and value of the capacitors need to be tuned for minimal output ripple. The 8 stage multiplier shown generates with 3.5 Volts RMS (5V p-p) and 1 kHz and no output load ca 78 Volt with 3.7 V ripple.
- Calculators for Cockroft-Walton multipliers can be found on the internet
<http://www.extremelectronics.co.uk/calcs/index.php?page=cwvoltage.php>



* http://www.allaboutcircuits.com/vol_3/chpt_3/8.html

Diode forward characteristics

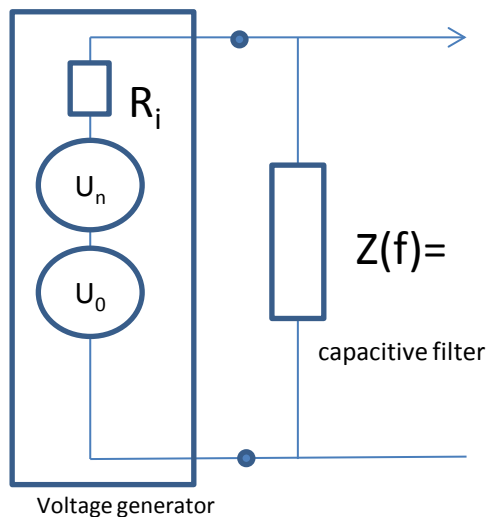


Compilation: Rolf Suessbrich, Dortmund, 6 / 2011

- Summary : at 1mA forward current , standard Si diodes (1N4148) have ~ 0.6 V drop, Schottky diodes ~ 0.25 V. The BE diode characteristics of a Si Transistor is steeper than that of a Diode when CB is connected

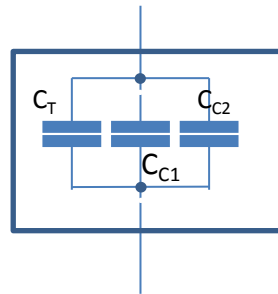
DC noise filtering

- Voltage sources are generally composed of DC voltage U_0 plus a parasitic noise voltage U_n with a large frequency spectrum.
- Capacitors are connected in parallel with Voltage Sources for noise suppression, each capacitor covering a different range.
- For example a parallel capacitor of
 - 1 tantalum 47 μF (C_T)
 - 1 ceramic X7R of 100 nF (C_{C1})
 - 1 ceramic NPO 1 nF (C_{C2})
 represents a low impedance $\sim 1 \text{ OHM}$ over the frequency range between 200 Hz and 200 MHz
- For DC and low frequency components $Z(f)$ has a very high value.

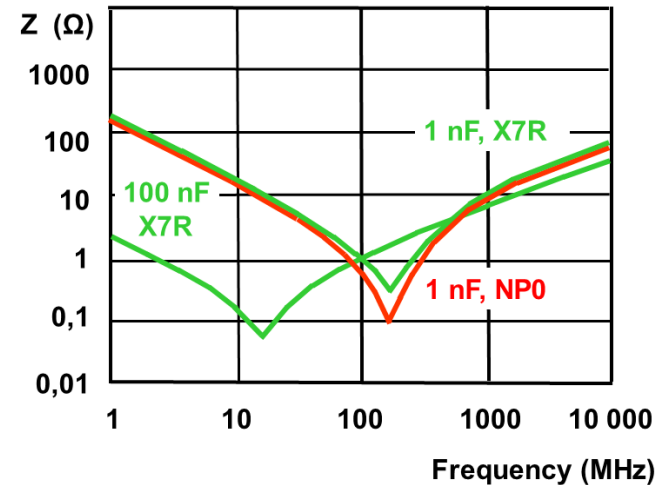


$$U(f) = U_0 + U_n \frac{Z}{R_i + Z}$$

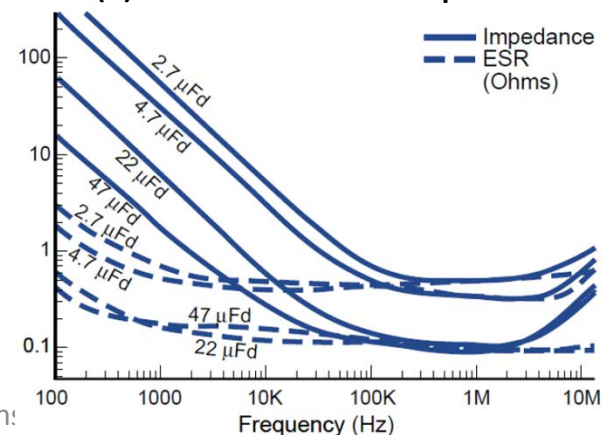
for $f \rightarrow \infty$ $Z \rightarrow 0$, $U_n \rightarrow 0$



$Z(f)$ of ceramic capacitors

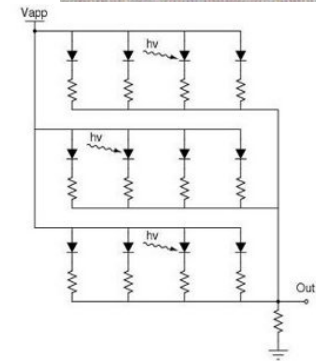
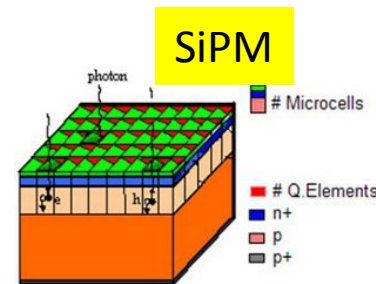
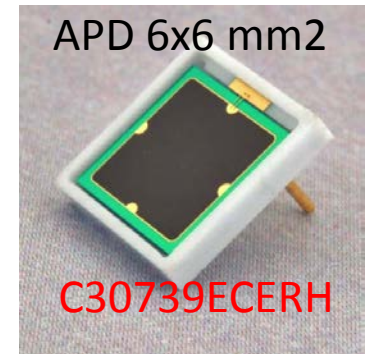
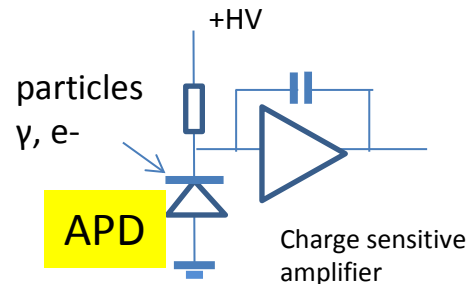
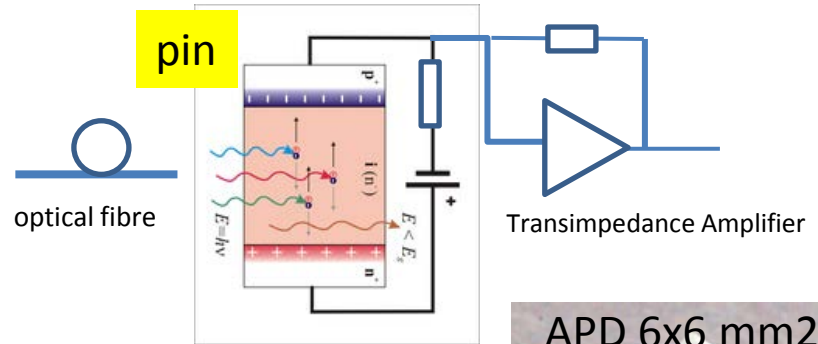


$Z(f)$ of tantalum capacitors



Detector diodes

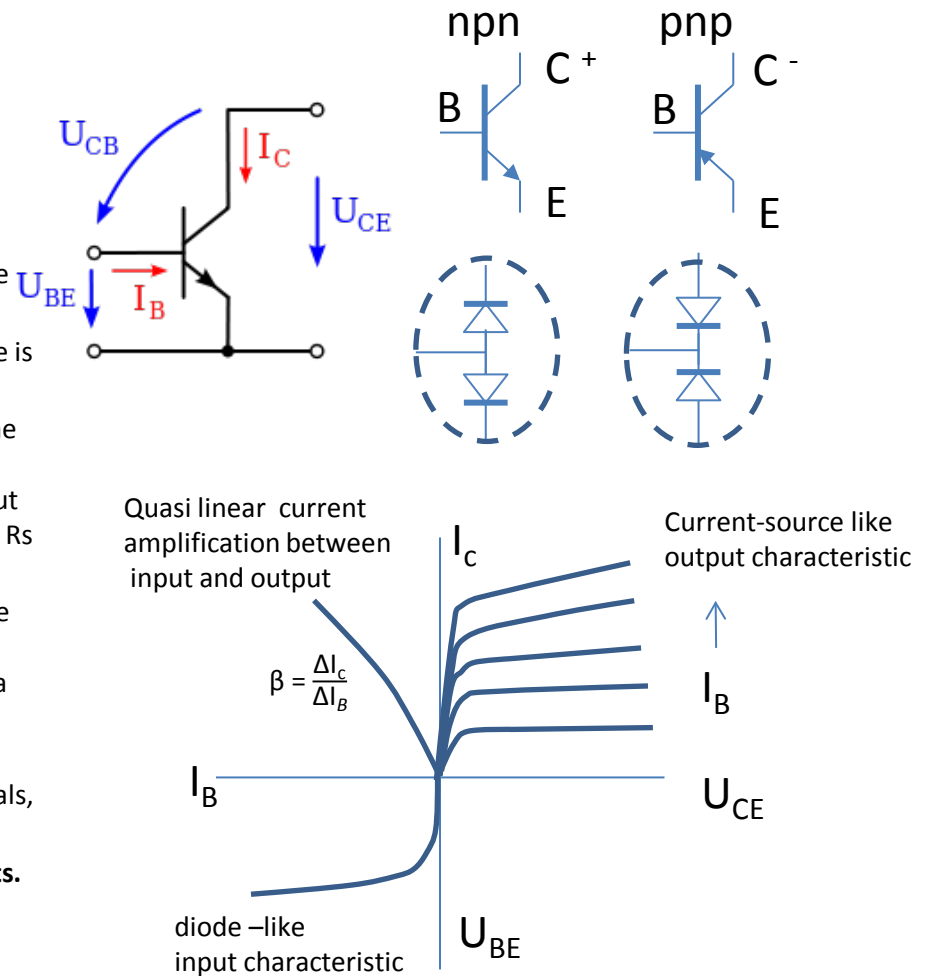
- **Pin diodes** have an **additional conductive i-layer** in between the p-n layer. The depletion depth is therefore much larger and in analogy to the plate capacitor, the capacitance is reduced.
- Photons of sufficient energy (wavelength 850 nm) that enter the depletion layer create electron- hole pairs and therefore small voltage impulses $\Delta U = \Delta Q / C$. Pin diodes are therefore used as **photodetectors** for conversion of optical signals from a optical fibres into an electrical signal.
- **Avalanche Photo Diodes (APD)** have in addition intrinsic signal amplification (50..100) due to application of high voltage (300-400V). APD's were initially used for electromagnetic Calorimeters, meanwhile also for telemetry and light amplifiers.
- **Silicon photomultipliers (SiPM's)** are like APDs but **segmented in x and y** into many individual diode cells. Each individual cell is operated in Geiger mode, resulting in single photon sensitivity like photomultipliers and very high timing resolutions down to 100 ps.



(bipolar) Transistors



- Bipolar transistors are either
 - $np + pn \Rightarrow$ **npn**
 - or
 - $pn + np \Rightarrow$ **pnp**
 doped transition of semiconductor material, normally Silizium, as if two diodes were connected with their same poles together however the middle layer is very thin.
- The external poles are named **Emitter** and **Collector**, the middle pole is named **Basis**
- The electrical current I_C from Collector to Emitter is a function of the much smaller current I_B through the Basis \rightarrow Emitter junction. The current-voltage relation between Basis and Emitter is like a **diode** but with larger differential resistance. The diff. input resistance is $r_{diode} + R_s$
- To first approximation $I_C = \beta I_B$ is a liner relation with $\beta = O(100)$. Hence a transistor is a **current amplifier**. For constant I_B currents, the collector current I_C is constant and almost independent of the U_{CE} Voltage between Collector and Emitter. This is the characteristic of a **current source**.
- Bipolar transistors exist in many different forms for many different applications: audio frequency, high frequency, switching, small signals, power usw.
- Bipolar transistor are also part of specific function **integrated circuits**.
- Discrete transistors** are mainly used for power and control applications.



Simple transistor amplifier

- The picture shows a simple voltage amplifier with npn transistor with current amplification factor β . Without input voltage ΔU_{in} the resistor divider R1,R2 at the Transistor base generates a fixed U_{BE} Voltage and with the input characteristics a constant current I_{BE} is defined which generates a collector current $I_C = \beta I_B$. One needs to consult the specification of the transistor to set these values. With a given I_C current the Collector resistor R_C can be chosen such that the optimal working point Voltage at the collector U_w is set: $U_w = U_0 - R_C I_C$. Best is to set $I_w = U_0/2$ such that an amplified signal has the maximum +- swing around U_w .

- The saturation Voltage $U_{CE(SAT)}$ which increases with I_b limits the linear output range to values in the order of 1 V. At $U_{CE(SAT)}$ the transistor is in its lowest output state .

- A small AC input voltage Δu_{in} gets superimposed via a capacitor C . The amplified output AC voltage is

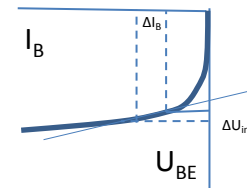
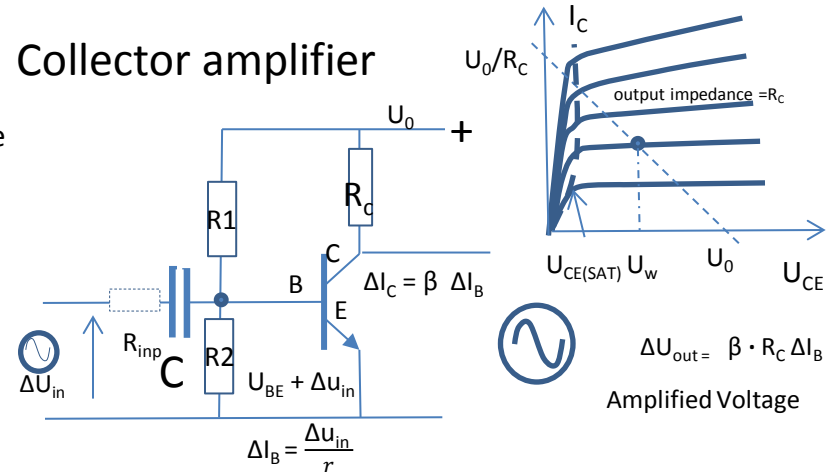
$$\Delta U_{out} = \beta \cdot R_C \Delta I_B = \beta \cdot R_C \frac{\Delta U_{in}}{r+R_S}$$

where $r+R_S$ is the differential input resistor plus a base-internal series resistor

- The input capacity C_D of the B-E diode is parallel to r . The replacement AC circuit add the capacity C_{BC} of the diode between Base and Collector. A current source of strength $I_C = \beta \cdot I_B$ drives current through the collector resistor.

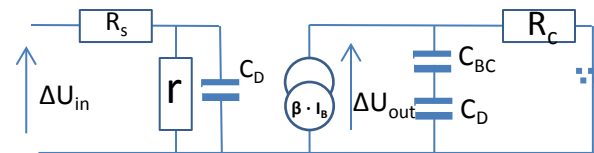
- The maximum input time constant given by $C_D \cdot r$ which is typically in the order of 1 ns, the output time constant to discharge the serial $C_D + C_{BC}$ is however slower O(10) since R_C is much larger than r .

Collector amplifier



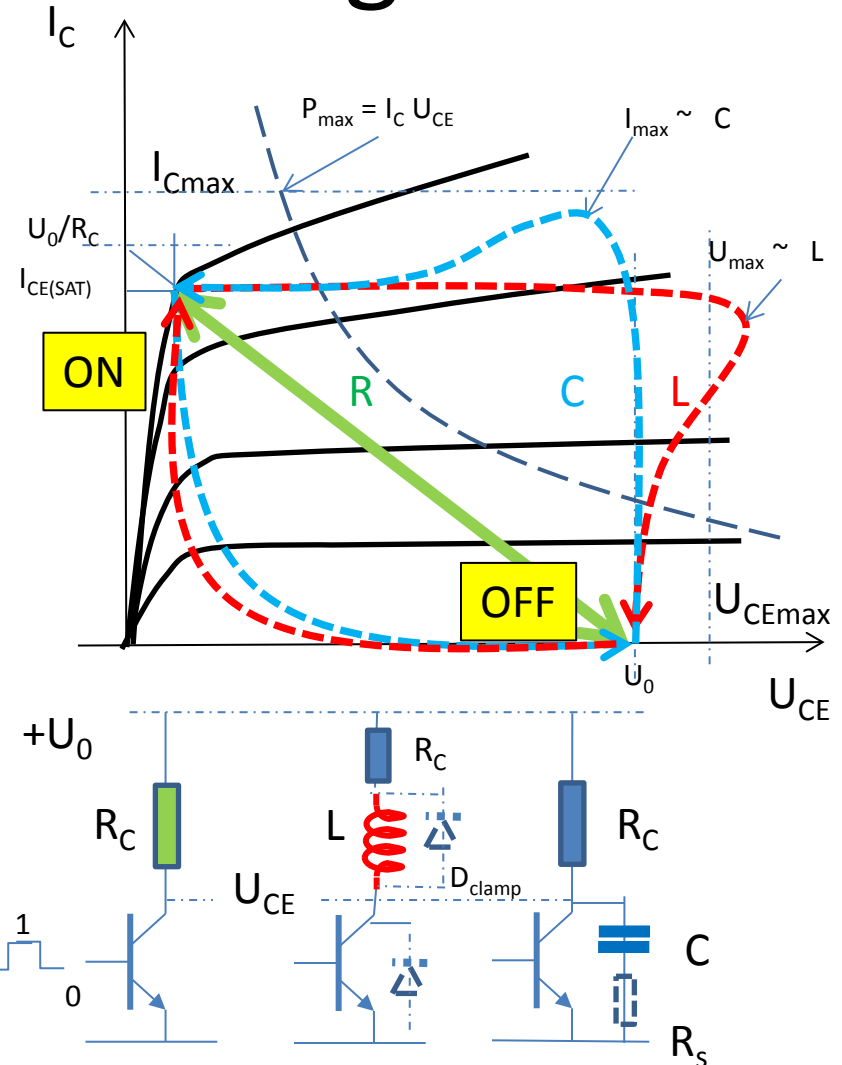
r = differential B-E diode resistance + R_S
i.e. NOT linear over large ΔU_{in}
-> make ΔU_{in} small, or add series input resistor or use feedback (see later)

AC replacement circuit for amplifier



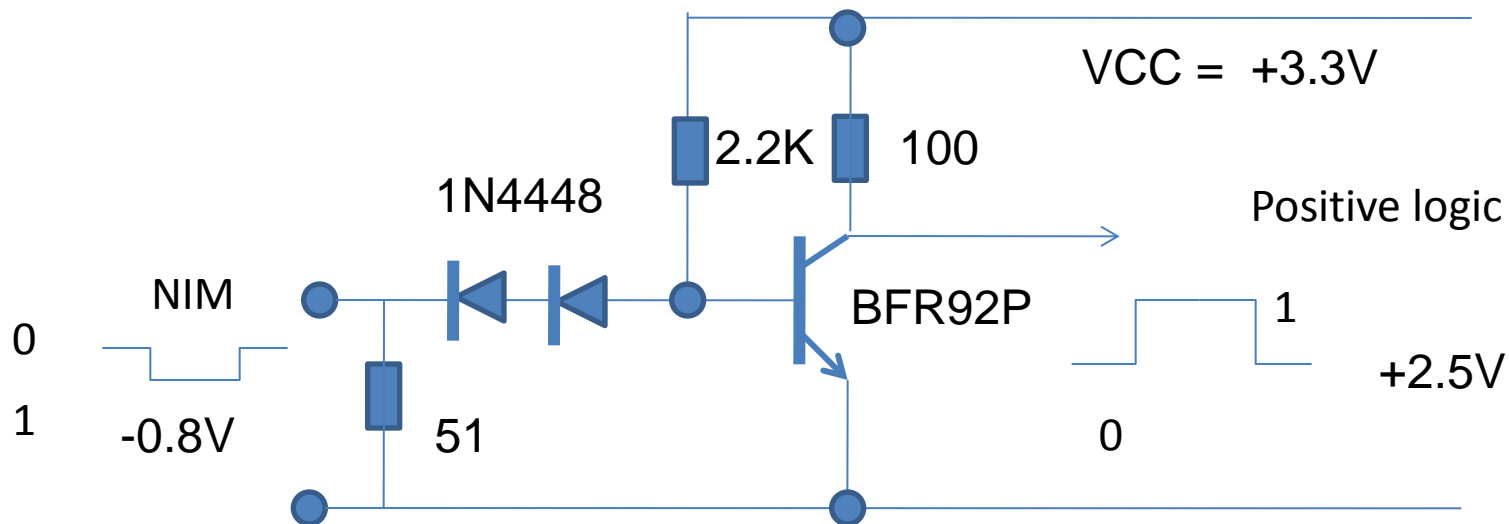
Transistor switching

- For switching applications, typically an npn transistor is driven via a bipolar input current "0" and "1" at the Base into a Collector Voltage output state "ON" and "OFF". "ON" is at $U_{CE(SAT)}$ where the transistor is completely saturated at its lowest status $O(\sim 1 \text{ VOLT})$. "OFF" is the cutoff status of the I_{CE} current, where the collector voltage U_{CE} practically equals the supply voltage U_0 . The output state is quasi binary, either ON or OFF. The transition path in the I_C - U_{CE} quadrant of the intermediate state between ON and OFF is depends however on the type of load impedance.
- For **resistive loads R**, the transition from OFF to ON follows the linear path $U_{CE} = U_0 - I_C \cdot R_C$.
- For **capacitive loads C**, the OFF-> ON transition features a strong discharge of the capacitor over a small $R_{CE(ON)}$ transistor resistivity, resulting in an initial fast rise of I_C and a slow change of voltage U_{CE} . With fast switching transition, the overpassing of the maximum P_{max} hyperbola is possible but must stay below the absolute limits I_{Cmax} and U_{CEmax} . Since I_C increases with C , a small protection resistor R_s can be put in series with C . The ON->OFF transition re-charges C with time constant $C \cdot R_C$ at an initial current of U_0/R_C .
- For **inductive loads L**, the OFF-> ON transition starts with a small I_C current opposing a voltage change over the coil and filling it up with magnetic energy. When switched from ON-> OFF, the self inductance maintains its energy via an EMF voltage which is opposed to the change of the current such that $U_C = U_0 + U_L$ can become significantly higher than U_{CEmax} and the transistor can get destroyed. The standard protection is a clamp diode D_{clamp} across the coil and polarized such as to quench the reverse EMF voltage below $\sim 0.7V$ by consuming its energy.
- Some switching transistors contain clamp diodes in parallel to UCE in order to protect against reversed voltage spikes, however the main protection principle for inductive loads is that the EMF energy of the coil must be consumed locally at the coil.



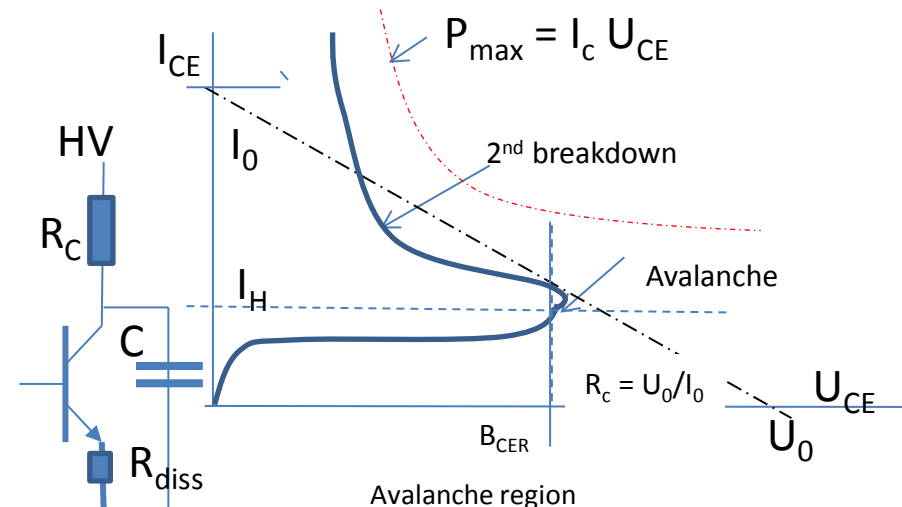
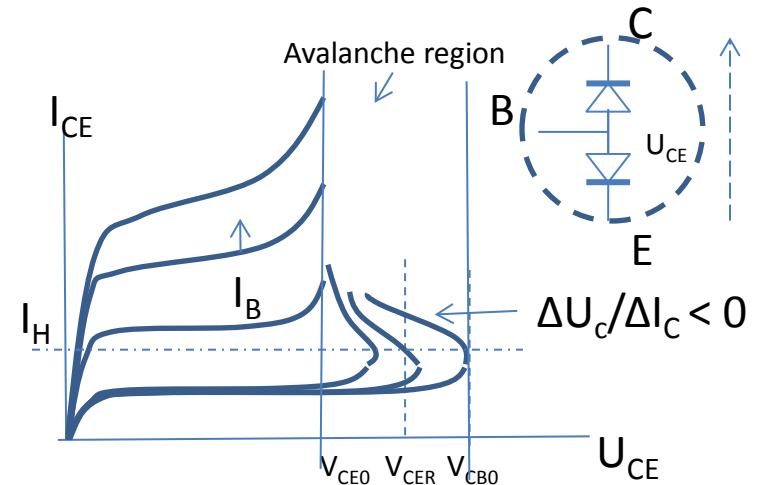
NIM -> TTL via switching transistor

- NIM signals are common in Nuclear Electronics for fast pulses over coaxial cables of 50 OHM impedance. The “0” status is at GND level, the “1” status is at “-0.8 V” (forward bias Voltage of a Si diode).
- Standard digital electronics (FPGA’s etc) work however with positive voltages like +3.3Volt. The conversion of a NIM pulse into a positive binary voltage can be implemented using a single transistor.
- The Basis of a fast npn transistor gets positive biased into Collector saturation by the 2 x 0.7 V bias of two successive Si diodes. These diodes are connected in series with a 50 OHM termination resistor of the input NIM pulse. Upon arrival of a negative (-0.8V) NIM pulse, the transistor bias is reduced to less than 0.7 V and which drives the transistor into the “OFF” status, hence the collector voltage rises quickly to the VCC level.
- When the NIM pulse ends, the transistor falls back into the OFF status with a low collector voltage.



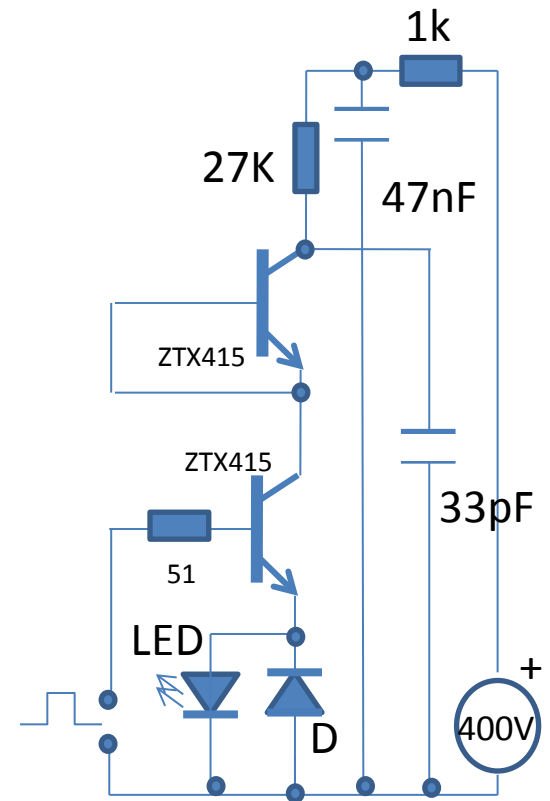
Avalanche transistors

- Breakdown of the characteristics of a bipolar transistor I_C versus reverse bias voltage U_{CE} is governed by thermal instability, tunnelling effect and avalanche multiplication. The breakdown voltages V_{CB0} (Emitter open) V_{CER} (Base reverse biased) and V_{CEO} (Base open) impose an upper limit on U_{CE} . Above these characteristic voltages, avalanche amplification makes the collector current rise due to charge carrier multiplication in the high field of the reverse biased C-B junction.
- When the I_{CE} current is increased above I_H , a 2nd breakdown occurs, characterized by a sharp increase of the collector current I_C and a negative differential resistance $\Delta U_C / \Delta I_C < 0$.
- When the maximum power capability P_{max} is overpassed for a more than a short time, the transistor junctions gets thermally destroyed.
- As protection against destruction, transistors are selected for avalanche operation, a high collector resistor $R_c = U_0 / I_0$ is used and the input pulse must have a limited duration. The avalanche switching has applications for very fast pulse generation, normally in combination with a discharge capacitor C.
- When the input trigger current at the Basis is strong enough to generates a collector current of more than I_H , an avalanche transistor (like to ZTX415 with $U_{CEO} = 100V$) can discharge up to 160 A within a few ns from a HV-charged capacitor. However a dissipation element (R_{diss}) is needed to contain the capacitor energy ($\frac{1}{2} Q^2 / C$) and avoid energy transmission as high frequency radiation.



Nanosecond LED pulser

- The (simplified) schematics shows a very high- intensity, nanosecond LED light pulser, as used in Calorimeters for multi-channel calibration.
- A 33 pF capacitor is charged to 400V corresponding with $Q=U \cdot C$ to 13 nanoCoulomb. Its discharge through a tandem of Avalanche transistors through a blue LED corresponds to a ca. 10 A pulse if the discharge time takes a $\Delta t \sim 1$ ns.
- The lower transistor Avalanche is triggered by a positive current pulse at the base of the lower transistor such that its collector current reaches the threshold current I_H . The sudden rise of U_{CE} over the second transistor generates an avalanche also in the second transistor.



Emitter follower

- The schematic shows an emitter follower circuit: the Collector is kept at a constant voltage U_0 and a load resistor R_E is inserted in the Emitter line. With R_1 and R_2 one can set a base current such that $I_C = \beta I_B \sim I_E$ and with an emitter resistor R_E a static working point is defined as $U_B = U_{RE} + U_{EB}$.

- Regarding only AC changes and the fact that the forward diode voltage U_{EB} remains basically constant:

$$\Delta U_B \sim \Delta U_{RE}$$

- In fact the emitter follower is an **amplifier with gain 1** due to 100% negative feedback
- The equivalent circuit shows all Δx values that change in small letters (r_{IN} , i_B , u_{IN})
- The following properties can be derived

$$u_{IN} = r_{IN} \cdot i_B = i_B \cdot R_S + (\beta+1) \cdot i_B r + R_E(\beta+1) \cdot i_B$$

neglecting the small term $(\beta+1) \cdot i_B r$

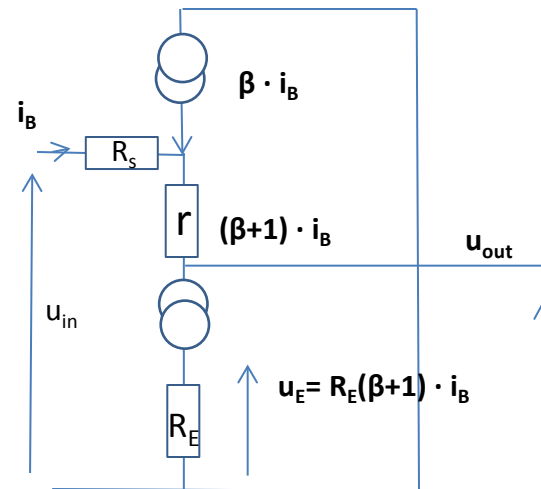
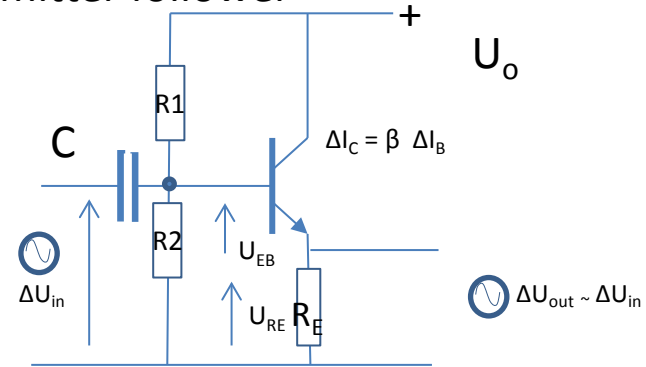
$$r_{IN} = R_S + R_E(\beta+1)$$

the input resistance r_{IN} is increased by large

$$r_{OUT} = u_{OUT} / ((\beta+1)i_B) = \frac{R_E(\beta+1)i_B}{(\beta+1)i_B} = R_E$$

the output resistance is selectable as R_E

Emitter follower

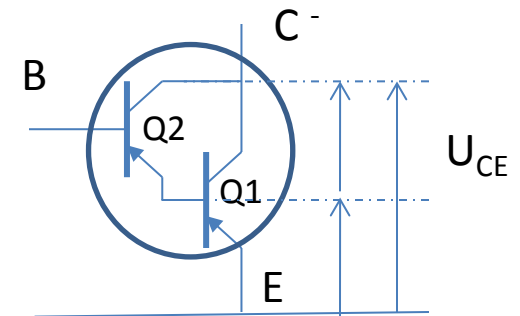


Darlington transistor

- Two transistors connected in an emitter-follower configuration with common collector contact are named Darlington. The total current gain β of the circuit equals the product of the current gain of the two transistors.

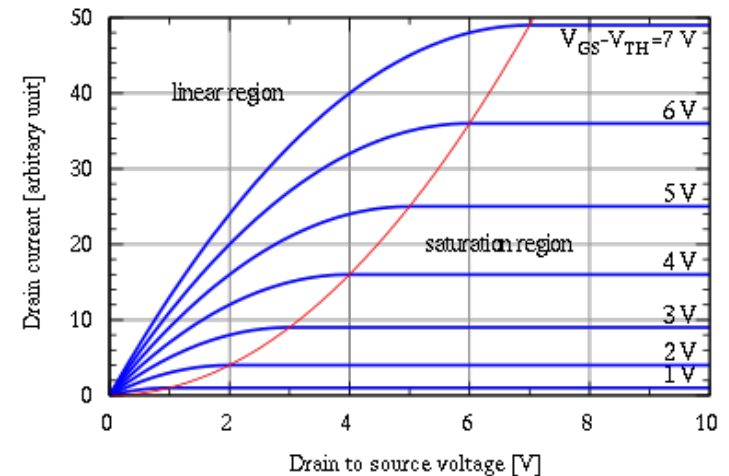
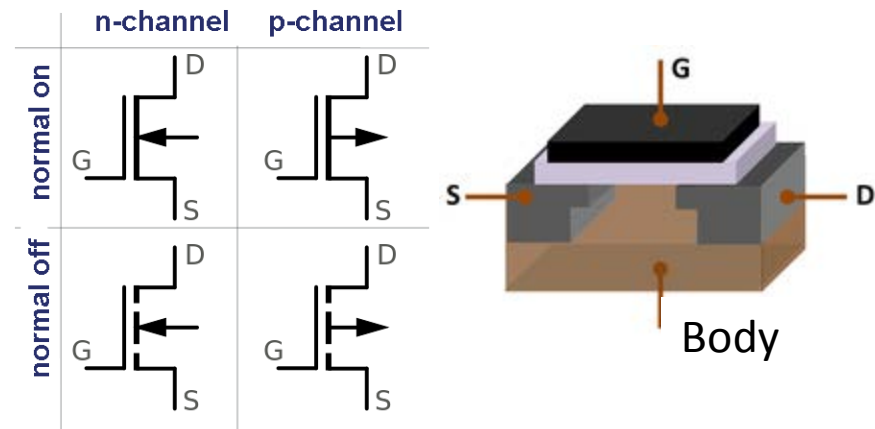
$$\beta_{AB} = \beta_A \cdot \beta_B$$

- The saturation voltage U_{CE0} of the pair equals the forward bias voltage of transistor Q1 plus the saturation voltage of transistor Q2 hence significantly larger than for a single transistor resulting in larger on-state power dissipation.



MOSFET transistor

- **Field effect transistors (FET)** are much more common than bipolar transistors. They consist of channels with N-doped (electrons), or P-doped (holes) with 2 electrodes Source (S) and Drain (D). The Gate (G) field relative to the Drain influences the resistance of the S-D channel. Enhancement FETs increase the channel conductivity with increasing Gate field, depletion FETs reduce it.
 - When a positive voltage (V_{GS}) is applied the gate of an **n-channel MOSFET** the insulating (inversion) layer starts to fill with electrons.
 - The threshold voltage (V_{th}) is the voltage at which there are sufficient electrons in the inversion layer to make a low resistance conducting path in the channel.
 - An n-channel MOSFET is composed of p-type Silicon with has positively charged carriers (holes). The n-channel MOSFET can be in **3 different modes**:
1. When the gate voltage is below V_{th} the FET is **turned off**
 2. $V_{GS} > V_{th}$ and $V_{DS} < (V_{GS} - V_{th})$ the FET is in **linear mode** The MOSFET operates like a resistor, controlled by the gate voltage relative to both the source and drain voltages
 3. $V_{GS} > V_{th}$ and $V_{DS} \geq (V_{GS} - V_{th})$ the FET is in **saturation mode**: The switch is turned on, and a channel has been created, which allows current to flow between the drain and source
- The output characteristics I_{DS} versus U_{DS} shows the 2 conducting regions with U_{GS} as parameter.



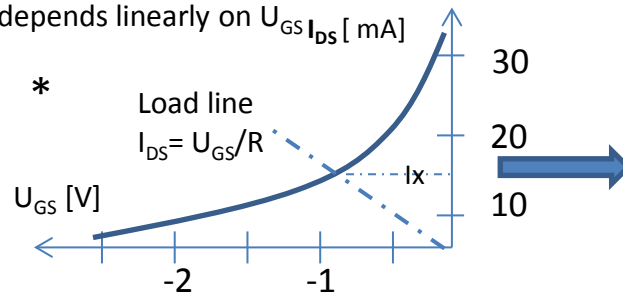
JFETs

- Junction FETs (JFET) have a reverse biased PN junction which generates a high input impedance. (The MOSFET has an isolated gate with poly-silicon) The JFET can only be operated in the **depletion mode**. In a JFET, if the gate is forward biased, excess-carrier injection occurs and the gate current is substantial. Thus channel conductance is enhanced to some degree due to excess carriers but the device is never operated with gate forward biased because any gate current is undesirable.
- JFETs have lower noise than bipolar transistors. Bi-FET input stages are found in very low noise OPA's down to $1.8 \text{ nV}/\sqrt{\text{Hz}}$
- The channel current is a function of the input voltage via

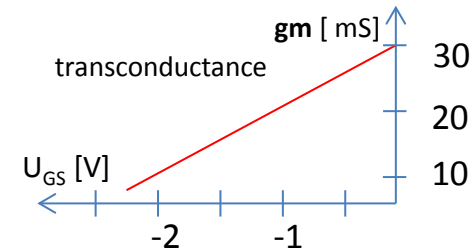
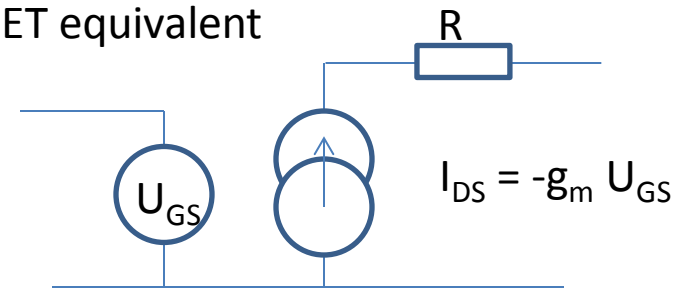
$$I_D = -g_m U_{GS}$$

with the **transconductance** g_m

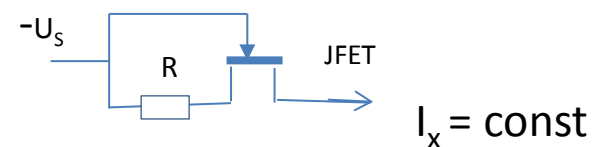
g_m depends linearly on U_{GS}



JFET equivalent



Self-biased current source



* <http://socrates.berkeley.edu/~phylabs/bsc/PDFFiles/bsc4.pdf>

Serial Feedback

- The **emitter follower is an example of serial feedback:**
- An open loop amplifier with **gain A** is described by

$$U_{out} = A \cdot U_{IN}$$

- With a serial feedback applied, the **feedback fraction** is

$$k = \frac{U_{out}^*}{U_{out}} \leq 1 \quad (* \text{ means "with feedback"})$$

- The gain A^* with feedback is obtained as

$$U_{out} = A \cdot U_{IN} = A^* \cdot (U_{IN} + k \cdot U_{out})$$

$$\Rightarrow A^* = \frac{A}{1+kA}$$

- The **input resistance** with feedback is

$$R_{IN} = \frac{U_{IN} + kA \cdot U_{IN}}{I_{IN}}$$

$$R_{IN}^* = R_{IN} (1+kA) \quad \text{.. increased by factor } (1+kA)$$

- The input voltage with feedback is effectively

$$U_{IN} = Q/C_{in} - kA \cdot U_{IN}$$

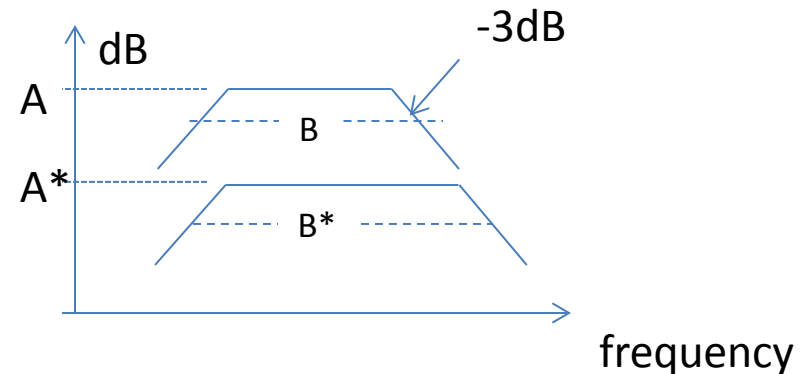
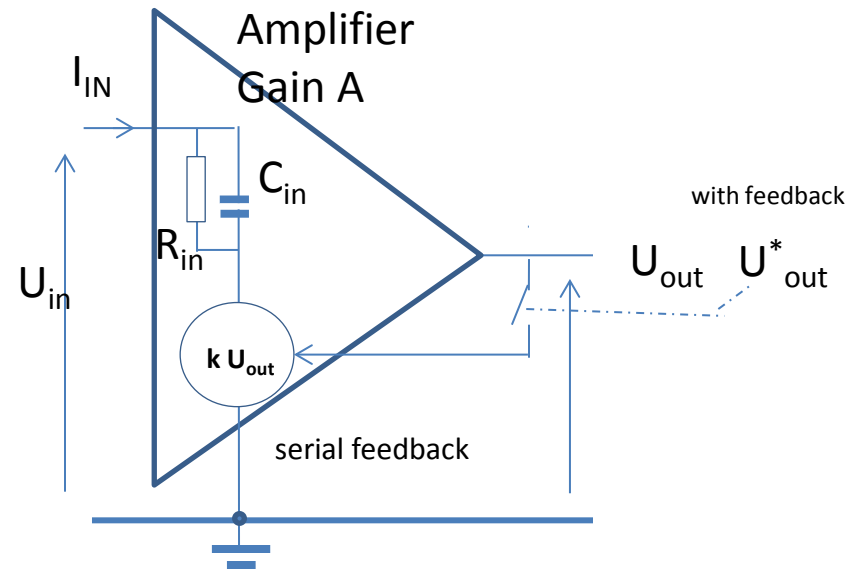
- Consequently the **input capacity** with feedback is

$$C_{in} = \frac{Q}{U_{IN}} \frac{1}{1+kA} \quad \text{.. reduced by the factor } \left(\frac{1}{1+kA} < 1\right)$$

- The reduction of the input capacity increases the maximum input frequency and with this the bandwidth B to B^*

- The **product of bandwidth and gain** is constant with and without feedback.

$$B \cdot A = B^* \cdot A^*$$



Charge sensitive preamplifier (CSP)

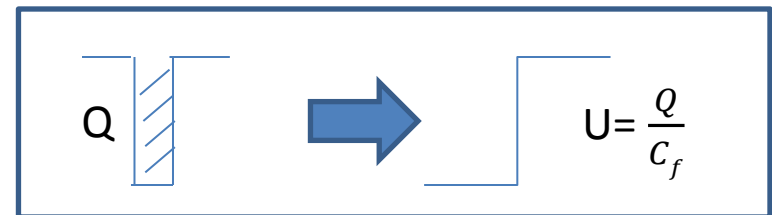
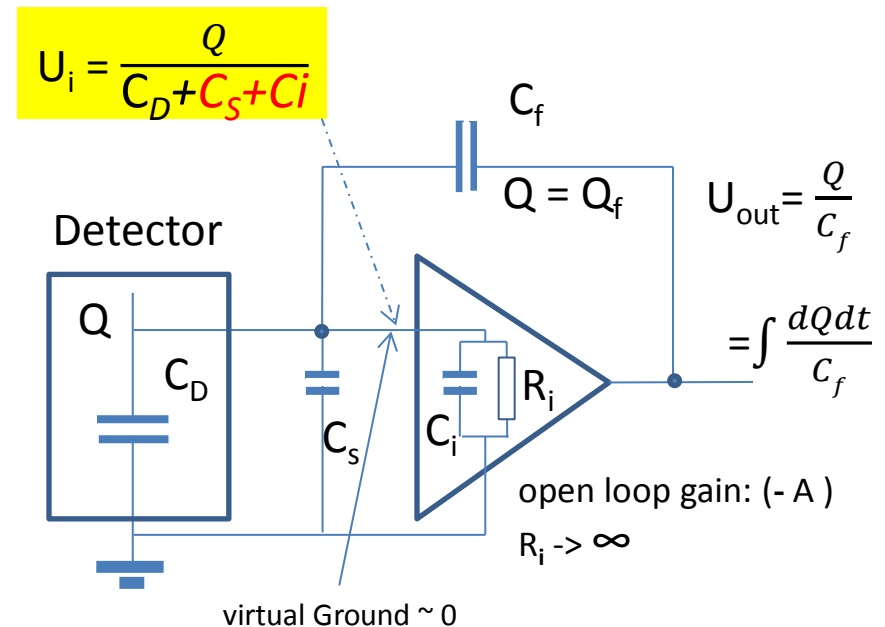
- Avalanche detectors generate **charge Q** in O(1ns).
- The corresponding **Voltage signal** on the detector capacitance **decreases relative to noise with capacitance.**
>Minimize Cs (stray) and Ci (amplifier) capacitances!
- Detector capacitances vary with strip geometry etc. >Make amplified voltage signal independent of varying detector capacity C_D
- The Charge amplifier applies a **feedback capacitor C_f** from output to input of an inverting amplifier of gain $-A$. For very large input resistance, all detector charge Q flows on the feedback capacitor C_f .
- $U_{out} = U_{CF} + U_i = U_i (A+1)$. The input voltage is therefore very small $U_i = U_{out}/(A+1)$ and therefore named **virtual Ground**.
- The output voltage is the voltage between the output and (virtual) Ground

$$U_{out} = \frac{Q}{C_f}$$

- The **voltage/charge conversion** gain is $1/C_f$. With $C_f = 1\text{pF}$ the VC gain is 1mV/fC i.e. 1 millivolt per 6250 electrons
- The effective input capacitance becomes large

$$C_i = C_f (A+1)$$

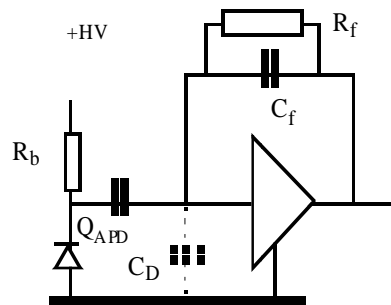
- The capacitance variations of the detector are eliminated



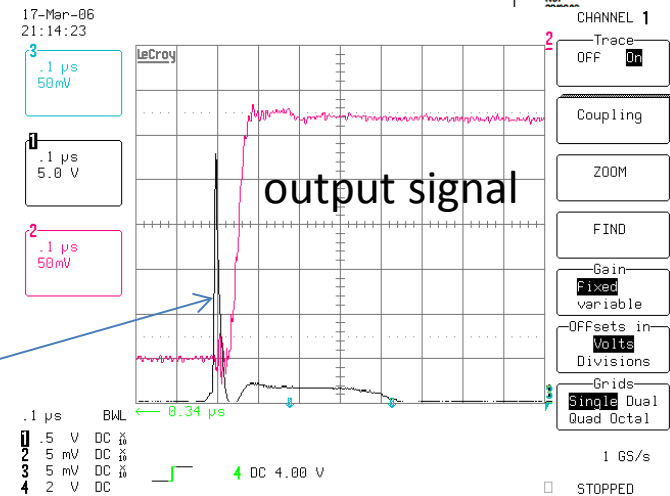
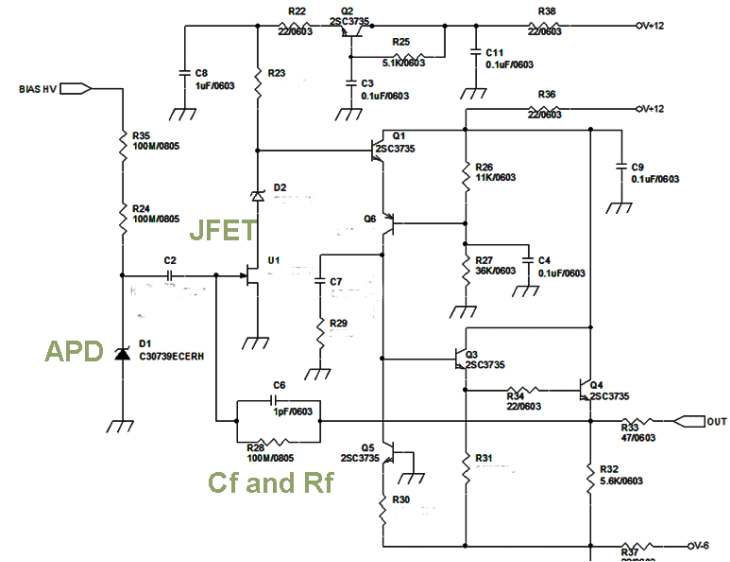
* http://www-physics.lbl.gov/~spieler/SLAC_Lectures/PDF/Sem-Det-II.pdf

CSP in detectors

- Charge sensitive amplifiers for detector may require a **large dynamic range**
- A small feedback capacitor provides large **charge /voltage gain**. With $C_f = 1\text{pF}$ and $U=Q/C$ the gain is **$1\text{mV}/\text{fC}$** , where 1 fC is a charge of 6250 electrons.
- A dynamic range requirement of (14 bit = 1/16000) the **maximum signal** output exceeds the possibilities of integrated frontend chips which are powered by $O(2.4)$ V. Therefore discrete CSP implementations are common.
- JFETs are used to achieve low noise



Input signal



Parallel feedback

- The Charge sensitive amplifier is an example of parallel feedback
- It consist in feeding back a part of the output current to the input. The feedback current is

$$I_f = kA \frac{U_{IN}}{R_{IN}}$$

where the feedback fraction $k = \frac{U_{out}^*}{U_{out}}$

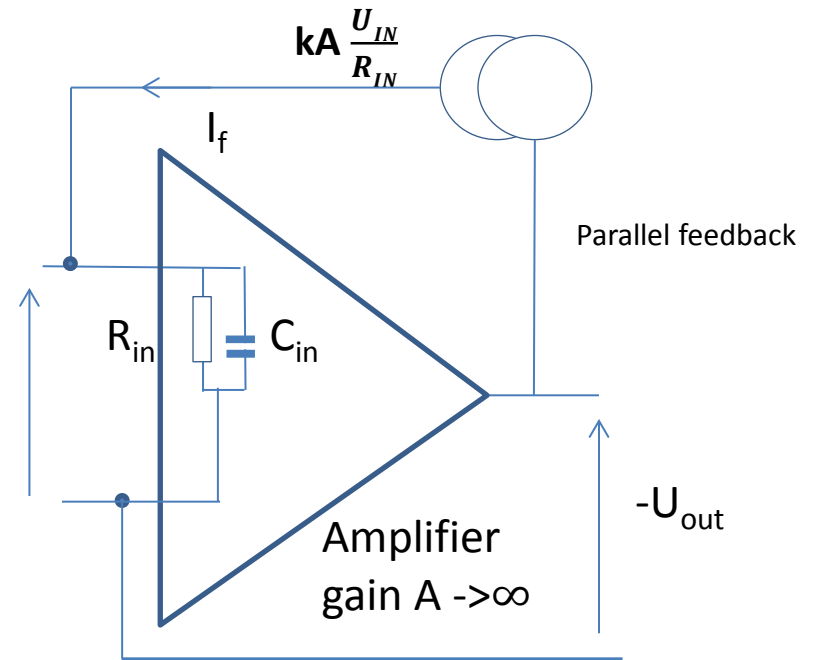
A is the open loop gain

- The **effective input resistance R_{in}^*** reduced

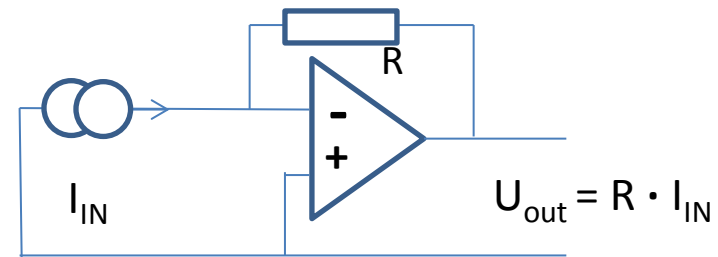
$$R_{in}^* = \frac{U_{IN}}{I_{IN} + kA I_{IN}} = \frac{1}{1+kA} R_{IN}$$

- The **effective input capacitance is increased**

$$C_{in}^* = (1 + kA) C_{in}$$



Example of current feedback: transimpedance amplifier

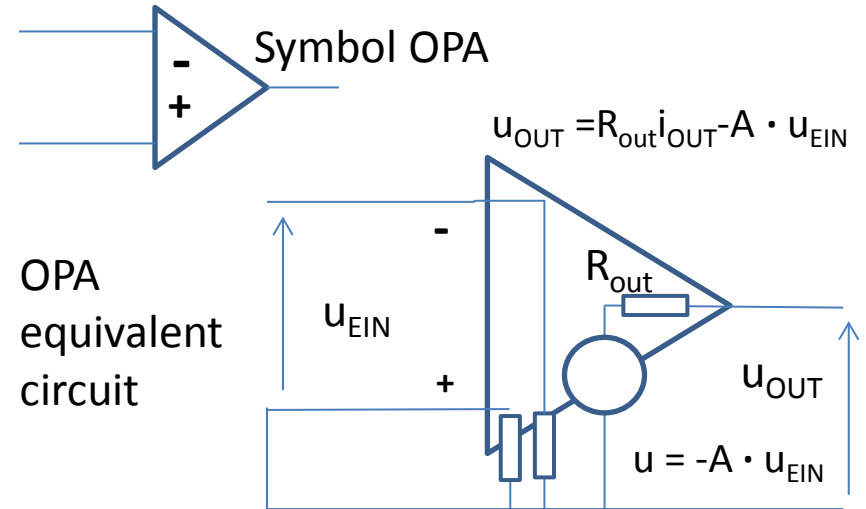


Operational Amplifiers (OPA)

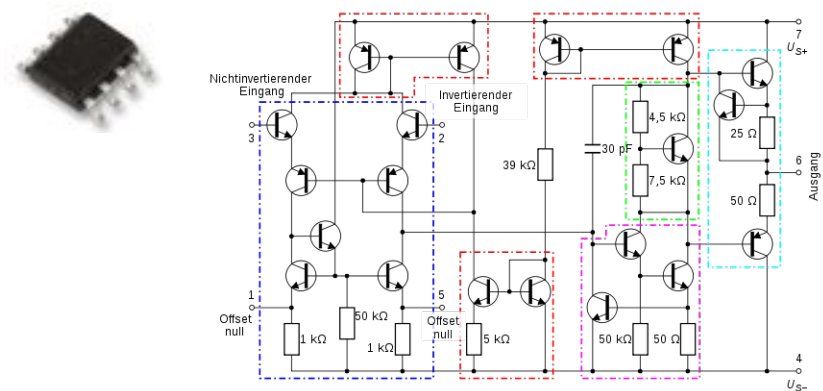
- OPAs are completely integrated amplifier systems with which are mainly characterized by open loop amplification A , Input resistance R_{IN} , Output resistance R_{OUT} , bandwidth, slew rate, max Voltage.
- R_{IN} , is generally very high ($> 10\text{ M}$) and R_{OUT} is generally low $O(50\text{ Ohm})$.
- OPA's have generally differential inputs, designated (+) and (-) such that the output voltage is $U_{OUT} = A \cdot \Delta U_{IN}$ where $\Delta U_{IN} = (U_+ - U_-)$
- Since A is generally a very large number $O(10.000)$ the differential input voltage ΔU_{IN} is very small $O(\mu\text{V})$.
- The common mode rejection CMR is the ability of an OPA to keep the output voltage constant when the differential inputs are connected ($\Delta U_{IN}=0$) and a common Voltage U_{CM} is applied. CMR is a large number.

$$CMR = \frac{U_{CM}}{u_{OUT}} A$$

- Another non ideal OPA property is the fact the output voltage u_{OUT} is not exactly zero when the differential input voltage is zero. Precision OPAs have therefore a an external compensation resistor.



Example of an integrated OPA



OPA's with parallel feedback

- The upper circuit shows an **inverting amplifier** with linear **voltage gain A_{inv}**
- With $I_E \sim 0$ and $U_E \sim 0$ follows $I_{IN} = U_{IN}/R1 = -U_{OUT}/R2$

$$A_{inv} = U_{OUT} / U_{IN} = - \frac{R2}{R1}$$

- The symmetry resistor R is calculated to generate a symmetrical input current (though almost zero) as

$$R = \frac{(R_2 + R_I)(R_1 + R_{IS})}{(R_2 + R_I) + (R_1 + R_{IS})}$$

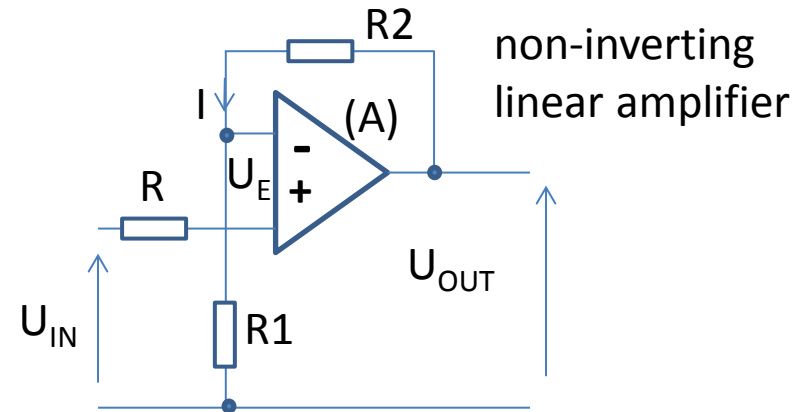
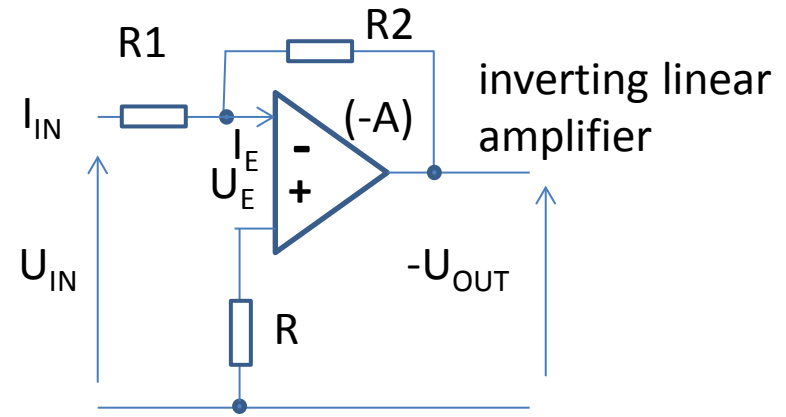
where R_{IS} is the internal resistor of the input voltage source and R_I is the internal resistance of the output Voltage

- The lower circuit is a non inverting amplifier with linear gain A_{plus}
- With $U_E \sim 0$ and $I = U_{OUT}/(R2+R1) = U_{IN}/R1$ follows

$$A_{plus} = U_{OUT} / U_{IN} = \frac{R2+R1}{R1}$$

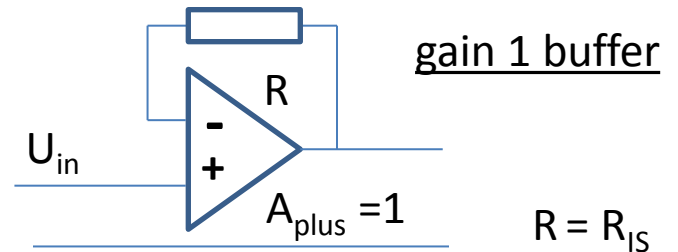
- The symmetry resistor R is calculated like above as

$$R = \frac{R1 (R_2 + R_I)}{R_1 + (R_2 + R_I)} - R_{IS}$$



More OPA circuits

- The **gain 1 buffer** can for instance be used to measure **very high ohmic voltages**. For example the measurement of 5kV with a 10M HV probe has a probe current of 0.5 mA. This current however presents a load on many HV supplies and changes the voltage to a different, lower value. A unity-1 amplifier with very high ohmic input allows to use a 5 GOHM/5 MOHM voltage divider with 1 uA load current to measure the high voltage at ratio 1/1000.



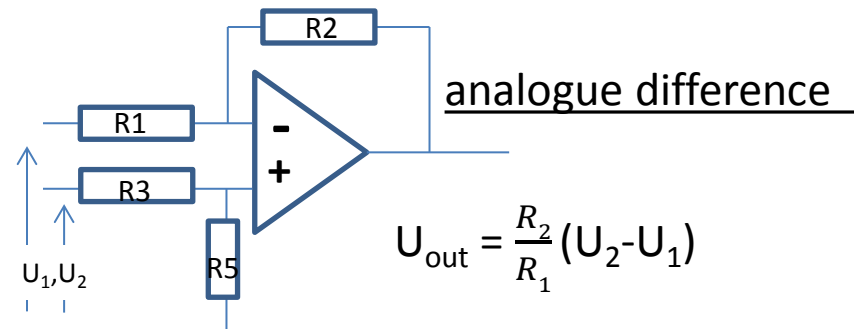
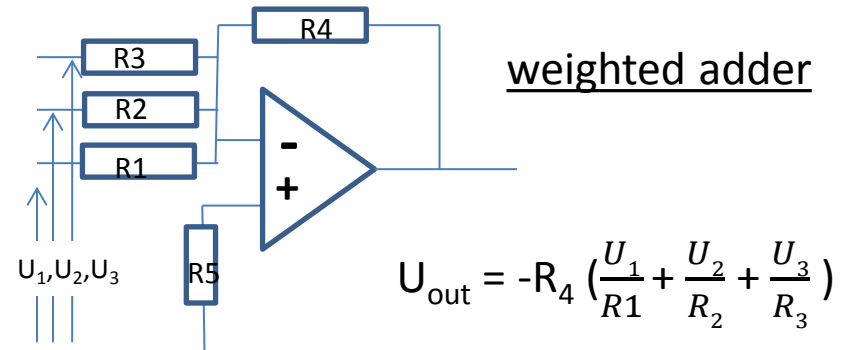
- The **weighted adder** allows adding high ohmic voltages with a weight given by the resistors R1, R2, R4. The symmetry condition for R5 is

$$1/R5 = 1/R1 + 1/R2 + 1/R3 + 1/R4$$

- The **analogue difference circuit** may be used to measure differential signals over a symmetrical cable. Coherent **pickup noise of the cable** is eliminated by the difference measurement.

- An example is readout of a **wheatstone bridge** with a desired gain R2/R1. The symmetry condition is

$$1/R1 + 1/R2 = 1/R3 + 1/R4$$



Frequency dependent feedback

- The **voltage differentiator** circuit provides a more precise differentiation compared to an RC highpass.

The frequency is limited by f_c

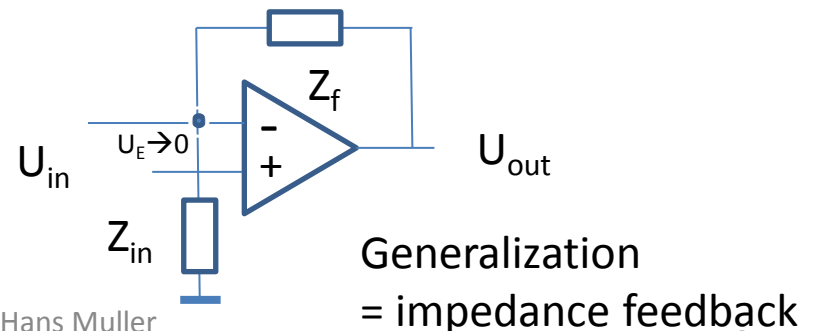
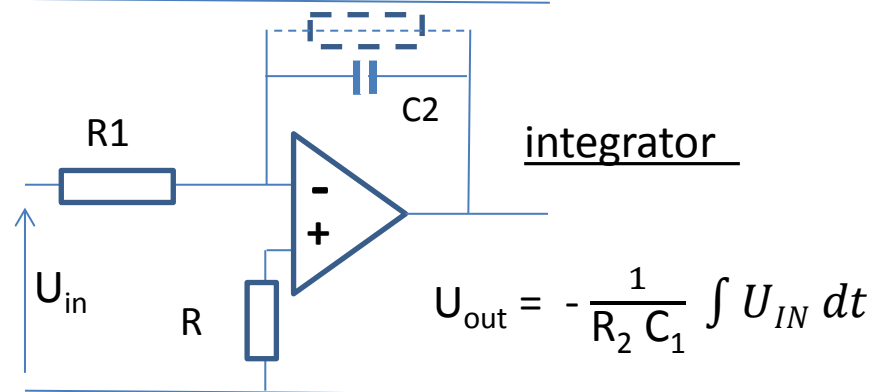
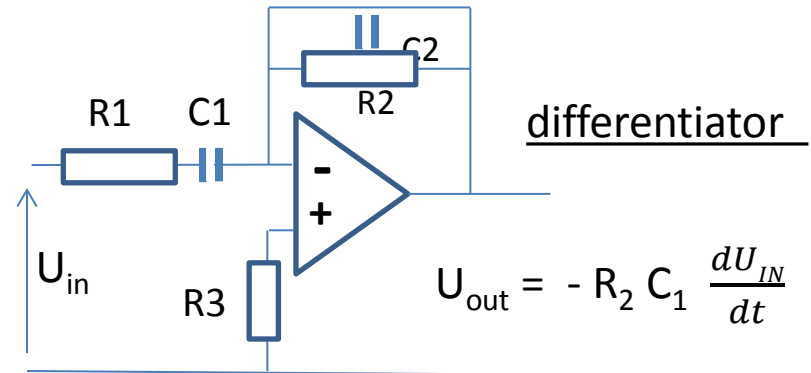
$$f_c = \frac{1}{2R_1 C_1} = \frac{1}{2R_2 C_2}$$

- The **voltage integrator** circuit is useful for sensor electronics where signals need to be integrated in time. In order to discharge the integrator a high ohmic resistor is placed in parallel to the integrating capacitor C_2 .
- In general with **Impedance feedback** and putting

$$U_E \rightarrow 0$$

$$U_{out}/U_{in} = \text{effective gain}$$

$$\beta = 1/k = \frac{Z_f + Z_{in}}{Z_{in}}$$



Logarithmic current amplifier

- The diode feedback shown provides a logarithmic current to voltage conversion. For OPA's with very low leakage current the input current I_{in} is almost the same as the diode current I_D . With the diode equation

$$I_{IN} \sim I_D = I_0 \left[\exp\left(\frac{e \Delta U}{n k T}\right) - 1 \right]$$

the diode current is an exponential function of the diode voltage ΔU which is equal to the output voltage of the amplifier over its virtual ground.

- In forward direction and very low values of I_0

$$I_{IN} \sim I_0 \exp\left(\frac{e \Delta U}{n k T}\right) \quad \text{with} \quad \frac{e}{n k T} = 38.85 \quad \text{for } n=1 \text{ at } 25^\circ\text{C}$$

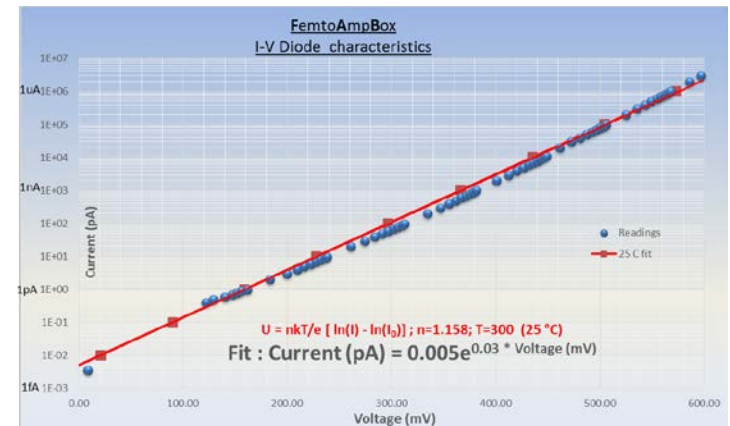
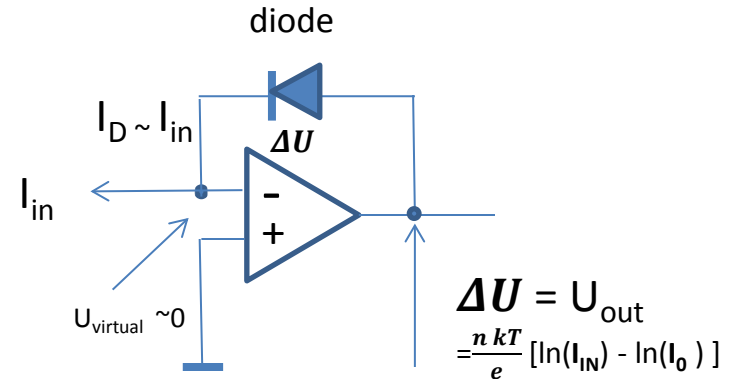
- Taking the natural logarithm of the diode equation

$$\ln(I_{IN}) = \ln(I_0) + \frac{e \Delta U}{n k T}$$

follows:

$$\Delta U = \frac{n k T}{e} [\ln(I_{IN}) - \ln(I_0)]$$

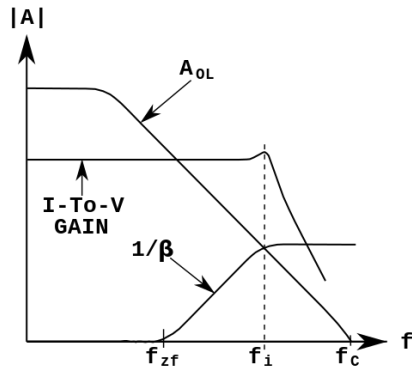
- Diodes with very low leakage current I_0 down to $O(10 \text{ fA})$ allow measuring ultra-weak currents at the output of the OPA as voltage signals of $O(0.1 \text{ V})$ hence very accessible levels. It turns out that JFETs, operated in the normally forbidden forward Gate-Source bias mode are diodes with very low leakage current.



Diode amplifier at 25 °C, Diode with $n=1.158$, $I_0 = 5 \text{ fA}$

Transimpedance amplifier (TIA)

- TIA's convert current to voltage at high gain and are therefore important for conversion of signals from current sources to a voltage signal.
- Typical current sources are detectors like photodiodes, pin diodes, PMs, SiPMs, GM's etc

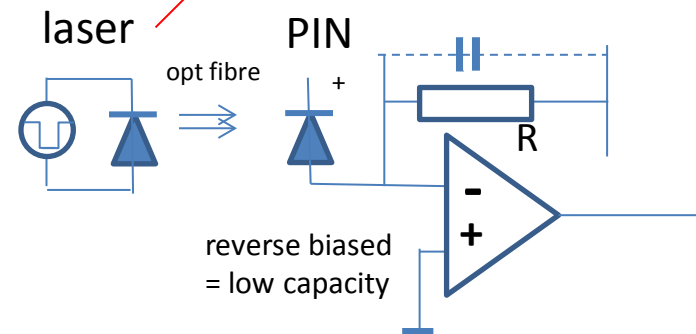
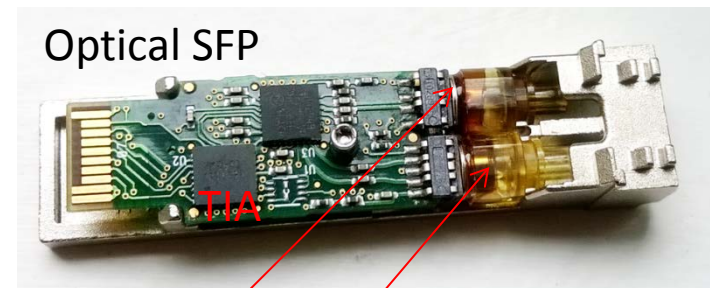
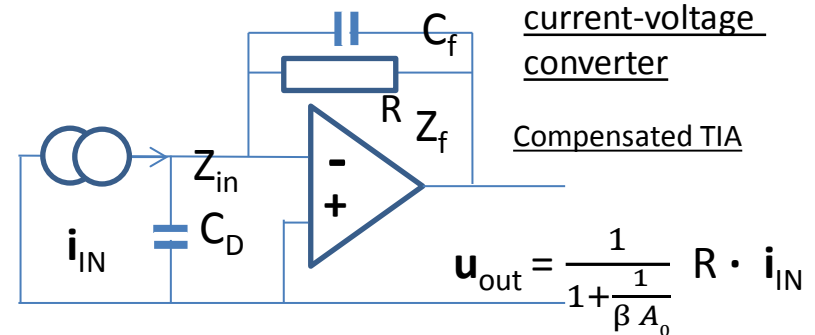


- With increasing frequency the feedback factor $\beta = \frac{Z_{in}}{Z_f + Z_{in}}$ is increasing whilst the open loop gain is decreasing

- The frequency behaviour of TIA's at high frequency needs to add the detector capacity C_D and a **compensation feedback** capacitance C_f in order to suppress ringing and to keep the I-to-V gain constant up to a high frequency point f_i . The optimum compensation capacitor is calculated by more in-depth analysis as or should be tuned for best response

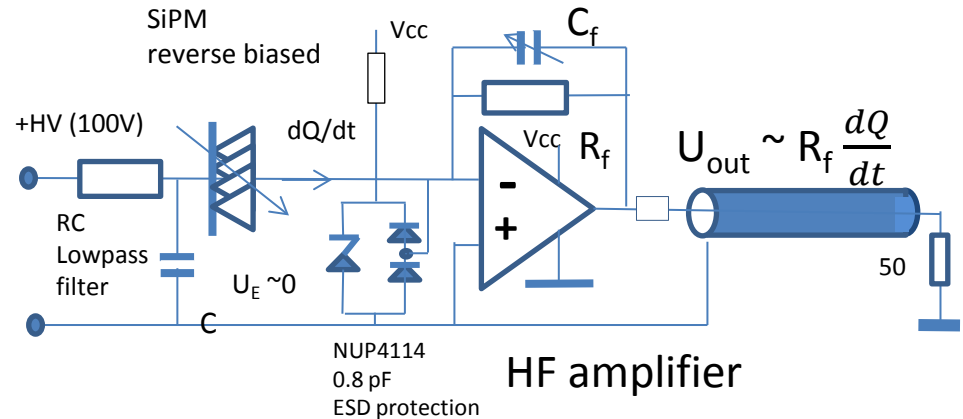
$$C_f = \sqrt{\frac{C_D}{BW R f}} \quad \text{where BW the bandwidth}$$

- http://en.wikipedia.org/wiki/Transimpedance_amplifier



TIA amplifier for SiPM

- SiPMs operate in GM mode and generate very fast charge avalanches with electron multiplication $M \sim 10^4 \dots 10^5$
- The (+) reverse bias voltage can be connected on the SiPM via an efficient RC noise filter which needs to have a fast capacitor to the amplifier ground at the cathode.
- The detector capacitance is in series with C, hence the SiPM charge is divided over C hence C should be a capacitor with low inductance and low capacitance.
- The SiPM is virtually grounded via the input of the OPAMP.
- The charge flow dQ/dt is converted by the TIA into a voltage pulse which can get transmitted via 50 OHM cable.



- The gain is proportional to the feedback resistor R_f
- The feedback capacitor C_f should be tuned to suppress oscillations at to increase the gain at high frequencies
- An ESD protection diode is recommended to protect the virtual ground input of the operational amplifier

Low noise electronics

- The **Signal to Noise Ratio** SNR is defined as

$$\frac{S(f)}{N(f)} = \frac{\text{RMS signal voltage}}{\text{RMS noise voltage}}$$

- Noise sources add up **under the root** as follows:

$$U_{\text{rms}} = \sqrt{U_{\text{rms1}}^2 + U_{\text{rms2}}^2 + \dots + U_{\text{rmsn}}^2}$$

- Noise units are **spectral densities** dP/df given in rms volt or ampere per “root Hertz” $\text{V}/\sqrt{\text{Hz}}$ or $\text{A}/\sqrt{\text{Hz}}$. These need to be related with a **frequency range** Δf in order to obtain the measurable noise level.

- The **rms of the shot noise current** is

$$I_{\text{sh}} = \sqrt{2qId}$$

I_d is the bias current, k_s the Boltzmann constant, T_a the absolute temperature, q the electron charge

- The **rms of the thermal noise voltage** on a series resistor R_s is

$$U_{\text{th}} = \sqrt{4k_s T_a R_s}$$

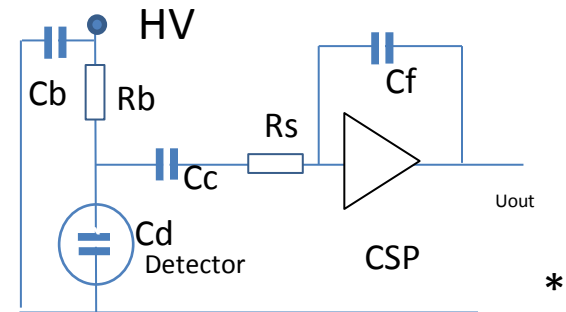
or rms of the current of a parallel resistor R_b

$$I_{\text{th}} = \sqrt{\frac{4k_s T_a}{R_b}}$$

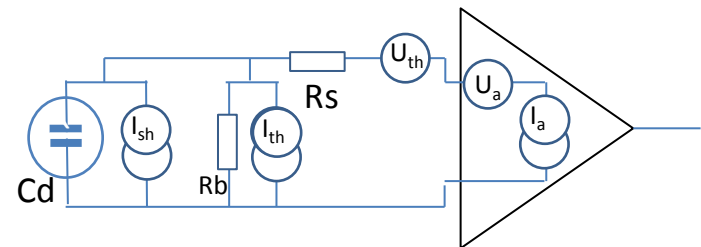
- The **rms of the flicker noise voltage** is

$$U_n = A_f / f$$

where A_f is a noise coefficient of the order $10^{-10} \dots 10^{-12} \text{V}^2$



- A typical detector frontend is depicted above: the detector itself has a capacitance C_d , Detector Bias Voltage is applied through R_b , the bias Voltage is AC grounded through C_b , the coupling capacitor to the preamplifier is C_c and any parasitic resistance in the input of the CSP is named R_s .
- The equivalent circuit for noise analysis includes current and voltage noise sources. Leakage current of a reverse biased detector diode produces shot noise current I_{sh} parallel to the detector. The resistors produce thermal noise, represented as either parallel current or serial voltage. The amplifier itself can be described with serial noise voltage U_a and parallel noise current I_a



* Review of particle Physics, Chapt 24, H.Spieler, Particle detectors

Shapers

- A **shaper** is in the simplest case a **combined RC and CR circuit** with equal time constants τ which corresponds to a center frequency f_c with the **shaping time constant** $\tau_0 = 1/(2\pi f_c)$.
- Detectors are typically measuring charge, therefore the **equivalent noise charge ENC** for the simplified case of a MOS transistor input, no leakage current and no series resistor component one can derive*

$$ENC^2 = \frac{4K \cdot T}{q^2 \cdot R_b} \cdot F_p \cdot \tau + \frac{4K \cdot T}{q^2} \cdot \frac{2}{3 \cdot g_m} \cdot F_s \cdot \frac{C_d^2}{\tau} + C_d^2 \cdot const$$

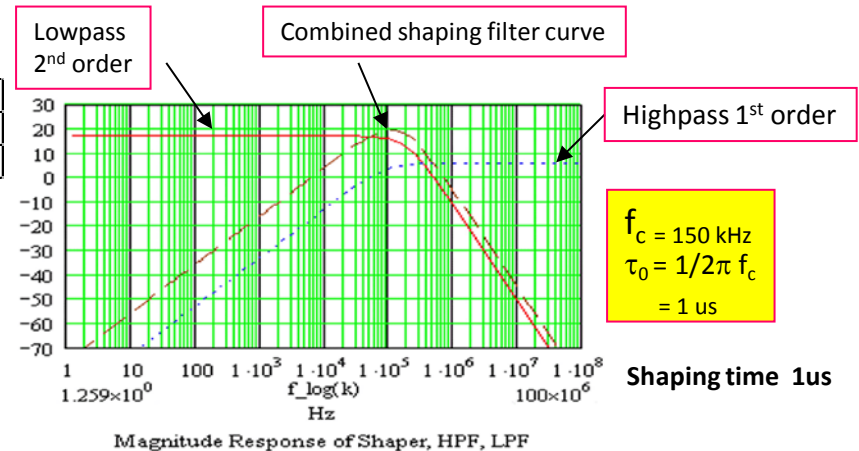
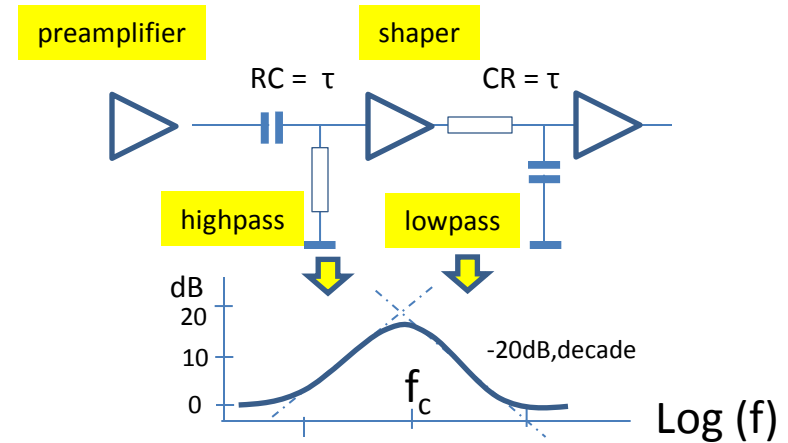
Where τ is the equal time constant of the shaper that follows the preamplifier. g_m is the gain of a MOS transistor $g_m = \Delta I_{DS} / \Delta V_{GS}$. The noise figures are for RC constants of order n are:

n	1	2	3	4
Fs	0.92	0.84	0.95	0.99
Fp	0.92	0.63	0.51	0.45

Low and high passes of first order n=1 have a bode transfer curve of + - 20 dB per decade. **Low and high passes of order n have transfer curves of n · 20 dB / decade.**

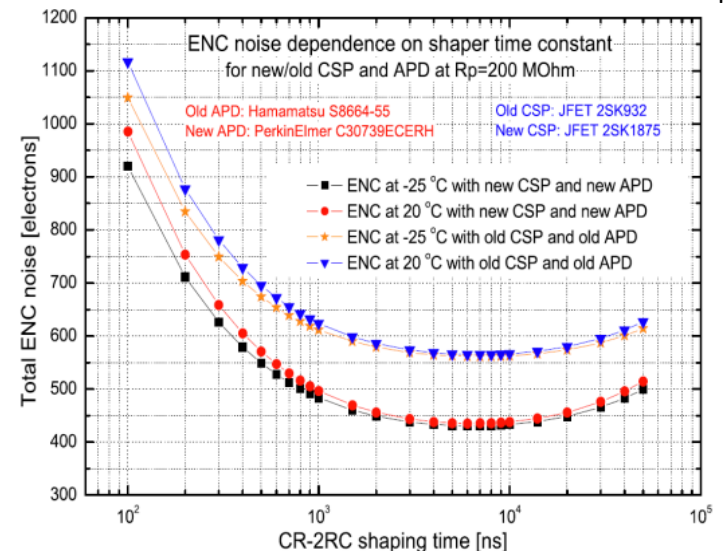
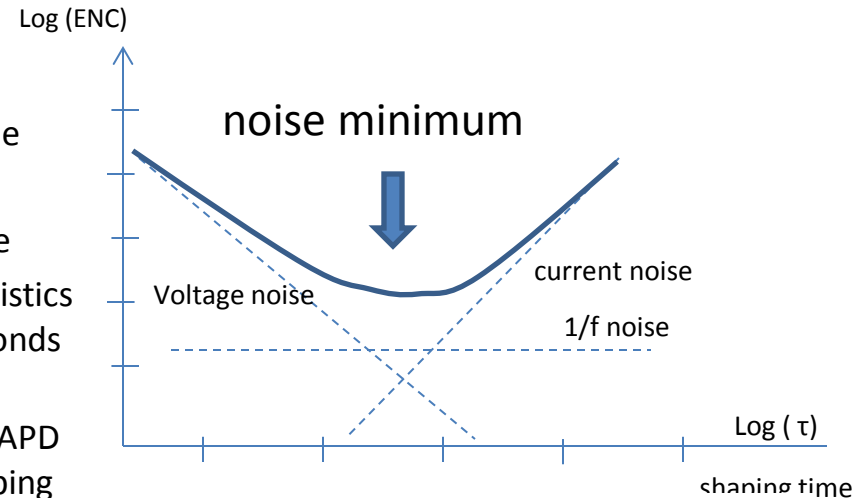
- The bode plot shows a **RC-CR2 shaper** with 1st order highpass-followed by a 2nd order lowpass. The shaping time τ_0 is 1 μ s.

* F. Anghinolfi, CERN Jan 2002 , "Analog signal processing"

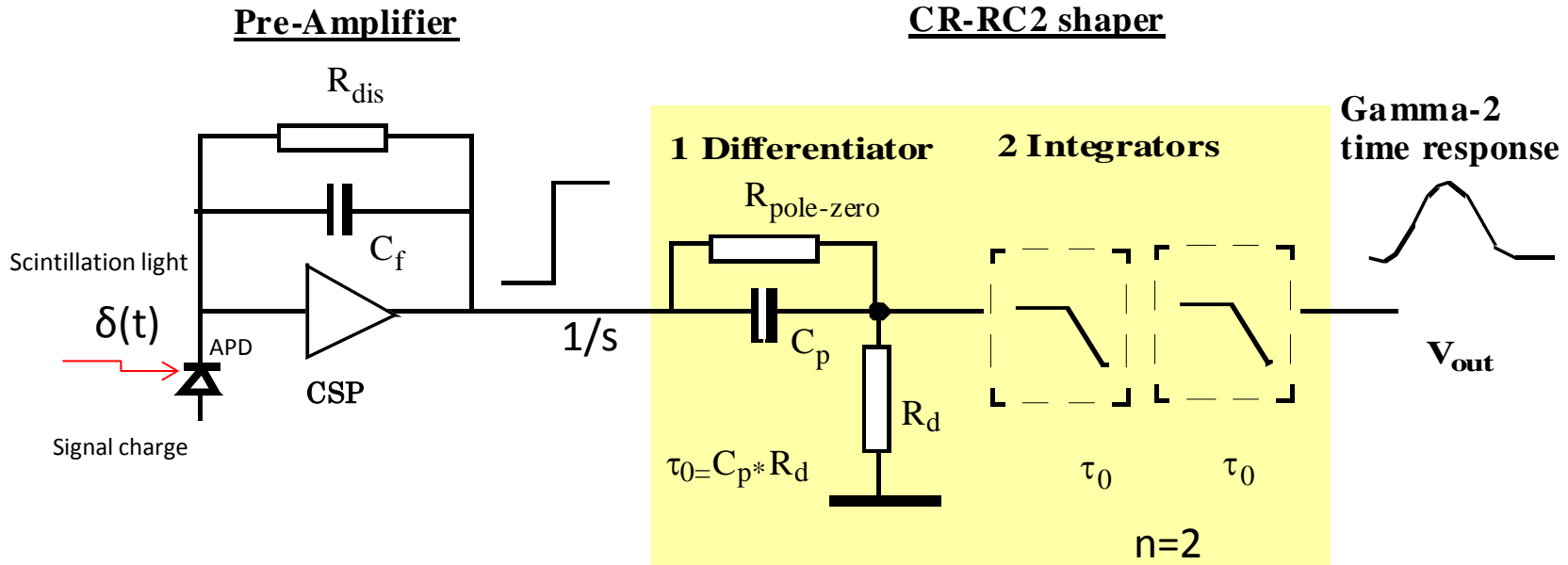


Noise minimization

- The formula for ENC has 3 components:
 - voltage noise that decreases with shaping time
 - $1/f$ noise independent of shaping time
 - current noise that increases with shaping time
- With a given detector capacity C_D and gain characteristics of the preamplifier input, a **noise minimum** corresponds to a specific shaping time.
- The graph shows the real case of shaping time for an APD detector frontend of the ALICE EMCAL detector, a shaping time of 1 μ s promises an ENC noise minimum of ca 500 electrons.
- The choice of the shaping time is a tradeoff between lowest noise and readout rate requirements. A large shaping time for detectors with high capacitance achieves lower noise however the corresponding large shaping time generates a long peaking time of the output signal.



Full readout chain



Laplace analysis of the whole chain:
decomposed in several operators

In general: the peaking time corresponds to order n of the low pass

Pole zero cancellation

$$H_{shaper}(s) = \frac{1}{s} \left[\frac{s\tau_0}{1+s\tau_0} \right] \times \left[\frac{q/C_f}{1+1/(R_{dis} \cdot C_f)} \right] \left[\frac{A}{1+s\tau_0} \right]^n$$

step function differentiator CSP discharge pole-zero RC integrator n-th order

$n=2$

➔

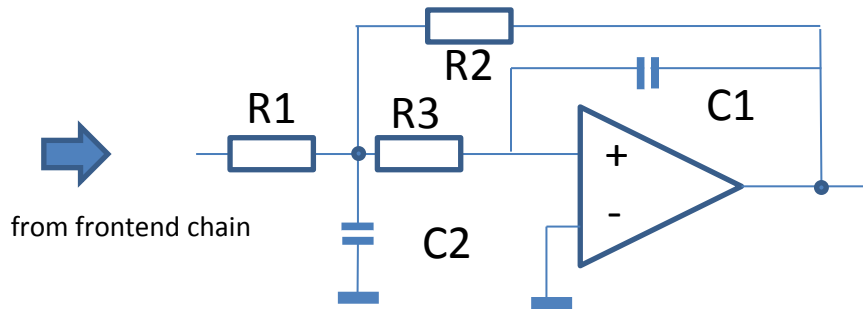
Time domain solution:

$$V_{out}(t) = \left[\frac{4Q \cdot A^2}{C_f} \right] \cdot \left[\frac{t-t_0}{\tau} \right]^2 \cdot e^{-2 \frac{t-t_0}{\tau}}$$

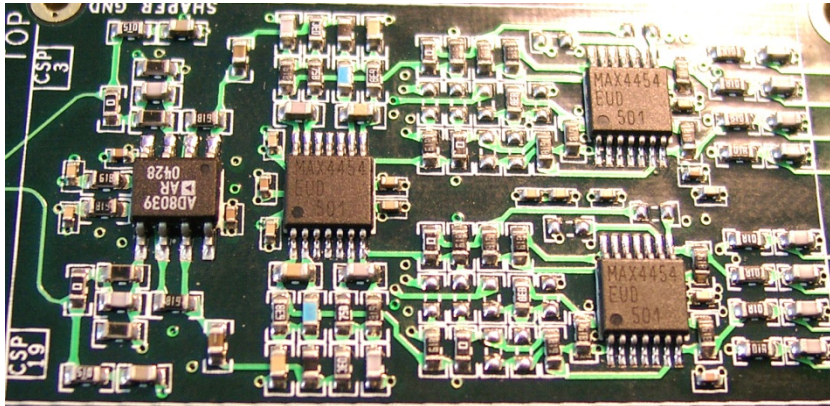
Gamma-2 (t) function

Shaper implementation and test

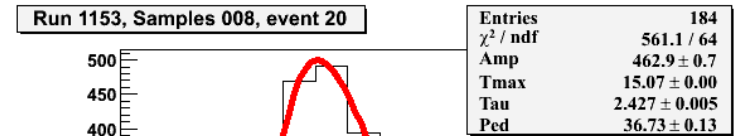
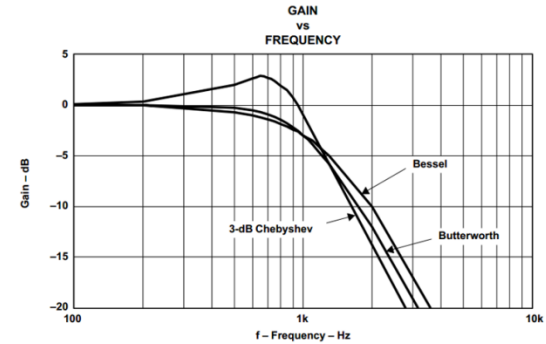
“Sallen-key” or “MFB” 2nd order lowpass filter *



R and C can be calculated* for user-defined cutoff frequency (shaping time), filter gain and Bessel filter characteristics



2nd order shaper implementation with low noise OPA's



Offline data
Fit with Gamma-2

$$V_{\text{out}}(t) = \left[\frac{4Q \cdot A^2}{C_f} \right] \cdot \left[\frac{t - t_0}{\tau} \right]^2 \cdot e^{-2 \frac{t - t_0}{\tau}}$$

Notice: a shaping time of 1 us with 2nd order shaper results in 2 us peaking time t_p , in general with filter order n $t_p = n \cdot \tau_0$

DC downstep conversion

- Duty cycle D is the ratio of the time when a periodic switch is "on" over the time period T

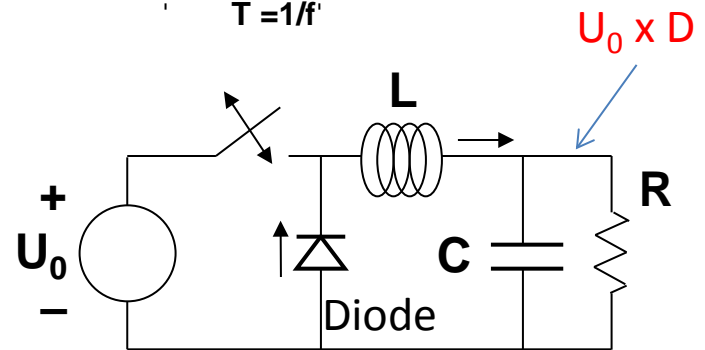
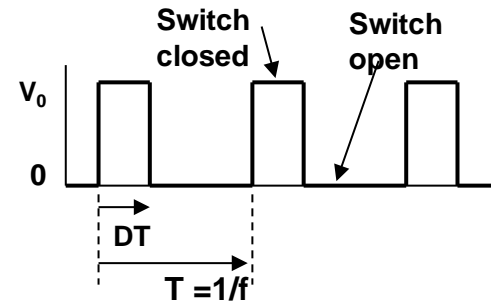
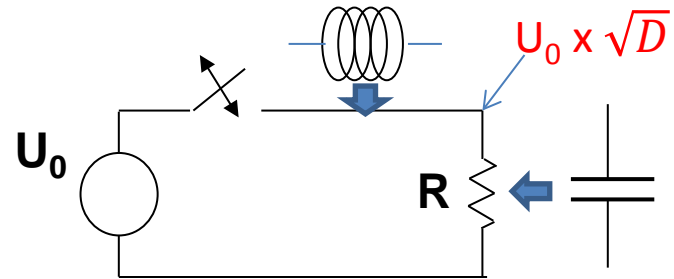
$$D = \frac{DT}{T} \leq 1$$

- A switch that opens and closes a DC voltage U_0 over a resistance R , generates an pulsating voltage over R with an RMS average of

$$U_{\text{rms}}^2 = \frac{1}{T} \int_0^{DT} U^2 dt = \frac{U^2}{T} DT = D \cdot U^2$$

$$U_{\text{rms}} = U_0 \sqrt{D} < U_0$$

- Adding a capacitor C parallel to R smoothes the ripple voltage over R . But there are big current spikes di/dt when the switch closes.
- Inserting an inductance in series with the load resistor will inhibit the current spikes but when the switch opens it will create large voltage spikes - dU/dt
- By adding a diode, when the switch opens, the current through L continues to flow and there is no large voltage spike. With sufficiently large frequency f and a large C , the ripple becomes small.



DC-DC Buck Converter

DC-DC buck converter

- A buck converter transfers “bucks” of electric charge from a Voltage source U_0 to an output capacitor for a different output voltage U_{out} . With a given duty cycle ($D = DT/T < 1$) of the buck switching, the output voltage on the capacitor C is lower than the input voltage. With ideal switches and ideal components this DC-DC voltage transformation is loss-free, realistically up to 90% efficiency can be achieved.

- Explanation top: when the switch SW is closed the Diode is reverse biased (off) and for the closed switch time fraction of time DT:

$$U_L = U_0 - U_{out}$$

- Explanation bottom: When the switch is opened, the diode is forward biased, hence like a closed switch, and during the fraction of time (1-DT) :

$$U_L = -U_{out}$$

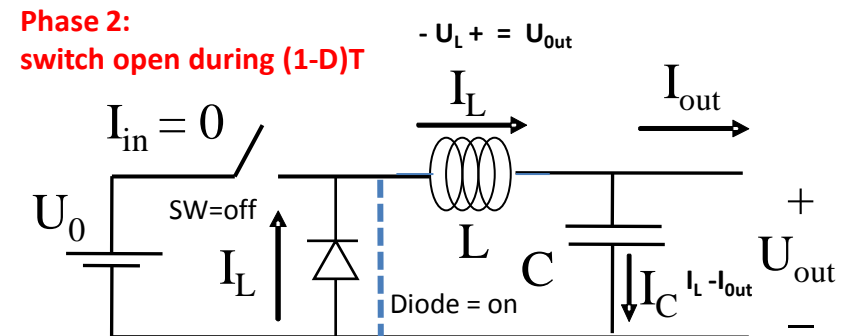
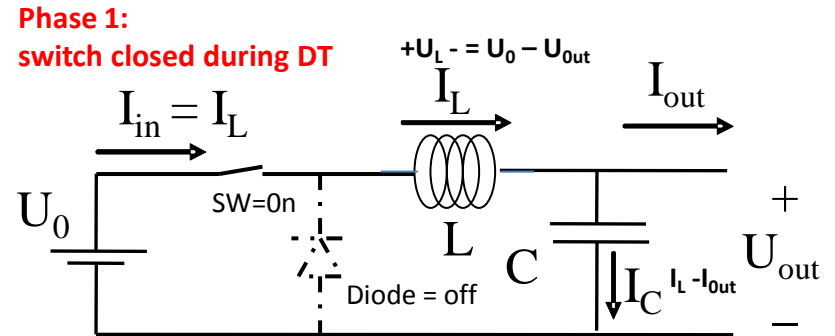
- Since the periodic average Voltage over L or C is zero and with $D = \frac{DT}{T}$

$$1/T \int_0^T U_L dt = 0$$

$$\rightarrow D \cdot [U_0 - U_{out}] - (1-D) \cdot U_{out}$$

$$\rightarrow U_{rms,out} = D \cdot U_0$$

- For a buck converter as shown, the output voltage is the input voltage multiplied by the duty cycle D**



Since Energy between input and output is conserved

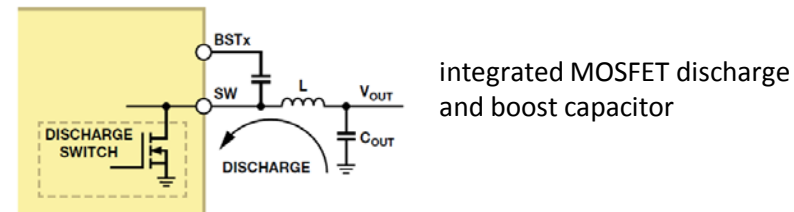
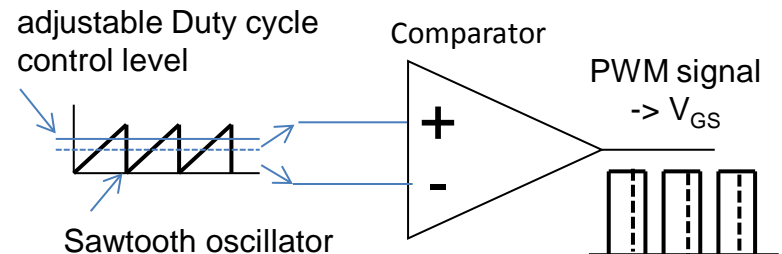
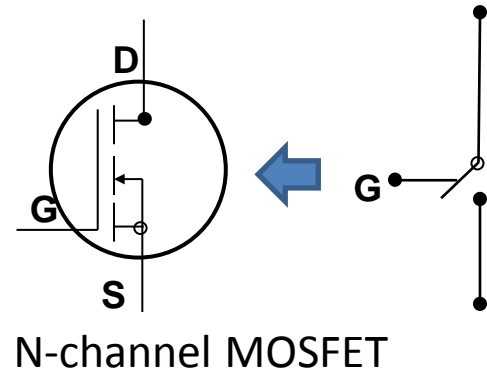
$$P = U_0 \cdot I_{in} = U_{out} I_{out} = \text{const}$$

$$\rightarrow I_{rms,out} = I_{in} / D$$

The output current is higher than the input current by inverse of the duty cycle

Practical Buck converters

- In practical Buck converters the switches are replaced by power MOSFETs as high-speed, voltage-controlled switches that operate above the 20kHz audible range.
- The MOSFET requires a steering pulse of $V_{GS} > V_{th}$ to be driven into saturation mode (like a closed switch). The intermediate, linear mode is undesired and lossy. The transit time from on – off must be small compared to the switching period T in order not to limit the switching frequency.
- For making the duty cycle variable, the steering voltage V_{GS} is generated from a fixed frequency saw-tooth oscillator followed by a Pulse Width Modulator (PWM), implemented as comparator with a variable analogue duty cycle control level.
- Integrated buck-boost converters contain the PWM and MOSFET circuitry for several programmable output voltage channels up to several Ampere at switching frequencies up to 1.5 MHz using air-coils, hence can be used even in magnetic fields where transformers fail.



Commercial buck converters*

PWM Mode*

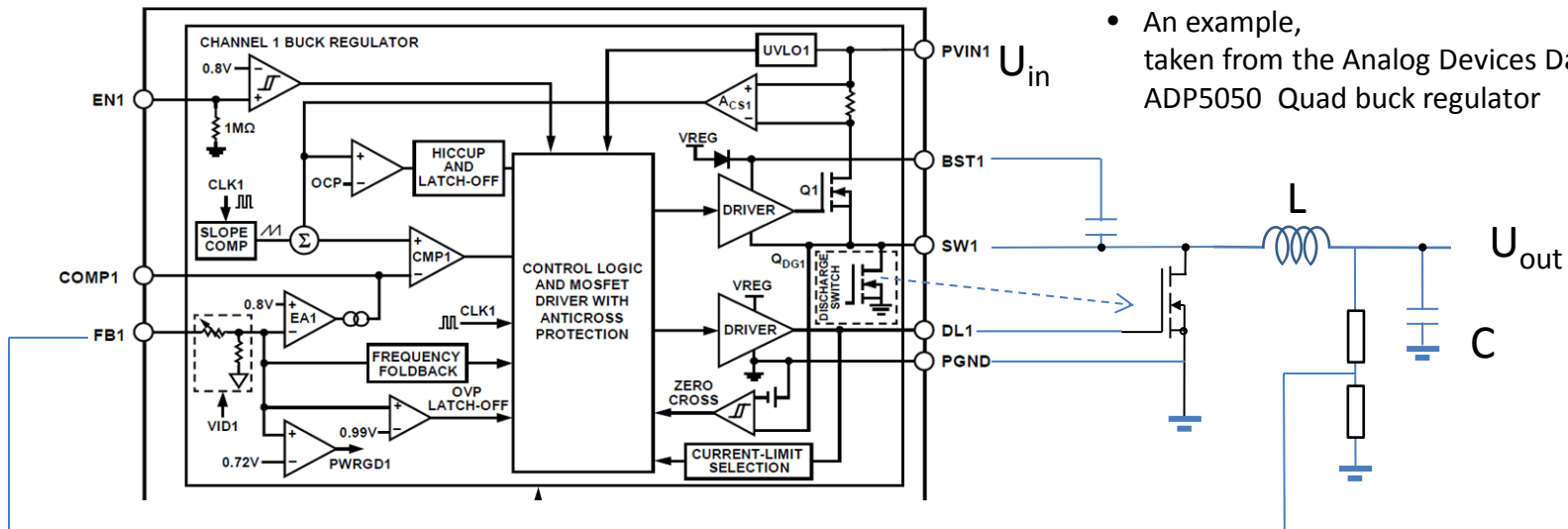
In pulse-width modulation (PWM) mode, the buck regulators in the ADP5050 operate at a fixed frequency; this frequency is set by an internal oscillator that is programmed by the RT pin. At the start of each oscillator cycle, the high-side MOSFET turns on and sends a positive voltage across the inductor. The inductor current increases until the current-sense signal exceeds the peak inductor current threshold that turns off the high-side MOSFET; this threshold is set by the error amplifier output.

During the high-side MOSFET off time, the inductor current decreases through the low-side MOSFET until the next oscillator clock pulse starts a new cycle. The buck regulators in the ADP5050 regulate the output voltage by adjusting the peak inductor current threshold.

PSM Mode*

To achieve higher efficiency, the buck regulators in the ADP5050 smoothly transition to variable frequency power save mode (PSM) operation when the output load falls below the PSM current threshold. When the output voltage falls below regulation, the buck regulator enters PWM mode for a few oscillator cycles until the voltage increases to within regulation. During the idle time between bursts, the MOSFET turns off, and the output capacitor supplies all the output current.

The PSM comparator monitors the internal compensation node, which represents the peak inductor current information. The average PSM current threshold depends on the input voltage (V_{IN}), the output voltage (V_{OUT}), the inductor, and the output capacitor. Because the output voltage occasionally falls below regulation and then recovers, the output voltage ripple in PSM operation is larger than the ripple in the forced PWM mode of operation under light load conditions.



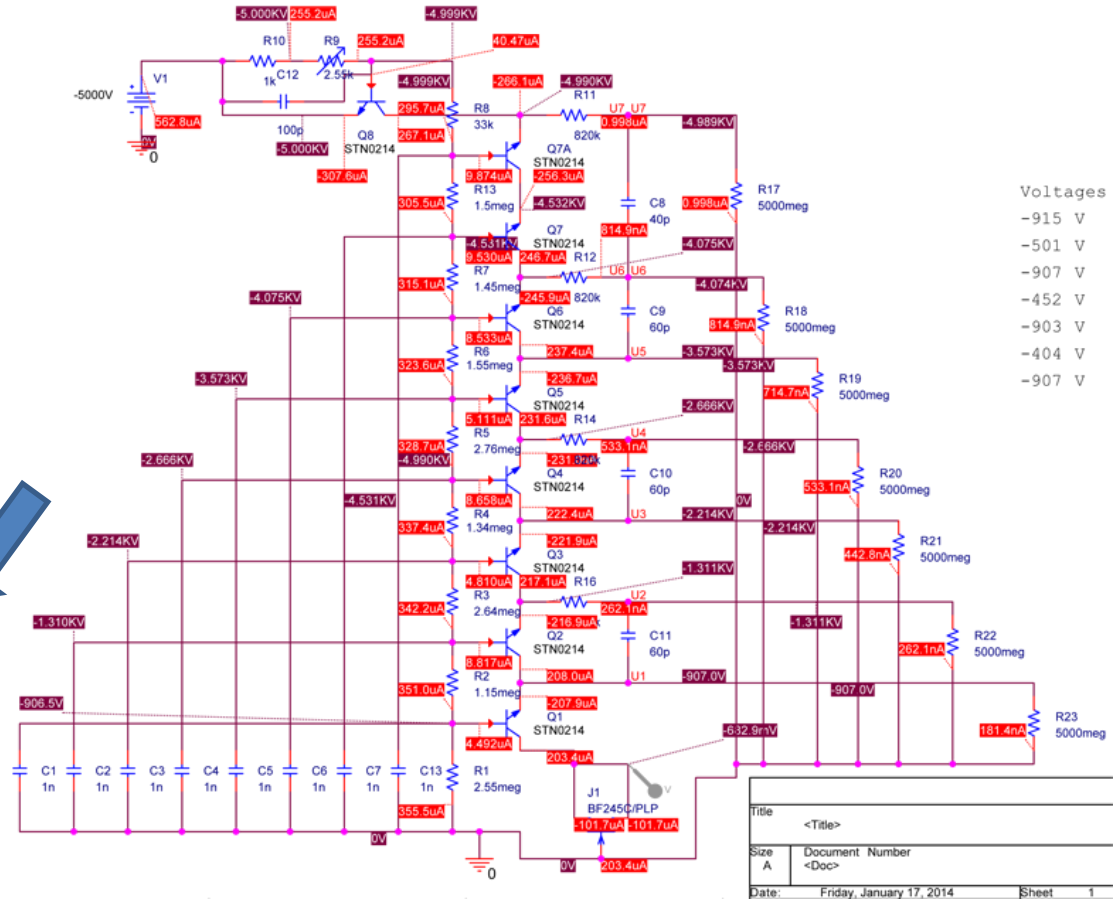
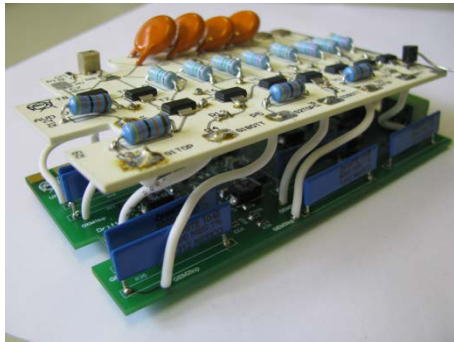
- An example, taken from the Analog Devices Data Sheet ADP5050 Quad buck regulator

Appendix

Design tools: PSPICE

- PSPICE simulation of AVD prototype Nr. 1:
- 5000V voltage down-divider for triple GEMs

AVD proto under test



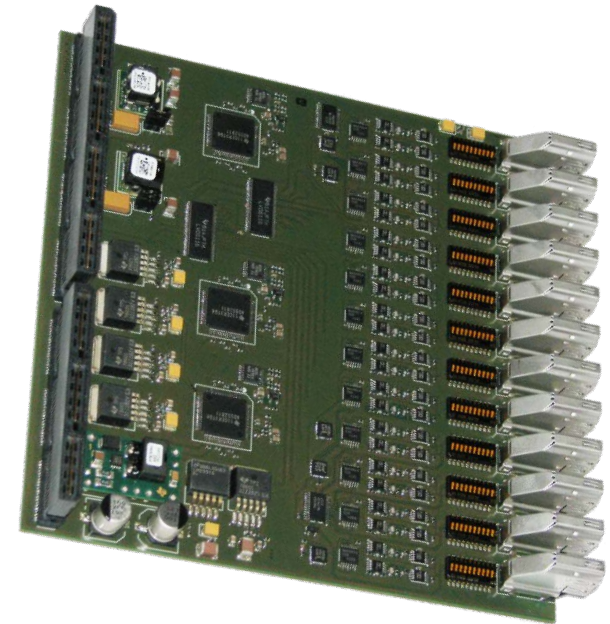
Design tools: Altium Designer

- Design view of the new ADC mezzanine for SRS ATCA

2013 Design view

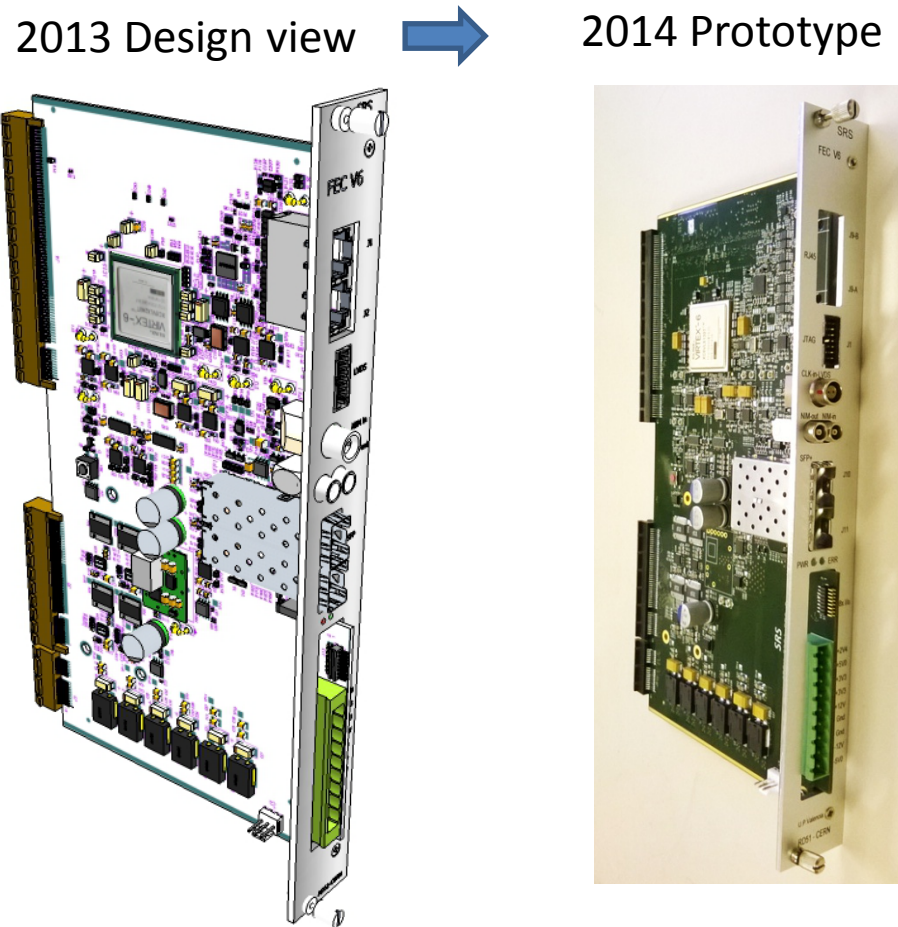


2014 product



Design tools: Sketchup 2013*

- 3-D design tool used for SRS electronics
- New FEC card V6



* free version available <http://www.sketchup.com/download/all>

Books and Tutorials

- *The Feynman lectures on physics* (3 ooks Vol 1,2,3) are on http://en.wikipedia.org/wiki/The_Feynman_Lectures_on_Physics
- Maxwells books: “A treatise on Electricity and Magnetism” VOL I and VOL II are available as e-book (pdf) on <https://archive.org>
- A students guide to Maxwell Equations, Daniel Fleisch (also available at the CERN reception Kiosk)

Electronics related equations and models:
<http://www.ece.uci.edu/docs/hspice/index.html>

Tutorials on the internet:

- <http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>
- <http://www-g.eng.cam.ac.uk/nms/lecturenotes/Slides1.pdf>
- <http://www.southalabama.edu/engineering/ece/faculty/akhan/Courses/EE334-Spring04/Lectures%20slides/ECE%20334-Lecture%2037-ECL.pdf>
- http://www-physics.lbl.gov/~spieler/SLAC_Lectures/PDF/Sem-Det-I.pdf
- http://www-physics.lbl.gov/~spieler/SLAC_Lectures/PDF/Sem-Det-II.pdf
- ELEC 2002 F. Anghinolfi in https://hr-training.web.cern.ch/hr-training/tech/elec2005_electronics_in_high.htm
- <http://ecee.colorado.edu/~bart/book/book/>

Micro Pattern Gas Detectors by Werner Riegler:

<https://indico.cern.ch/event/283113/session/0/contribution/1/material/slides/0.ppt>

Specialized books:

Grundgebiete der Elektrotechnik 1, 9th edition, H. Clauser / G Wiesemann => Books.Google.ch

Advances in High Voltage Engineering, A. Haddad, D.F. Warne,
http://books.google.ch/books?id=_lt13860YAwC&hl=de

Bob York 2009 “ *Frequency response and Bode plots*”

<http://download.ebooks6.com/Frequency-response-pdf.html>

Jerald Graeme , *Photodiode Amplifiers—Op Amp Solutions*, ISBN = 0-07-024237-X =>
www.amazon.com