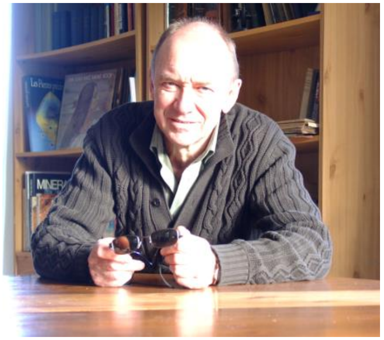
Detector Electronics
Part 1

Basics



Rev 080314

History of electronics (1)

as part of physics discoveries over the last 260 years

1707 -1783 Leonhard Euler

➔ considered as one of the most important mathematicians ever as inventor of mathematical analysis and the number theory. Mathematical symbols for natural base e, irrational number π and the symbol Σ for sum of series were introduced by Euler.

1750 Benjamin Franklin

➔ Proved that lightning is electricity. Labelling electricity Plus and Minus. First to discover the principle of conservation of charge

1781-1814 Simeon Denis Poisson

➔ mathematician with major contributions to physics: Poisson Distribution, Poisson Equation, Poisson Integral etc

1749 – 1824 Pierre Simeon Laplace

➔ Minister under Napoleon but as mathematician and physicist Inventor of Laplace equation and harmonic functions as solutions of this equation. More known for theLaplace operator and Laplace transforms, though these are based on Eulers mathematics and wre developed in detail by the hungarian mathematician Józeph Miksa Petzval

1785 Charles-Augustin de Coulomb

➔ determined that the magnitude of the electric force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them (Coulombs Law)

1800 Alessandro Volta

➔ First galvanic pile battery (zinc and copper in an sulfuric acid as electrolyte)

1820 Hans Christian Oersted

➔ Discovers that electric current influences a compass needle. Shows that electric current generates circular magnetic field lines around a current in a wire

1822 Joseph Fourier

➔ Invented harmonic Analysis known as Fourier transforms and Fourier Integrals in his « Théorie analytique de la chaleur »

1827 Andre-Marie Ampere

➔ Based on Oersteds discovers Ampere's law which relates electric current with magnetic field for static currents

1827 George Simon OHM

➔ OHM's first law in his Book "Die galvanische Kette, mathematisch bearbeitet" which describes theory of electricity : Electromotive force on the ends of a circuit is product of electric current and resistance

1831/32 Michael Faraday

➔ discovers induction of current from one current ring to another (first transformer). Discovers that ring wires rotating in a magnetic field generate a current (first dynamo generator)

1865 Karl Friedrich Gauss

➔ Mathematical genius who proved fundamental theorems and brought advances to geometrical geometry . He formulated the Gauss law which is equivalent with one of the Maxwells equations

1864 James Clark Maxwell

➔ unified theory of magnetism and electricity. 4 Maxwell equations become foundation of Classical Electrodynamics. Electromagnetism was born but still implied an "ether" which transmits the forces in form of waves

1888 Heinrich Hertz

➔ Confirms Maxwells theory that forces are transmitted by waves and extends the concept to radio waves of any wavelength. First radio transmitter (30 m). 1925 Nobel price physics with James Franck for discovery of the laws governing the impact of an electron upon an atom

1892 -1895 Hendrik Antoon Lorentz

➔ Explanation of X-rays as vibrations of electrons in a hypothetical ether. Developed Lorentz transformations with local time variable which laid foundations for Einstein's Relativity theory. Improved description of Maxwells electrodynamics Lorentz Force law (right hand rule) describes the magnetic force on a current-carrying wire. 1902 Nobel Price physics with Pieter Zeeman

1885-1900 Guglielmo Marconi

➔ Refined Hertz radio transmitter to build the first telegraph with Morse code transmission over increasing distances, in 1900 first transatlantic wireless transmission . 1909 Nobel Price Physics with Karl Ferdinand Braun for development of wireless telegraphy

1896 Wilhelm Roentgen

➔ Discovery of X –rays generated by 2 plates (anode +, cathode -) in a vaccum tube and with electric field applied.. The X-rays transit though matter and allow Roentgen to photograph the bones of the hand of his wife. The discovery has very big impact in applications in medicine. 1st Nobel price in Physics in 1901

1882/1897 Joseph John Thomson

➔ Model of Atom with Electrons, consisting of a sphere containing positive charges and negative electrons. Proves that cathode rays consist of electrons that are much lighter than the atom. Birth of particle physics: electron is 1st Subatomic particle. 1906 Nobel Price Physics for conduction of electricity in gases

History of electronics (2)
as part of physics discoveries over the last 250 years

1893 Arthur E. Kennelly
➔ Paper on Impedance (the measure of the opposition that a circuit presents to a current when a voltage is applied) with formulation based on complex numbers

1898 Karl Ferdinand Brown
➔ Invention of crystal diode rectifier (cats whisker detector) as thin wire that touches a semiconducting material. Nobel price 1909 with Guglielmo Marconi. Inventor of the Brown tube, a predecessor of the Cathode Ray Tube

1897-1901 John Sealy Townsend
➔ Discovered and described avalanche multiplication caused by ionisation of gas molecules, basis of all gas detectors

1903 Henri Becquerel, Pierre and Marie Curie
➔ Discovered spontaneous radioactivity of materials (Radium, Polonium, Uranium)
1903 Nobel Prize in Physics with Pierre Curie and his wife Marie Skłodowska-Curie
Extracted Polonium and Radium, a metal that emitted blue light!

1905 Albert Einstein
➔ 3 Publications , Photoeffect, Browns Movement and Relativity theory. The latter states that electromagnetic waves travel at speed of light in the vaccum (there is no ether as Maxwell thought). 1921 Nobel price physics for his services to theoretical physics in particular for explanation of the photoelectric effect

1911 Robert Millikan
➔ Measure of electric charge of an electron via oil drop deflection in a static electric field. 1923 Nobel Price physics for work on the elementary charge of electricity and on the photoelectric effect

1911 Ernest Rutherford
➔ Detects that uranium emits 2 types of radiation: α (helium atoms) and β (electrons). X-rays already identified as non-deflectable X-rays by Paul Villard were named third type of radiation γ (photons) by Rutherford. Exposed a target to beam of α particles to find a deflection pattern and backscattering, concluding on concept of atomic kernels which are much smaller than the full atom.

1913 Johannes Stark
➔ 1919 Nobel Prize for Physics for discovery of the Doppler effect (basis of Radar) in canal rays and the splitting of spectral lines in electric fields (Stark effect).

1911 Charles Thomson Rees Wilson
➔ Invention of the cloud chamber to make ionizing particles visible in the vapour.
1927 Nobel Price in physics for this discovery

1914 Max von Laue
➔ 1914 Nobel Price physics for detection of diffraction of X-rays by crystals like electromagnetic light waves. Interpretation that Roentgen's X-rays are electromagnetic waves (of shorter wavelenght)

1925 **Julius Edgar Lilienfeld**
➔ inventor of the FET transistor and patent holder of the electrolytic capacitor

1928 **Harry Nyquist**
➔ Important work on Johnson –Nyquist noise generated by agitation of charge carriers in a electrical conductor and independent if Voltage is applied. Originator of the Nyquist-Shannon Theorem

1931 Ernest O. Lawrence
➔ Development of the first cyclotron, a first 4-inch cyclotron generating 80 kV protons and laying the grounds for High Energy Physics. Accelerators. 1939 Nobel price in Physics for development of the cyclotron

1947 John Bardeen, Walter Brattain, William Shockley
➔ Inventors of the bipolar transistor , 1956 Nobel price in physics for researches on semiconductors and their discovery of the transistor effect

1957 Gordon Gould
➔ Invention of the Laser as optical amplification of light using optical resonators

1962 James R. Biard and Brian Pittman , Nick Holonyak Jr.
Biard and Pittman patent titled "Semiconductor Radiant Diode" the first infrared GaAs LED, Holonyak published the first visible light LED

1965 Richard Feynman
➔ Book "The Strange Theory of Quantum Electrodynamics" QED (1965 Nobel price physics with Julius Schwinger and Sin –Itiro Tomonaga)

1987 Karl Alexander Muller
➔first HTS High Temperature Supraconductor,1987 Nobel price for breakthrough dicovery of supraconductivity in ceramic materials

1992 Georges Charpak
➔ 1992 Nobel price in Physics for invention and development of particle detectors, in particular the multiwire proportional chamber" @ CERN

The fundamental lows

- **Euler's formula**

  $e^{i\emptyset} = \cos(\emptyset) + i \cdot \sin(\emptyset)$

- **<u>Coulombs law</u>** (electricity)

  $F_e = K \frac{q_1 q_2}{r^2}$

- **<u>Charge conservation</u>** [1]

  $\oiint j\, dS = -\frac{d}{dt}(Q)$

- **<u>Ampere's law</u>** [2]

  $\oint B\, dl = \mu I$

- **<u>Lorentz force law</u>** [3]

  $\mathbf{F} = q(\boldsymbol{E} + \boldsymbol{v}\, x\, \boldsymbol{B})$

- **<u>Laplace equation</u>** [4]

  $\Delta U = \boldsymbol{\nabla}^2 U = 0$

- **<u>Ohm's law</u>**

  $U = R \times I = R \times \frac{dQ}{dt}$

- **Poisson equation**

  $\boldsymbol{\nabla}^2 U = -\frac{\rho}{\epsilon_0}$

- **<u>Maxwell's equations</u>** [3,5] of classic electrodynamics

  (1) $\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$ (= Gauss law)

  (2) $\boldsymbol{\nabla} x \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$

  (3) $\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$

  (4) $c^2\, \boldsymbol{\nabla} \mathrm{x} \mathrm{B} = \frac{\partial \boldsymbol{E}}{\partial t} + \frac{\boldsymbol{j}}{\varepsilon_0}$

<u>Electrostatics:</u> $\frac{\partial \boldsymbol{E}}{\partial t} = 0$ and $\frac{\partial \boldsymbol{B}}{\partial t} = 0$

<u>Charge conservation:</u> follows[5] from (4)

$\boldsymbol{\nabla} \cdot \boldsymbol{j} = -\frac{\partial \boldsymbol{E}}{\partial t}$

1 surface integral $\oiint$

2 Line integral $\oint$

3 Operators "div" ($\boldsymbol{\nabla} \cdot \boldsymbol{B}$), "curl" ($\boldsymbol{\nabla}\mathrm{x}\mathbf{B}$) and $\boldsymbol{\nabla} \cdot \boldsymbol{\nabla}$ and are vector notations of partial differential equations

4 Laplace operator $\Delta = \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}$ invented by Laplace around 1800

5 The div of a curl is always zero : $\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\mathrm{x}\mathbf{B}) = 0$

Electrostatic potential

- Two insulated charges q_1 and q_2 of opposite polarity are attracted by the force F as described by Coulomb's law:

  $F = k_e\, q_1 q_2 / r^2$
- One can separate the Coulomb equation into a single charge that feels a force F generated by an electric field E from the other charge.

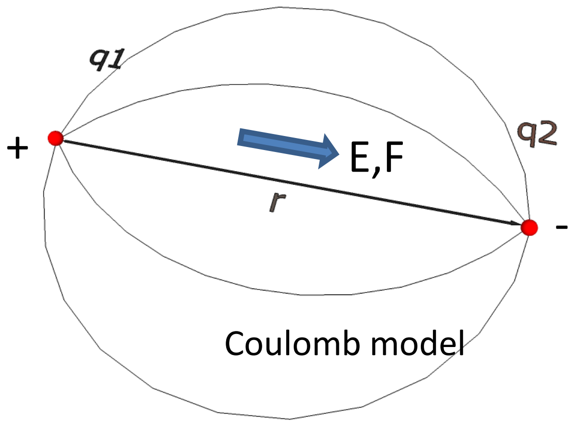
  $E = k_e\, q_1 / r^2$ with $F = q_2 E$
- The Force F is a vector that follows electric field lines stretching from one charge to the other. The work done[1] moving a unit charge from a to b along against the field E corresponds to energy $W = -\int_a^b E ds$ and this is equal to the difference of the electrostatic potential

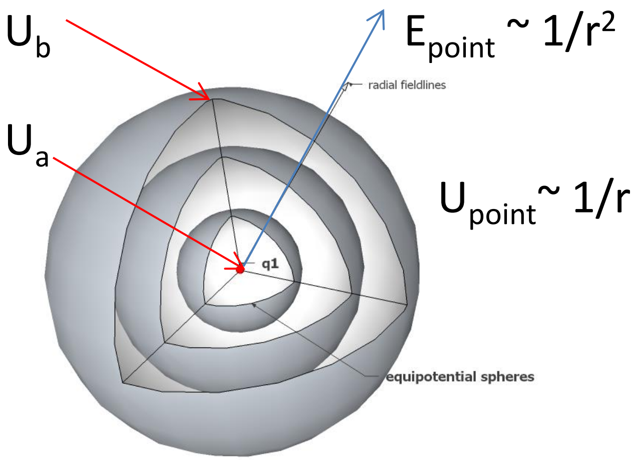
  $W = -\int_a^b E ds = U_b - U_a$
- The integral $\int E ds$ results in a 1/r dependence of the potential from the point charge.
- Taking the inverse operation, $E = -\frac{\partial U}{\partial x}$ or in general

  $E = -\boldsymbol{\nabla} U$

  the **electric field is fully defined by the electrostatic potential**
- In the radial field of a point charge the potential difference ΔU is constant equipotential spheres at distance r and the electric field strength is $E = (U_b - U_a)/L$ where L is the displacement E is measured in (Volt/m)
- **1 eV** is the energy of one electron, moved along a constant potential difference of $W = Ub - Ua = 1$Volt[1].

1 Note: This is analogue to the potential energy of a mass object that falls down in the gravitational field of the earth. The height L is analogue to the Voltage.





Radial field for a point charge in a field where q_2 is placed very far away

charges in conductors

- Generalization to N charges is based of the principle of superposition of vector forces.The potential U at any point in space U(1) is the sum of contributions from all other charges in the closed space volume (2):

  $$U(1) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(2) dV_2}{r_{12}}$$

  where ρ is the density of all charges in the volume V
- The vector sum of all electric fields E through a closed surface is called electric Flux Ø. Gauss law states that the flux though any closed surface is equal to the charge inside, divided by the dielectric constant ε_0

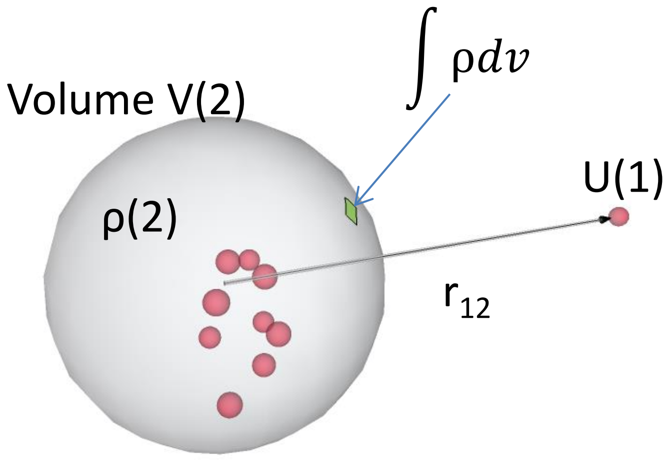
  $$\emptyset = \oiint E dA = \frac{Q}{\varepsilon_0}$$

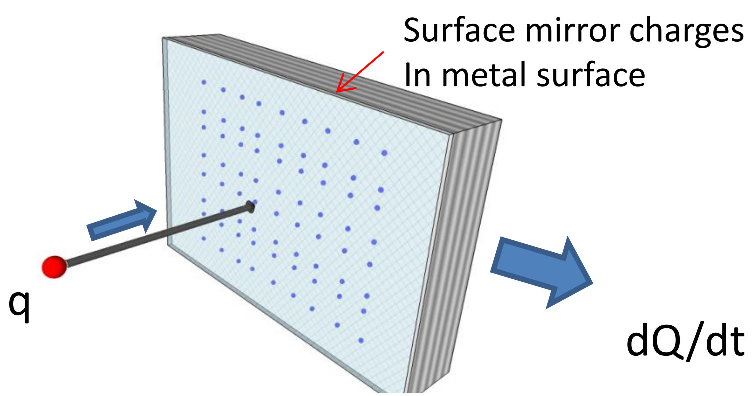
  where $Q = \sum_{all\ inside\ volume} qi$

  this means the sum of all electric fields of a closed surface is

  $$\emptyset = \frac{Q}{\varepsilon_0}$$
- Free electric charges inside a conductor are in an an equipotential plane i.e. feel no electric field. An external charge interacts with the charges at the surface and the field lines are perpendicular to it. With the Gaussian law, the sum of fields at the conductor surface is

  $$E = \frac{\sigma}{\varepsilon_0}$$ where σ is the surface charge density
- The field of external charges with conducting spheres can be described by mirror charges behind the sphere, though the real charge is only in the conductor surface.
- The movement of external charges is balanced by opposite movement of charges at the surface of the conductor which add up to a common current I=dQ/dt signal which can be measured with appropriate instrumentation.



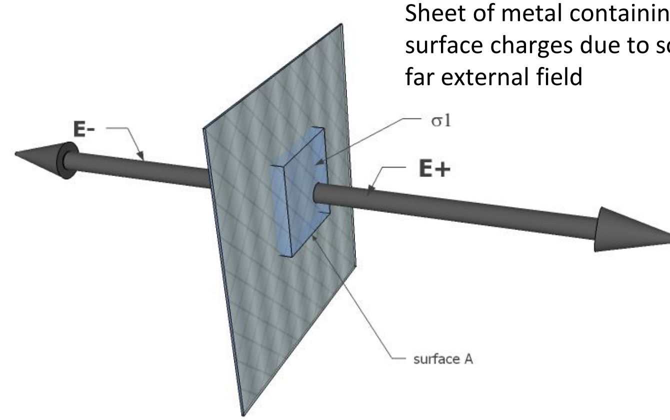


Parallel Plate Capacitor

- According the Gauss law, if we have a sheet of metal the closed surface integral over the electric field vector E is equal to the contained surface charges Q/A = σ

$$E= \oiint E dA = \frac{Q}{\varepsilon_0} = \frac{\sigma}{A\,\varepsilon_0}$$

Sheet of metal containing surface charges due to some far external field



- If we choose a rectangular box as the surface , only the top and bottom surfaces A contribute to the integral:

$$EA + EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0}$$

- Applied to parallel plates separated at distance d the vectors fields **E** add up such that they cancel outside and double inside:

$$\mathbf{E}_{inside} = \mathbf{E_1} + \mathbf{E}_2 = \frac{\sigma}{\varepsilon_0}$$

$$\mathbf{E}_{outside} = 0$$

- U = U1 – U2 is the potential energy required to move a single charge from one plate to the other against the constant field E

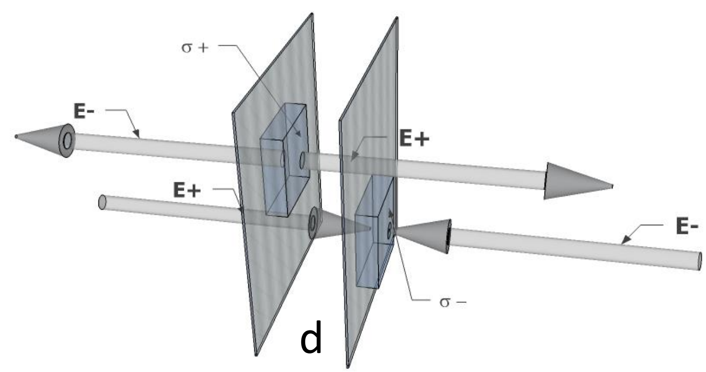
$$U = \int E ds = Ed = \frac{\sigma}{\varepsilon_0} d = \frac{d}{A\varepsilon_0} Q$$

By defining $C := \frac{\varepsilon_0}{d} A$ and naming this geometrical constant "Capacity" with unit 1Farad = 1F we get

U = Q/C

The **Voltage on a capacitor is proportional to its surface charge and inversely proportional to the capacity**.

Parallel plates:
E fields cancel outside and double inside



The capacity is proportional to the surface of the sheets and Inversely proportional to their distance. Big capacitors have a large surface and are separated at very small distance (oxide layers)

Some relevant fields

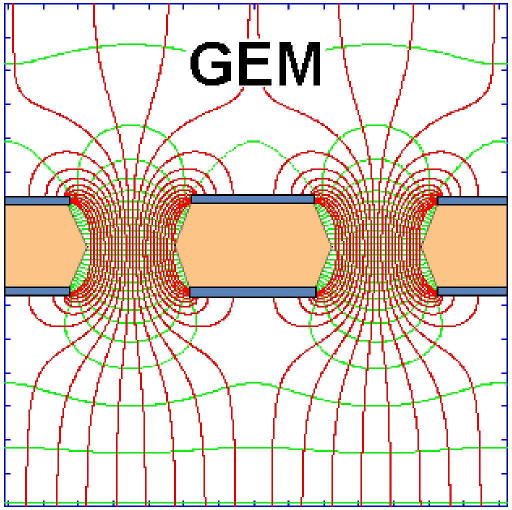
- Fields are defined as **derivatives of the electrostatic potential E=-∇U**. In general any **field is a solution to Laplace equation**

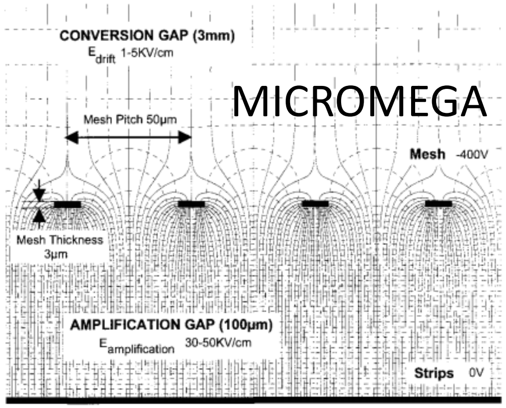
  $$\Delta U = \nabla^2 U = 0$$

  a vector notation of a partial differential equation of second order… we will not try to get into this !
- Solutions to the Laplace equation are normally based on boundary conditions given by some 3-dimensional equipotential sphere.
- We had a simple example of the r = const sphere for a point charge. The field solution for more complex equipotential metal spheres require numerical methods, like Garfield (Rob Veenhof , author and RD51 member !), Maxwell, etc
- Mathematics helps: any 3-dimensional potential function may be split into a two functions F (xy) which are combined as real and imaginary part of the 3-dimensional function like

  $$z^2 = x^2 + y^2 + 2xy \Rightarrow F(xy) = (x^2 + y^2) + i\,(2xy)$$

  The real and imaginary part fulfil the Laplace equation separately with 2 separate orthogonal field-line solutions. The solution for the above z^2 potential is a quadrupole field.
- For RD51, two fields are particularly relevant:
- a.) electrostatic field of **Gas Electron Multipliers** (GEM). A single electron is accelerated passing the high fields in the holes of a GEM foil. Usually triple stacks of several GEM foils make up for an electron amplification of $M = m_1 \cdot m_2 \cdot m_3$. With individual gains m_x of Order (10), the total gain of triple GEMs is in the order of 1000 … 2000.
- b.) electrostatic fields in a Micromega are 1.) drift gap and 2.) amplification gap. The amplification gap d is chosen such that the electron amplification $M = e^{ad}$ reaches its optimal value (a is the 1st Townsend coefficient). Gains between M= 4000 and 10000 are common.





Map of the electric field lines source [2]

More on capacitors

- A **parallel plate** of 1cm^2 and d=1 mm has a capacity of

  $C = \frac{\varepsilon_0}{d} A$

  $= (10^{-2}\text{m})^2 \text{ x } (10^{-3}\text{m})^{-1} \text{ x } 8.854 \text{ x } 10^{-12}\text{ F/m}$

  $= 8.85$ pF

  When charged to 1 Volt, a 1 pF capacitor contains

  Q= C U = 10^{-12} Coulomb = 6.25 Million electrons

- The work to transfer 1 charge element dQ from one plate to the other is dW = V dQ = Q/C dQ
- The total **stored energy** in the capacitor is:

  $$W = \int_0^Q \left(\frac{Q}{C}\right) dQ \ = \frac{\mathbf{1}}{\mathbf{2}} \frac{\boldsymbol{Q^2}}{\boldsymbol{C}}$$

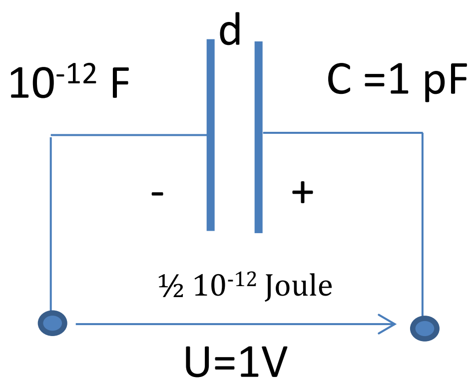
- the stored energy in above example (1 pF, 1V) is

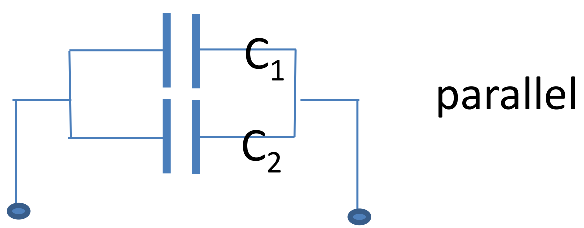
  $W_{1pF,1V} = ½ \, (10^{-12} \text{ C})^2 / \ 10^{-12}\text{ F}$

  $= ½ \, 10^{-12}$ Joule

- Connecting **capacitors in parallel** is like increasing the surface A and the capacity increases in proportion. The capacity of N parallel connected capacitors is the sum of each individual capacity. Technical capacitors of up to 1000 uF = 1 mF are based on electrodes separated by very thin gap (Oxide layers)
- Connecting 2 **capacitors in series** is like doubling the gap d. For two equal capacitors the result is ½ of their value.
- In general, series of capacitors are added as

  $\mathbf{C^{-1} = C_1^{\ -1} + C_2^{\ -1} + C_3^{\ -1}}$





C = C1 + C2

$$\frac{1}{C} = \frac{1}{C1} + \frac{1}{C2} + \frac{1}{C3}$$

Dielectric constant

- Farady observed an **increase** of capacitance by inserting an **insulator** in the empty gap of parallel plate capacitor The coefficient of the increased capacitance relative to the a vacuum gap is called the **dielectric constant** κ.
- From C= $\frac{\varepsilon_0}{d}$A for the parallel plate capacitor, ε_0 is the **dielectric constant of the vacuum** with κ =1.
- Inside **insulator materials**, the Atoms get slightly **polarized** by the electric field that is applied. This generates a net **surface charge** $\boldsymbol{\sigma_{pol}}$ on both sides of the insulator.
- The polarization charge per volume at the insulator surface is proportional to the number N of charges on the conductor surface. The volume is the insulator surface A multiplied by the thickness of the displacement δ . The charge/surface is then

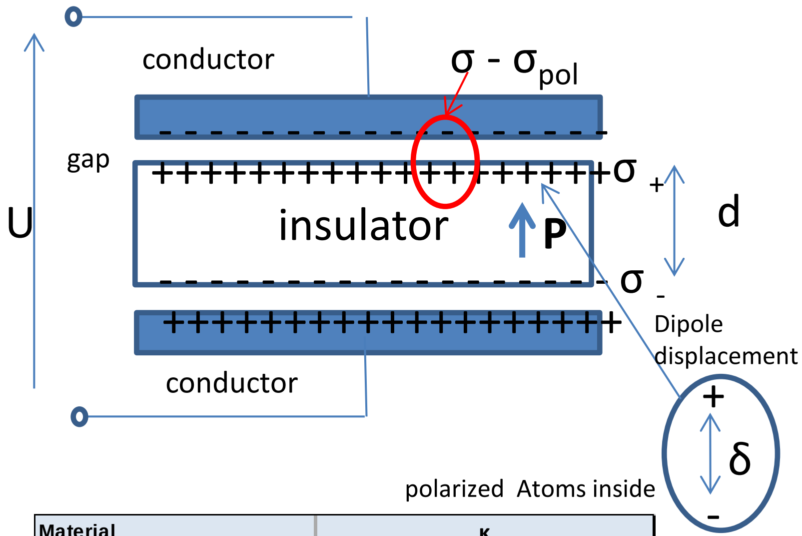
  $\sigma_{pol} = N\, q\, \delta$
- With the Gaussian law, the field in the insulator surface that fills the gap between conductor and insulator is

$$E = \frac{\sigma - \sigma_{pol}}{\varepsilon_0}$$

$$U = Q/C = E\, d = \frac{\sigma - \sigma_{pol}}{\varepsilon_0}\, d$$

$$C = \frac{\varepsilon_0}{d} \frac{1}{\sigma - \sigma_{pol}} = \kappa \frac{\varepsilon_0}{d} A$$

- The capacitance C of the parallel plate capacitor is **increased by the relative electric constant κ** (see table).



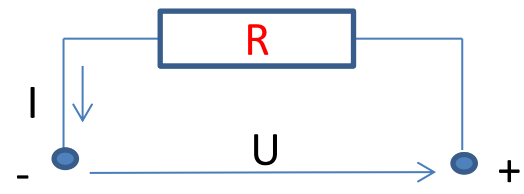
Material	κ
Vacuum	1
Air	1.00058986
PTFE/Teflon	2.1
Polyethylene	2.25
Epoxy +fibreglass (FR4)	4.7
Polyimide (kapton)	3.4
Polypropylene	2.2–2.36
Polystyrene	2.4–2.7
Paper	3.85
Pyrex (Glass)	4.7 (3.7–10)
Rubber	7
Diamond	5.5–10
Graphite	10–15
Silicon	11.68
Water	88, 80.1, 55.3, 34.5 (0, 20, 100, 200 °C)
Conjugated polymers	1.8–6 up to 100,000
Calcium copper titanate	>250,000

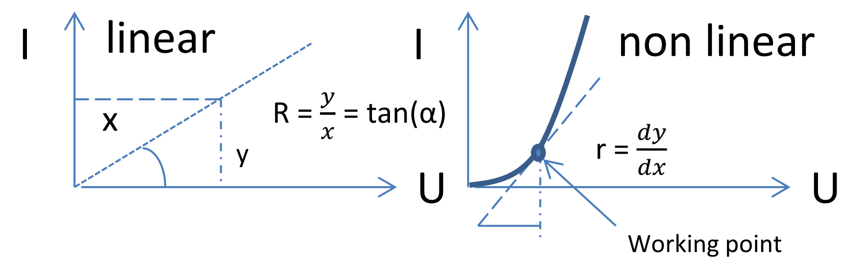
Resistance

- Resistance R is a measure how much current I can flow when a voltage potential U is applied to a material. The difference between copper, steel, and rubber is related to their microscopic structure and electron configuration, and is quantified by a property called **resistivity.**
- OHMs law is intuitive, the current through a resistor is proportional to the Voltage applied.

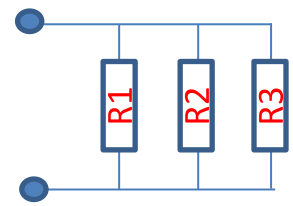
  U = R · I = R · dQ/dt

  The proportional constant is the resistivity R
- A more detailed characteristic of a resistor is the **I versus U relation**. An ideal resistor is characterized by line that passes through the U/I origin and is linear. The ohmic resistor curve is independent of the voltage polarity. The value of the resistor is equivalent to **tan α** of the **characteristic curve**. A high value corresponds to a low resistor and a low value to a high resistor value.
- **Non linear U/I** curves characterize semiconductors like diodes. The resistor concept is applied for these as differential resistor **r** at a given **U/I** working point.
- A typical diode characteristics (shown as non linear example) has an exponentially rising current for linearly increasing voltage. Semiconductor characteristics are in addition strongly dependent on voltage polarity.





R = R1 + R2 +R3



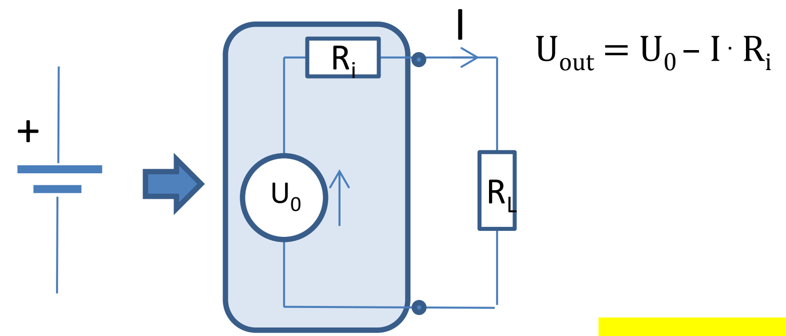
$$\frac{1}{R} = \frac{1}{R1} + \frac{1}{R2} + \frac{1}{R3}$$

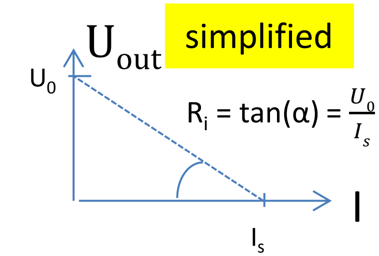
DC Voltage source

- An **ideal Voltage source U_0** provides a constant Voltage, independent of the load resistor
- A **real Voltage source** provides a lower Voltage the higher the load current.
- The simplified equivalent of a real Voltage source is an ideal Voltage source in series with a **very low internal resistor R_i**
- Without load current (I =0) the open loop voltage is equal to the internal ideal voltage source U_0
- The equivalent internal resistor R_i can in principle be measured as the slope of the U/I characteristics.
- Real Voltage sources have non-linear U/I characteristics, therefore the differential resistance

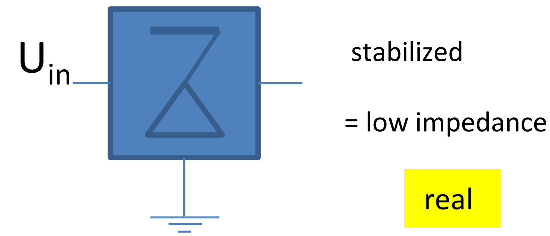
  $\mathbf{r_i} = \frac{\Delta U}{\Delta I}$ is a more realistic characterization
- Electronic regulators have low differential resistance and deliver their specific voltages up to a limit beyond which the voltage sharply decreases in case of short circuits
- Voltage sources below 48 Volt are not dangerous for humans, even if they can deliver tens of Amperes like car batteries.

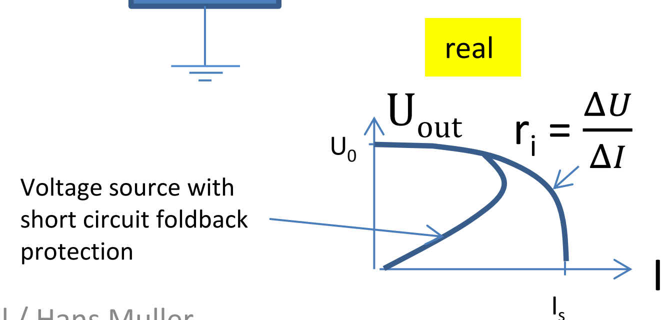
Battery



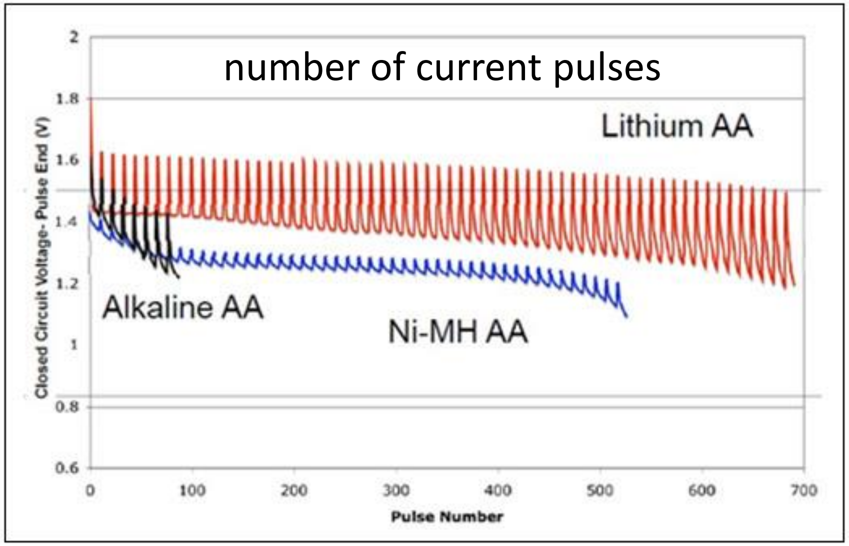


Electronic voltage source





Batteries are voltage sources



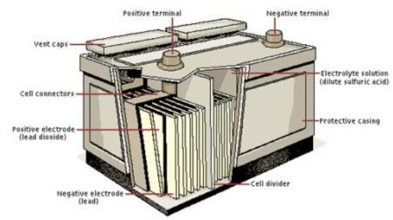
Battery type	Carbon-zinc	Alkaline	Lithium	NiCd	NiMH
Capacity mAh AA	400-1,700	1,800-2,600	2,500-3,400	600-1,000	800-2,700
AAA	~300	800-1,200	1,200	300-500	600-1,250
Nominal [V]	1.5	1.5	1.5	1.2	1.2
Discharge Rate	Very low	Low	Medium	Very high	Very high
Rechargeable	No	No	No	Yes	Yes
Shelf life	1-2 years	7 years	10-15 years	3-5 years	3-5 years
Leak resistance	Poor	Good	Superior	Good	Good

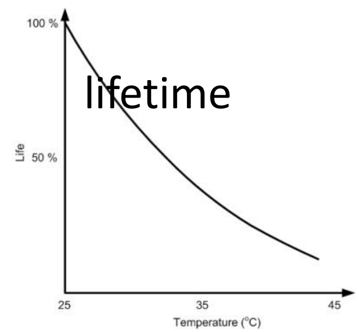
Cell type	IEC label	Example	Voltage
Mercury-oxide-zinc	MR	MR52	1,35 V
zinc-air	PR	PR41	1,4 V
Alkali-Mangan	LR	LR44, L1154	1,5 V
silveroxid-zinc	SR	SR44, SR1154	1,55 V
Lithium-mangandioxid	CR	CR2032	3,0 V
Lithium-carbon-monofluorid	BR	BR2016	3,0 V

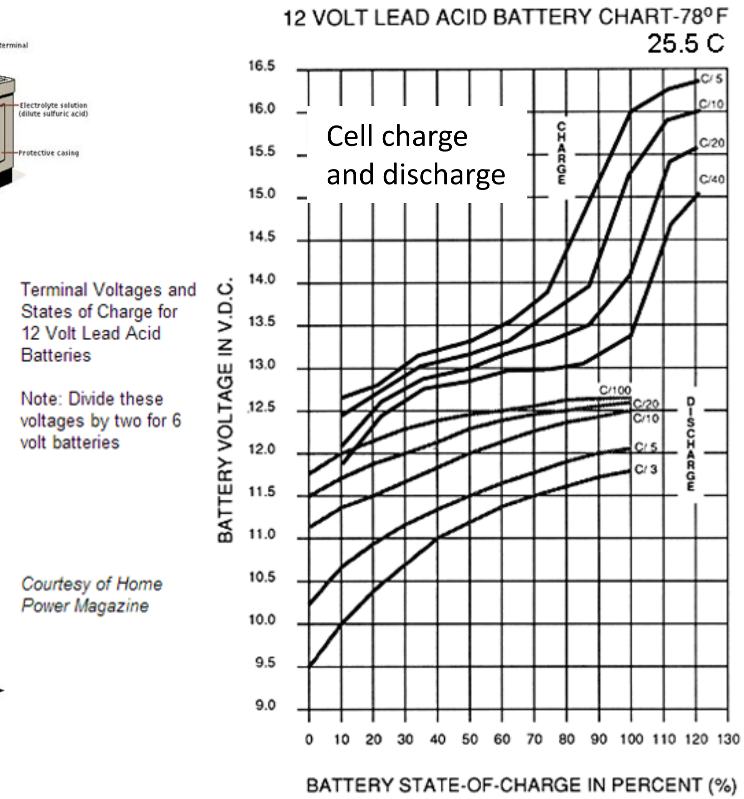
* http://batteryuniversity.com/learn/article/examining_loading_characteristics_on_primary_and_secondary_batteries

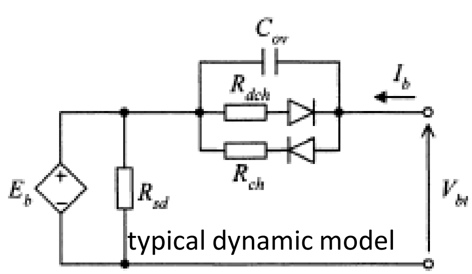
Lead Acid Batteries

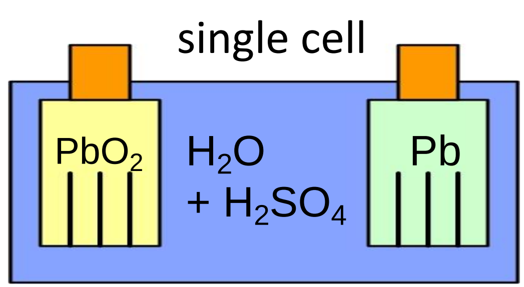
- Lead-Acid is the most inexpensive type of battery (used in cars) and rated for 20% discharge but not suitable for frequent / deeper discharges like Photovoltaics. A discharge rate like C/20 means discharge at a current equal to 1/20th of its total capacity C. Battery capacities are rated at +27C. Lower temperatures reduce amp-hour capacity significantly. Higher temperatures result in a slightly higher capacity, but this will increase water loss and decrease the number of cycles in the battery life
- A single battery cell has ~ 2V, hence 12V batteries are composed of 6 cells.
- Many dynamic electrical models have been published, a typical example is shown below.
- Internal resistances and lifetime are temperature dependent. The electrolyte, partially water, can freeze. If left in a deep discharged condition for a long time, the battery becomes "sulfated“.
- Specific gravity of the fluid is a good indicator of the state of the charge. Chargers should be equipped with temperature sensors to change their charge settings with temperature. Batteries may loose 5% / month of their capacity, depending on temperature and cell chemistry. The higher the temperature, the faster the self-discharge.











States of Charge, Specific Gravities, Voltages, and Freezing Points for Typical Deep Cycle Lead Acid Batteries

State of Charge	Specific Gravity	Voltage per Cell (volts)	Voltage of 12V (6 cell) Battery	Freezing Point (°F)	(C)
Fully Charged	1.265	2.12	12.70	-71	-57
75% Charged	1.225	2.10	12.60	-35	-37
50% Charged	1.190	2.08	12.45	-10	-23
25% Charged	1.155	2.03	12.20	+3	-16
Fully Discharged	1.120	1.95	11.70	+17	-8.3

http://polarpowerinc.com/info/operation20/operation25.htm

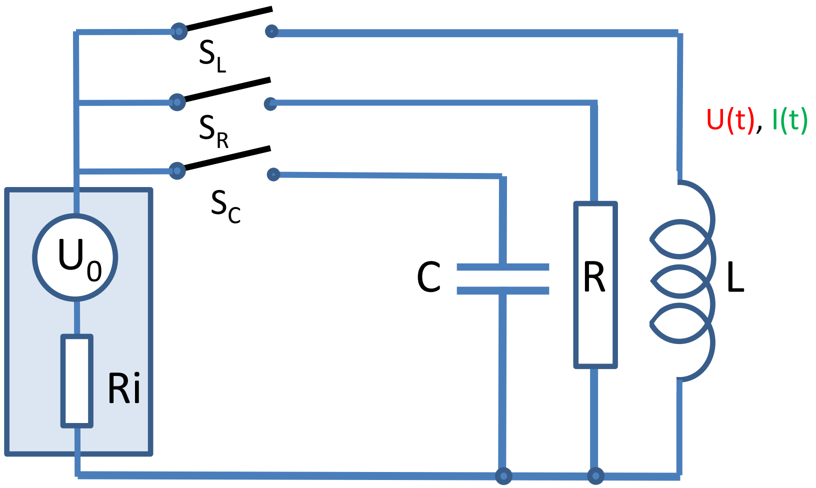
L,R,C phases

- Electronics is constructed from the elements Inductance L, Resistance R and Capacitance C
- Their electronics behaviour is mutually orthogonal
- Applying a step voltage U_0 at time t=0 via a switch

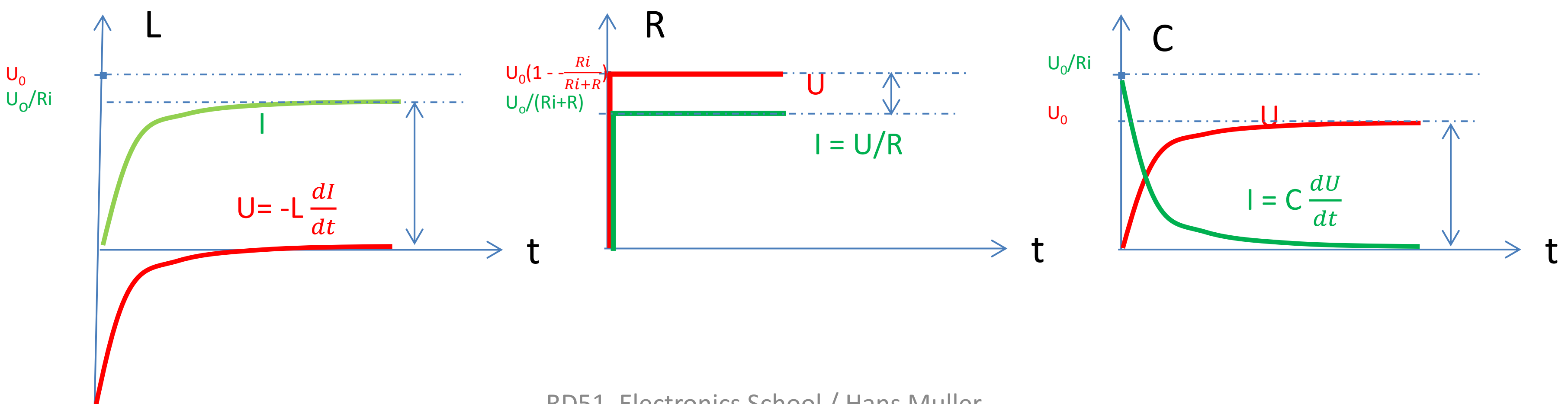
  R - constant voltage and current are in phase (0°)

  C - initial high current decays to zero whilst the voltage rises to $U_{0,}$ current precedes the voltage (+90°)

  L - initial maximum voltage U_0 decay is followed by a rising current , voltage precedes the current (-90°)



- Orthogonality is manifested by observing that for harmonic time functions I ~ cos($2\pi \frac{t}{T}$) where one period T =1/f means 2π or 360°, the phase relation is
- **Phase (L) :** $U = -L \frac{dI}{dt}$ ➔ $\frac{d}{dt}\cos(\omega t) = -\sin(\omega t) = \cos(\omega t - 90°)$
- **Phase (R) :** I = U/R ➔ $\cos(\omega t + 0°)$
- **Phase (C)** $I = C \frac{dU}{dt}$ ➔ $\frac{d}{dt}\cos(\omega t - 90°) = -\sin(\omega t - 90) = \sin(\omega t + 90°)$



Periodic currents through L and C

- Capacitors relate the change of an applied voltage with a current. Integration over time results in

  U(t) = U(t_0)+ 1/C $\int_{t_0}^{t_o+t} I(t)dt$
- A periodic voltage is the same after one period T

  U(t_0+T) = U(t_0)

  U(t_0+T)-U(t_0) = 0 = 1/C $\int_{t_0}^{t_o+t} I(t)dt$

  ➔ 1/T $\int_{t_0}^{t_o+t} I(t)dt$ =0

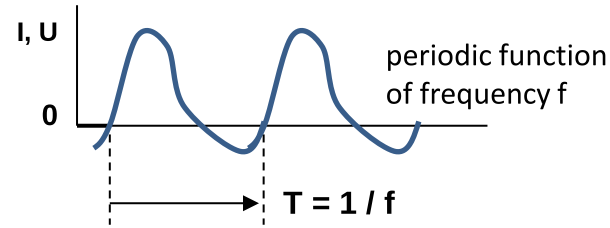
  The average current for a periodic voltage applied to a capacitor is zero
- Inductances relate the change of an applied current with an inverse voltage

  I(t) = I(t_0)+ 1/L $\int_{t_0}^{t_o+t} U(t)dt$
- A periodic current is the same after one period T

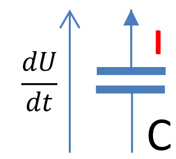
  I(t_0+T) = I(t_0) with the same argument as above

  ➔ 1/T $\int_{t_0}^{t_o+t} U(t)dt$ =0

  The average voltage for a periodic current applied to an inductance is zero

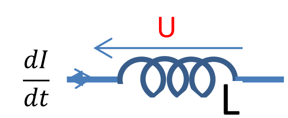


I = C $\frac{dU}{dt}$



1/T $\int_{t_0}^{t_o+t} I(t)dt$ = **0**

U= -L $\frac{dI}{dt}$



1/T $\int_{t_0}^{t_o+t} U(t)dt$ = **0**

Current sources

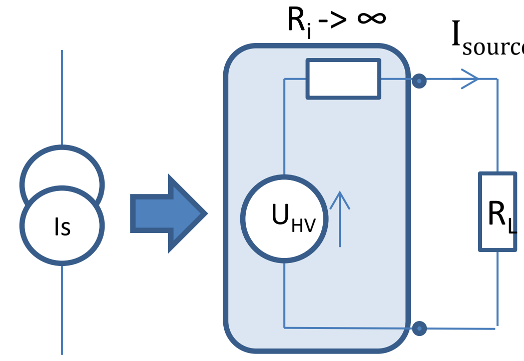
- An **ideal current source** provides a constant current, independent of the load resistor
- A **real current source** provides a lower current when the load resistor increases
- The simplified equivalent of a real current source is an ideal high Voltage source U_0 in series with a **very high internal resistor** R_i
- The short circuit current Ic is equal $U_{HV/}$ Ri
- Without load resistor the external voltage is Uo (can be very high!)
- Example of current sources are lightnings, welding transformers, cathode ray devices (TV), accelerators, photomultipliers, and all **avalanche detectors**

- **Detector physics is more concerned with current sources than with voltage sources.**

- In low voltage applications, current sources for a low voltage range are made with transistors or operational amplifiers. This is possible via a high differential resistance of semiconductors within in a limited voltage range.

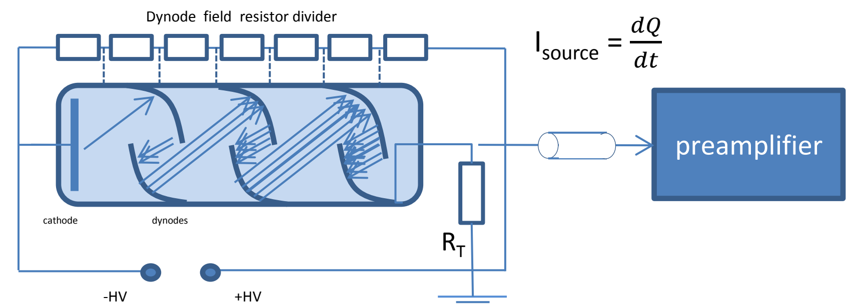
- Current sources can be dangerous for humans unless they are limited to the low voltage range below 48V.

- Big capacitors are like current sources.

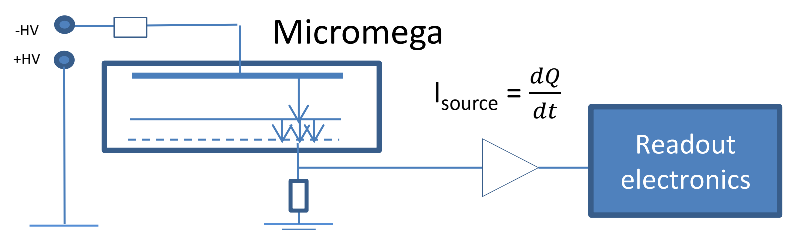


$I_{source} = U_{HV} / R_i = const$

Photomultiplier



Micromega



Kirchhoff Networks

- <u>Kirchhoff's rules</u> define the added Voltages in a loop and currents in a node:

$$\sum_{around\ closed\ loop} \boldsymbol{U_i} = \mathbf{0}$$

$$\sum_{in\ a\ node} \boldsymbol{I_i} = \mathbf{0}$$

- These rules are expressions of the Maxwell equations for electrostatics with $\frac{\partial \boldsymbol{E}}{\partial t} = 0$ and $\frac{\partial \boldsymbol{B}}{\partial t} = 0$
  The work integral around a closed loop of a field E is 0
  W= $-\oint Eds$=0 and the charge is conserved $\boldsymbol{\nabla} \cdot \boldsymbol{j} = 0$

- The **resistor divider** is a simple application of Kirchhoff:
  - The current sum into the NODE is zero, hence
    the ingoing current I is the same as the outgoing (I= I1= -I2 , I3=0)
  - The Voltage loop means U2 + U1 = Uo.

    $\mathrm{I \cdot R2 = U2}$, $\mathrm{I \cdot (R1 + R2) = U0}$ $\Rightarrow \mathrm{I} = \frac{U2}{R2} = \frac{U0}{R1+R2}$

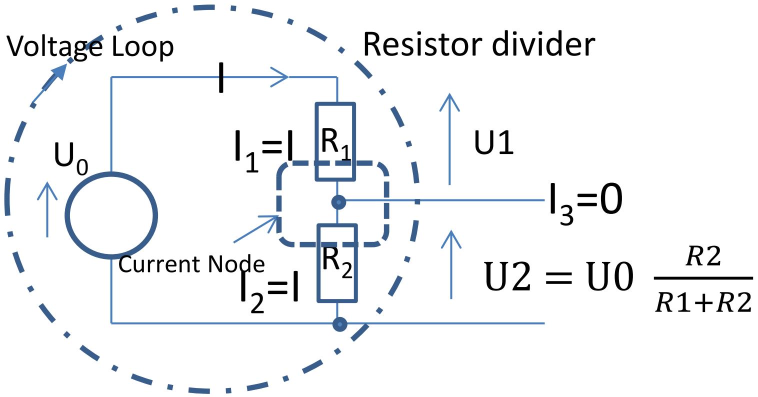
    $$\mathbf{U2 = U0}\ \frac{\boldsymbol{R2}}{\boldsymbol{R1+R2}}$$

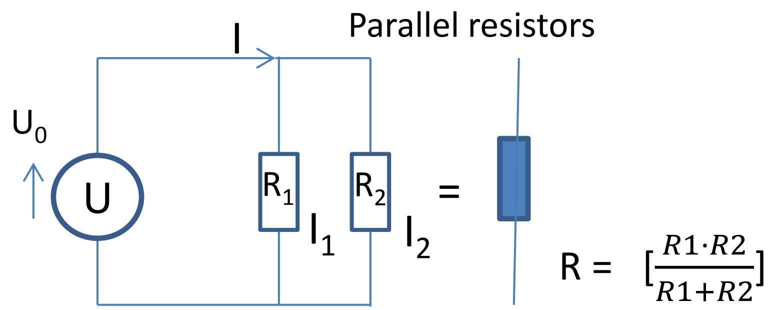
- A **parallel resistor** network

  Node: I1+ I2 = I Loop: U_0= I1 · R1 = I2 · R2

  U_0 = I · R= $(\frac{U0}{R1} + \frac{U0}{R2})$ R ; R= U_0 / $(\frac{U0}{R1} + \frac{U0}{R2})$;

  $$\mathbf{R} = [\frac{\boldsymbol{R1 \cdot R2}}{\boldsymbol{R1+R2}}]$$





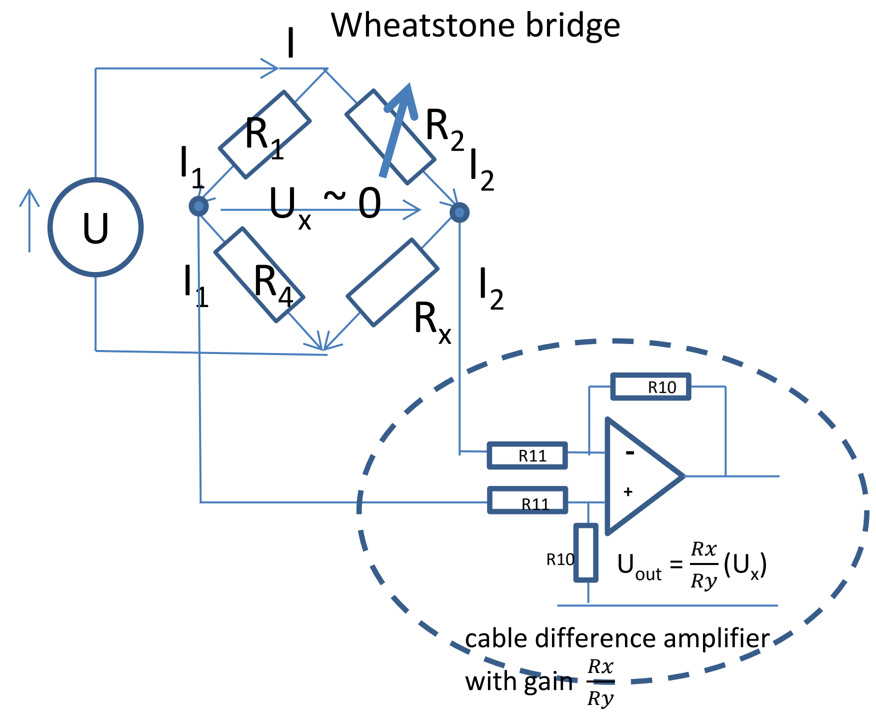
Wheatstone brigde

- The Wheatstone Bridge circuit is a network for precision measurement. With 3 known resistances R1,R2,R3 the value of a fourth impedance Rx can be determined by adjusting R2 until Ux =0 at this point:

  $I1 \cdot R1 - I2 \cdot R2 = 0$
  $I1 \cdot R4 - I2 \cdot Rx = 0$

  $$R_x = R_2 \cdot R_4 / R_1$$

- Using an Operational Amplifier to measure U_x, the Wheatstone bridge can measure and amplify very small changes in resistance, for example from temperature- or light sensitive resistors.
- There are many different applications* of bridge circuits for instrumentation amplifiers, scales, stress gages etc



http://newton.ex.ac.uk/teaching/CDHW/Sensors/an43.pdf

Resistor properties

- Resistors with a uniform flow of charge have a specific **resistivity** $\rho = R\,\frac{A}{l}$ (ohm m)

  R is the electrical resistance (OHM) , l is the length of the resistive material and A is the cross section area of the resistor.

  The conductivity is $\sigma = \frac{1}{\rho}$ (1/ ohm m)
- Resistivity is normally **temperature dependent:**

  $$\frac{\Delta R}{R} = \alpha\,\Delta T$$

  α is the temperature coefficient
- For pure metals, α is positive since with increasing temperature the number of collisions with atoms increase. A positive coefficient is named **PTC** for positive temperature coefficient, a negative one can be achieved with semiconductor materials and is named **NTC**.
- **Thick-film resistors** are very common in electronics and have a temperature coefficient > +- 50 ppm/°C. A homogeneous film of metal alloy is deposited on a high grade Al_2O_3 ceramic substrate and laser-trimmed to achieve the desired precision.
- **Thin-film resistors** (more expensive) are much thinner resistive layers that are sputtered on a substrate. They are available down to 0.5 ppm/°C with +-0.02 % tolerance and large temperature range -55 to +200 °C . They exhibit a much **lower current noise** spectrum (< - 30 dB, 0.1 ppm/V).
- Some applications require carefully selected resistors. For example the resistors in the anodes of avalanche detectors should be of high stability and low noise.
- Ohms law is defined originally for metals at a specific temperature. Resistors are to some extent non-linear with Voltage, this non-linearity is specified as **voltage coefficient**.

**

Characteristics type resistors	tolerance %	temperature coefficient	voltage coefficient
Carbon composite	+-10	+400 to -900ppm/C	350 ppm
carbon film	+-5	+100 to -700 ppm/C	100 ppm
metal film	+-1	+100 ppm/C	1 ppm
metal oxide	+-5	+350 ppm/C	variable
wirewound	+-5	+-70% to +- 250 %	1ppm

*

Material	Resistivity ρ (ohm m)		Temperature coefficient per degree C	Conductivity σ x 10^7 /Ωm
Graphene	1	x10^-8		
Copper	1.68	x10^-8	0.0068	5.95
Tungsten	5.6	x10^-8	0.0045	1.79
Iron	9.71	x10^-8	0.00651	1.03
Platinum	10.6	x10^-8	0.003927	0.943
Lead	22	x10^-8	...	0.45
Constantan	49	x10^-8	...	0.2
Carbon	3..60	x10^-5	-0.0005	...
Germanium	1-500	x10^-3	-0.05	...
Silicon*	0.1-60	...	-0.07	...
Glass	1-10000	x10^9	...	...
Quartz	7.5	x10^17	...	...

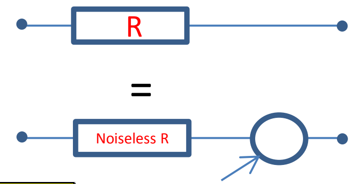
*http://en.wikipedia.org/wiki/Resistor ** Douglas Self "Small Signal Audio design"

Resistor Noise

- Resistors create thermal, current and flicker noise
- **Johnson noise**, **Nyquist noise**, **thermal noise** is the electronic noise generated by the thermal agitation of the charge carriers inside an electrical conductor, which happens <u>regardless of any applied voltage. It is a property of the resistor and temperature</u>
- The equivalent of a resistor R at temperature T is a noiseless resistor in series with an rms voltage source per frequency interval Δf :

  $$V_{rms}^2/\Delta f = 4ks\ BT_a \cdot R$$

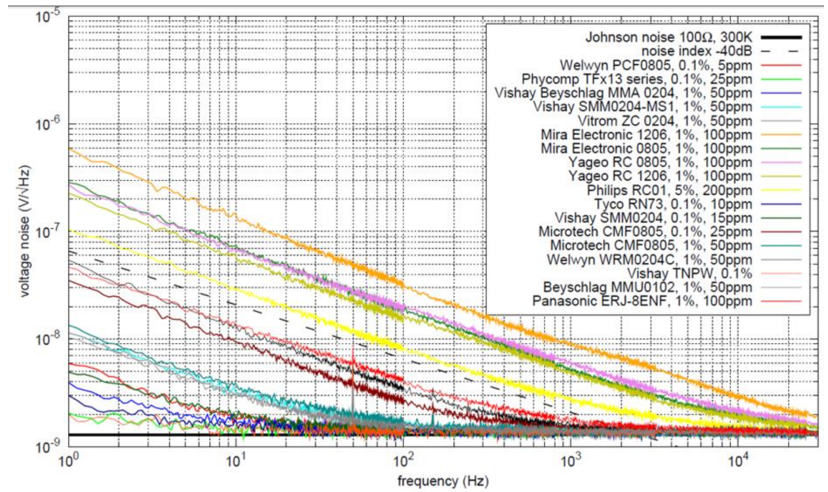
  k_B Boltzmann constant and T_a absolute temperature above T_a = T+273.15 °C
- For a **1M** resistor the mean square Johnson noise voltage per $\sqrt{Hz}$ is 0.13 uV. For a bandwidth of **1MHz** the RMS noise voltage is V_{rms} = (0.13 uV/ $\sqrt{Hz}$) x $\sqrt{1MHz}$= **0.13 mV**
- **Shot noise, current noise** is purely based on electron statistics, temperature and frequency independent and **important only for low noise electronics** at very low charge densities
- **1/f noise, flicker noise** is predominant at **low frequencies** and in particular at DC, at higher frequencies it gets overshadowed by the Johnson noise. The "**noise corner**" is called the frequency at which the Johnson noise starts to overshadow the 1/f noise. This **excess noise** is therefore **present at low frequency** and a DC effect which varies with resistor types.
- A measure against excess noise is AC coupling and bandwidth limitation towards low frequencies
- Thin film resistors exhibit a much lower excess noise than thick film carbon resistors.
- MOSFET have noise corners much higher frequencies (up to 1 GHz) than bipolar transistors.



$V_{rms}^2/\Delta f = 4kBTaR$

Resistor Excess noise type of resistor	noise uV/v
carbon film TH	0.2-0.3
metal oxide TH	0.1-1
thin film SM	0.05 -0.4
bulk metal foil	0.01
wirewound TH	~0

Current noise of surface-mount devices (SMD) up to a Max. power dissipation of 1 W *



https://dcc-llo.ligo.org/public/0002/T0900200/001/current_noise.pdf

Ohmic heat

- According Joule's first law the energy delivered to a resistor gets converted into ohmic heat. The amount of Ohmic heat energy is

$$W = U \cdot I = U^2/R = I^2 \cdot R$$

(independent of polarity)

For alternating current (AC) the average power is

$$\mathbf{W_{AC} = U_{rms}^{\ 2} / R}$$

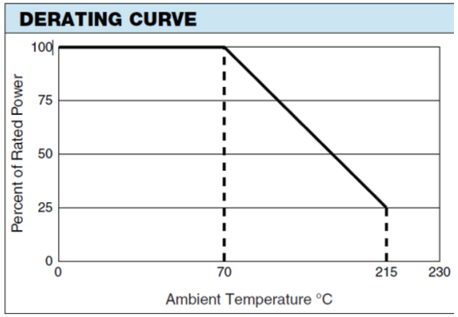
for sinus wave : $U_{rms} = U_o \times 1/\sqrt{2}$ (Uo = max amplitude)

- Power ratings of SMD resistors are specified for an ambient temperature (typ. 70°C) below which the resistor is 100% safe from burn damage. Smaller size resistors like 0402 can only cope with power dissipation of max 0.1 Watt. The derating curve specifies up to which temperature the resistor can cope with its rated power.
- Power loss in resistive power transmission lines is proportional to the conductor resistance and the square of the current. Using low current at high voltage reduces the joule heating loss in long conductor lines.
- In a light bulb, only a small fraction (less 5%) of the electric energy gets converted into light radiation, the rest of the energy is converted in ohmic heat.

common SMD sizes and power ratings

Standard size	metric L (mm)	metric W (mm)	Height (mm)	typ. power rating (W)
"0402"	1	0.5	0.32	0.063
"0603"	1.6	0.8	0.45	0.125
"0805"	2	1.2	0.6	0.2
"1206"	3.1	1.6	0.6	0.5

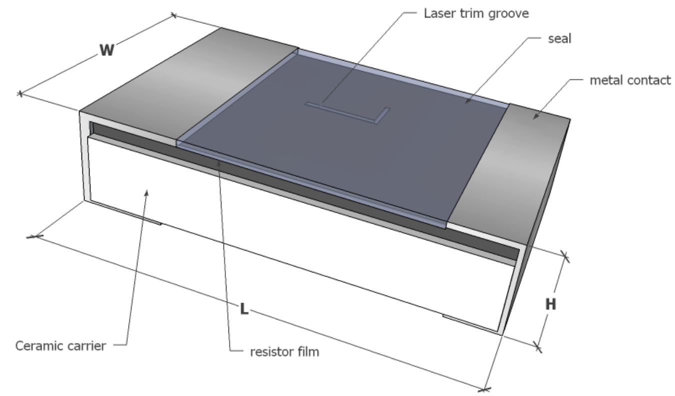
The **SMD size code** refers to US units
1/10 inch = 0.254 mm, example :
0603 means:
06 x 0.254 mm length (~1.53 mm)
03 x 0.254 mm width (~0.76 mm)



Resistor types

Axial resistor codes			
1st. two Digits	Multiplier-	Tolerance-	Temp. Co-eff.
Black 0	Black 1	-	-
Brown 1	Brown 10	Brown 1%	Brown 100
Red 2	Red 100	Red +2%	Red 50
Orange 3	Orange 1K	-	Orange 15
Yellow 4	Yellow 10K	-	Yellow 25
Green 5	Green 100K	-	Green 0.5
Blue 6	Blue 1M	-	Blue 0.25
Violet 7	Violet 10M	-	Violet 0.1
Grey 8	Not Used	-	-
White 9	Not Used	-	-
-	Silver 0.01	Silver+10%	-
-	Gold 0.1	Gold +5%	-
Memorizer:			
Big Ben Realized Only Yesterday; Girls, Boys, Very Good Work Saves God			

Surface mount resistor (SMD)



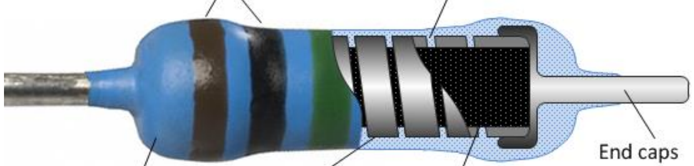
Power resistor (wirewound)

up to 50 W when cooled

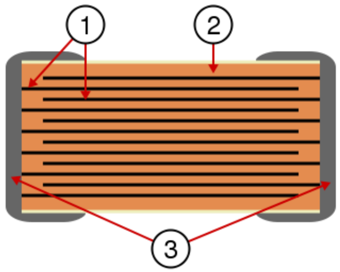
R

Axial resistors

(not for high frequency)



Ceramic capacitors



1 electrode layers
2 ceramic dielectric
3 solder contacts

- Multi-Layer Ceramic Capacitor” (**MLCC**) are metal/ceramic stacks in the common SMD sizes (0201, 0402, 0603, 0805, 1206,1210, 1812) and are available from 0.5 pF up to 47 uF
- **Class 1** capacitors have a temperature coefficient that is typically ~ linear with temperature. They have very low electrical losses with a dissipation factor of approximately 0.15%. They undergo no significant aging processes and the capacitance value is nearly independent of the applied voltage. Typ. application: **frequency defining circuits where stability is essential**

  NP0/CG/C0G with an α ±0•10^{-6} /K and α tolerance of 30 ppm are technically of great interest
- **Class 2** ceramic capacitors have **high density per volume** but lower accuracy and stability with a dissipation factor up to 2.5%. Typical application: **buffer, by-pass and coupling**

  Z5U capacitor : +10 °C to +85 °C, capacitance change at most +22% to −56%.

  X7R capacitor : −55 °C to +125 °C , capacitance change of at most ±15%

Ceramic capacitors Class 1

IEC/EN 60384-8/21 and EIA-RS-198

Ceramic names	Temperature coefficient α 10^{-6} /K	α-Tolerance 10^{-6} /K	Sub-class	IEC/ EN- code	EIA code
P100	100	±30	1B	AG	M7G
NP0	0	±30	1B	CG	C0G
N33	−33	±30	1B	HG	H2G
N75	−75	±30	1B	LG	L2G
N150	−150	±60	1B	PH	P2H
N220	−220	±60	1B	RH	R2H
N330	−330	±60	1B	SH	S2H
N470	-470	±60	1B	TH	T2H
N750	−750	±120	1B	UJ	U2J
N1000	−1000	±250	1F	QK	Q3K
N1500	−1500	±250	1F	VK	P3K

Ceramic capacitors Class 2

Letter code low temperature	Number code upper temperature	Letter code change of capacitance over the temperature range
X = −55 °C (−67 °F)	4 = +65 °C (+149 °F)	P = ±10%
Y = −30 °C (−22 °F)	5 = +85 °C (+185 °F)	R = ±15%
Z = +10 °C (+50 °F)	6 = +105 °C (+221 °F)	S = ±22%
	7 = +125 °C (+257 °F)	T = +22/−33%
	8 = +150 °C (+302 °F)	U = +22/−56%
	9 = +200 °C (+392 °F)	V = +22/−82%

Tantalum capacitors

- Sintered Tantalum powder, surrounded by an electrolyte, forms on the surface of an oxide (Ta_2O_5) with a relative dielectric of κ = 26 which is quite big. According C = κ $\frac{\varepsilon_0}{d}$ A such capacitors can have very high capacity (up to 1 mF). The oxide is only maintained when the electrolyte is polarized negative (cathode) and the contact to the Tantalum is positive (anode). **Tantalum capacitors** are preferably used for DC voltage applications.
- **Bipolar Tantalum capacitors** can be made by connecting two capacitors at their same poles to make a series capacitor. If the 2 capacitors have the same value the non-polarized value is ½ of the individual capacity.
- An alternating ripple Voltage (superimposed to the polarization DC Voltage) on the capacitor generates an alternating current I_{AC} which generates ohmic heat at the capacitor's (**ESR = Equivalent Series Resistor**).
- If conduction electrons are the dominant loss

$$\mathbf{ESR} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\kappa}\, \omega^2 C}$$

where σ is the conductivity and

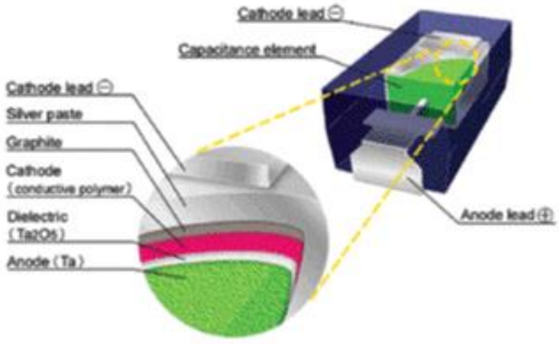
κ the relative electric constant

- The ESR resistor represents energy loss in the dielectric by alternating polarization of the atomic dipoles

$$\mathbf{W_{AC} = I_{AC}^2 \cdot ESR = U_r^2 / ESR}$$

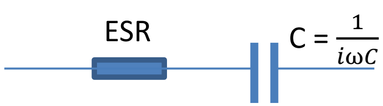
- The maximum power dissipation of the capacitor W_{max} which is specified for the different capacitor sizes and defines the maximum ripple current through the capacitor

$$I_{r.max} = \sqrt{W_{max}/ESR}$$

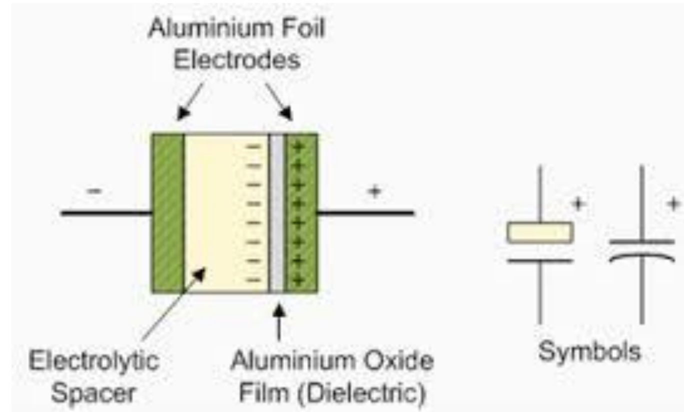


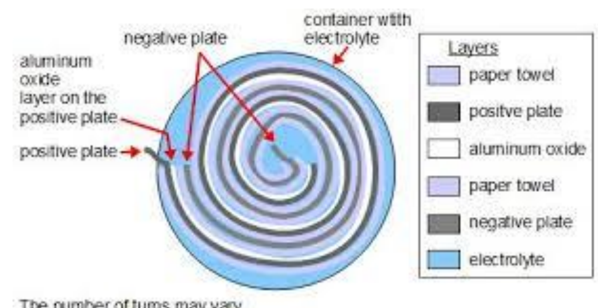
Tantalum sizes and max power dissipation

Size	L mm	B mm	H mm	Marking AVX	Marking Kemet	Max power dissipation mWatts @ 25°C
805	2,05	1,35	1,2	R	R	
805	2,05	1,35	1,5	P	-	
1206	3,2	1,6	1,0	K	I	
1206	3,2	1,6	1,2	S	S	60
1206	3,2	1,6	1,8	A	A	75
1411	3,5	2,8	1,2	T	T	70
1411	3,5	2,8	2,1	B	B	85
2312	6,0	3,2	1,5	W	U	90
2312	6,0	3,2	2,0	F	-	
2312	6,0	3,2	2,8	C	C	110
2917	7,3	4,3	1,5	X	W	
2917	7,3	4,3	2,0	Y	V	125
2917	7,3	4,3	3,1	D	D	150
2917	7,3	4,3	4,0	-	Y	
2917	7,3	4,3	4,3	E	X	165
2623	7,2	6,0	3,8	-	E	200
2924	7,3	6,1	3,8	V	-	

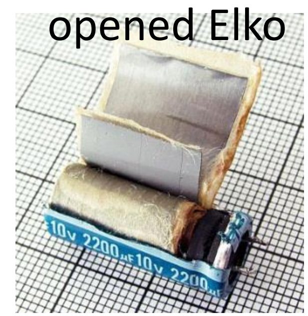


Electrolytic capacitors





- **The electrolytic capacitor** uses an electrolyte (an ionic conducting liquid) as one of its plates to achieve a **larger capacitance per unit volume** than other types. Most electrolytic capacitors are **polarized**, so they cannot be used with AC signals unless there is a DC polarizing bias voltage.
- They are used primarily in **power supplies** to filter DC from **low frequency AC** , but also to store charge for a power subsystem on a PCB, or to filter low frequency signals. For a given size, the working voltage is inversely proportional to the maximum capacity, i.e. 470 uF at 6.3 Volt may have the same housing as 47 uF at 63 Volt.
- The (ESR Equivalent Series Resistor) and dissipation factor are significantly inferior to other types of capacitors, the leakage current is higher and working life is shorter. Computer-grade lifetimes are up to 10.000 h at 100 °C (~ 1 year ! of permanent use). One of the effects of aging is an increase in ESR although a capacitance meter will not find any fault , ESR meters can measure the ESR even in circuit.



opened Elko

SMD: **plus** pole mark

Cans: **minus** pole mark

defective, leaking

Inductance

- Ampere's law says that a current I generates a magnetic B field around it : $\oint B\, ds = \frac{I}{\varepsilon_0\, c^2}$
- The line integral around $a\ wire\ \oint B\, ds$ equals B · 2πr

  B= $\frac{1}{4\pi\varepsilon_0\, c^2}\frac{2I}{r}$ the B field is ~ 1/r

  [$\frac{1}{4\pi\varepsilon_0\, c^2}$ = 10^{-7} in MKS Units]
- In a solenoid of length L it can be shown that the field inside adds up to:

  $B_0 = \frac{n\, I}{\varepsilon_0\, c^2}$; $n = \frac{N}{L}$ number of turns per length L
- Two solenoid which share the same field B_0 but with different number of turns N1 and N2 , a current I1 is applied to coil 1:

  $B_0 = \frac{1}{\varepsilon_0\, c^2} \cdot \frac{N1\, I_1}{L}$

- With **Maxwell's 2nd equation** in integral form, also called **Faraday's law** there is an **induced voltage U_2 in the second coil:**

  $U_2 = \oint E ds = - \frac{\partial}{\partial t} \oiint B\, da = - \frac{\partial(\,flux\ B\ through\ surface)}{\partial t}$

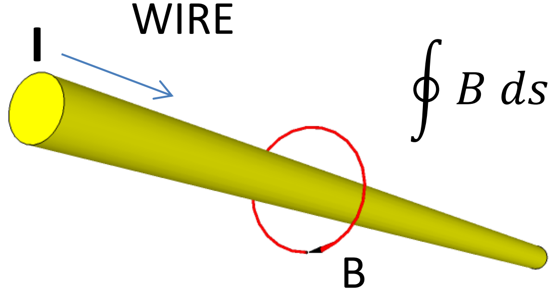
  this reverse voltage is due to a **change of magnetic flux** through the surface S. With the solenoid geometry this is simply

  $U_2 = - N_2 \cdot S \frac{dB_0}{dt}$

  $U_2 = - \frac{N1 \cdot N2\, S}{\varepsilon_0 c^2\, L} \frac{dI_1}{dt} = M21 \cdot \frac{dI_1}{dt}$

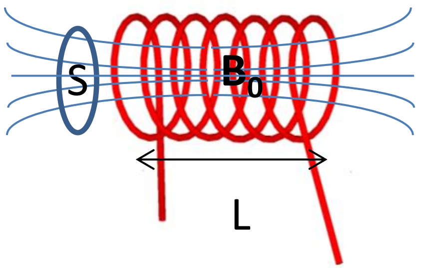
- The **mutual Inductance M21** of 2 coupled coils relates the induced reverse voltage in a secondary coil to the change of current in the primary coil.

WIRE

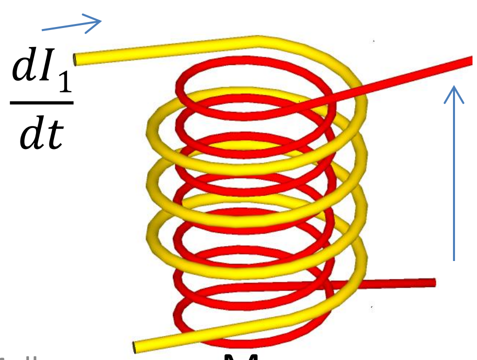


$$\oint B\, ds = \frac{I}{\varepsilon_0\, c^2}$$

solenoid



Mutual inductance



$U_2 = M_{21} \cdot \frac{dI_1}{dt}$

M_{21}

Self Inductance

- The change of current through a single coil produces via the **magnetic flux change** a **reverse voltage** also in **itself.**

  $\mathbf{U = - L \frac{dI}{dt}}$
- The negative sign indicates that the induced voltage opposes the change of current like a kind of inertia: **Lenz's law** states that an induced current has a direction such that its magnetic field opposes the change in magnetic field that induced the current.
- The **(self) inductance** of an **air solenoid** is

  $\mathbf{L = \mu_0 \frac{S N^2}{l}}$ [H]

  μ_0 is the susceptibility of the vacuum

- The power or energy per time unit is P = U · I = dE/dt or

  dE/dt = - L $\frac{dI}{dt}$ · I after integration the **stored energy in a coil** is

  $\mathbf{E = ½ L \cdot I^2}$

- An ideal inductor would have no resistance or energy losses. **Real inductors** have resistance from the metal wire which appears as a series resistance with the ideal resistance . The resistance converts electric current through the coils into heat, thus causing energy loss. The **quality factor *Q*** of an inductor is the ratio of its inductive reactance to its resistance at a given frequency, and is a measure of its efficiency. The higher the Q factor of the inductor, the closer it approaches the behaviour of an ideal, lossless, inductor. High Q inductors are used with capacitors to make resonant circuits in radio transmitters and receivers.

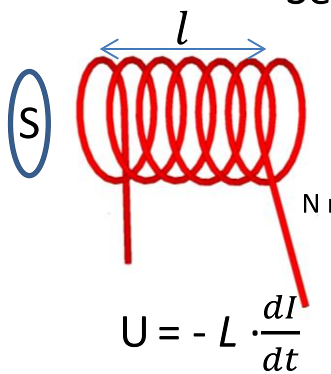
  Q= $2\pi f \frac{Energy\ stored}{energy\ lost}$= **ω** L/R

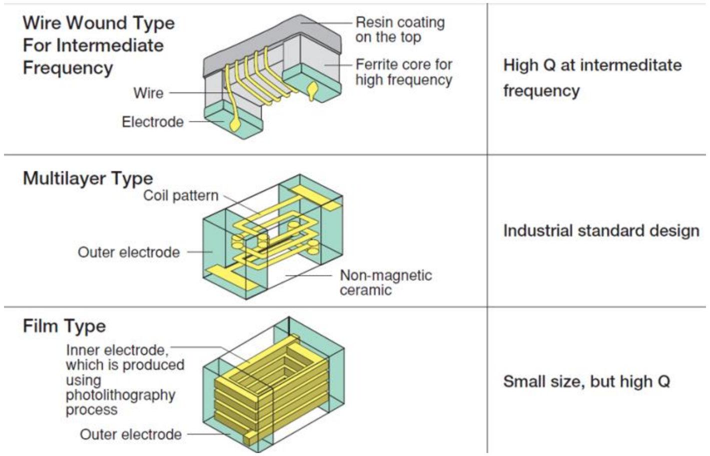
  when looking at the imaginary Impedance plane for
  an inductive impedance L and ohmic resistor R

  Q= 1/ tan(Ø) becomes large for an inductance with no ohmic resistance

- Chip inductors exist between 1 nH and 1 mH, they have typical Q factors between 10 and 80. Due to parasitic capacitance in the leads, inductors have a self resonance frequency, hence coils must be chosen according their frequency characteristics.

Self inductance





Example of chip inductors

Note: keep distance between chip inductors to avoid coupling of signals !

Skin effect

- The current I through a wire generates a magnetic field outside **and inside**. For DC, the current density inside the wire is constant, however for alternating currents the magnet field H around the wire axis changes creates eddy current loops along the wire, distributed radially around the wire axis.
- The density of the eddy currents is **higher towards the interior**. Using the Lorentz right hand rule, the direction of the eddy currents is opposed to the generating current I and therefore reduces the net current towards the inner part of the wire. In good conductors like copper, the current density J_s at the surface decreases exponentially with distance x from the wire surface towards the axis

  $J = J_s\, e^{\frac{-x}{\delta}}$ where δ is the effective skin depht

  $$\boldsymbol{\delta} = \sqrt{\frac{2\rho}{\omega\mu}}$$

  ρ is the specific resistance [Ohm m]

  $\mu = \mu_0\, \mu_r$ is the absolute permeability

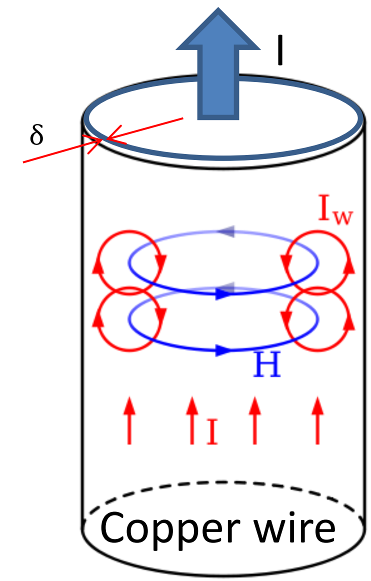
  $\omega = 2\pi f$
- Due to the depletion of the net current in the inner part the effective cross section of the wire is reduced, increasing the effective Impedance $Z > R_{DC}$
- Bad conductors have larger skin depth than good ones. For example the skin depth of Silicium is so large that it does not have influence even at high frequencies

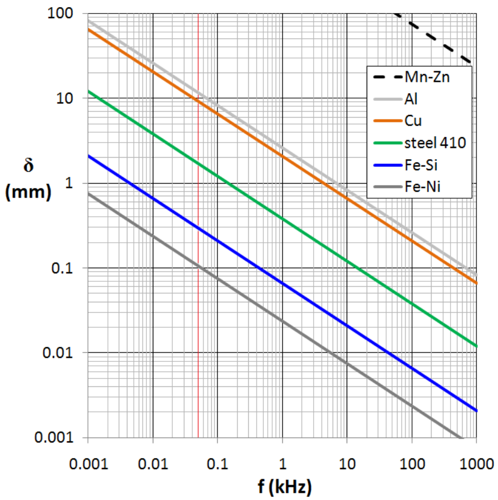
$\rho_{copper} = 1{,}78 \cdot 10^{-2}\ \Omega\, \frac{mm2}{m}$

$\mu_{r,\ copper} = 1$

Skin depth Copper *

frequency	δ depth copper
50 Hz	9,38 mm
60 Hz	8,57 mm
1 kHz	2,10 mm
5 kHz	0,94 mm
10 kHz	0,66 mm
50 kHz	0,30 mm
100 kHz	0,21 mm
500 kHz	0,094 mm = 94 µm
1 MHz	0,066 mm = 66 µm
10 MHz	0,021 mm = 21 µm
100 MHz	6,6 µm
1 GHz	2,1 µm
10 GHz	0,7 µm
100 GHz	0,2 µm





* http://de.wikipedia.org/wiki/Skin-Effekt

Ferro-magnetics

- Magnetization of ferromagnetic iron is described by 4th Maxwell equation

$$\nabla \mathrm{x} \mathrm{H} = \frac{j}{c^2 \varepsilon_0} \quad \text{with} \quad \mathbf{B} = \mathbf{H} + \frac{M}{\varepsilon_0 C_2}$$

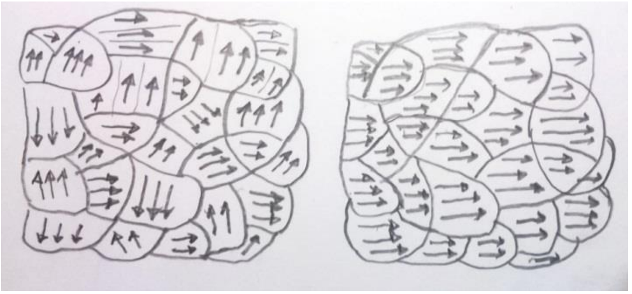
- The magnetic field **B** in ferromagnetic iron is the sum of a small magnetizing field **H** and of the resulting magnetic moment M inside the metal. **H** can be created by a current **I** through a primary inductance with N windings around the metal. Its internal micro-dipoles line up and create a macroscopic polarization ***P***. An alternating **H** field changes **P** and the internal current movement due to $\frac{\partial P}{\partial t}$ makes up for effective currents that generate a much larger **B** field than the magnetizing field **H**.
- *Note: this does NOT work in a external magnetic field ! Don't put transformers or motors inside strong magnets*

- The non-linear hysteresis relation between **B** and **H** demonstrates: initial magnetization due to a small **H** creates a large **B** which reaches saturation when all dipoles are lined up. When **H** is removed many of them remain to make a large retention field **B** when **H**=0.

- A complete cycle of H consumes energy due to the force required to change the retained polarizations. Only when small H fields are applied the hysteresis effect is small and can be approximated by a linear relation **B** = μ**H** where μ is the relative permeability of the iron. The self inductance of a ferromagnetic solenoid is

$$\mathbf{L} = \mu \frac{N^2}{\varepsilon_0 c^2} \frac{S}{l} \qquad \left(\frac{S}{l} \text{ ratio of coil cross section and length}\right)$$

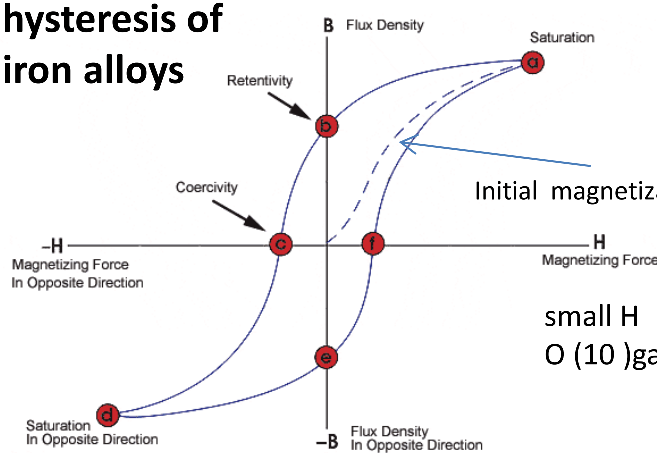
H=0 , B = 0 H>0 , B >> 0



very large B

O(10.000) gauss)

hysteresis of iron alloys



Initial magnetization curve

small H
O (10)gauss)

Transformers

- A transformer is in the simplest case a ferromagnetic iron ring with N1 primary turns and N2 secondary turns. Both coils share the same magnetic flux. The Voltage generated by changing magnetic flux can be measured at the second coil

  U2 = $-\frac{d}{dt}$(magnetic flux) = -M $\frac{d}{dt}$ (I_{prim})

  with sinoidal magnetic flux = N · B · A sin(2πf)

  the main transformer equation is *

  $U2_{rms}$ ~ 4.44 · N · B · A · f

  Note: high B fields are needed at low frequencies , at high frequencies air coils are sufficient

- The Voltage generated by changing magnetic flux can be measured at the second coil

  U2 = $-\frac{d}{dt}$(magnetic flux) = -M $\frac{d}{dt}$ (I_{prim})

- The mutual inductance is M = k $\sqrt{L1\, L2}$ where $k \leq 1$. If the magnetic flux equals L = M and with $L_1 \sim N_1^2$ and M ~ N1 · N2

  => $\mathbf{U_2/U_1 = - N_2/N_1}$ (phase 180°)

- The ratio of windings **γ= N1/N2** defines the ratio of the primary and secondary voltages and currents

  $U_1/U_2 = \gamma$ Voltage transformation

  $I_1/I_2 = 1/\gamma$ Current transformation

  The impedance of one side is transferred as

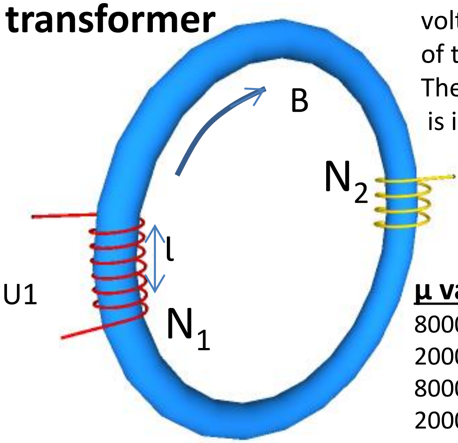
  $Z_2 = \gamma^2 Z_1$

  A resistor R on the secondary side transforms like

  $Z_{prim} = \gamma^2$ R

- **Impedance transformers** are used to adapt impedances of transmission lines, for example 100 Ohm differential to 50 Ohm requires N1/N2 = $\sqrt{2}$ with R = 50 OHM

ring transformer



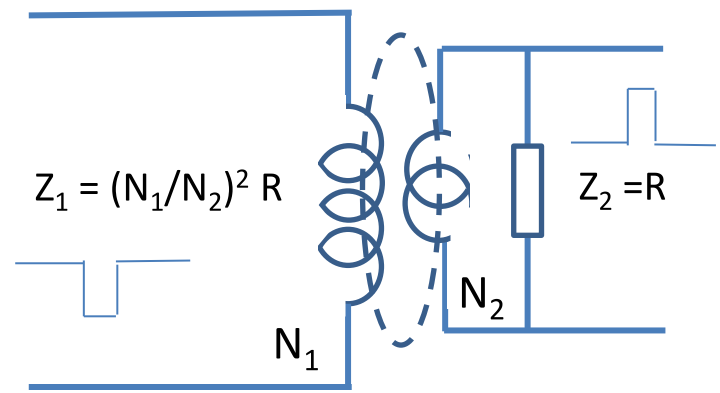
voltage ratio is the ratio of the windings.
The phase of the voltages is inverted

U2 = - U1 $\frac{N2}{N1}$

µ values

8000 (Permalloy)
20000 (Mu metal)
80000 (Nanoperm)
200000 (pure annealed iron)

impedance / pulse transformer



*http://de.wikipedia.org/wiki/Transformator

Risetime LR

- A real inductance consists of an **ideal self inductance L** connected in series with a **parasitic resistor R**. According **Kirchhoff's rule**, when a Voltage U_0 that is applied (via a switch) to the real inductance:

  $$U_0 = I \cdot R + L\frac{dI}{dt}$$

- The solution of this differential equation is:

  $$\mathbf{I(t) = \frac{U0}{R}\,[\,1\text{- }exp(\frac{-Rt}{L})]}$$

  after a <u>time constant</u> t = π = L/R

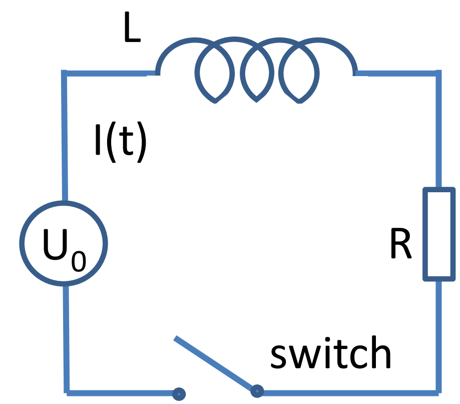
  $I(t{=}\pi) = \frac{U0}{R}\,[\,1\text{- }1/e] = \frac{U0}{R}\,[\,1\text{- }0.36788]$

  $I(t{=}\pi\,) = 0.6321\ U_0/R$ => (63.2% U_0/R)

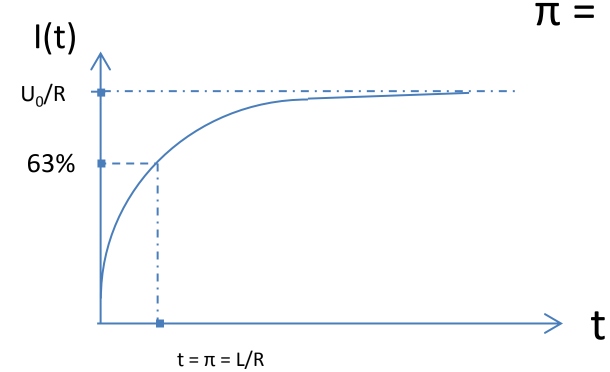
  after time t -> ∞

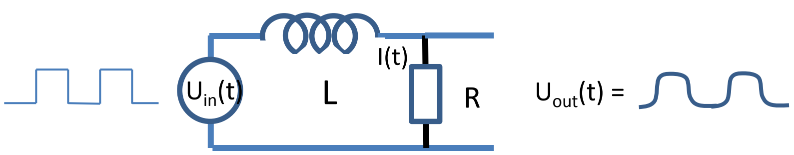
  $I(t\text{-}>\infty) = U_0/R$

- Interpretation: the reverse Voltage generated in an inductor by change of current effectively prevents the current from rising (or falling) much faster than the time L/R.
- All lumped elements possess some self-inductance, as well as some resistance, and thus have a **<u>finite rise-time</u>**. When powering up a circuit, the current does not jump up instantaneously to its steady-state value.
- Applying a rectangular shape AC voltage U_0(t) to an inductance means that the current through a resistance R cannot rise or fall faster than the time L/R, the leading and trailing edges of the signal get smoothed out.



π = L/R





Impedance

- A generalization of Ohms law **U = I · R** for alternating Currents (AC) is the notation:

  $\mathbf{U}_{AC} = \mathbf{I}_{AC}\,\mathbf{Z}$

  where **U,I** and **Z** are complex numbers

- Z is named **Impedance** as generalized resistance for AC electronics. Written as a complex number, **Z** is a vector in the X, R plane, R is the real part and X is the imaginary part.

  $\mathbf{Z} = \mathbf{R} + i\ \mathbf{X}$

  $|Z| = R^2 + X^2$ and $\tan(Ø) = X/R$

- With Eulers formula one can write complex numbers as harmonic functions

  $\mathbf{e^{is} = x + iy = \cos(s) + i \cdot \sin(s)}$

  with $\cos^2(s) + \sin^2(s) = 1$

- For a given oscillation frequency f, sinus waves are repetitive in time after a period time T = 1/f when =0, 2π, 4 π... Therefore by defining ω = 2π/T = 2π · f , one can write for any time t: **s = ωt** with the circular frequency ω to get the **very important complex notation of periodic functions:**

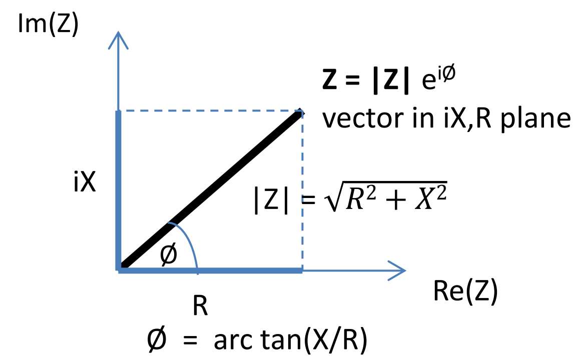
  $$\mathbf{e^{i\omega t} = \cos(\omega t) + i \cdot \sin(\omega t)}$$

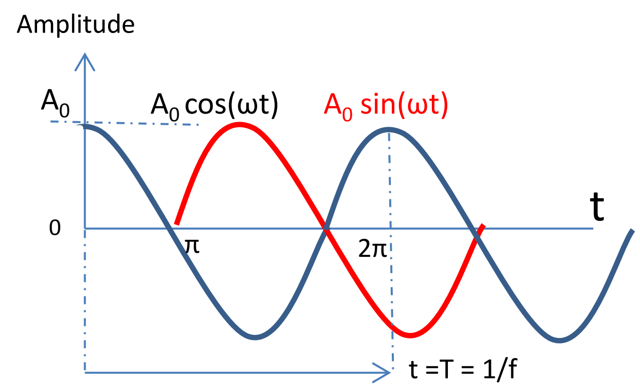
- In general, Voltage, Current and Impedances can be described as

  $\mathbf{U, I, Z = A_0}\, e^{i\,\omega t}$

  which implies that they are harmonic functions of a given frequency with Amplitude A_0

Complex Impedance





RMS Power

- the **instantaneous electric power P** in an AC circuit is given by **P=UI** where U and I are the instantaneous voltage and currents, assumed sinusoidal here. In general U and I have a **phase difference Ø**

  $U=U_0 \sin(\omega t)$ and $I=I_0 \sin(\omega t-Ø)$

  $P = U_0 I_0 \sin(\omega t) \sin(\omega t-Ø)$

- By integrating over one period T can show that the average electric power.

  $P_{av} = \frac{U_0 I_0}{2} \cos(Ø)$

  cos(Ø) is called **power factor**

- The **root-mean-square** (or **quadratic mean**) is a statistical measure of the magnitude of a time-varying quantity. For n discrete measurements x_i , the RMS is

  $RMS_{discrete} = \sqrt{1/n(x_1 2 + x_2 2 \; ... + xn^2)}$

  for continuously varying measures f(t), the RMS becomes:

  $RMS_{continuous} = \lim_{t\to\infty} \left(\sqrt{\frac{1}{T}\int_0^T f(t)^2 \, dt} \right)$

- For sinusoidal currents or voltages, integration over one period T is sufficient and it can be shown that

  $\mathbf{U_{rms} = U_0 / \sqrt{2}} \quad \mathbf{I_{rms} = I_0 / \sqrt{2}}$

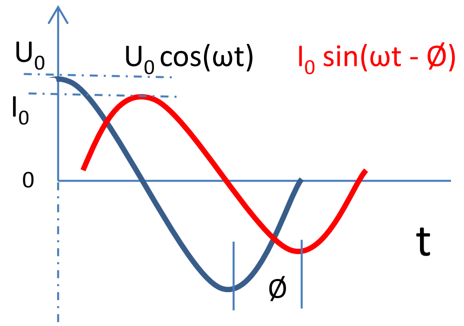
  $\mathbf{P_{average} = U_{rms} \cdot I_{rms} \cos(Ø)}$

  real resistors R , cos (Ø) =1 [Ø = 0 °]

  pure capacitors or inductances cos(Ø) = 0 [Ø= 90°]

- **On pure capacitors**, the **Current leads the Voltage by 90°**

U and I of an inductance



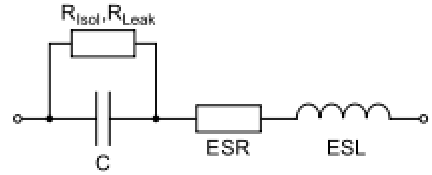
Example: 220V AC

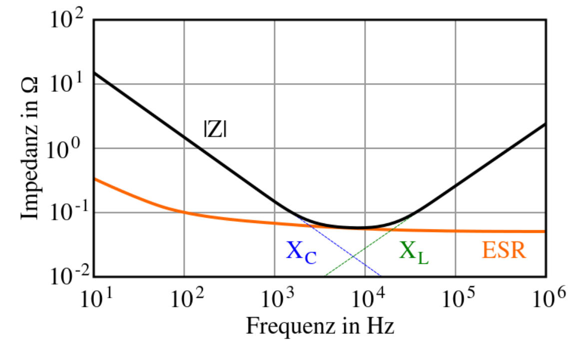
- 220 V is RMS
- peak amplitude is $U_0 = 220 \times \sqrt{2}$ = **311 V**
- The real energy (to be paid) is 220V x I_{rms} x cos(Ø)
- Motors or Neon lights integrate a phase shifting capacitor for cos(Ø) compensation

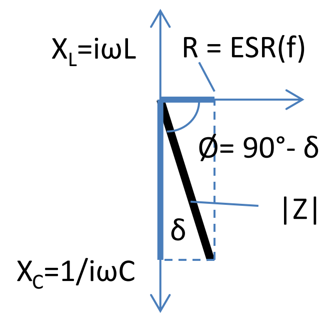
ESR and ESL

- Real capacitors consist of an ideal component C and parasitic components: **ESR** = **e**quivalent **s**eries **r**esistance, **ESL =** equivalent series inductance, $\mathbf{R}_{leak}$ = leakage resistor in parallel to C. The magnitude of the impedance is with $|Z|$ = $\sqrt{R^2 + (Xc + XL)^2}$ = $\sqrt{ESR^2 + (1/\omega C + \omega L)^2}$. The minimum of Z is in the frequency range where $\frac{1}{\omega C} = \omega L$. In this range **Z is equal to the ESR**.
- A parallel plate capacitor of superconducting electrodes separated by vacuum would give a **loss-free capacitance.** To improve the capacitance per volume an insulating material of higher dielectric constant needs to be inserted. The increase in C comes from the alignment of dipole charges (electrons, atoms or molecules) in the alternating electric field , involving motion in a viscous medium and, hence, frequency dependent **frictional losses**.
- This fact is normally expressed as an equivalent resistance **ESR** in series with the capacitor. Replacing the superconductors by normal materials introduces **other resistive** losses. This applies to all capacitors, whatever their dielectric. The loss factor is usually obtained by lumping all resistive losses into a **single ESR** value, obtained via measurements of either **capacitance plus tan δ** or **impedance Z_C plus phase angle ϴ.**
- For a **pure capacitor**, the waveform of the current leads the voltage by 90°. For any real capacitor the **phase angle ϴ** would be less than 90°, i.e. between that of a pure capacitor and a pure resistor. The difference between ϴ and 90° is the **angle δ**. The **cosine of ϴ is the power factor**: for low values, up to about 0.2 this is almost identical to **tan δ** which is easier to measure. (**100 x tan δ is the dissipation factor expressed as a percentage**). In approximation for small **R** and inductance **L**

  ESR = |Z| **tan δ ~ tan δ /ωC**
- **Industrial ESR values** are normally quoted for **100 kHz**, i.e. for $\omega = 0.63 \times 10^6$







Z = |Z| $e^{i\varnothing}$

$|Z| = \sqrt{R^2 + (Xc + XL)^2}$

$\cos(\varnothing) = \frac{R}{|Z|}$

$\tan\delta = ESR \cdot \omega C \sim \cos(\varnothing) = \frac{ESR}{|Z|}$

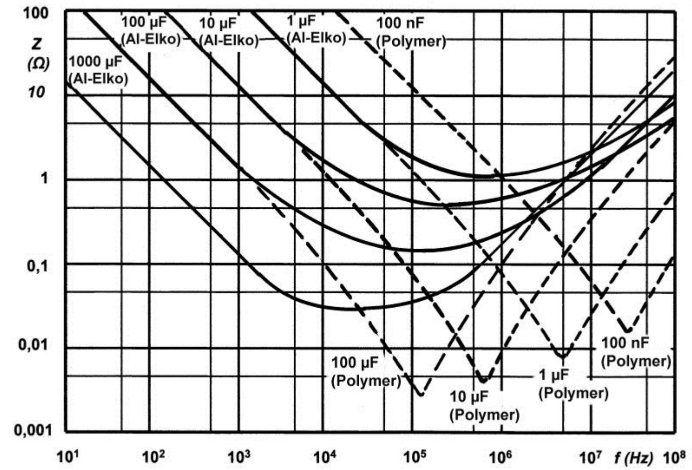
= Power factor

ESR = |Z| cos(Ø)

ESR measurement

- Industrial ESR figures are approximate (L=0) and valid for 100 kHz. ESR's are particularly important for electrolytic capacitors where high values up 20 Ohm are common. Polymer Elkos have a much lower ESR minimum than Aluminium Elkos and are therefore better for filtering low frequencies.
- A critical filter for high frequencies circuit requires calculation or measurement of true ESR at a given frequency.
- ESR meters are low cost devices for measuring ESR's at 100 kHz based on measurement of **C** and the loss angle **δ** in a range between 0.01 OHM and 20 OHM, for capacitances between 1 uF and 10.000 uF.

ESR

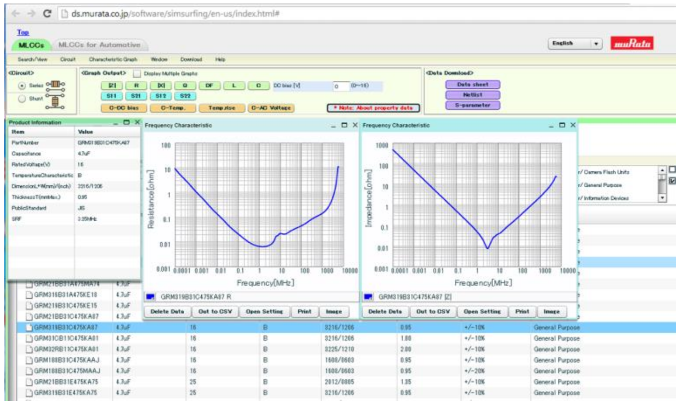


* http://de.wikipedia.org/wiki/Elektrolytkondensator

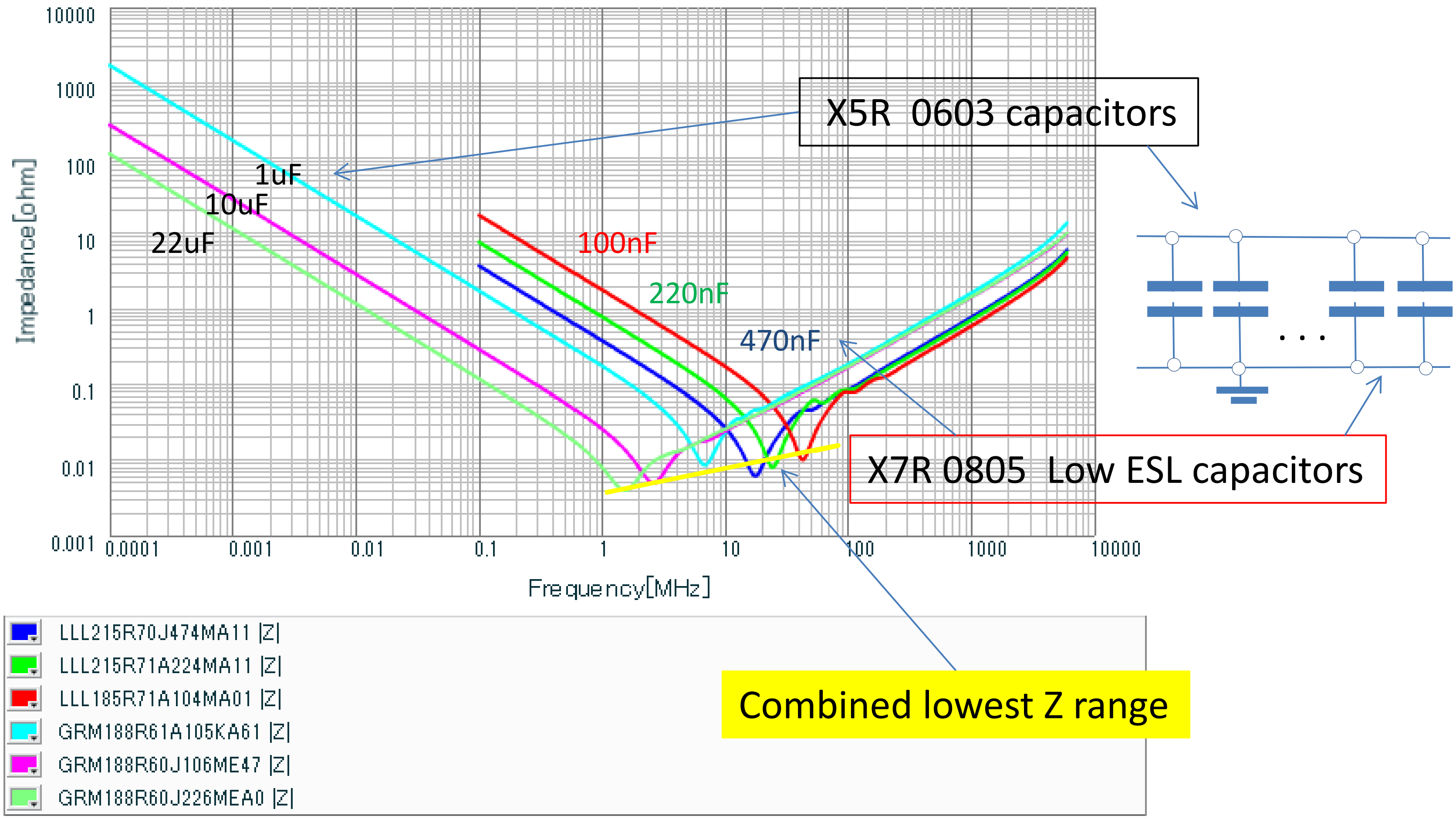
- *The free **WEB tool SimSurfing***

http://ds.murata.co.jp/software/simsurfing/en-us/index.html#

generates frequency-dependent ESR's and Z spectra of different types of capacitors. The picture shows a WEB calculation of ESR and Z of a specific 4.7 uF ceramic Murata capacitor.



Low-Z filters with MLCC capacitors



Lumped elements

- Electronic circuits can be described by 3 types of lumped elements

$$\mathbf{U_C = q \cdot \frac{1}{C}} \quad \text{=> capacitance C}$$

$$\mathbf{U_R = \frac{dq}{dt} \cdot R} \quad \text{=> resistance R}$$

$$\mathbf{U_L = \frac{d^2q}{dt^2} \cdot L} \quad \text{=> inductance L}$$

- Apply a voltage U(t) to 3 lumped elements L, R, C connected in series

$$U(t) = U_C + U_R + U_L = q \cdot \frac{1}{C} + \frac{dq}{dt} \cdot R + \frac{d^2q}{dt^2} \cdot L$$

- In complex notation

using $\frac{d}{dt} e^{i\omega t} = \mathbf{i\omega}\, e^{i\omega t}$ this becomes:

$$U = U_0\, e^{i\omega t} = q_0\, e^{i\omega t}\, [\frac{1}{C} + i\omega R + (i\omega)^2 L]$$

$$\mathbf{U} = [\frac{1}{C} + i\omega R + (i\omega)^2 L]\mathbf{q}$$

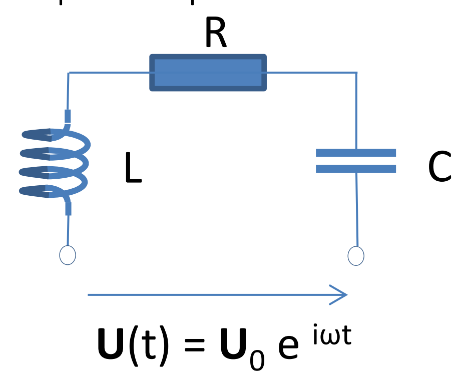
- With $I = dq/dt = i\omega q$ the complex number version of OHMs law is

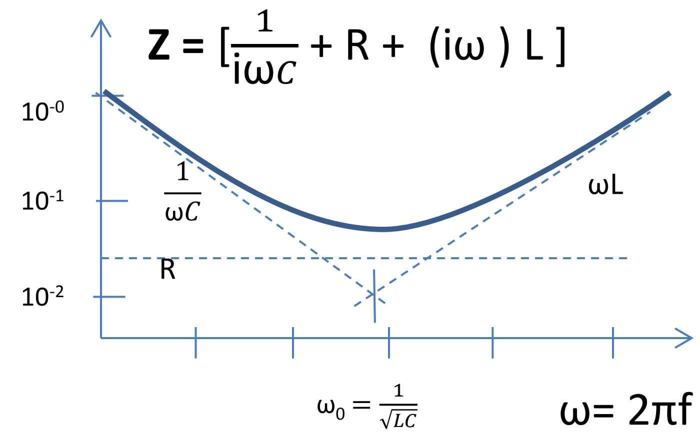
$$\mathbf{U = [\frac{1}{i\omega C} + R + (i\omega) L] \cdot I = Z \cdot I}$$

$$\mathbf{Z = [\frac{1}{i\omega C} + R + (i\omega) L]}$$

Z has a minimum at $\frac{dZ}{d\omega} = 0 \quad \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

Example of lumped elements





Impedance of lumped elements

- Impedances are in general written as complex numbers

  $Z = |Z| \exp(-\varnothing) = R + iX$

  $\varnothing$ is the phase between resistive R and L or C part

  Note: rules of number i: $i^{-1} = -i$, $i^0=1$, $i^1=I$, $i^2 = -1$
- The complex notation of impedance of 3 lumped elements for C, R, L

  Z = $[\frac{1}{i\omega C} + R + (i\omega)L]$

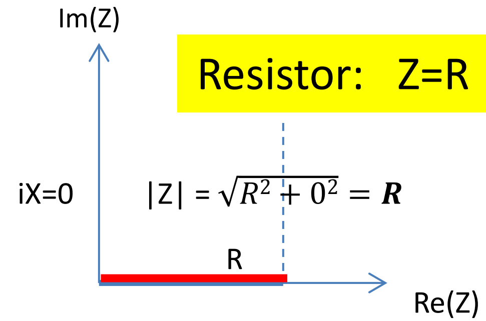
  identifies their individual impedances:
- The impedance of an ideal **resistor** is purely real and is referred to as a *resistive impedance*:

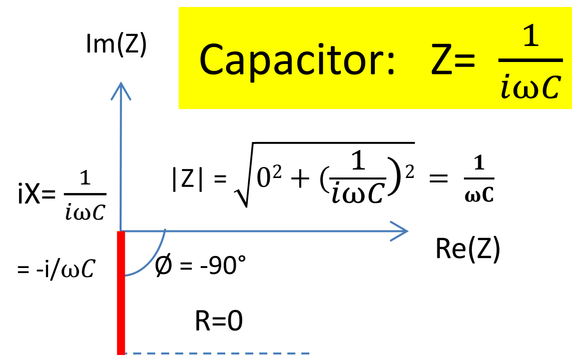
  $Z_R = R$

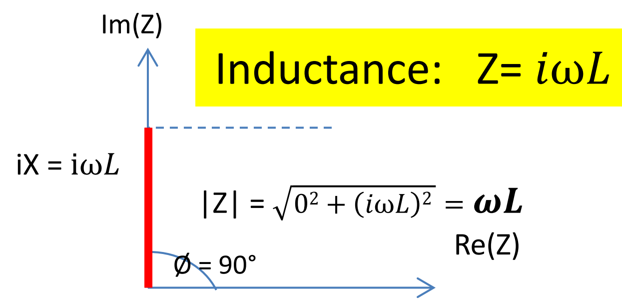
  In this case, the voltage and current waveforms are proportional and in phase
- Ideal **inductors** and **capacitors** have a purely imaginary *reactive impedance*:

  $Z_L = i\omega L$ and $Z_C = \frac{1}{i\omega C} = \frac{-i}{\omega C}$

  the impedance of inductors increases as frequency increases; the impedance of capacitors decreases as frequency increases







Serial and parallel impedances

- For serially connected Z_i the voltages U_i add up

  $\mathbf{U}(t) = \mathbf{U}_1(t) + \mathbf{U}_2(t) + \ldots U_i(t) = \mathbf{I}(t)\,(\mathbf{Z}_1 + \mathbf{Z}_2 + ..Z_i)$
  because the current $I(t)$ is the same

  $\mathbf{Z = Z_1 + Z_2 + \ldots Z_i}$

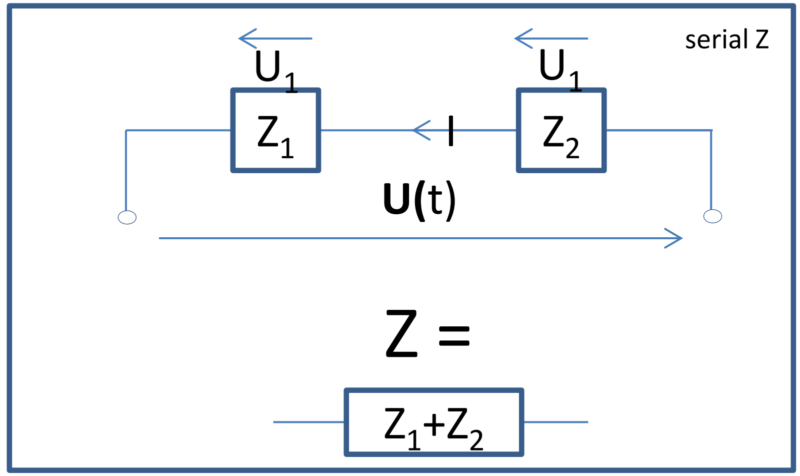
- For parallel connected Z_i the currents add up and the Voltage across each Z is the same.

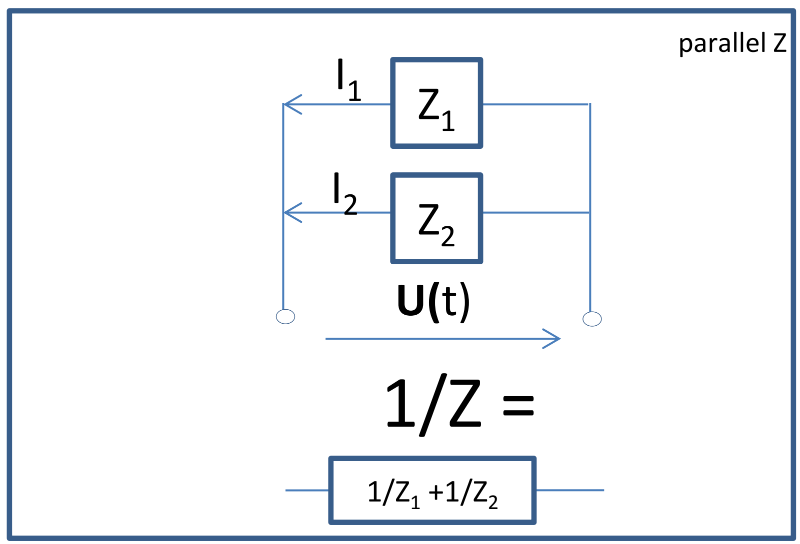
  $\mathbf{I}(t) = \mathbf{I}_1(t) + \mathbf{I}_2(t) + \ldots I_i(t)$

  $= \frac{U(t)}{Z1} + \frac{U(t)}{Z2} + \ldots \frac{U(t)}{Zi}$

  $\mathbf{I}(t) = U(t)\,[\frac{1}{Z1} + \frac{1}{Z2} + \ldots \frac{1}{Zi}] = U(t) / Z$

  $\mathbf{1/Z = [\frac{1}{Z1} + \frac{1}{Z2} + \ldots \frac{1}{Zi}]}$





RC low pass

- With $Z_R = R$ and $Z_C = \frac{1}{i\omega C}$ the voltage divider formula for **complex impedances** is:

  $$U_{out} = U_{in} \frac{Z_c}{Z_c + R} = U_{in}\left[\frac{\frac{1}{i\omega C}}{\frac{1}{i\omega C} + R}\right] = U_{in}\left[\frac{1}{1 + iR\omega C}\right]$$

- The **Bode-Diagramm** is a logarithmic plot of the mean voltage ratio $A(\omega_0) = \left|\frac{U_{out}}{U_{in}}\right|$ versus frequency. The diagram has two asymtotes: at low frequencies the impedance of C becomes very large and hence A-> 1 for DC (frequency $f = 0$). With increasing frequencies, A decreases with 6 dB/Oktave (or 20 dB/decade). The cutoff frequency ω_0 corresponds to the intersection of the asymtotes:

  $$A(\omega_0) = \left|\frac{U_{out}}{U_{in}}\right| = \frac{1}{\sqrt{1+(R\omega C)^2}} = \frac{1}{\sqrt{1+1^2}} = 1/\sqrt{2} = 0.707$$

  at ω_0 , the half power point $\left|\frac{U_{out}}{U_{in}}\right|^2$ is reached

- **With $Z = |Z|\, e^{(i\varnothing)} = R + iX = R - i/\omega C$**

  $\tan(\varnothing) = \tan(-R/\omega C) = -\tan(R/\omega C)$

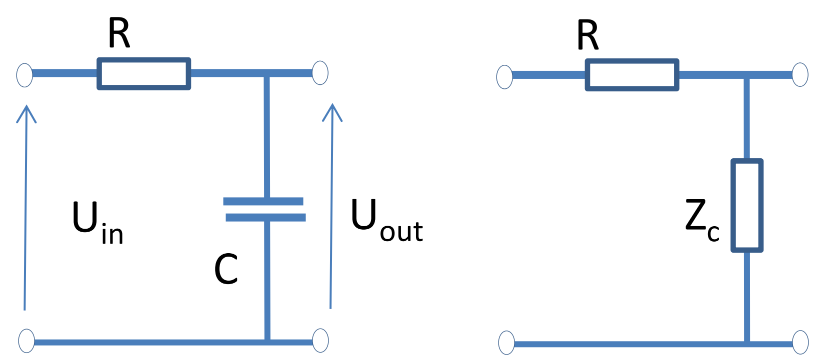
  the phase relation between the real and imaginary part $\varnothing = -\arctan(R/\boldsymbol{\omega C})$

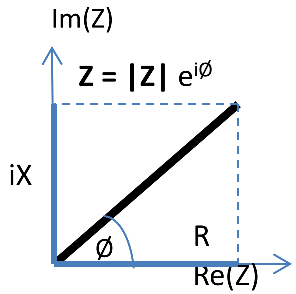
- **At $\varnothing$ = 45 degree**

  $iX = R = 1/\omega C$ hence $\boldsymbol{\omega}_{45°} = 1/RC = \omega_0$

  corresponding to the cutoff frequency

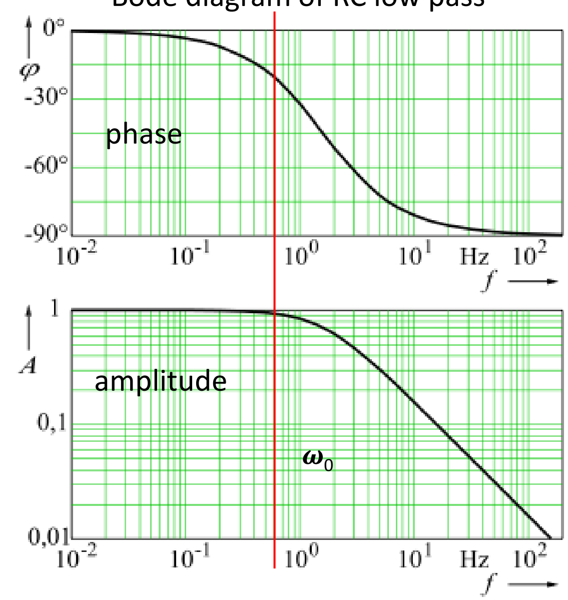
RC low-pass = voltage divider Z





$\varnothing = \arctan(X/R)$

Bode diagram of RC low pass



RC high pass

- Equally, the RC high-pass is calculated like a voltage divider.
  With $Z_R = R$ and $Z_C = \frac{1}{i\omega C}$

  $$U_{out} = U_{in}\,\frac{R}{Z_c + R} = U_{in}\left[\frac{R}{\frac{1}{i\omega C} + R}\right]$$

- At the cutoff frequency (also called 3dB point)

  $\omega_0 = 1/RC$ => ($f_0 = \frac{1}{2\pi\, RC}$)

  $$A(\omega_0) = \left|\frac{U_{out}}{U_{in}}\right| = \left|\frac{R}{\frac{RC}{iC} + R}\right| = \frac{R}{|R+R|} = \frac{1}{\sqrt{1^2+1^2}} = 1/\sqrt{2}$$

  In **deziBel units :**

  $\mathbf{P}_{\mathbf{dB}} = \left\{20\,log\,\frac{U_{out}}{U_{in}}\right\}$ $[\,dB\,]$ for ratios of Voltages

  $\left\{20\log\frac{U_{out}}{U_{in}}\right\} = \left\{20\log\left(1/\sqrt{2}\right)\right\} =$ **- 3dB**

  => ω_0, the CR network
  reduces the input amplitude by -3dB

- In general for RC or CR networks:

  Cutoff frequency corresponds to **-3 dB** attenuation

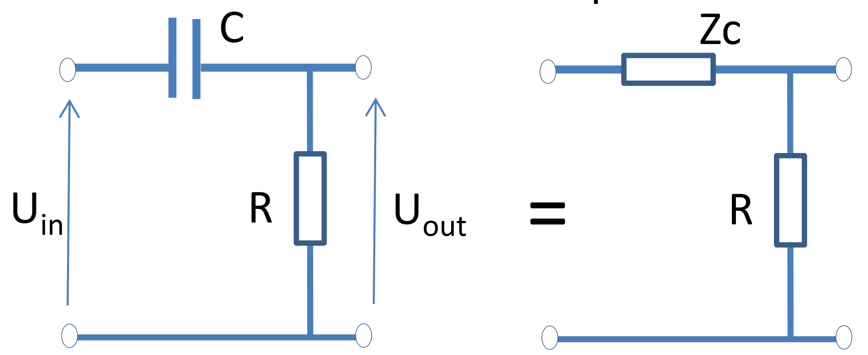
  CR or RC attenuation/**octave** (factor 2) is **-6dB**

  CR or RC attenuation /**decade** (factor 10) is **-20 dB**

- The RC or CR combination are named “low pass” and “high-pass” of 1st order. The 4th order RC or CR networks would be characterized by a Bode diagram of 4 x (-20 dB) = -80 dB per decade.

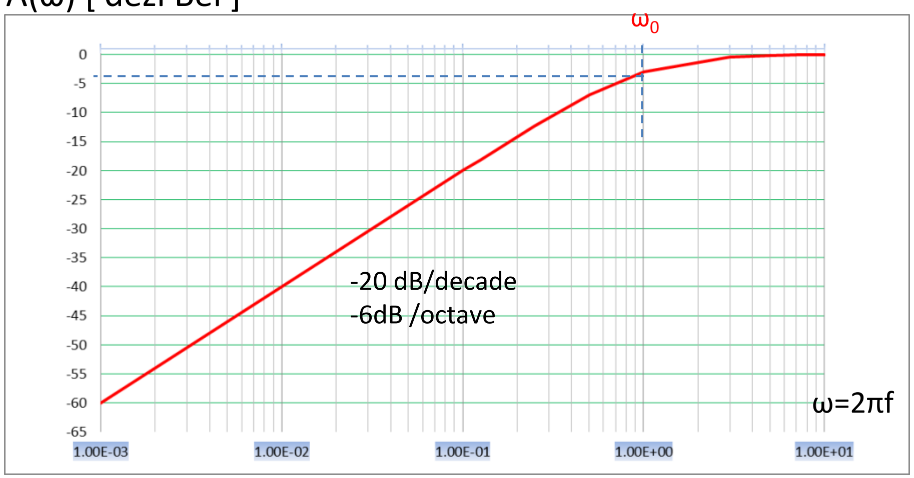
CR high-pass

voltage divider
= impedance divider



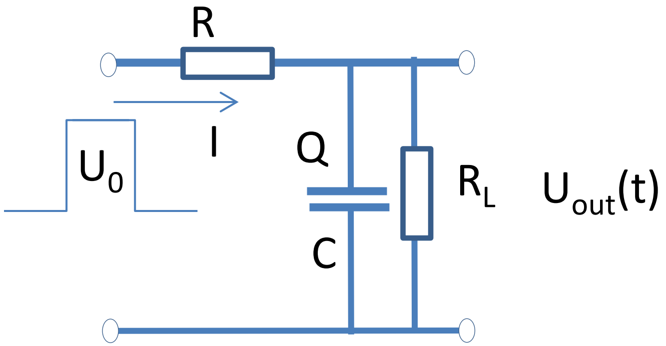
Bode Diagram (CR high pass)

A(ω) [dezi Bel]



Step pulse on loaded RC low pass

In the real world, a bandpass is connected to a termination resistor at the output. The influence of such a load resistor R_L on a low pass filter (R_L parallel to C) can be analyzed by the application of Kirchhoffs rules for the simple case of a step pulse input with amplitude U_0



$t > t_0$ $\quad U_o = I \cdot R + Q/C$

$dQ/dt + U_{out}/R_L = I$

take 1st derivative of U_o equation :

$0 = R \cdot dI/dt + 1/C\ (I - U_{out}/ R \cdot L)$

This results in a differential equation:

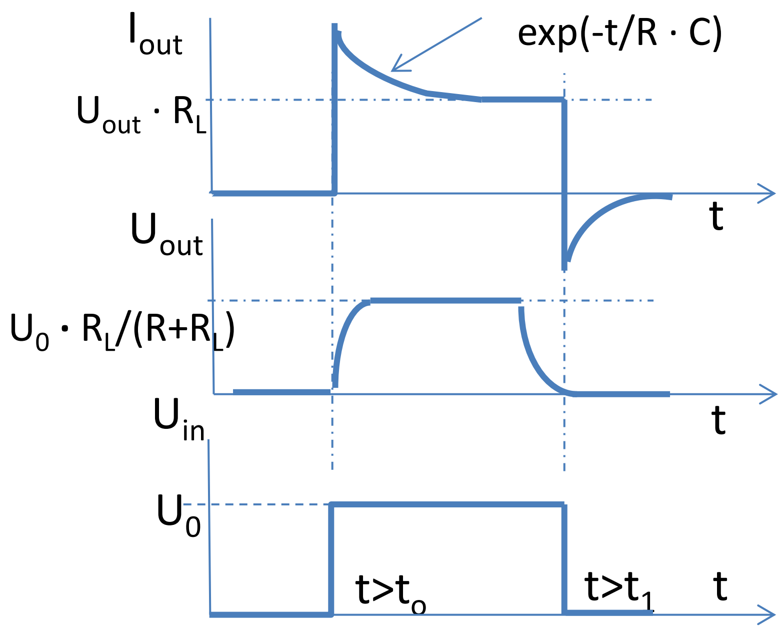
$$dI/dt + I/RC = U_{out} /(R \cdot C \cdot R_L)$$

With $I_0 = U_0/(R+R_L)$ the solution for the current

$$I = U_{out} / RL + I_0 \exp(-t/R \cdot C)$$

Correspondingly the output Voltage

$$U_{out} = R_L/(R+R_L) \cdot U_0 [1-\exp(-t/R \cdot C)]$$



RC transients

- The **RC high-pass** is frequently used to filter a low frequency signal from a high frequency ripple. The **RC low-pass** is used to remove a high frequency signal from a low frequency one.

- In pulse electronics **voltage steps** are very common. When sending a step through a low-pass it gets “**integrated**” in the sense that fast changes get smoothed out. This step behaviour can be derived :

$U_0 = IR + Q/C$

$\frac{dU_0}{dt} = 0 = R\,\frac{dI}{dt} + \frac{dQ}{dt}\,\frac{1}{C}$; $dQ/dt = I$

$\frac{dI}{dt} = -\,\frac{1}{RC} \cdot I$

with the rule that for an e -function

$\frac{d}{dt}\,[\exp\ u(t)] = \frac{d}{dt}\,u(t) \cdot [\exp\ u(t)]$

the solution of the above differential equation is

$I = I_0 \exp(\frac{-t}{RC})$

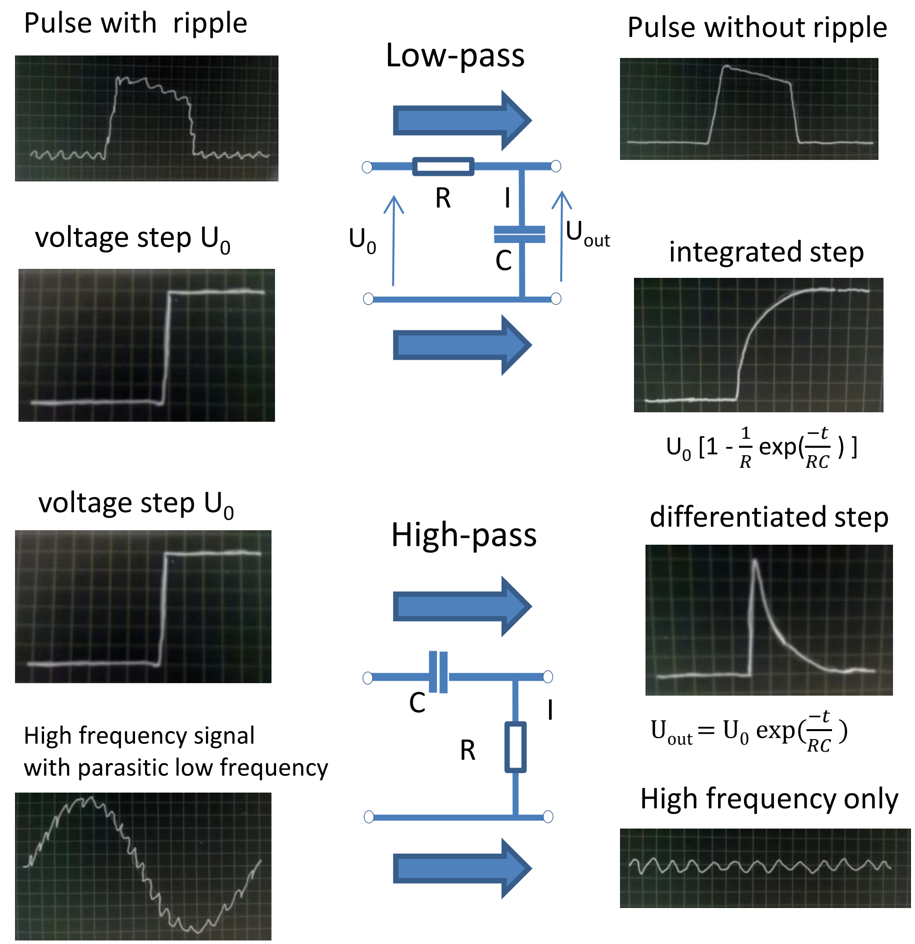
$\mathbf{U_{out} = U_0 - IR = U_0\,[1 - \frac{1}{R} \exp(\frac{-t}{RC})]}$

- Inversely, a voltage step gets “**differentiated**” by a high-pass in the sense that only fast changes are transferred

$U_{out} = IR = \frac{dQ}{dt}\,R = RC\,\frac{d(Uout - Uo)}{dt}$

analogue to the above:

$\mathbf{U_{out} = U_0 \exp(\frac{-t}{RC})}$



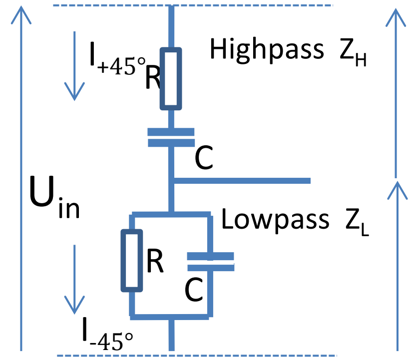
Wien Bridge

- The combination of a low and high pass with the same RC constant as a complex divider $\frac{Z_L}{Z_H+ZL}$ is basis of the Wien bridge. At the cutoff frequency $\omega_{45°}$ = 1/RC = ω_0 the current in the high pass is 45° advanced, and in the low pass lags by 45°
- At ω_0 the phase shifts of the current I cancel such that U_{in} and U_{out} have the same phase and $U_{out} = k\, U_{in}$ with

$$k = \frac{U_{out}}{U_{in}} = \frac{Z_L}{Z_H+ZL} = \frac{\left(R+\frac{1}{i\omega C}\right)^{-1}}{\left(R+\frac{1}{i\omega C}\right)+\left(R+\frac{1}{i\omega C}\right)^{-1}}$$

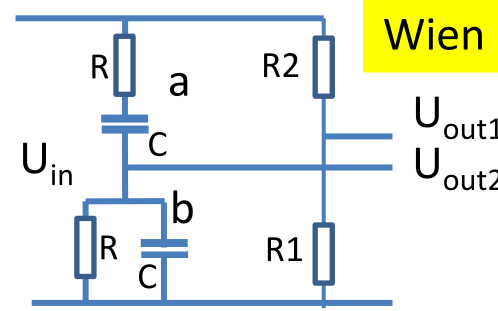
$$k = \frac{1}{3+i\left(\omega RC - \frac{1}{\omega RC}\right)}$$

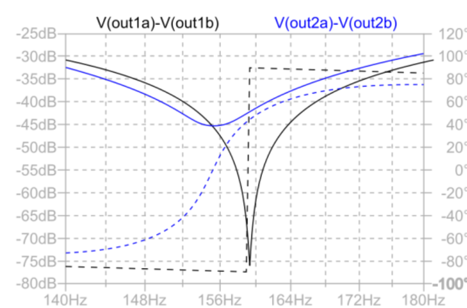
- A the frequency ω_0 the phases cancel and k becomes a real number (i=> 1) with $\left(\omega RC - \frac{1}{\omega RC}\right) = 0$, hence
- at ω_0, k = 1/3
- With a real resistor voltage divider of R1/(R2+R1) = 1/3, or $R_2 = 2R_1$ added as in a parallel Wheatstone bride, the Wien-Bridge has a cusp-like resonance behaviour for the differential output voltage U_{out1}-U_{out2} at $\omega_0 = 1/RC$.
- In general the Wien bridge is balanced when RC is equal in both filters a and b: $\omega_0 = \sqrt{1/RaCaRbCb}$. In this case the resistors R2 and R1 need to obey to the relation $\frac{Ca}{Cb} = \frac{R1}{R2} - \frac{Rb}{Ra}$
- Using non-ideal tolerances (5%) for R and C the resonance is less pronounced and shifted in frequency.



$U_{out} = k\, U_{in}$
at ω_0:
$U_{out} = 1/3\, U_{in}$

Wien bridge





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* http://de.wikipedia.org/wiki/Wien-Robinson-Br%C3%BCcke

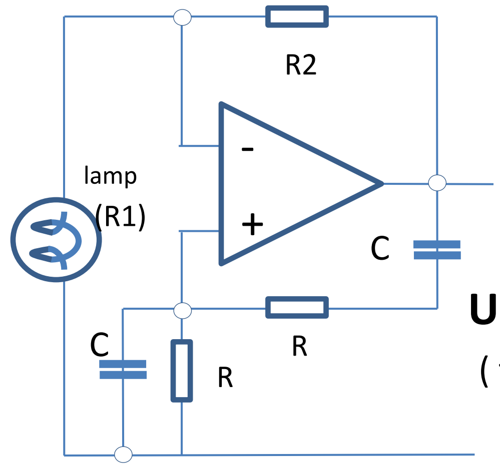
Sinus waveform

- The circuit shows the first stable and low distortion sinus oscillator of W.R. Hewlett (founder of Hewlett-Packard Co). It is a Wien bridge combined with a difference Amplifier. Negative feedback with increasing amplitude of the oscillation increase linearity. The Wien type High/Low pass filter defines the frequency at the zero phase shift point. The incandescent lamp increases resistance with temperature (which increases with Amplitude), and the increased resistance reduces the amplitude.
- The lamp was later replaced later by circuits that mimic the negative feedback for increasing amplitudes.
- The RMS Voltage of the sinus waveform is:

$$U_{RMS}^2 = \frac{1}{T}\int_0^T U_0^2\{\sin(\omega\, t)\}^2 dt$$

$$= \frac{U_0^{\ 2}}{T}\int_0^T \frac{1}{2}\{1-\cos(2\omega\, t)\}dt = \frac{U_0^{\ 2}}{2T}[T - \frac{\sin(2\omega T}{2\omega}]$$

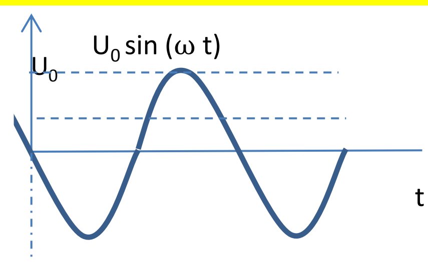
$$\mathbf{U_{RMS}} = \frac{U_0}{\sqrt{2}}$$



$$\mathbf{U}(t) = \mathbf{U}_0 \sin(2\pi f\ t)$$

$$(f = \frac{1}{2\pi RC} < 20\text{ kHz})$$

Sinus Oscillator as described by W.R. Hewlett in his master thesis 1939, Stanford Univ. Palo Alto 1939
" A new type Resistance Capacity Oscillator"



$$U_{RMS} = \frac{U_0}{\sqrt{2}}$$

Square Waveform

- A simple square wave generator is a comparator amplifier with positive feedback from a frequency-defining RC lowpass.
- Initially when the difference Δ of the (+ -) inputs of the differential amplifier with gain **A** is positive, the output of the amplifier, with $U_{out} = \Delta \cdot \mathbf{A}$ is at its maximum U_0.
- In the (+) branch, the voltage divider keeps the (+) input at the fraction Γ of the output voltage U_{out}. In the (-) branch the current from the output charges the capacitor C until it reaches the same voltage as the (+) input. At this point, the differential voltage Δ becomes negative and the output voltage switches to $(-U_0)$.
- Square waves with **duty cycle** $\mathbf{D = D_T/T}$ = 0.5 are a special case of the the rectangular waveform with any duty cycle D (0<D<1)
- The RMS of a rectangular waveform is

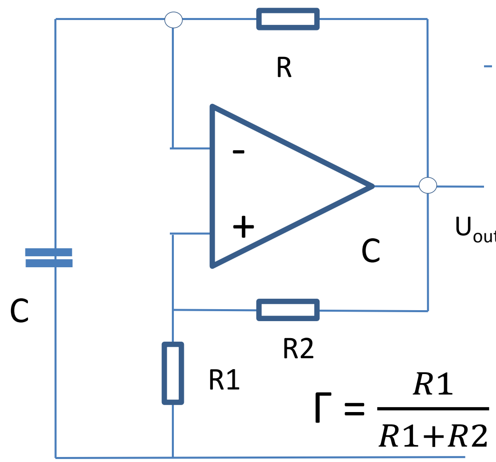
$$U_{RMS}^2 = \frac{1}{T}\int_0^{D_T} U_0^2 dt = \frac{U_0^2}{T} D_T = D\, U_0^2$$

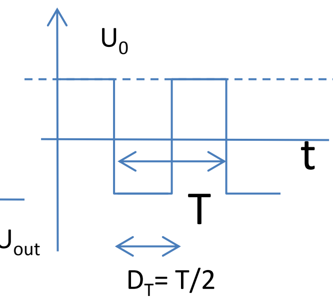
$$\mathbf{U_{RMS} = U_0\sqrt{D}}$$

- The square wave with D=0.5 is mathematically a superposition of a sine wave with frequency f = 1/T and an infinite series of odd-numbered multiples (3f, 5f, 7f…) of decreasing amplitude. (In practise, a sharp risetime of the square means a large number multiples up to the very high frequency range)

Note: a low-distortion Sinus generator can also be made from square wave generator, succeeded by an n-th order filter which suppress all multiples above the ground wave.

Square wave generator

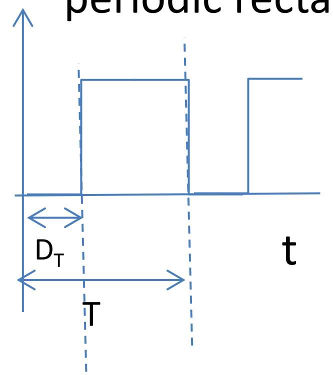




$$\Gamma = \frac{R1}{R1+R2}$$

$$T = 2RC\left[\frac{1+\Gamma}{1-\Gamma}\right]$$

periodic rectangular wave



$$\mathbf{U_{RMS} = U_0\sqrt{D}}$$

Sawtooth waveform

- A linear voltage ramp is created by a current source loading a capacitor

  I_s = const = $C\,\frac{dU_c}{dt}$

  $U_s = \frac{I_s}{C}\,t$

- A practical current source is limited to some maximum voltage. The example shows a precision current source using a 10V reference Voltage source (REF102) and a OPA with feedback -1.

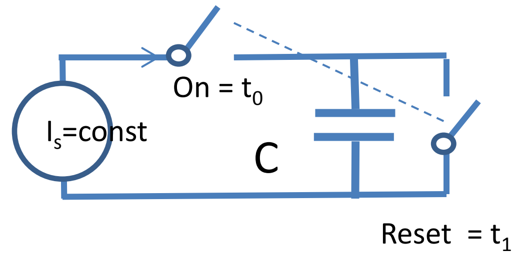
  I_s= 10V/R (R>1 kOHM)

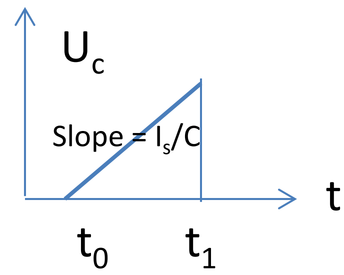
- Using a capacitor as load and electronic switches (MOSFETs) to charge and discharge the capacitor periodically, a periodic sawtooth waveform U(t) = $\frac{U_0}{T}$ (t-t_0) is generated.
- The RMS Voltage of the sawtooth is:

  $U_{RMS}{}^2 = \frac{1}{T}\int_0^T \left\{\frac{U0}{T}\,t\right\}^2 dt = \frac{U0^2}{3T^3} t^3 (0|T)$

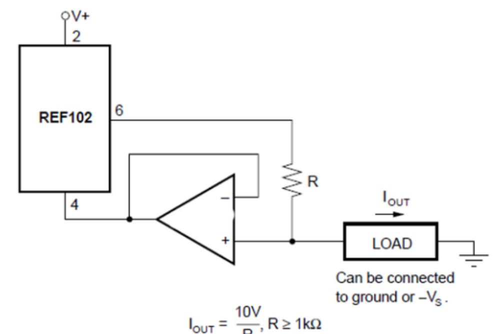
  $\mathbf{U_{RMS}} = \frac{U0}{\sqrt{3}}$

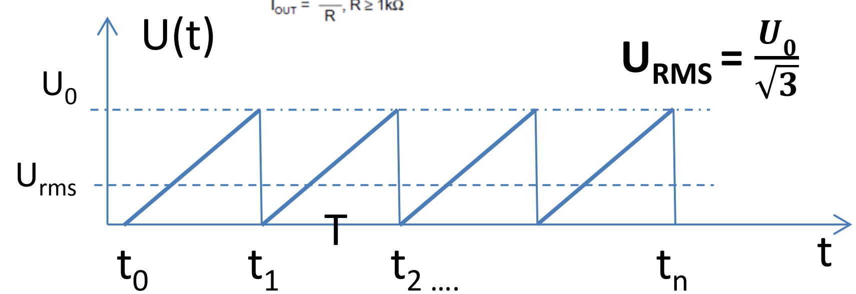
- Sawtooth waveforms, together with Sinus and Square waveforms are the most important waveforms for electronics and therefore available on most waveform generators.





A practical current source* max 10V





* Texas Instruments REF102 datasheet

Appendix part 1

Scales and Units

Prefix	Symbol	10^n	English word
yotta	Y	10^{24}	septillion
zetta	Z	10^{21}	sextillion
exa	E	10^{18}	quintillion
peta	P	10^{15}	quadrillion
tera	T	10^{12}	trillion
giga	G	10^{9}	billion
mega	M	10^{6}	million
kilo	k	10^{3}	thousand
hecto	h	10^{2}	hundred
deca	da	10^{1}	ten
		10^{0}	one
deci	d	10^{-1}	tenth
centi	c	10^{-2}	hundredth
milli	m	10^{-3}	thousandth
micro	µ	10^{-6}	millionth
nano	n	10^{-9}	billionth
pico	p	10^{-12}	trillionth
femto	f	10^{-15}	quadrillionth
atto	a	10^{-18}	quintillionth
zepto	z	10^{-21}	sextillionth
yocto	y	10^{-24}	septillionth

Note: The decimal scales are mankind's convenient choice due to our 10 fingers . An Extra-terrestic population would probably use a different base for their mathematics. For convenience working with binary Computers we also use dual and hexadecimal bases

Unit	Symbol	mks unit	equals
absorbed dose	Gy	gray (J/kG)	100 rad
activity	Ba	Bequerel (1/s)	37 $.10^{6}$ millicurie
capacitance	C	1 Farad (F) = 1 (C/V)	1 s/OHM
charge	q	1 Coulomb (C)	6.241 10^{18} electrons
current	I	1 Ampere (A)	1 C/s
electric field	E	(V /m)	
electric potential	U	Volt (V)	1 kg.m^2/ A.s^2
energy	W	1 Joule (J) = 1 Ws	6.2415 10^{18} eV
" "		1 eV	2.41768 10^{14} Hz (1)
" "		1 calorie	4.184 joule
frequency	f	1 Hertz (Hz)	1/s
force	F	1 Newton (N)	1 kg.m/s^2
inductance	L	1 Henry (H)	1 kg.m^2/A^2.s^2
length	l, d	1 meter (m)	39.3699 inch
magnetic field	B	1 Tesla (T)	10 kiloGauss
luminous flux	lm	1 lumen (Cd.sr)	
mass	m	1 kilogram (kg)	2.20462 lb (pound)
power	P	1 Watt (W)	1.34 10^{-3} hp (PS)
resistance	R	1 Ohm ()	1 kg.m^2/A^2.s^2
temperature	T	Kelvin (K)	
time	t	second (s)	
	(1) corresponding wavelenght of 1 eV => 1240 nm (fλ=c)		

Note: Units represent an agreement of one choice, also called a standard. But any other unit choice work as well , one just needs to adapt the constants in the equations which describe the laws of nature. The laws of physics however are the same for us as for an extra-terrestic physicist. Maybe they discovered more, or less laws than we.

Pseudo Units (Ratios)

- **Gain** <u>(ratio of voltage)</u>

characterises Voltage amplification

$$A = \frac{U1\ [Output\ signal]\ (V)}{U2\ [Input\ signal]\ (V)}\ \text{[number]}$$

Gains of succeeding stages multiply:

$$\mathbf{A_{total}} = \boldsymbol{A1 \times A2 \times A3}\ \ \textbf{[number]}$$

- **Signal over noise** (ratio of voltage)

measures signal level referenced to the noise level

$$S/N = \frac{signal\ rms\ (V)}{noise\ rms\ (V)}\ \ [\text{number}]$$

- **Gain** <u>(ratio of power)</u>

characterises Power amplification

$$P = \frac{P1\ [Output\ power]\ (W)}{P2\ [Input\ power]\ (W)}\ \ \text{[number]}$$

power gain may however scale over many decades and is therefore expressed as the log of the power ratios

$$P_{dB} = \left\{10 \log \frac{P1}{P2}\right\}\ \ [\ deciBel\ = dB\ ]$$

with $P \sim U^2/R$ $P_1/P_2 = U^2_1/U^2_2$

$$P_{dB} = \left\{20 \log \frac{U1}{U2}\right\}$$

dB gains of succeeding stages add:

$$\mathbf{P_{total}} = \boldsymbol{dB_1 + dB_2 + dB_3}\ \ \textbf{[dB]}$$

- **Signal over noise** (ratio of power)

$$\mathbf{S/N} = \left\{\boldsymbol{20\ log \frac{signal\ rms\ (V)}{noise\ rms\ (V)}}\right\}\ \ [\ dB]$$

Constants

Quantity	Value	SI unit	Symbol	Comment
electron charge	$1.60217733 \times 10^{-19}$	C	-e	
permittitivity of vaccum	$8.8541878 \times 10^{-12}$	F/m	ε_0	$\frac{1}{36\pi\, x\, 109}$ F/m
permeability of vaccum	$4\pi \times 10$-7	H/m	μ_0	
speed of light	*299792458*	m/s	c	
Planck Constant	$6.6260755 \times 10^{-34}$	J s	h	
wavelength of 1 eV	1.24×10^{-6}	m	λ_0	
Coulomb's constant	8.987×10^{9}	Nm^2/C^2	Ke	$1/4\pi\varepsilon_0$
Gravitational acceleration	*9.81*	m/s^2	g	
Boltzman constant	$1.3806488(13) \times 10^{-23}$	J/K	k_B	
1 Angstrom	10^{-10}	m	Å	
1 degree	*0.0174 radian*	°	π/180°	

Playing with electrical units

- An electric current I of 1 Ampere transports 1.6 x 10^{19} electrons per second.
- The number of stars in the Universe is > 10^{22}

  1 Ampere over 1000 seconds corresponds to 1.6 x 10^{22} electrons
- A car Battery of 50 Ah delivers 50 C/s x 3600 s= 180.000 Coulomb during 1 h. These are 1.12 x 10^{24} free electrons from a chemical electrolyse and generating 12 V electric potential between the Minus and the Plus pole.
- The energy stored in a 50 Ah battery of 12 V is

  50Ah x 12V =600 Watt h = 600 x 3600 Watt s = 2.16 x 10^6 Joule.

  With the well-know formula for the potential energy U of a mass M

  (U= Mgh) lifted by height h=1 m the potential energy of a 100 kg man climbing

  up 1 m is (mgh = 100 kg x 9.81 m/s^2 x 1m) = 981 joule. With the energy of the battery he can theoretically get lifted 2200 m high. In real, much of the energy gets lost by heat, an electric motor with 80% efficiency would get him up to 1760 m
- A current of 1 A delivered by a 12 V battery corresponds to an electric power of

  1A x 12V = 12 Watt = 12 J/s corresponding to a power of 12 x 6.24 10^{18} eV / s.

  This is 7.4 10^7 TeV ! But??? TeV ?? LHC ???
- The LHC generates colliding protons of 7 TeV energy each, which is 1.2 10^{-6} Joule. But each bunch contains 1.15 x10^{11} protons, that makes 1.29x10^5 Joule per bunch. And there are 2808 bunches, that makes a stored beam energy of ~ 360 MegaJoule ! The energy of 171 car batteries of 50 Ah.
- Old color TVs with CRT tube apply a voltage of 27 kVolt to accelerate electrons from a hot cathode wire to impinge on the screen and generate color dots for the pictures. We spent a lot of time at home in front of 27 keV electron accelerators.