Kinematic Constraints and MT2

-Calculational algorithms for MT2

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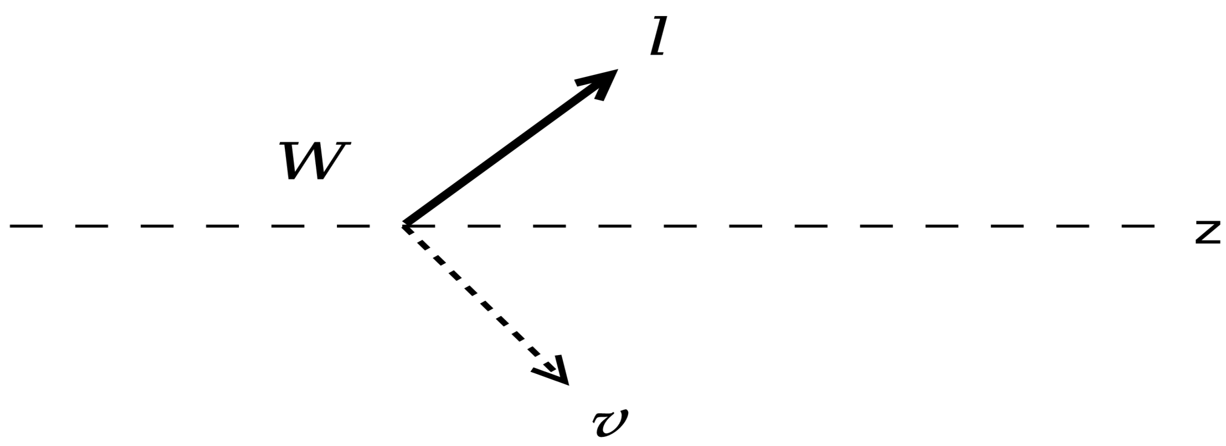
Outline

- Introduction to MT2
- MT2 as kinematic constraints
  - The bisection method for calculating MT2
- Extensions
- Conclusion

Transverse mass (MT)

- Transverse mass



$$\alpha_\ell = (E_T^\ell, p_x^\ell, p_y^\ell), \quad \alpha_\nu = (E_T^\nu, p_x^\nu, p_y^\nu)$$

$$E_T^\ell = \sqrt{(p_x^\ell)^2 + (p_y^\ell)^2 + m_\ell^2}, \quad E_T^\nu = \sqrt{(p_x^\nu)^2 + (p_y^\nu)^2 + m_\nu^2}.$$

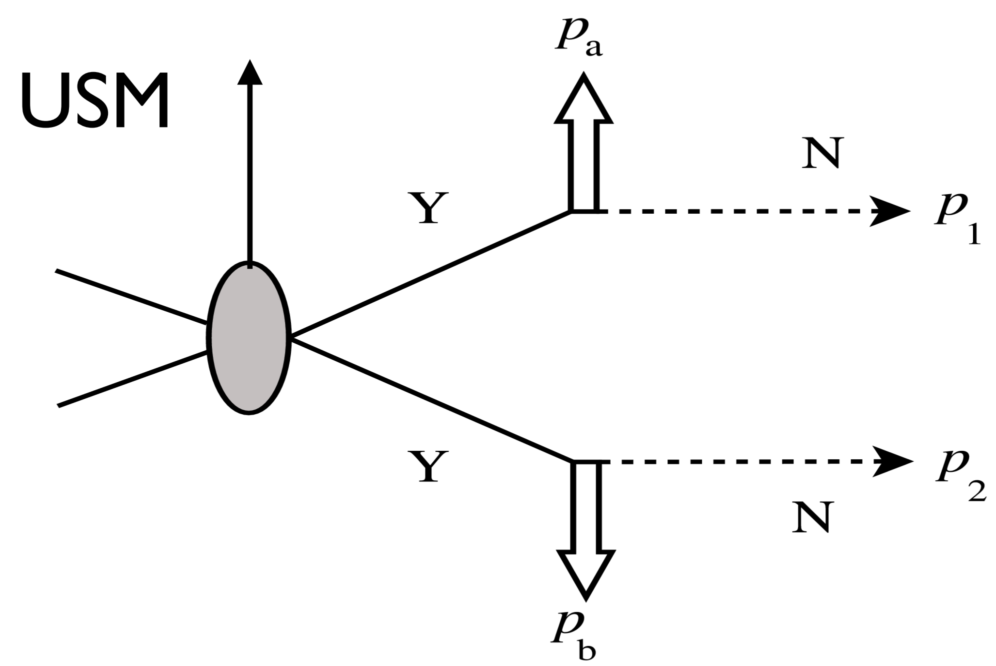
Transverse mass defined by

$$M_T^2 = (\alpha_\ell + \alpha_\nu)^2.$$

* MT is the ‘mass’ in 2+1 dimensions

From MT to MT2

Lester & Summers, 1999; Barr, Lester &Stephens, 2003



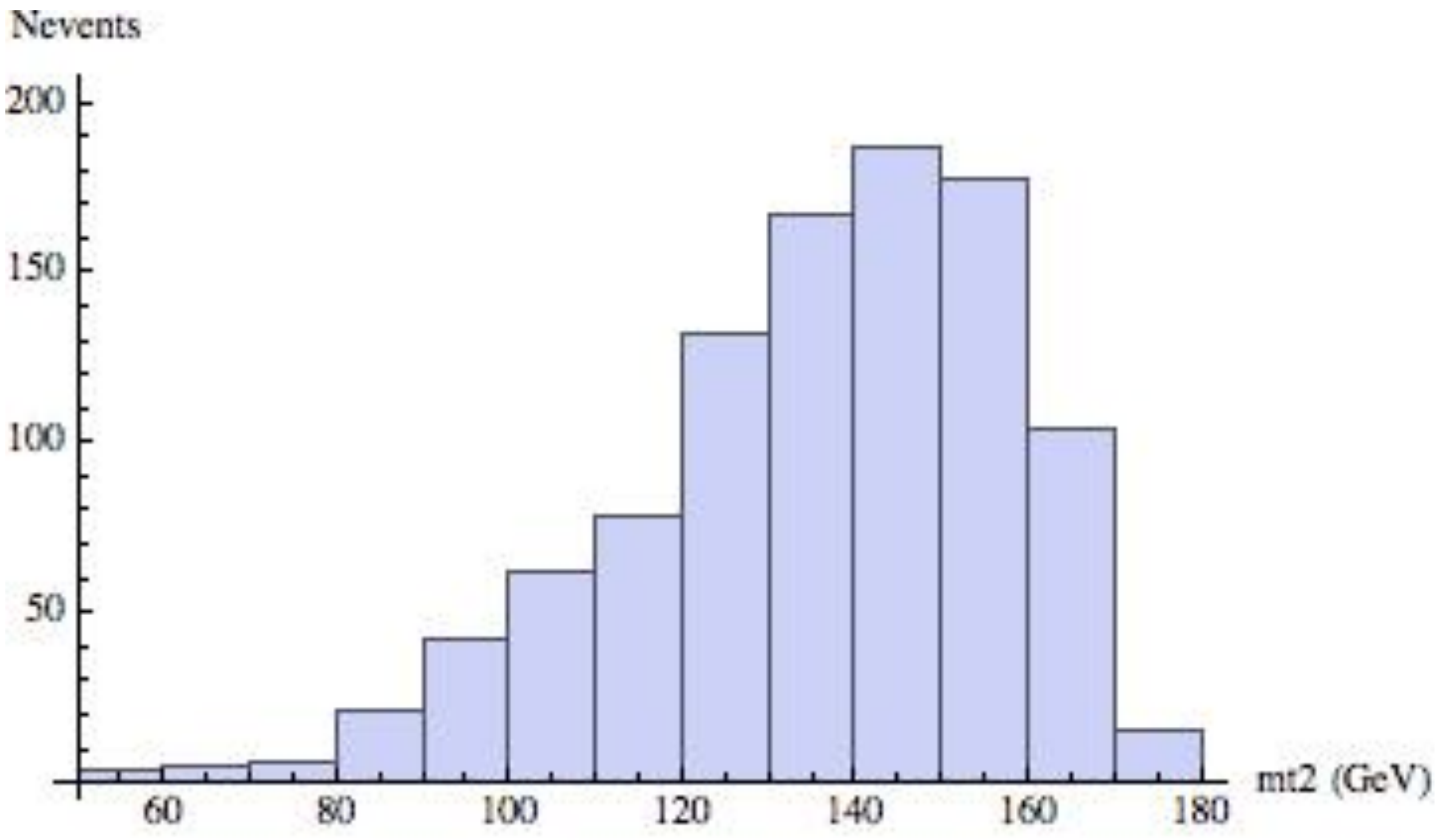
- Trial N mass, μ_N
- Consider all partitions of $\not{p}_T = \mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)}$.

$$M_{T2}(\mu_N) \equiv \min_{\mathbf{p}_T^{(1)} + \mathbf{p}_T^{(2)} = \not{\mathbf{p}}_T} \left[\max\{ M_T(1, a; \mu_N), M_T(2, b; \mu_N) \} \right]$$

MT2 is the larger of the two MT's, minimized over all partitions of the missing transverse energy.

*USM=upstream transverse momentum

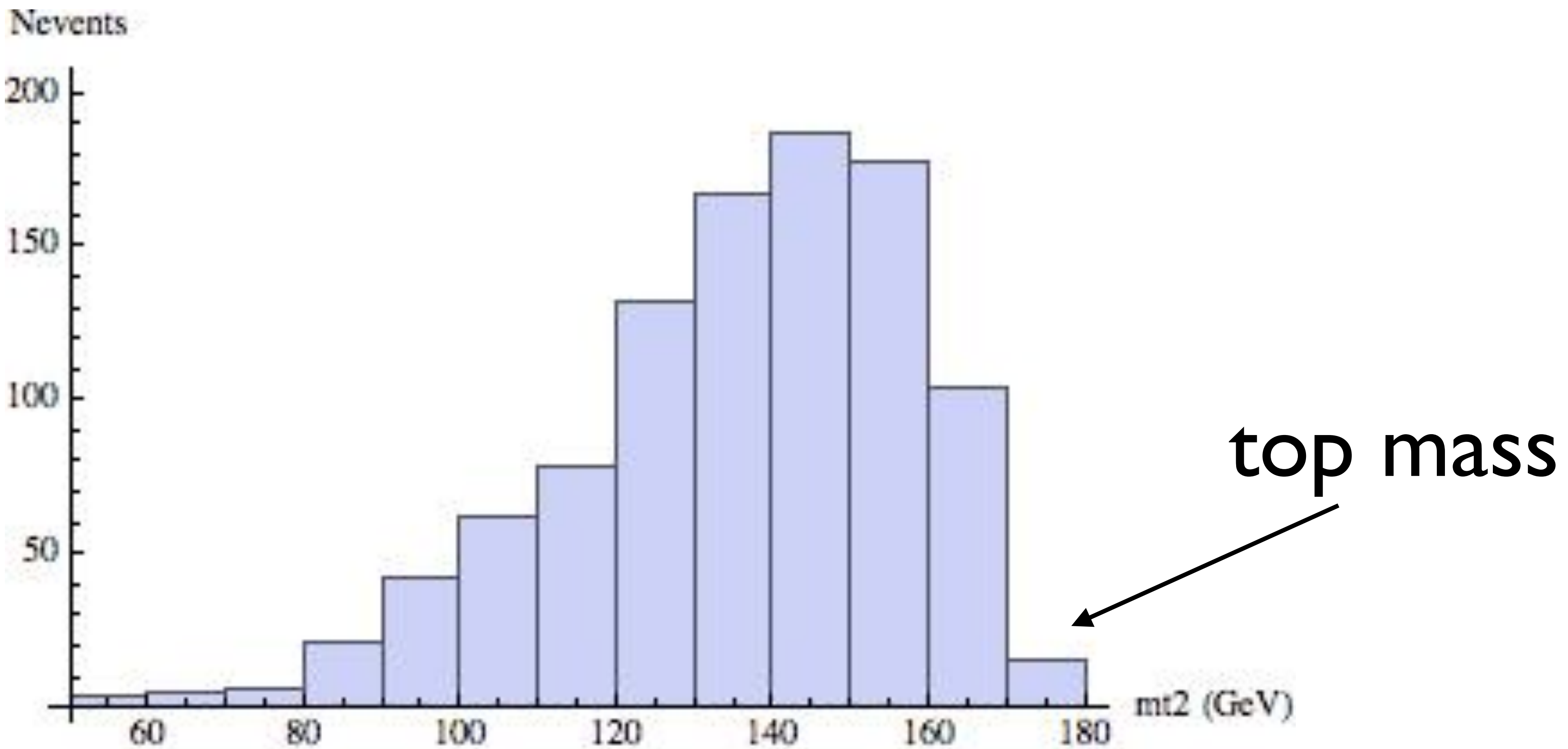
MT2 for Top decay



Missing particles: neutrinos
visible particles: b+lepton pairs

Tutorial: http://pages.uoregon.edu/zyhan/mc4bsm-2014/mc4bsm-2014.html

MT2 for Top decay



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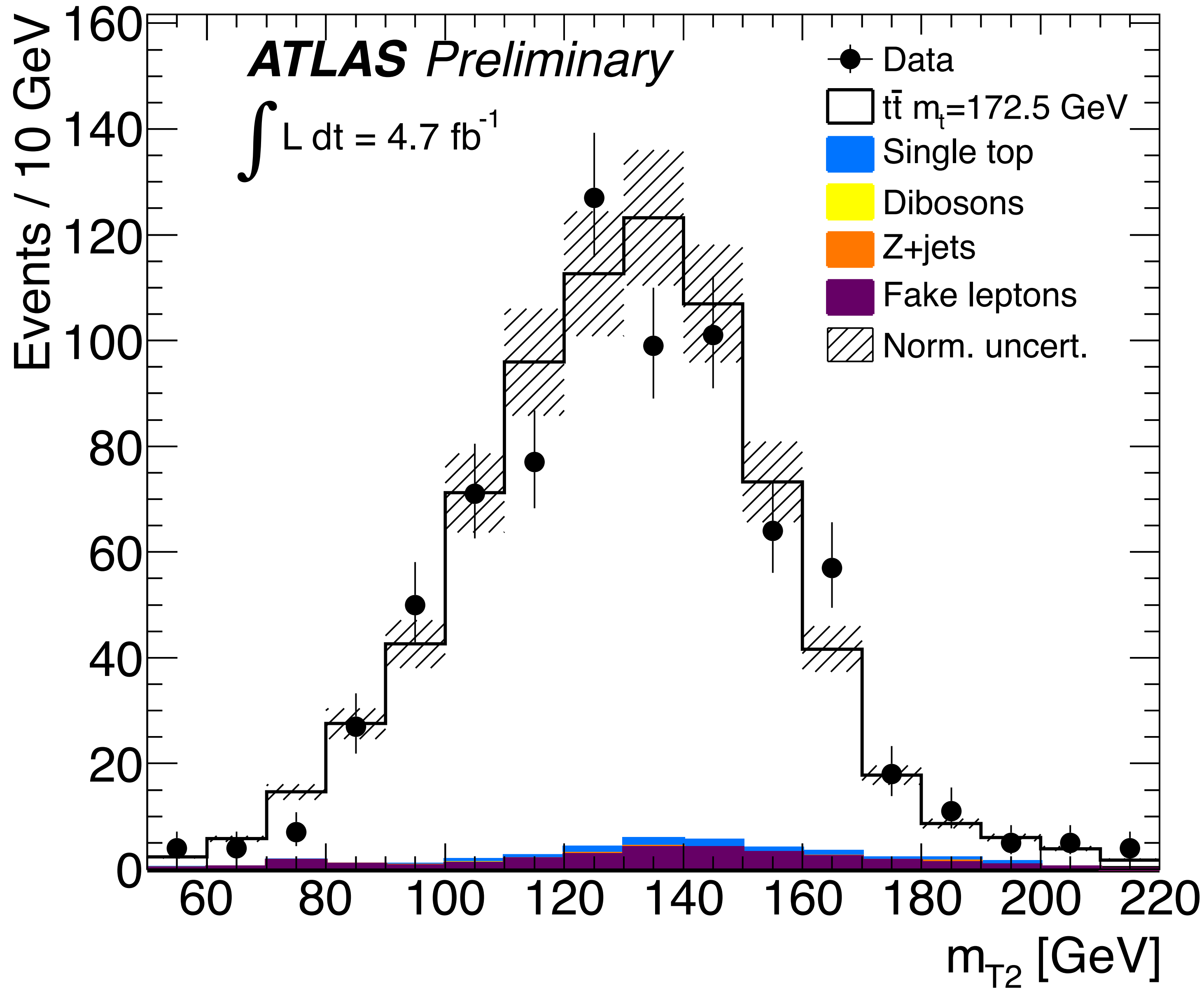
Top mass measurement

ttbar in the dileptonic channel



Tutorial

Step 1:

Make a new directory, for example, `top_mt2/`. Download the MT2 code here, and the example code, and events (generated by Doojin Kim) here . Extract all files to `top_mt2/`.

```
tar xvzf mt2-1.01a.tar.gz

tar xvzf mt2-mc4bsm.tar.gz
```

Step 2:

Compile and run the code:

```
g++ -o top mt2_bisect.cpp top.cc

./top
```

The result is a file that contains MT2 for 1000 events, `mt2.dat` .

Step 3:

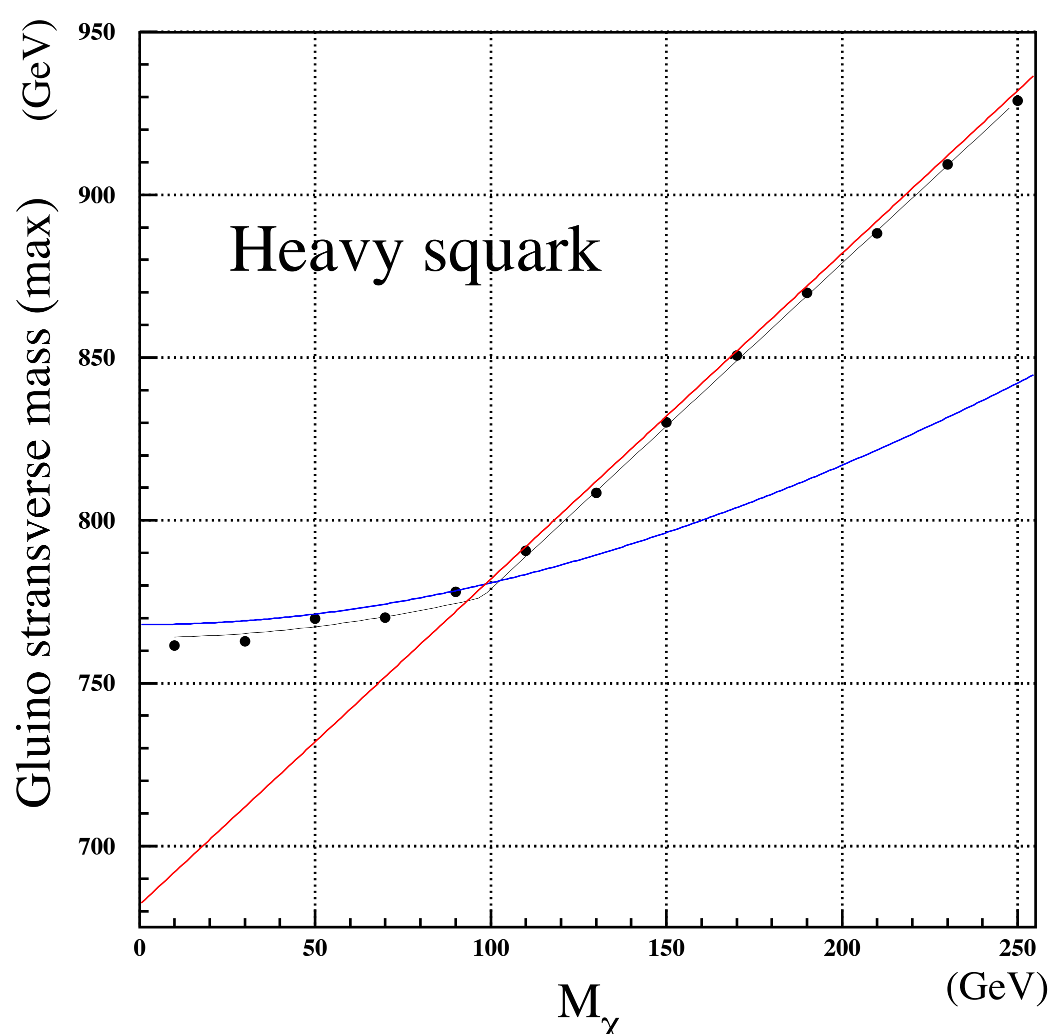
Plot the MT2 distribution, for example, in Mathematica. We see an endpoint at the top mass.

http://pages.uoregon.edu/zyhan/mc4bsm-2014/mc4bsm-2014.html

Use for mass determination

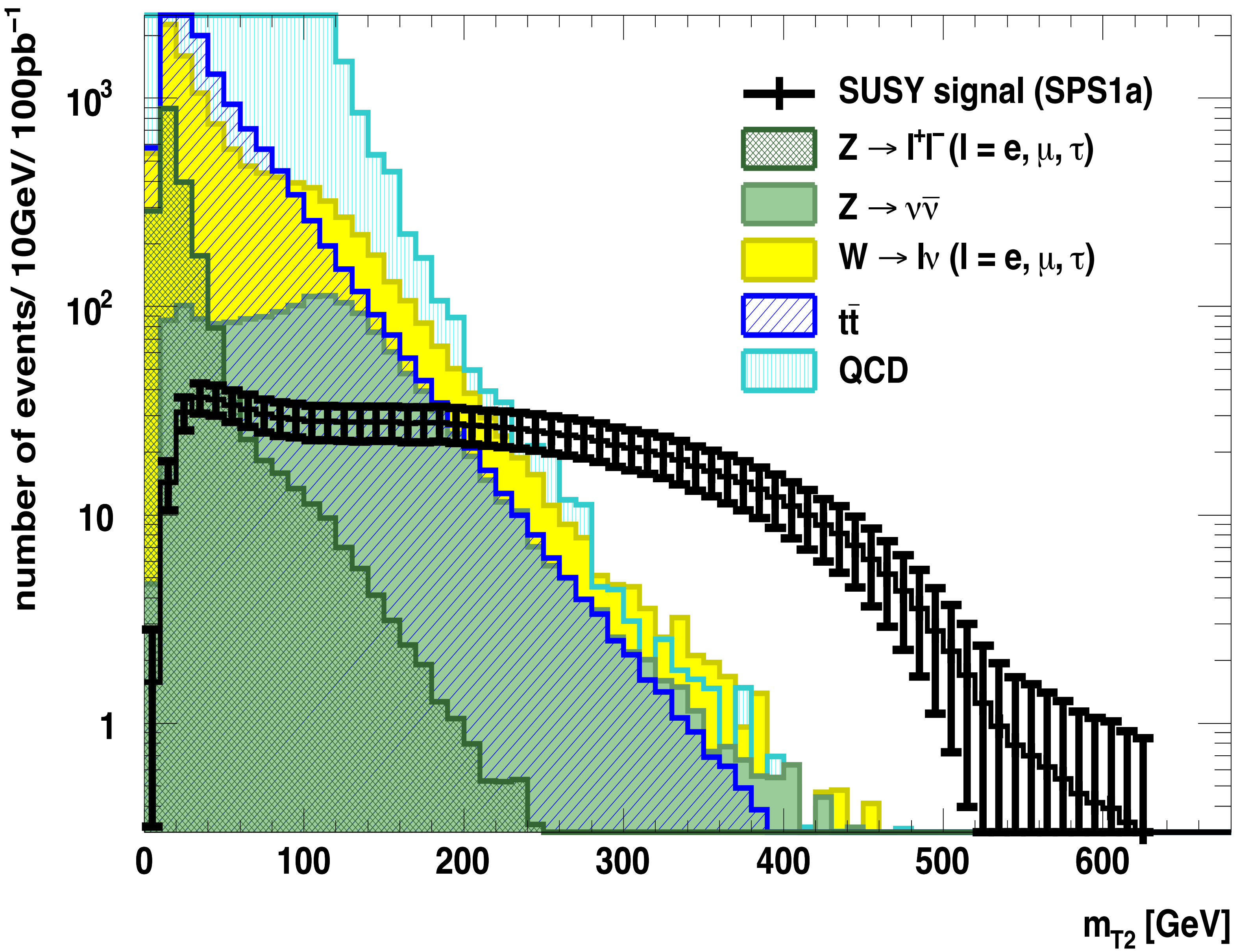
- As a function of the daughter particle's mass, one can use MT2 to determine mother particle's mass if daughter particle's mass is known.

- Possible to determine both masses



MT2 "kink"
Cho, Choi, Kim, Park, 2007

MT2 as a cut in SUSY search

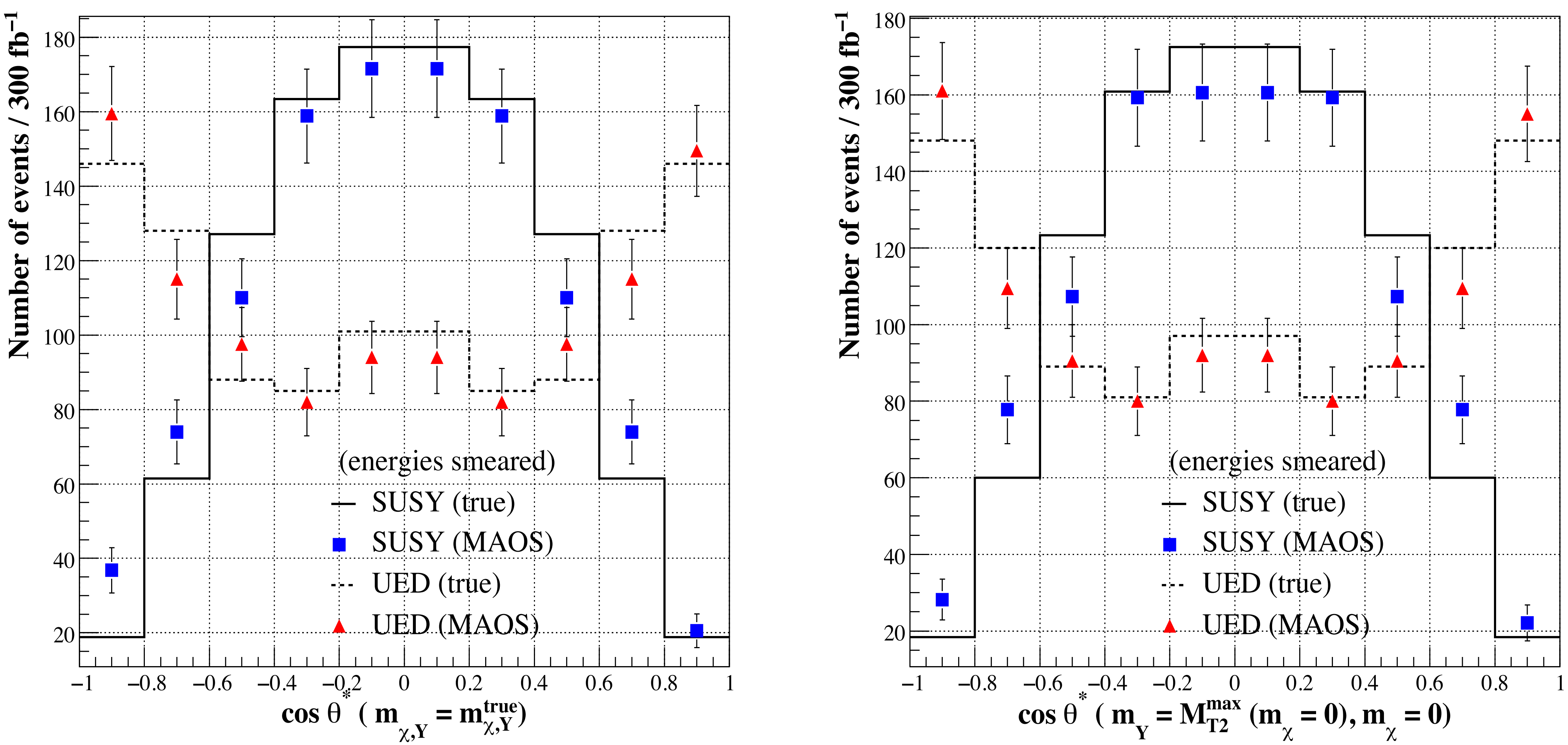


Barr & Gwenlan, 2009

MT2-assisted spin determination

(cho, choi, Kim & Park, 2008)

Slepton vs KK-lepton

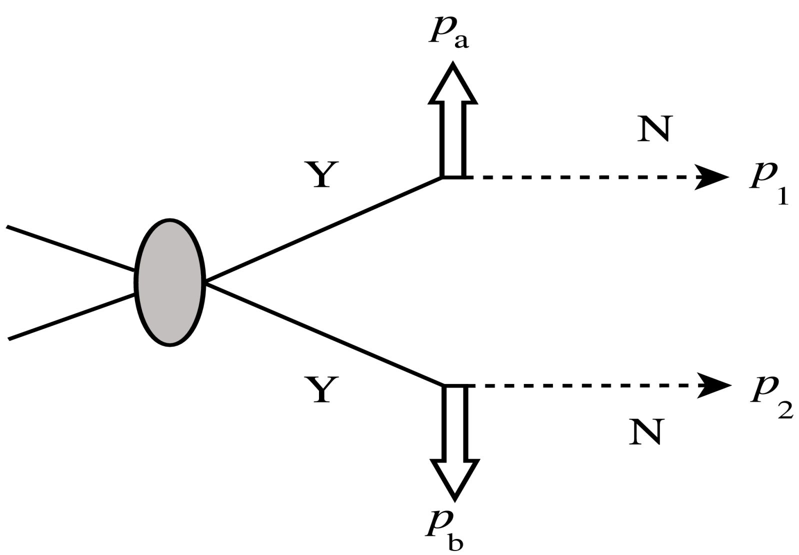


Calculating MT2

Refer http://www.hep.phy.cam.ac.uk/~lester/mt2/

- Special cases: analytical solutions exist
  - No upstream transverse momentum (Kong &Matchev; Barr &Lester)
  - Visible particles are massless, and assumed daughter particle mass is zero (Lester)
- Generic case:
  - Minimization using minuit — original approach (Barr etal)
  - Bisection method using kinematic constraints
    - Has a clear physical interpretation
    - faster and more accurate
    - doesn't need any external libraries (except C++ compiler)

Kinematic constraints



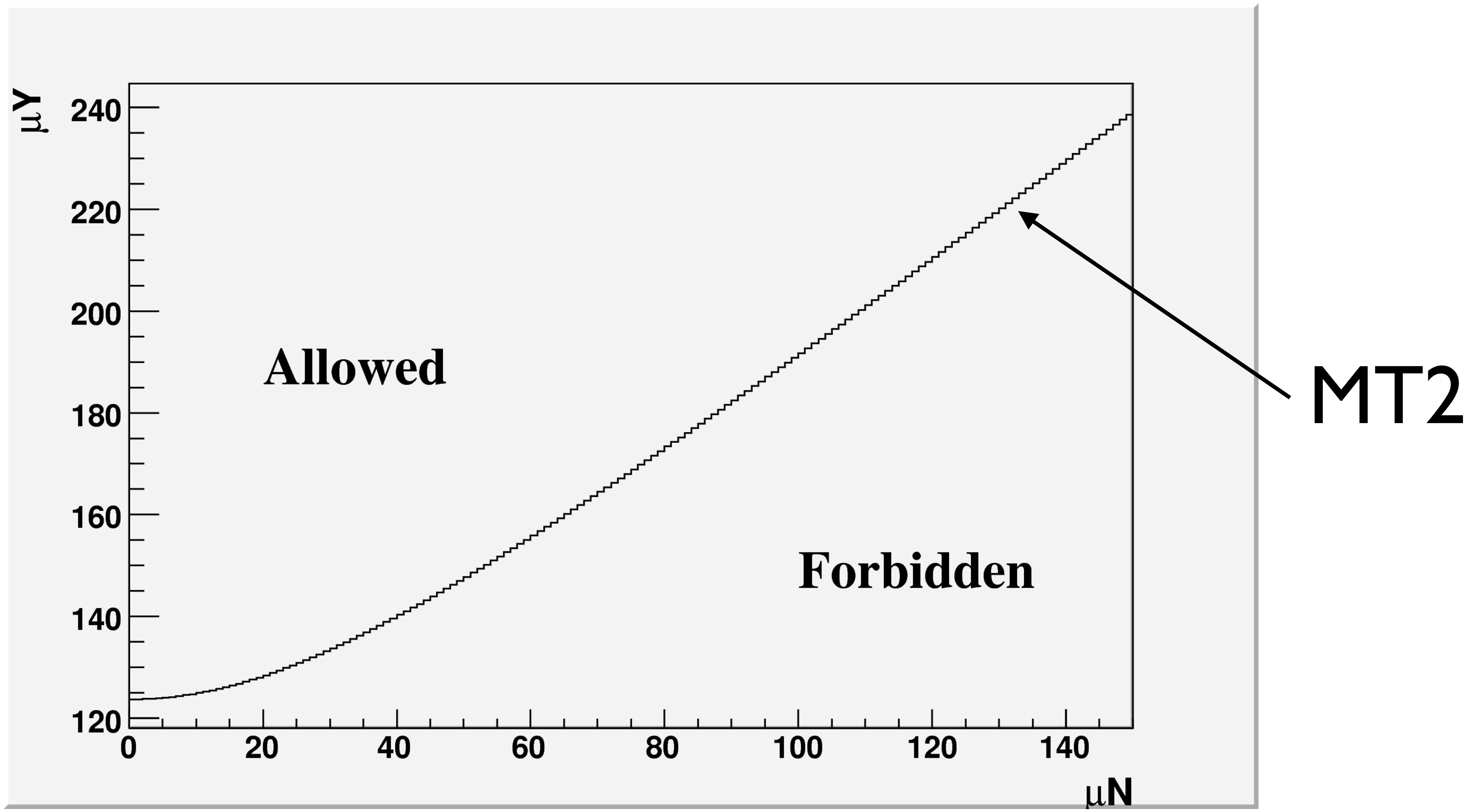
$$p_1^2 = p_2^2 = \mu_N^2,$$
$$(p_1 + p_a)^2 = (p_2 + p_b)^2 = \mu_Y^2,$$
$$p_1^x + p_2^x = \not{p}^x,\ p_1^y + p_2^y = \not{p}^y,$$

p is 4-momentum

MT2 is the smallest μ_Y that can satisfy the above kinematic constraints' —Cheng and Han, 2008

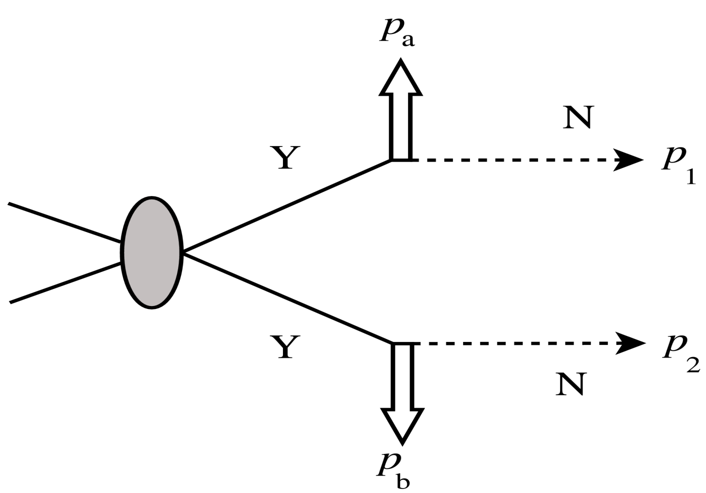
Polynomial (Kinematic constraints) methods are a super set of MT2, the latter is equivalent to the 'minimal' kinematic constraints.

MT2 as the boundary of the allowed mass region



Allowed = consistent with observed visible/missing momentum

Calculate MT2 from kinematic constraints

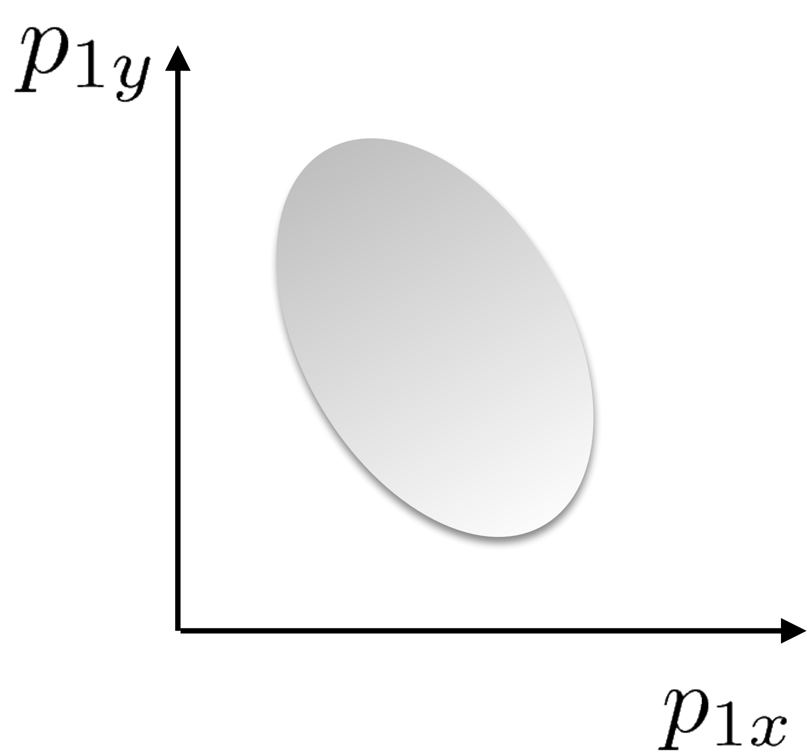


One decay chain:

$$p_1^2 = \mu_N^2,$$

$$(p_1 + p_a)^2 = \mu_Y^2$$

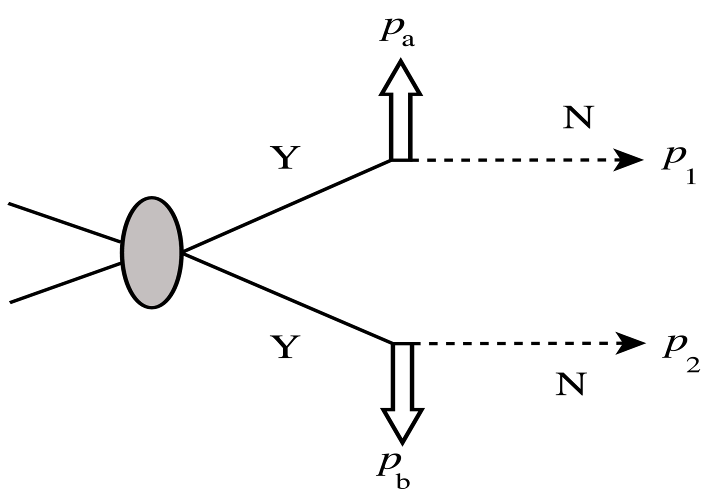
$$\Rightarrow [E_1(p_{1x}, p_{1y}), p_{1x}, p_{1y}, p_{1z}(p_{1x}, p_{1y})]$$



* Physical momentum->(p1x, p1y) within an elliptical region, for given (μ_N, μ_Y)
* ellipse expands for increasing mother particle mass
* ellipse becomes a point when

$$\mu_Y = m_a + \mu_N$$

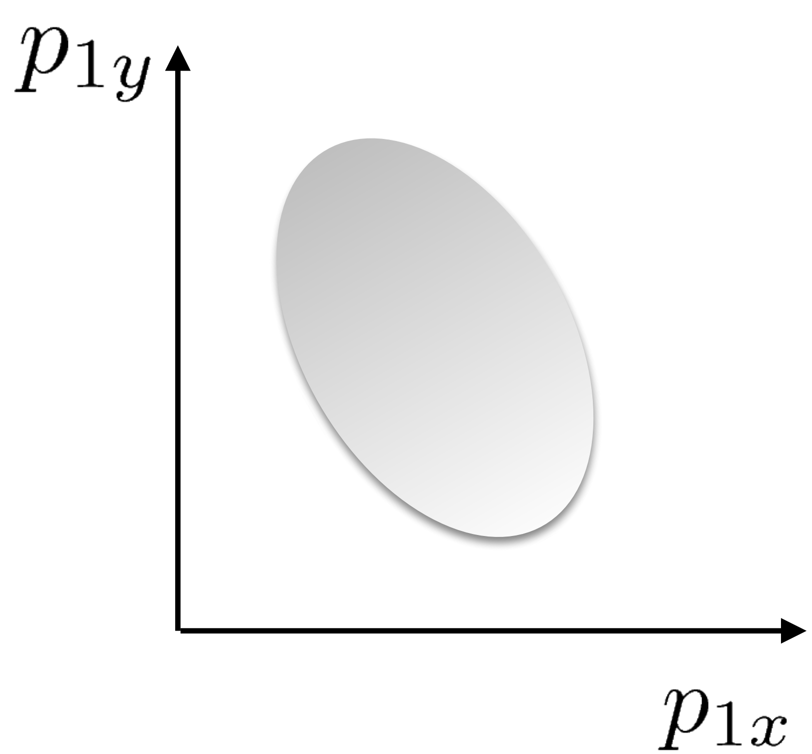
Two decay chains



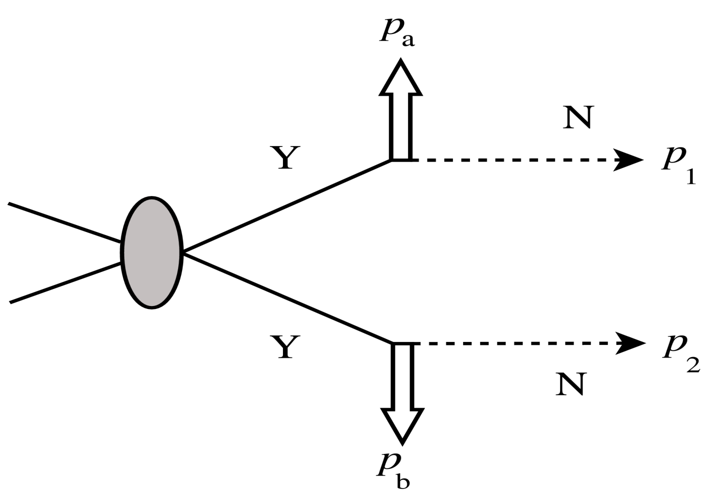
* Another elliptical region on (p2x, p2y) plane
* However,

$$p_1^x + p_2^x = \not{p}^x,\ p_1^y + p_2^y = \not{p}^y$$

Then we have another elliptical region on the (p1x,p1y) plane



Two decay chains



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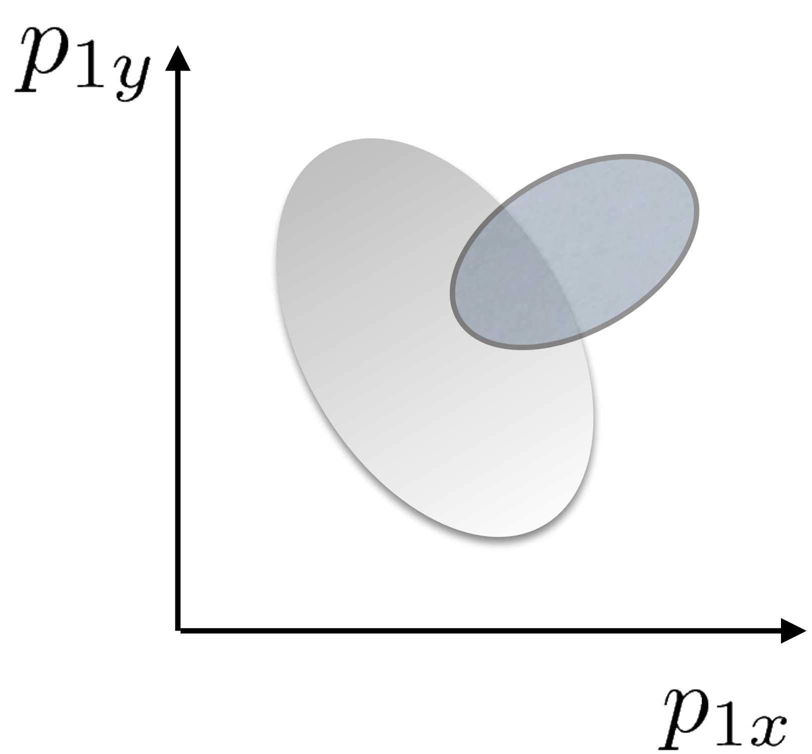


* $(\mu_N,\ \mu_Y)$ Consistent with the event if and only if two elliptical regions overlap

Balanced and unbalanced solutions

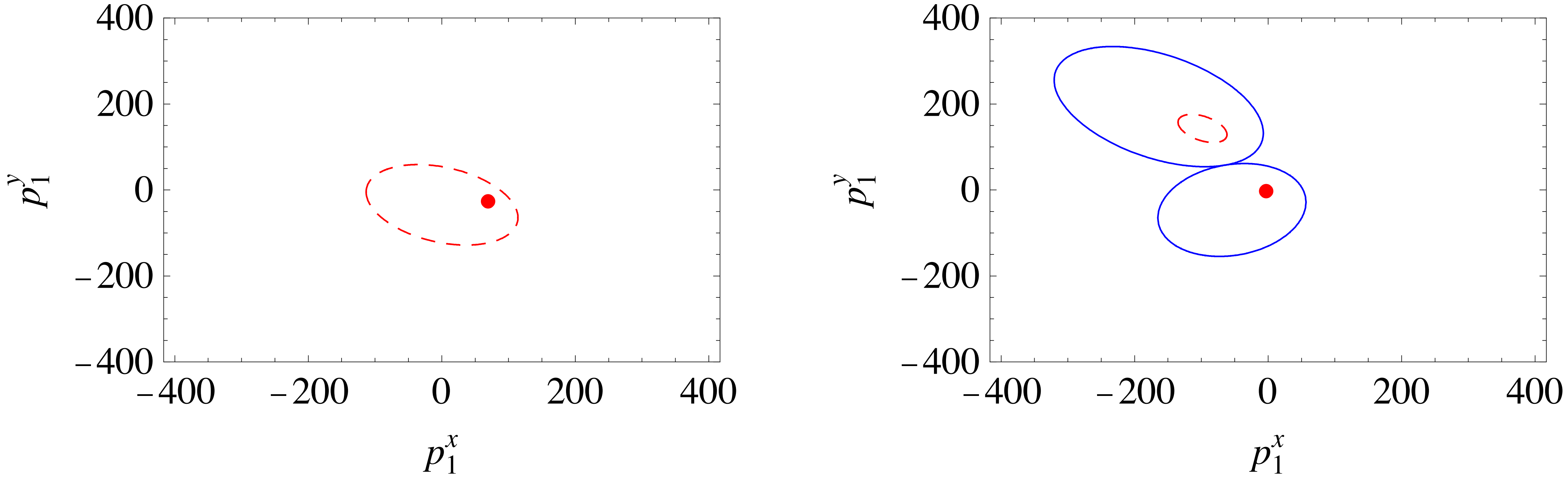


Figure 2: The unbalanced solution (left) and the balanced solution (right). The red (dashed) ellipse and the red point are the two ellipses when $\mu_Y = \mu_N + m_a$. For the unbalanced solution, the point is inside the red ellipse, and $m_{T2} = \mu_N + m_a$. For the balanced solution, the point is outside the red ellipse, and m_{T2} is given when the two ellipses (solid blue) are tangent to each other.

Last ingredients- Sturm sequence and bisection

- Unbalanced configuration is trivial
- Balanced configuration, find μ_Y when two ellipses are tangent:
  - two ellipses tangent -> discrimination =0 -> Solve 12th order polynomial equation: slow and not reliable. (No UTM case, 12th order polynomial simplify to 4th order—analytical sols)
  - test if two ellipses intersect using Sturm sequence (D. Eberly)
  - Bisection: start from an interval $(\mu_Y^{\min},\ \mu_Y^{\max})$, two ellipses are separate for $\mu_Y^{\min}$, intersecting for $\mu_Y^{\max}$. Bisect the interval, update whether we have a new lower bound or upper bound. Repeat until interval small enough.

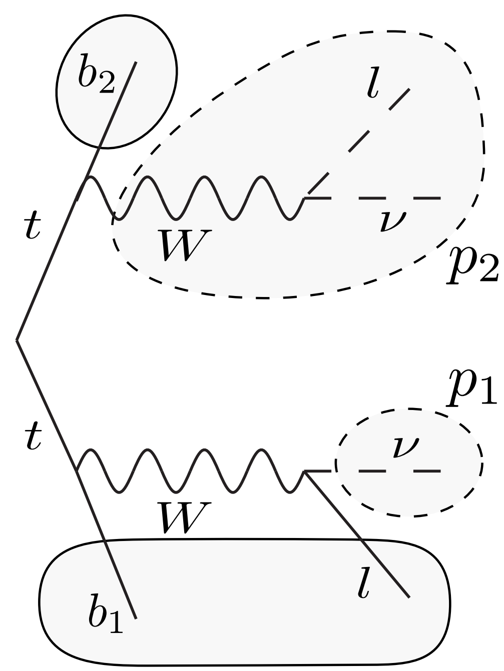
Performance

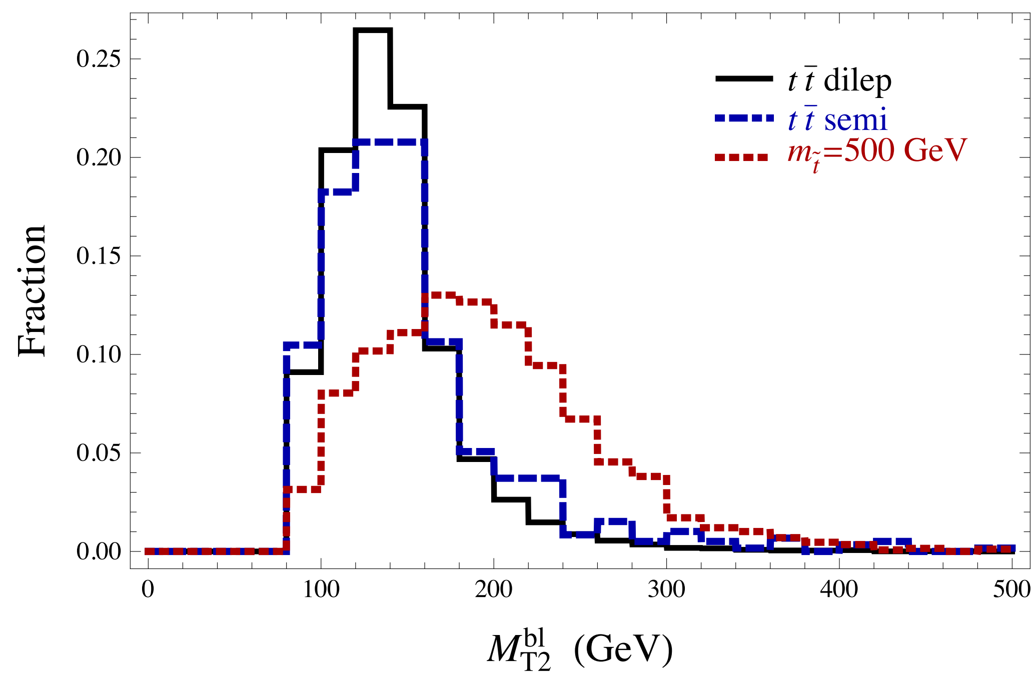
- 5-9 times faster than code using minimization (maintained by Barr and Lester)
- Stand alone, simple code structure, no external dependence.
- Gives more accurate results when small differences exist between our code and Barr and Lester — can be verified with Mathematica
- Can be easily extended to other cases.

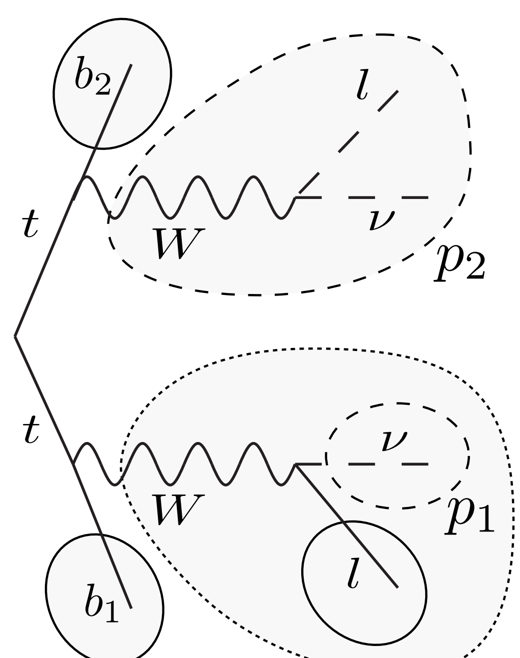
Extensions (Bai, Cheng, Gallicchio, Gu, 2012)

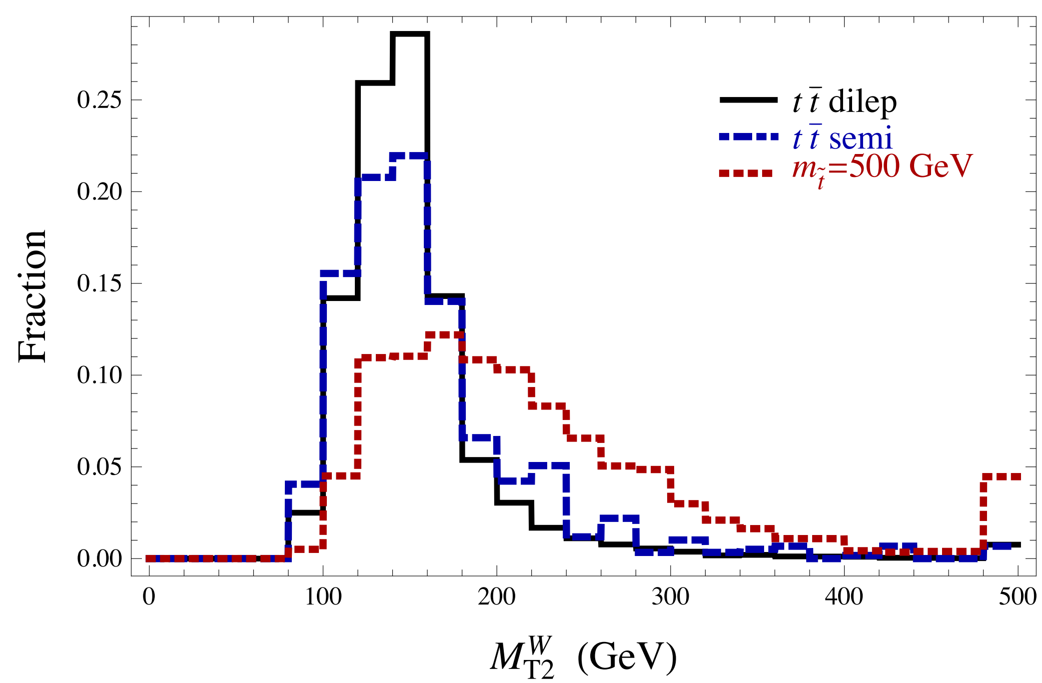
- Motivation: diletonic top decays, one lepton missing, as a background to single-lepton search of stops
- Different daughter particle masses, M_{T2}^{bl}
  - one side: b-l as visible particle, neutrino as missing; the other side: b as visible particle, W as missing
- With more constraints, M_{T2}^{W}
  - with W mass shell constraints, elliptical becomes an ellipse.

Stop search (Bai, Cheng, Gallicchio, Gu, 2012)

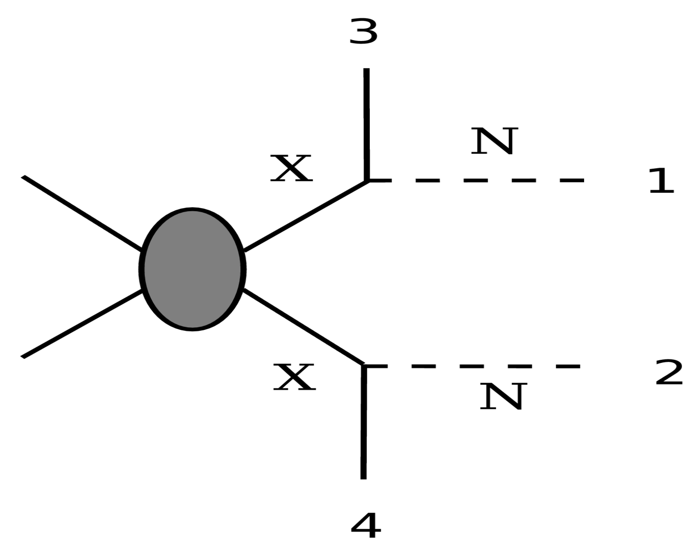


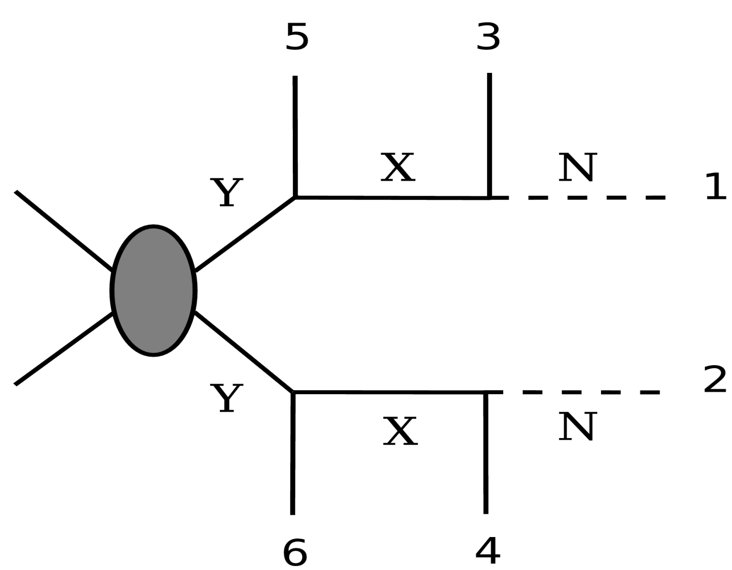


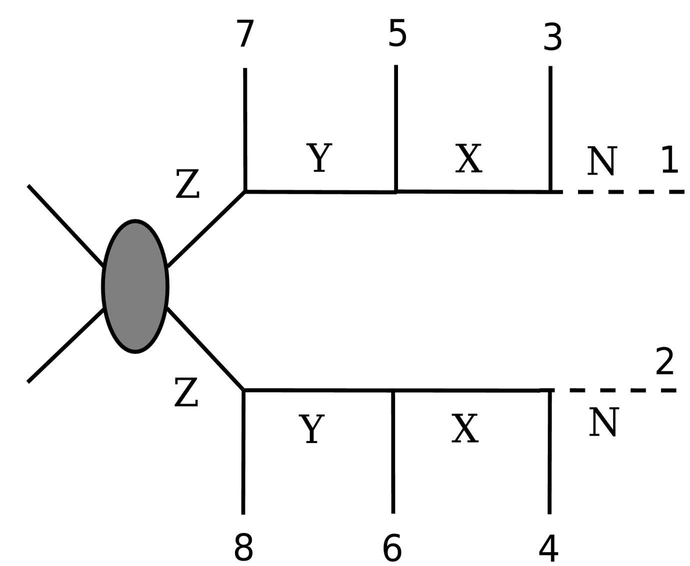




More particles, more constraints

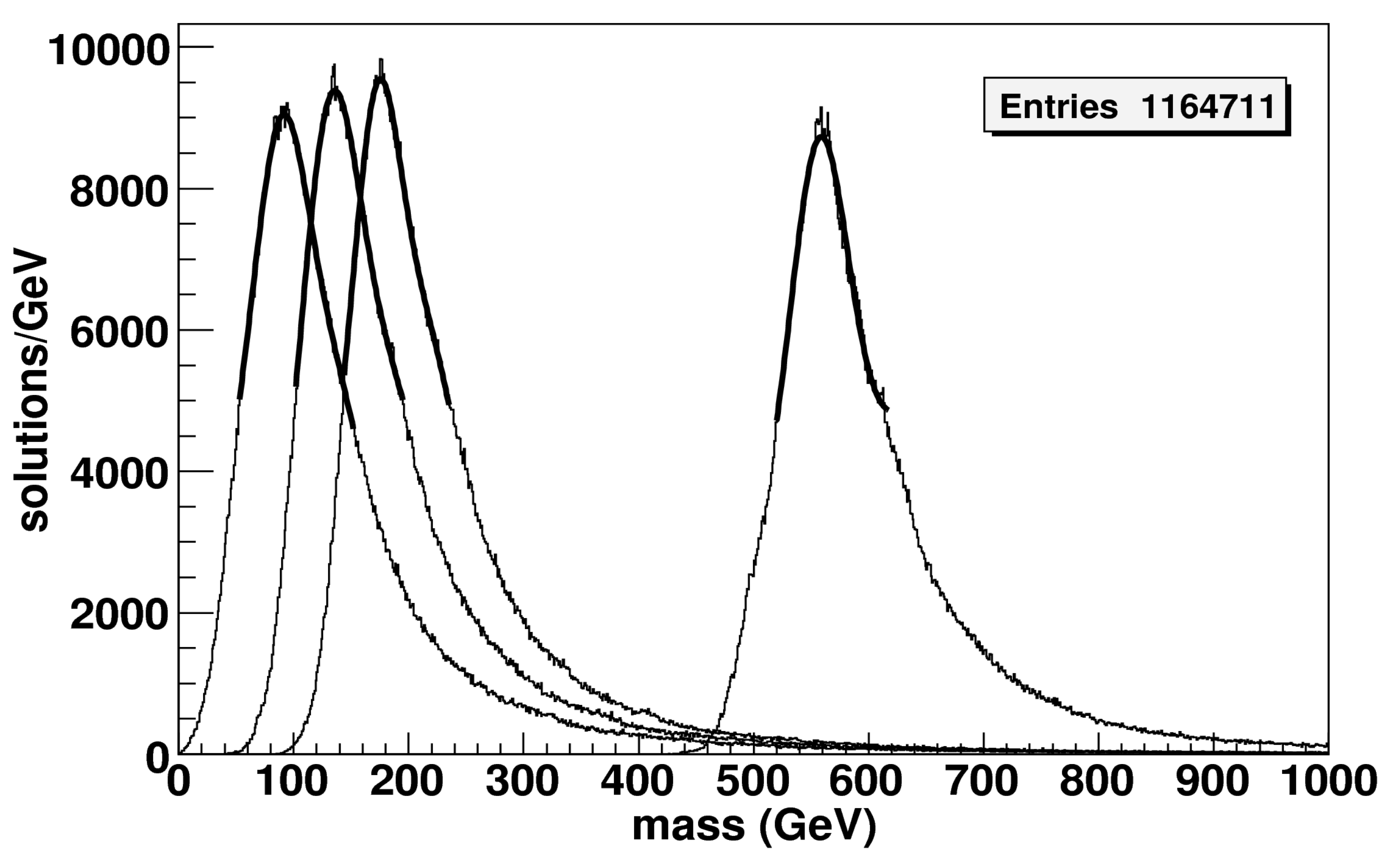






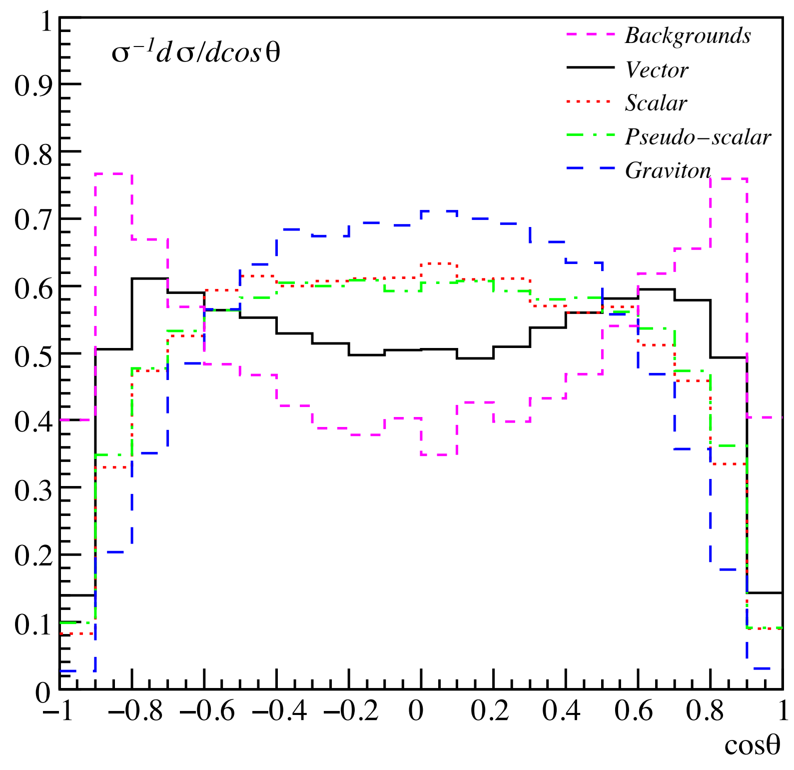
3 visible particles per decay chain

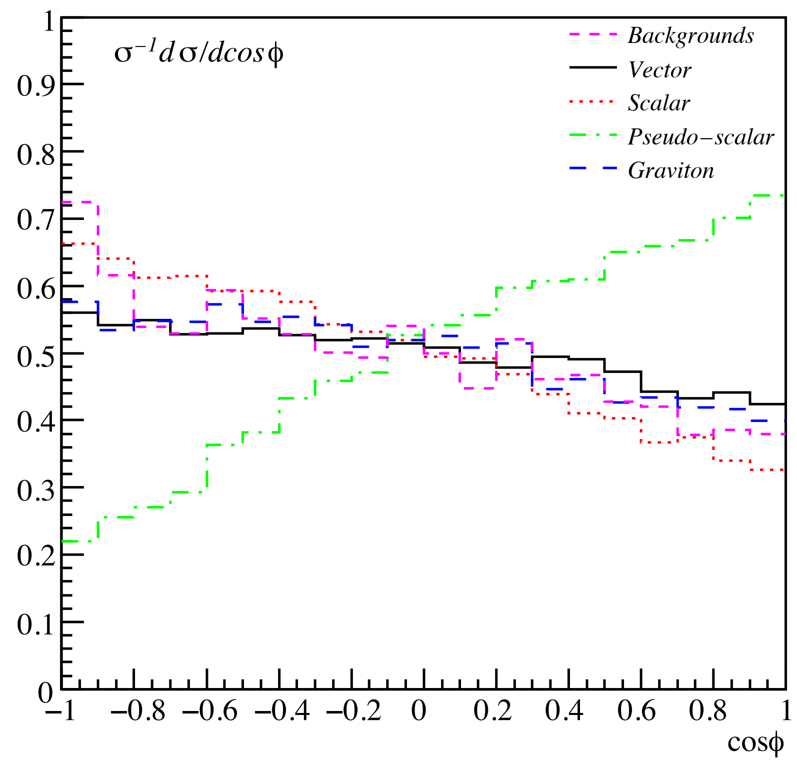
- Assumptions: two decay chains in the event involve the same particles; we have a sample of events containing the same decay chains.
- Can simultaneously determine all unknown masses.



2 Visible particles per decay chain

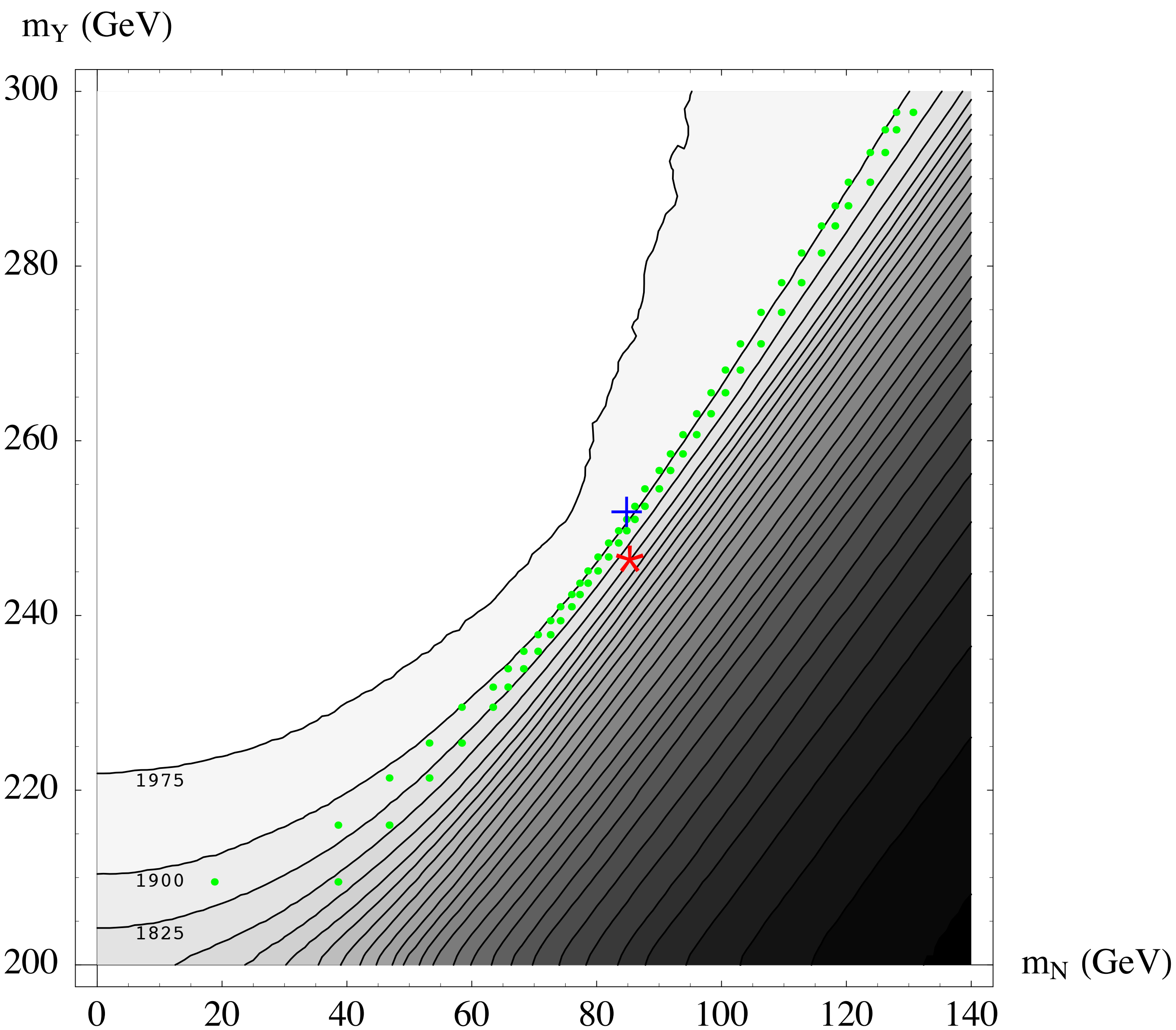
- Event reconstructable if all masses known, example: ttbar in the dileptonic channel
- Determine spin of ttbar resonance (Bai & ZH, 2008)





2 Visible particles per decay chain

- Can also be used for mass determination
  (Cheng, Gunion, Han, Marandella, McElrath, 2007)



Conclusion

- MT2 is useful for mass/spin determination in events with missing particles; can also be used as a cut to discover new physics
- MT2 can be interpreted as the "minimal" kinematic constraints imposed on an event with two one-step decay chains
- We obtain an algorithm for calculating MT2, which is the fastest and the most accurate.
- Kinematic constraints lead to more methods/ applications.