A storm in a “T” cup

(Overview of kinematic variables)

K.C. Kong

MC4BSM8

Center for Theoretical Physics of the Universe, Daejeon, South Korea, May 19-23, 2014

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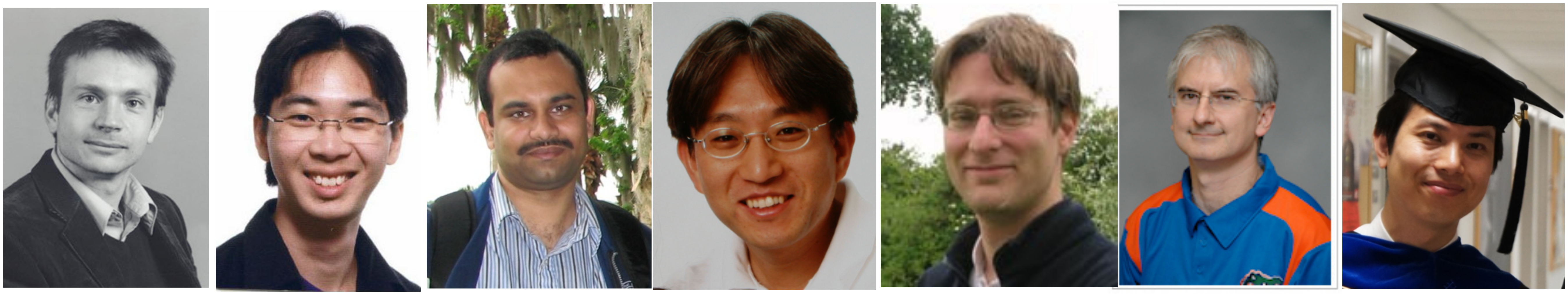

based on work with:

Fl OxBridge (flˈɒksˌbrɪdʒ) collaboration

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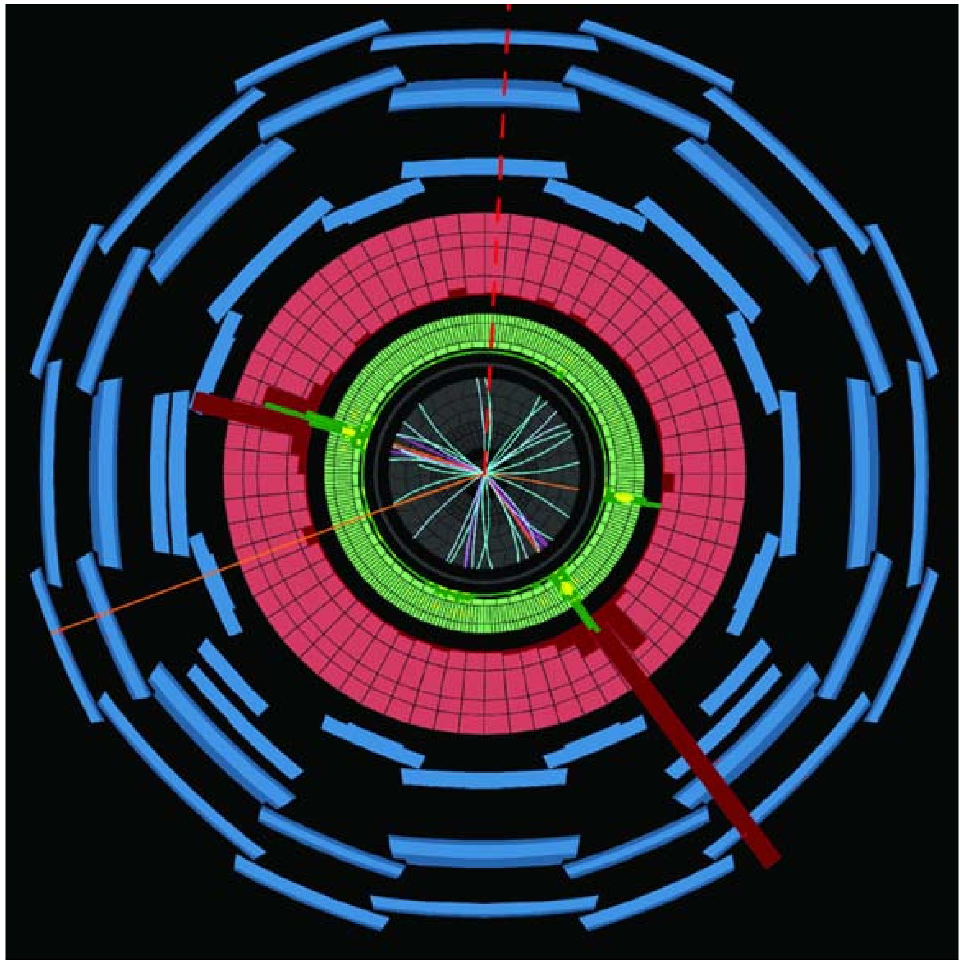
Recall there are some problems (?) in SM

See A. Weiler's talk

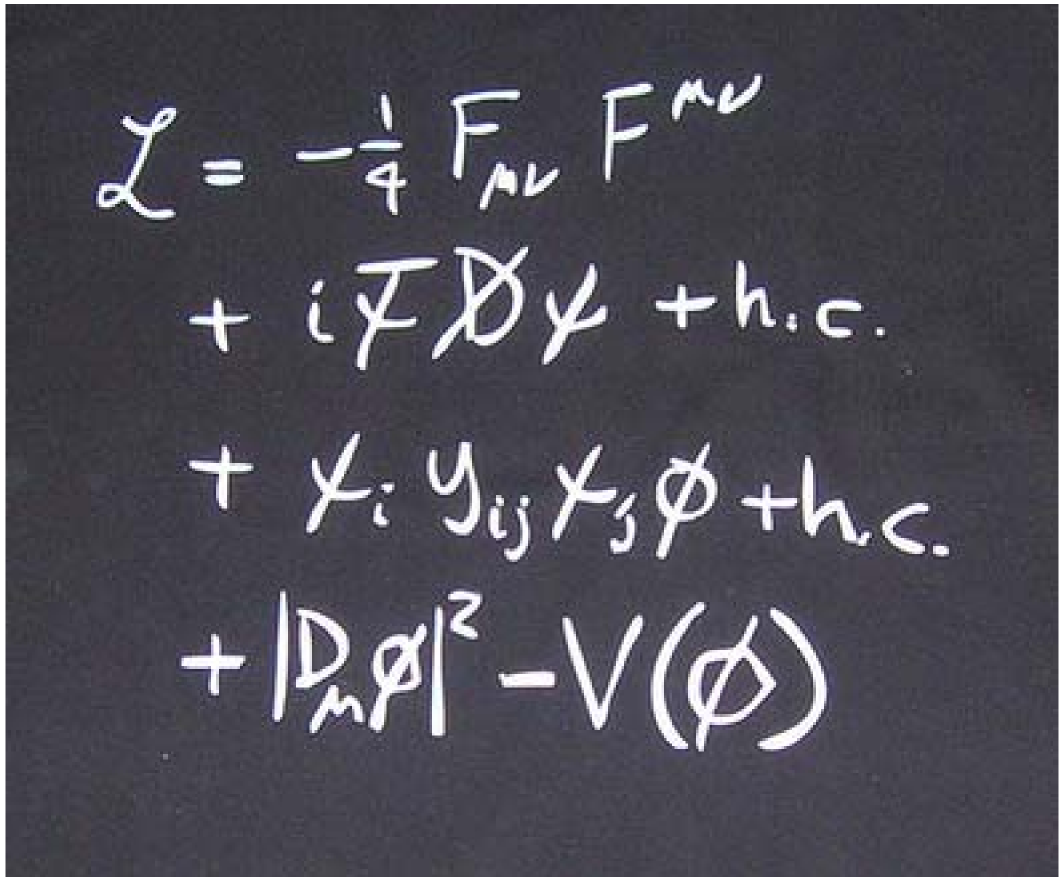
What are common (?) features of “solutions” to these problems?

- Big increase in particle content
- Longish decay chains
- Missing massive particles
- Large jet/lepton/photon multiplicity

The game



40 M / second over 10 years



$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &+ i \bar{\psi} \not{D} \psi + h.c. \\ &+ \psi_i y_{ij} \psi_j \phi + h.c. \\ &+ |D_\mu \phi|^2 - V(\phi)\end{aligned}$$

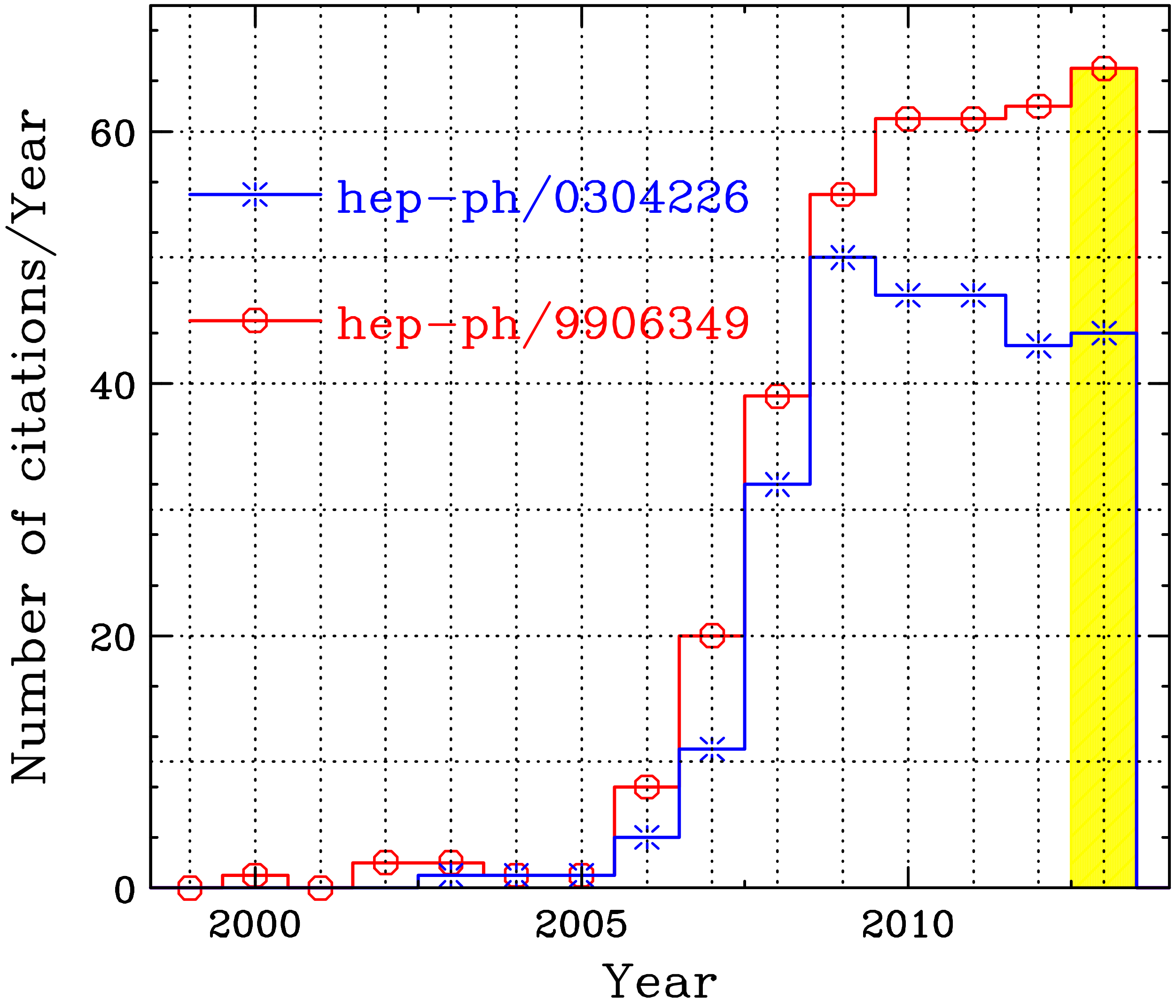
+ more terms…?

At some point, 5000 people will shout:

"We've found a …
[long pause]
… SOMETHING!"

A large collider of hadrons …
… not a collider of large hadrons

What is that *something*?
How hard is it to identify what was found?
What is the mass scale of the "thing"?
Can we measure it?
There were lots of ideas, especially for 7-8 years.



Do we care about masses?

- Common Parameters in the Lagrangian
- Interpretation
  - SUSY breaking mechanism, geometry of ED
- Prediction of new things
  - Mass of W,Z -> indirect top quark mass "measurement"
  - Masses of W/Z/t -> indirect measurement of the Higgs mass
- Expedites discovery - optimal selection

“mass measurement methods”

... short for ...

“parameter estimation and discovery techniques”

Some methods, variables...

Missing momenta reconstruction?	Mass measurements		Spin measurements
	Inclusive	2 symmetric chains	
None	Inv. mass endpoints and boundary lines		Inv. mass shapes
	M	Wedgebox	
Approximate	S	M M	As usual (MAOS)
Exact	?	Polynomial method	As usual

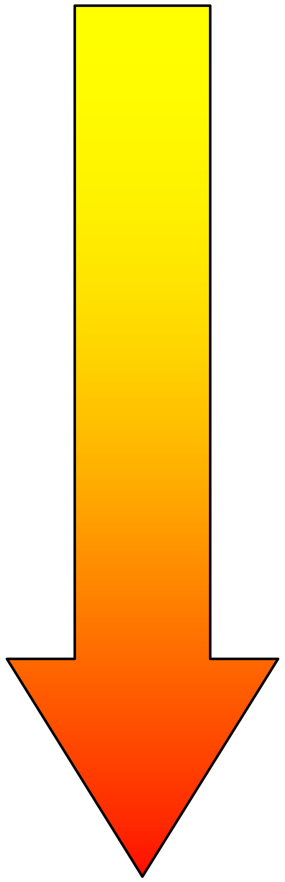
pessimism → **optimism**

pessimism → **optimism**

Types of Technique

Few

assumptions



Many

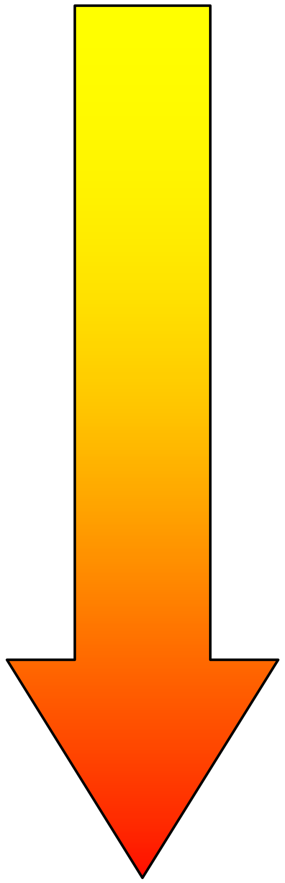
assumptions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Types of Technique

Vague

conclusions



Specific

conclusions

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
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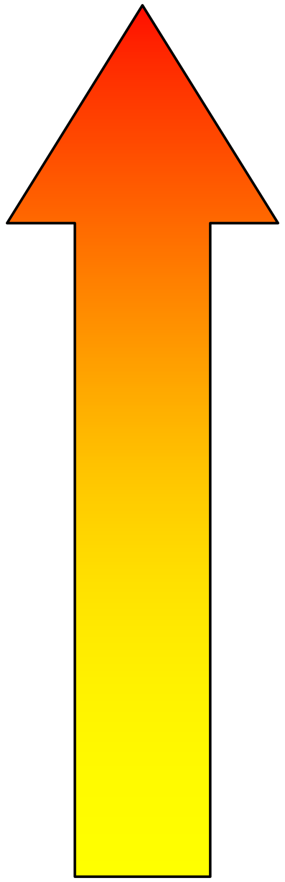
Types of Technique



Robust

Fragile

- Missing transverse momentum
- M_eff, H_T
- s Hat Min
- M_T
- M_TGEN
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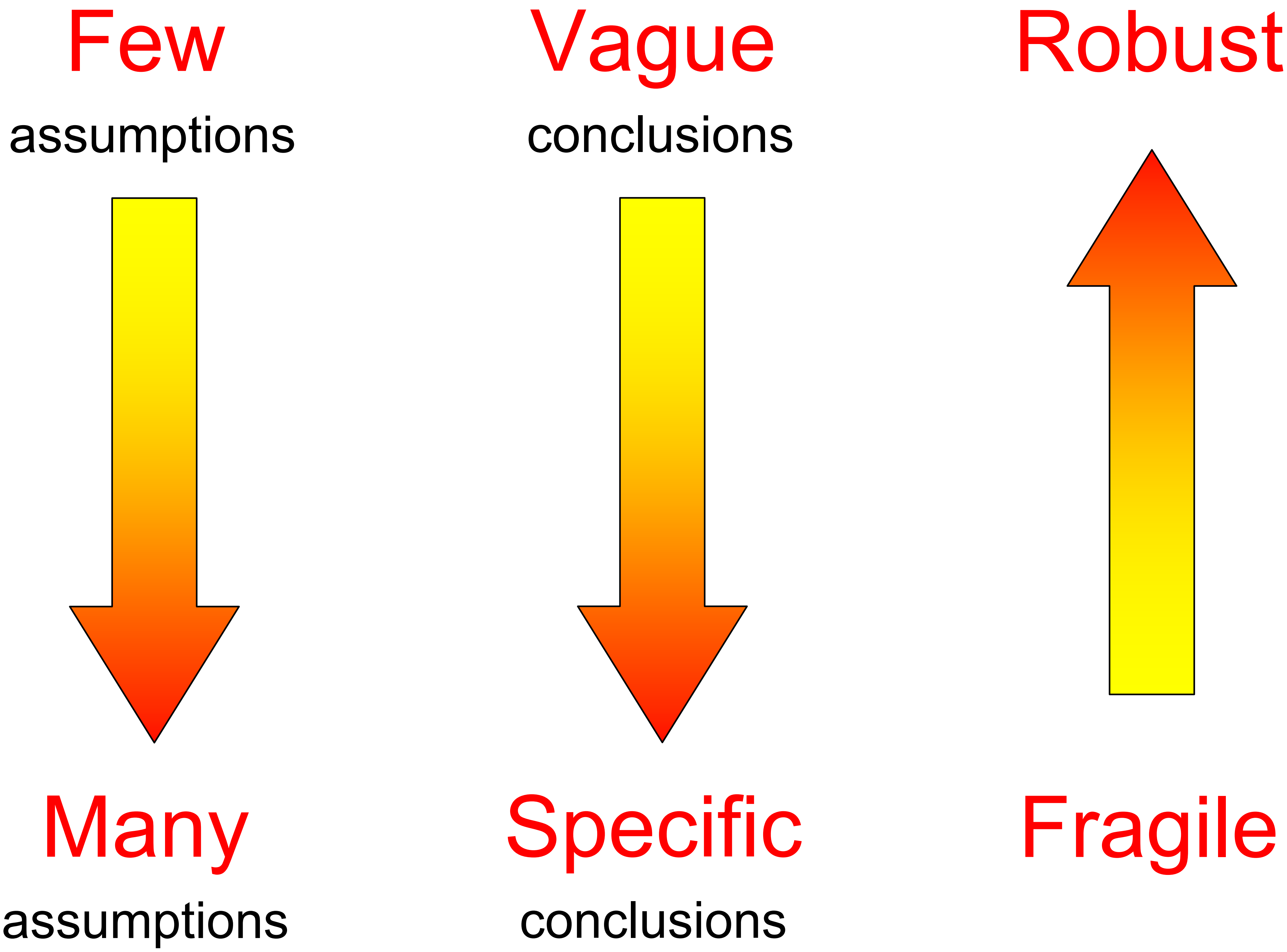
Interpretation : the balance of benefits



Few assumptions

Many assumptions

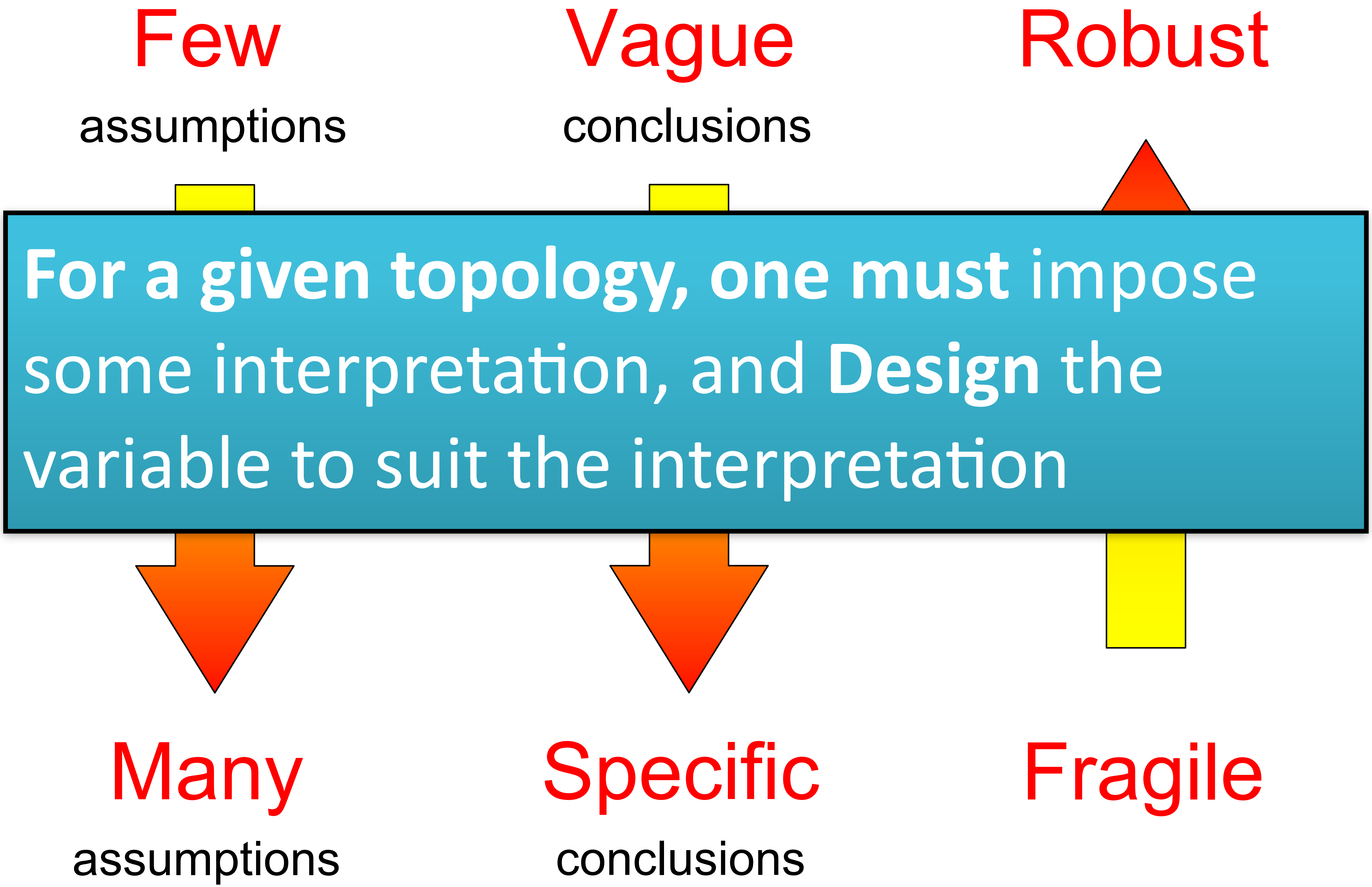
Vague conclusions

Specific conclusions

Robust

Fragile

Interpretation : the balance of benefits



Few

assumptions

Vague

conclusions

Robust

For a given topology, one must impose some interpretation, and **Design** the variable to suit the interpretation

Many

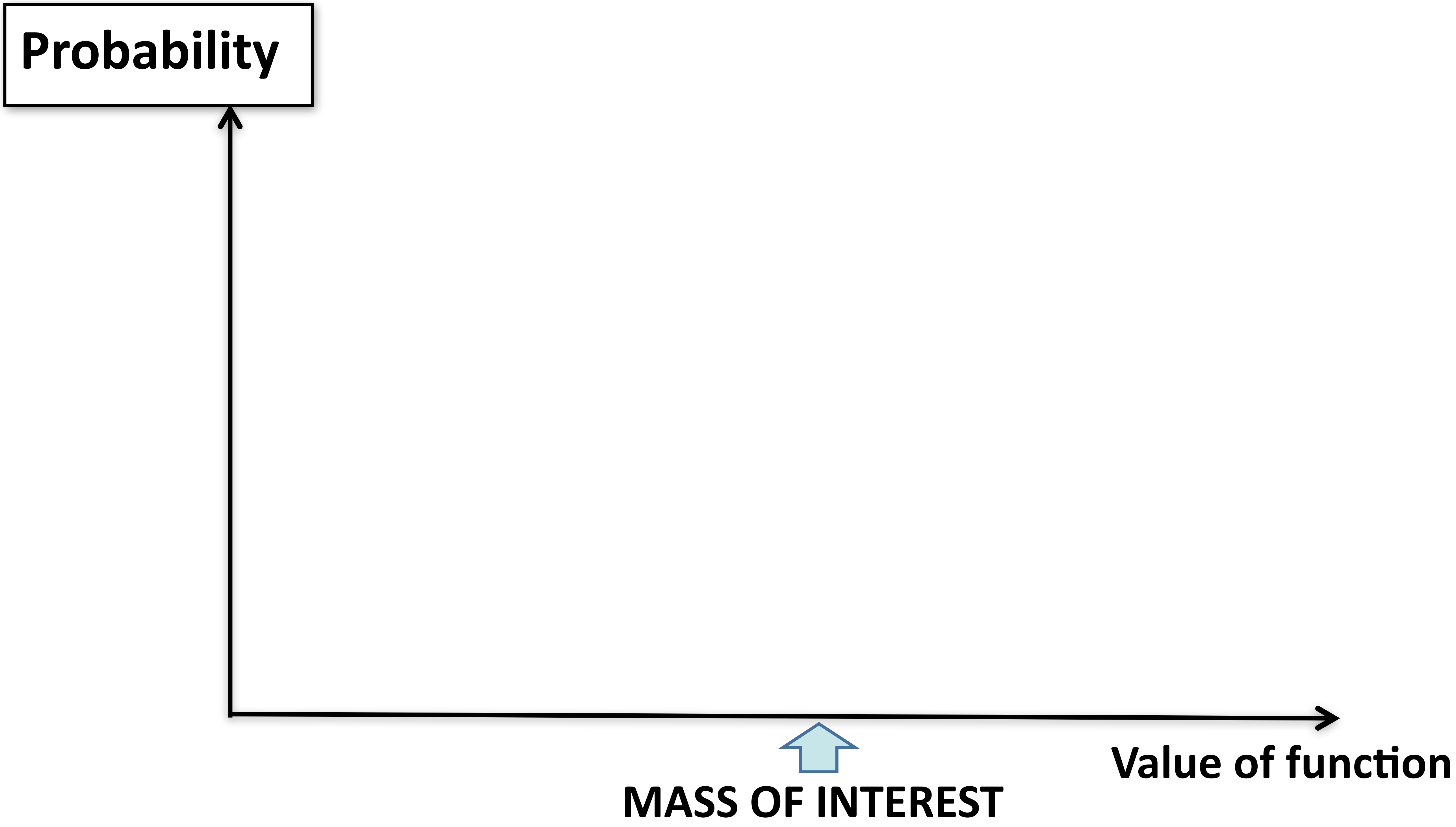
assumptions

Specific

conclusions

Fragile

Good vs poor variables



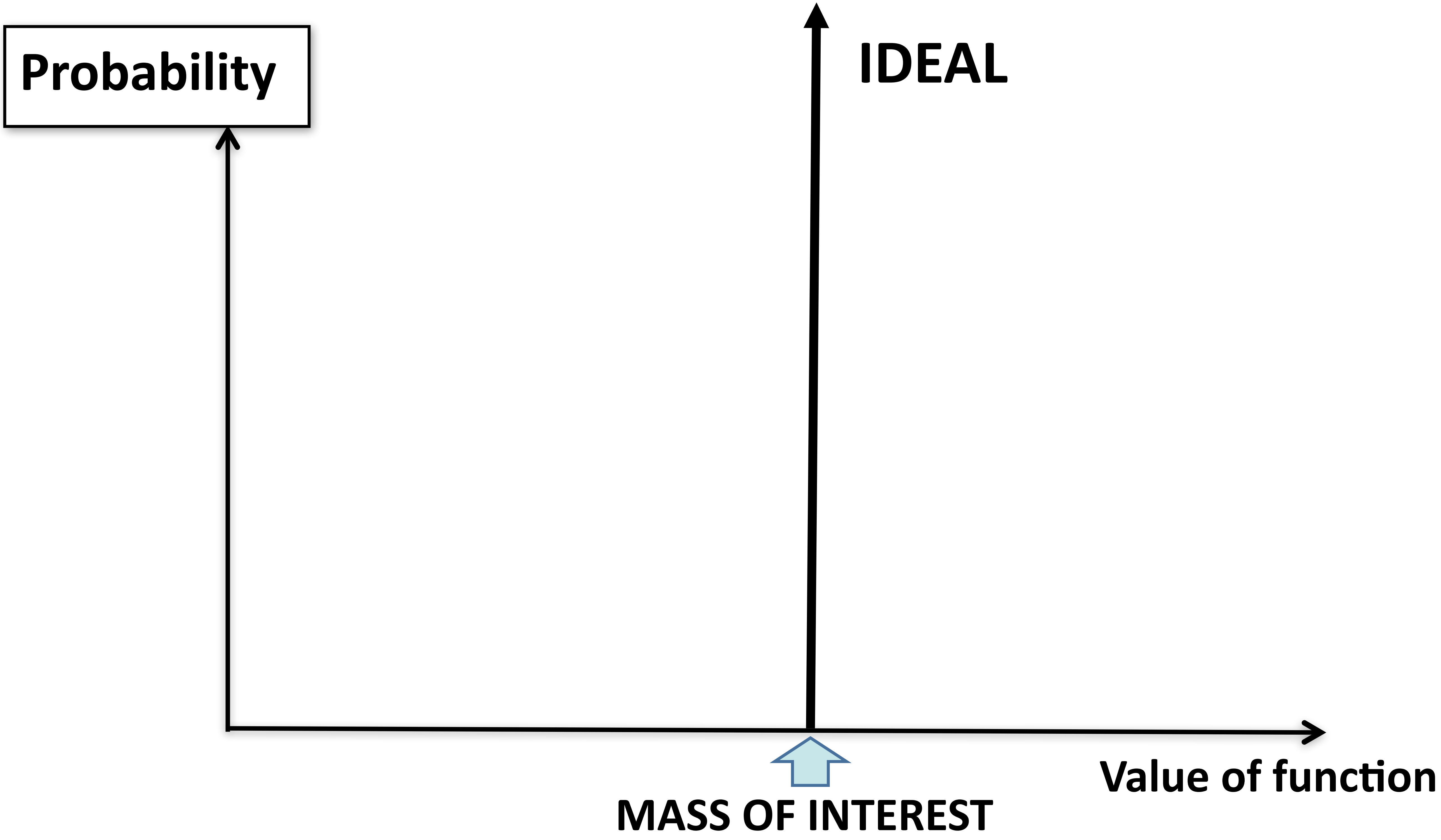
Probability

Value of function

MASS OF INTEREST

Taken from Alan Barr’s talk

Good vs poor variables



Probability

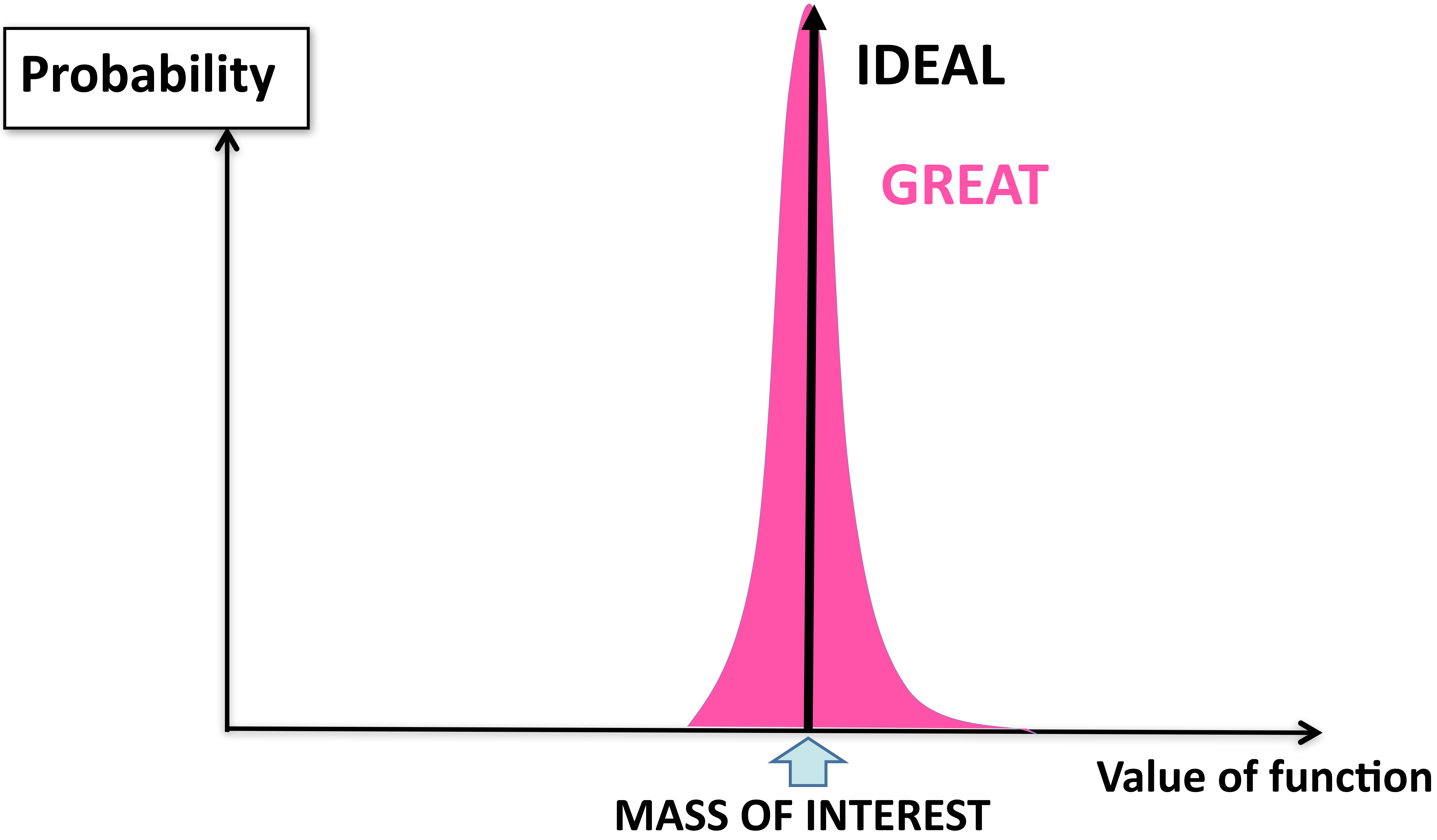
IDEAL

Value of function

MASS OF INTEREST

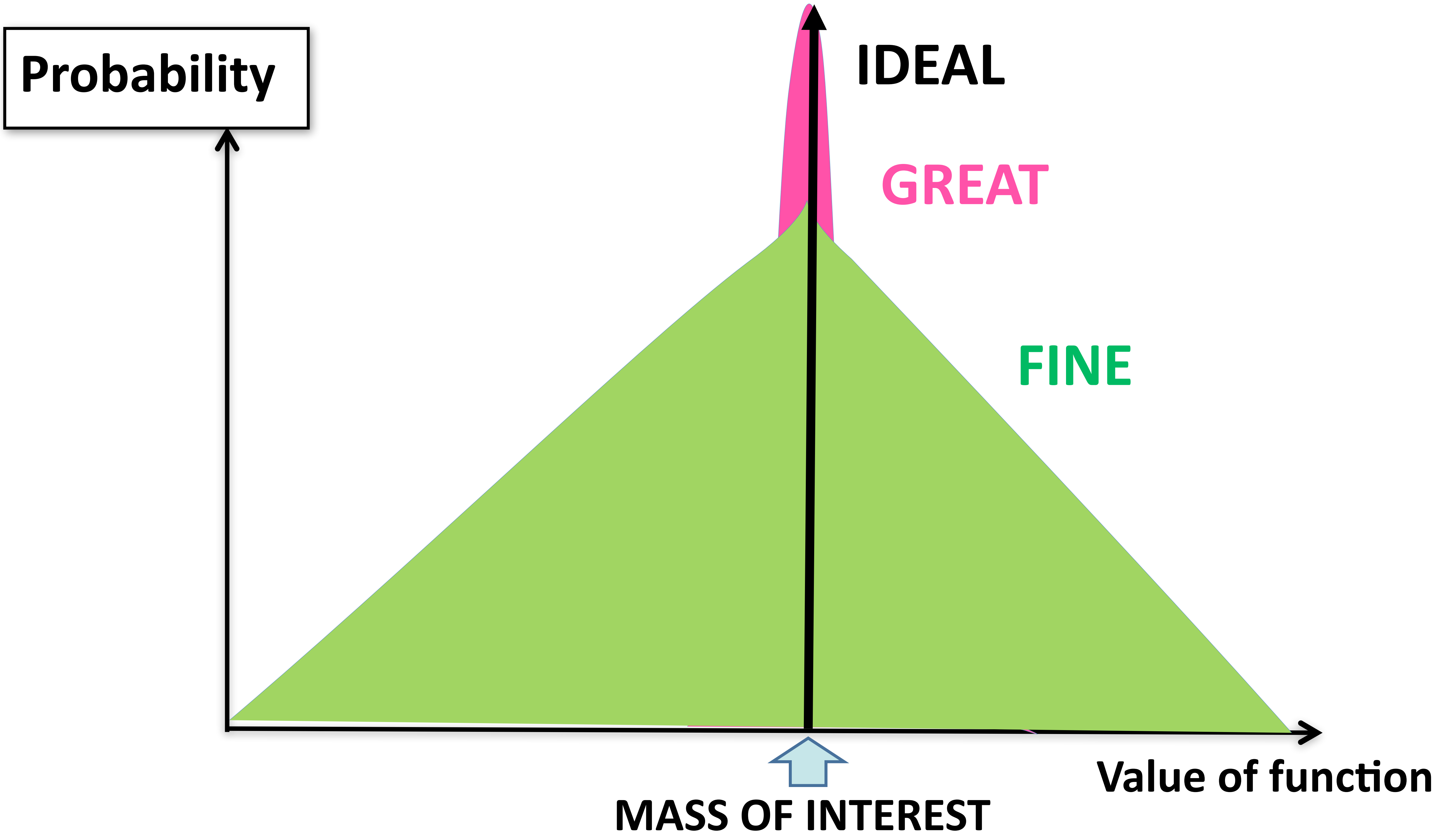
Taken from Alan Barr's talk

Good vs poor variables



Taken from Alan Barr's talk

Good vs poor variables



Probability

IDEAL

GREAT

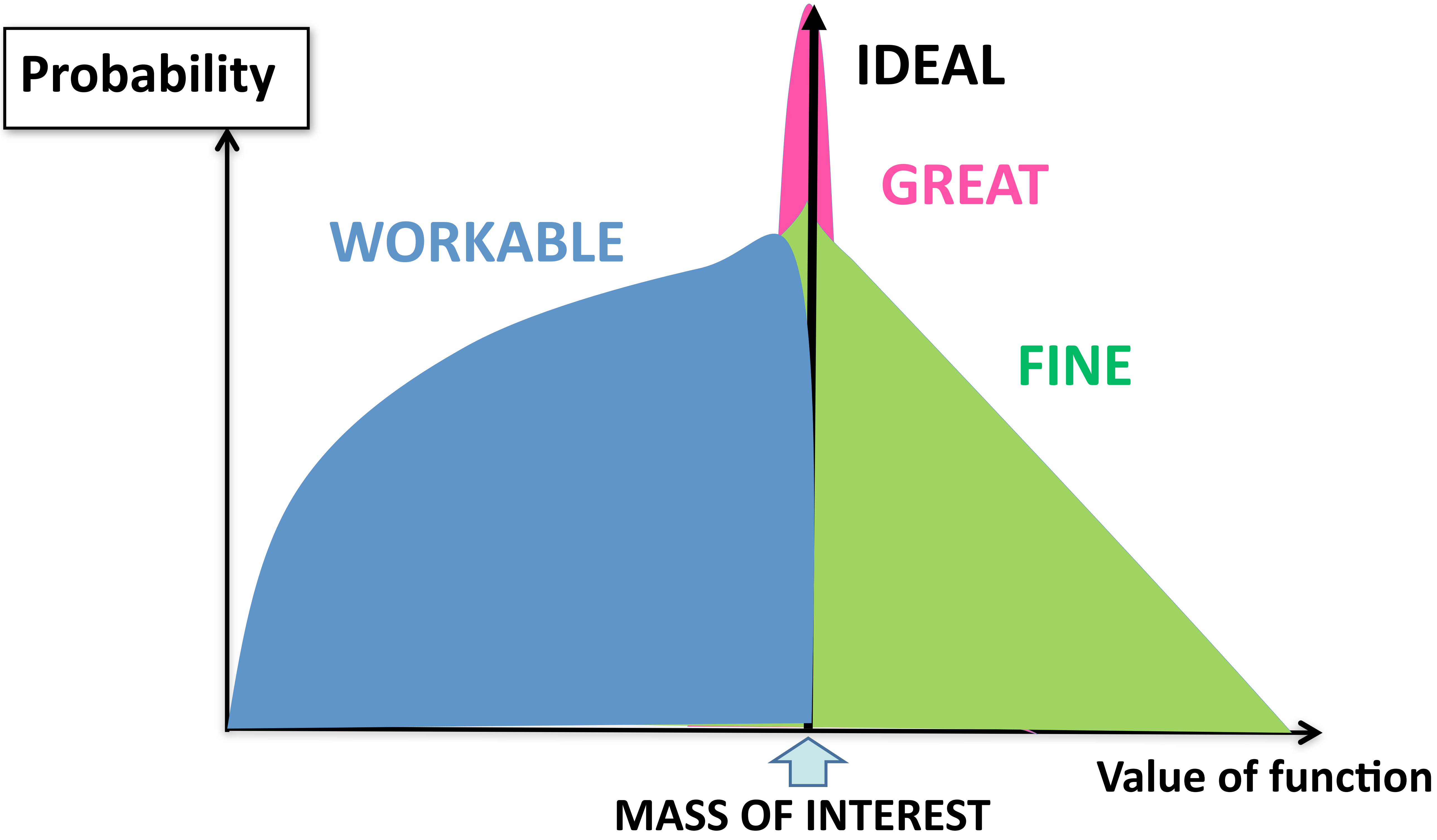
FINE

Value of function

MASS OF INTEREST

Taken from Alan Barr's talk

Good vs poor variables



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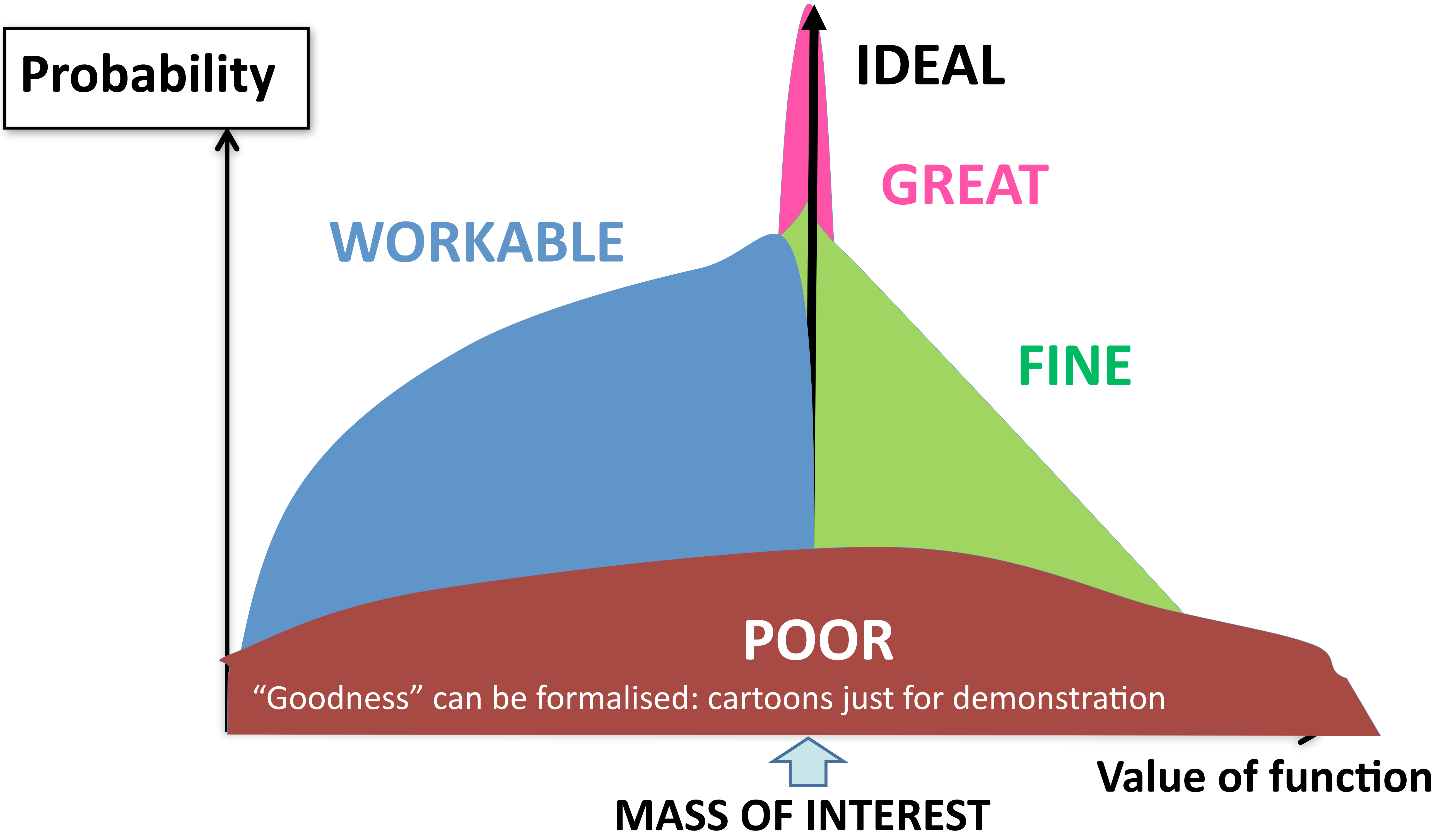
Good vs poor variables



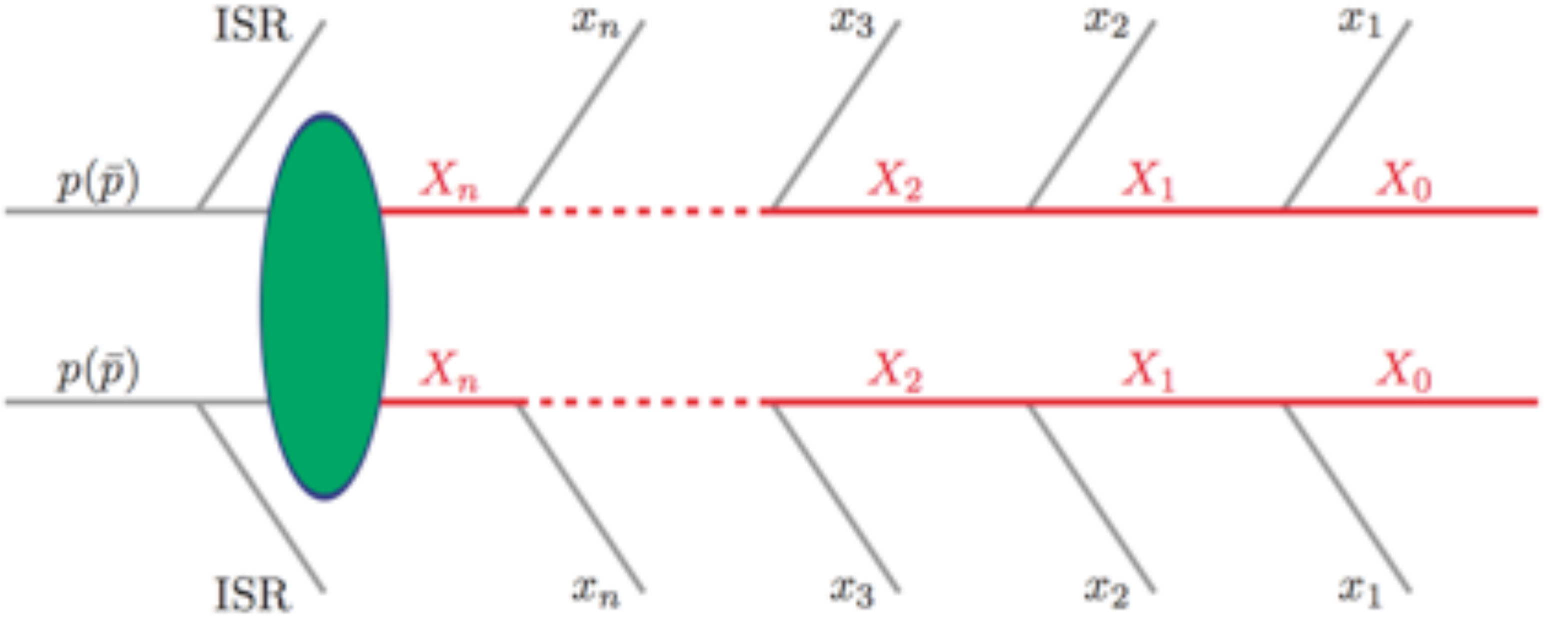
“Goodness” can be formalised: cartoons just for demonstration

Taken from Alan Barr’s talk

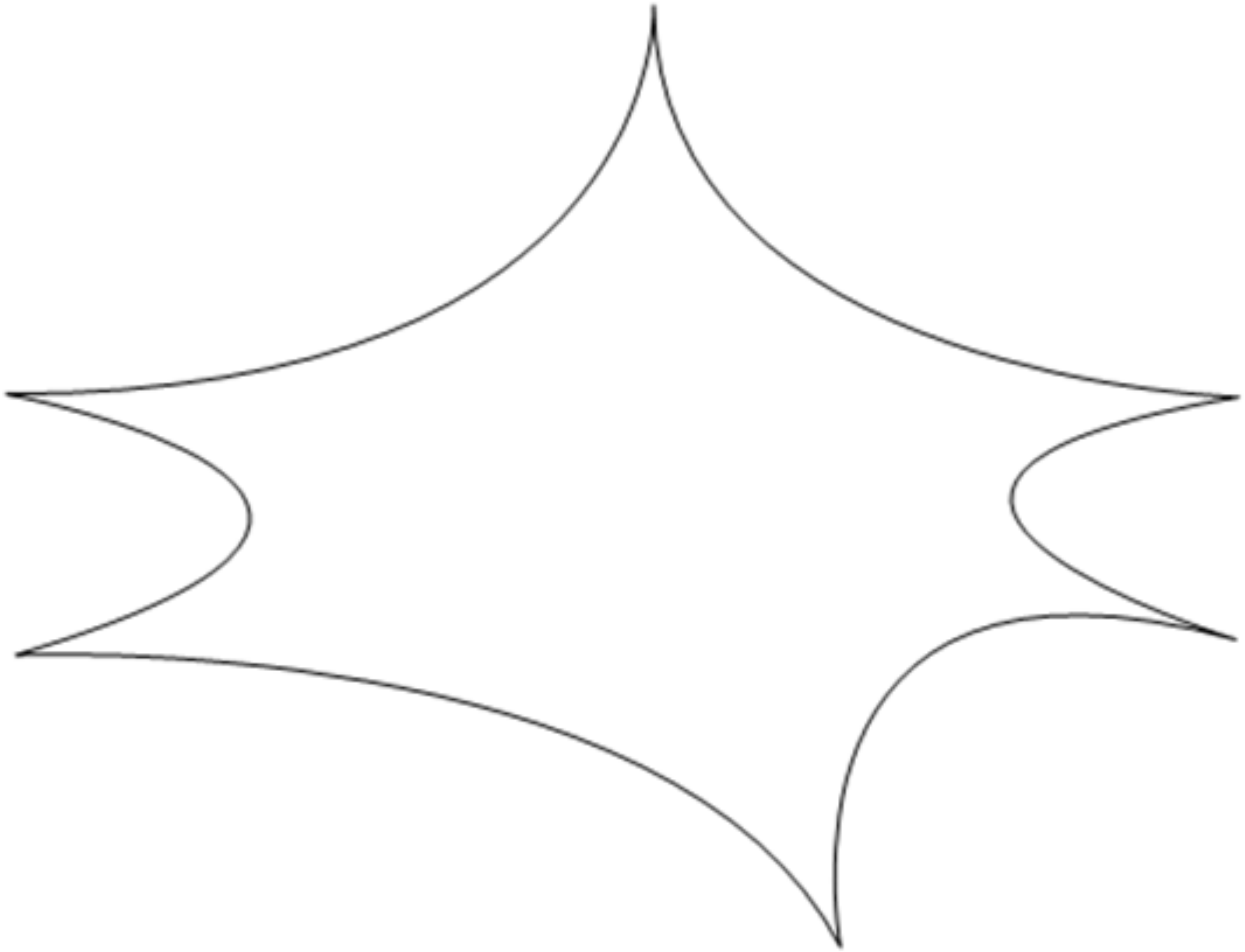
A storm in a "T" cup: the connoisseur's guide to transverse projections and mass-constraining variables, 1105.2977

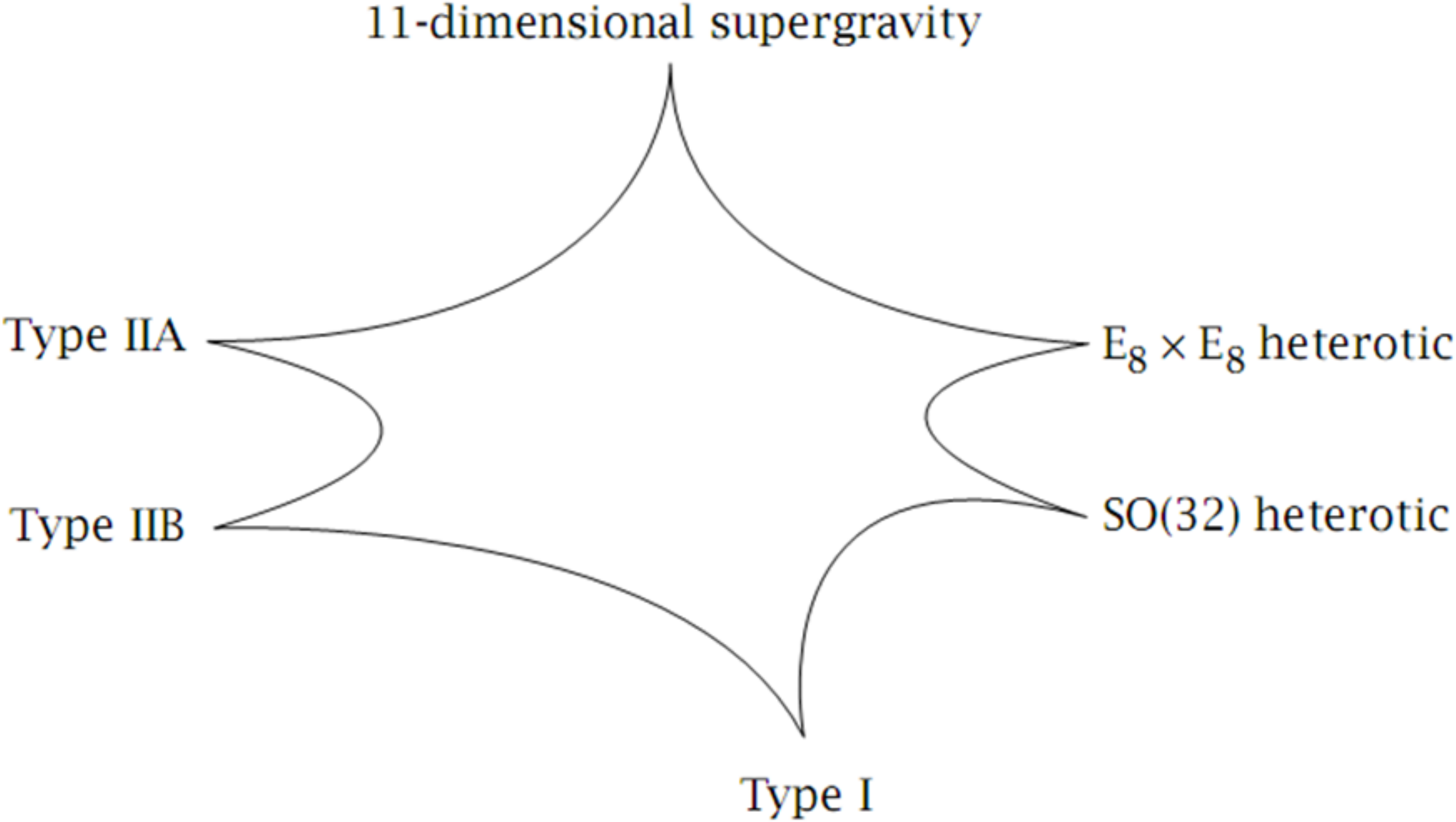
- 7 authors (3 ATLAS, 2 CMS, 2 Theory)
  - 3-2-2 to 5-1-1 (faculty/postdocs/students)
  - 4-3 to 5-2 (experimentalists/theorists)
- ~ 50 pages (in two columns)
- ~ 300 equations
- 14 figures
- ~60 references

Why so many variables?



- Generic SUSY-like event: (at least) two invisible particles. Exact reconstruction is difficult, especially for:
- Large n: combinatorial problem
  - H_T, missing E_T, M_{eff}, M_{TGen}, S_{min}
- Small n: lack of information problem
  - M_T, M_{T2}, $M_{T,ZZ}$, $M_{C,WW}$, M_{2C}, M_{T2perp}, $M_{T2parallel}$
- Note the common feature in many of these variables
  - the index “T”





11-dimensional supergravity

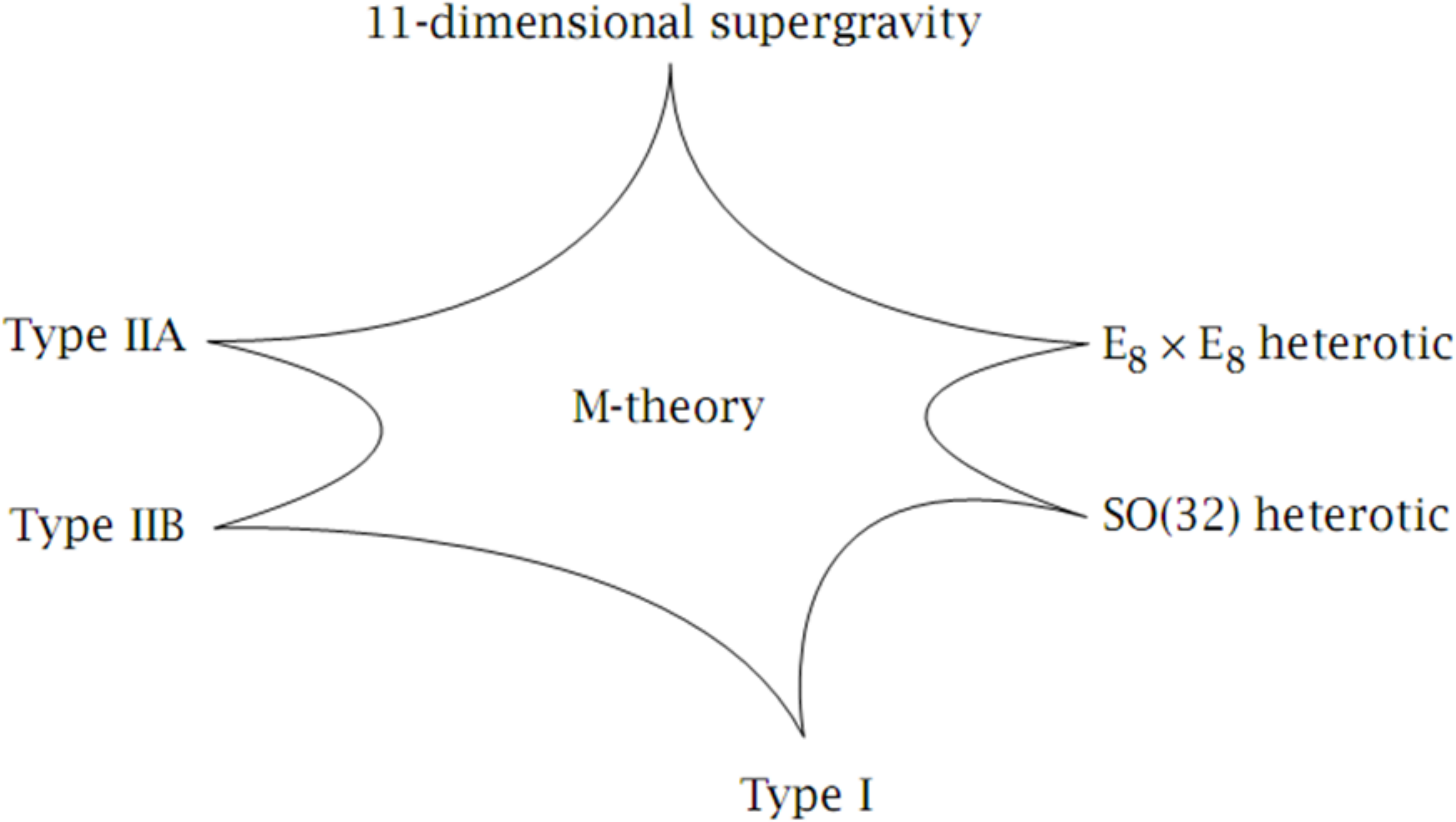
Type IIA

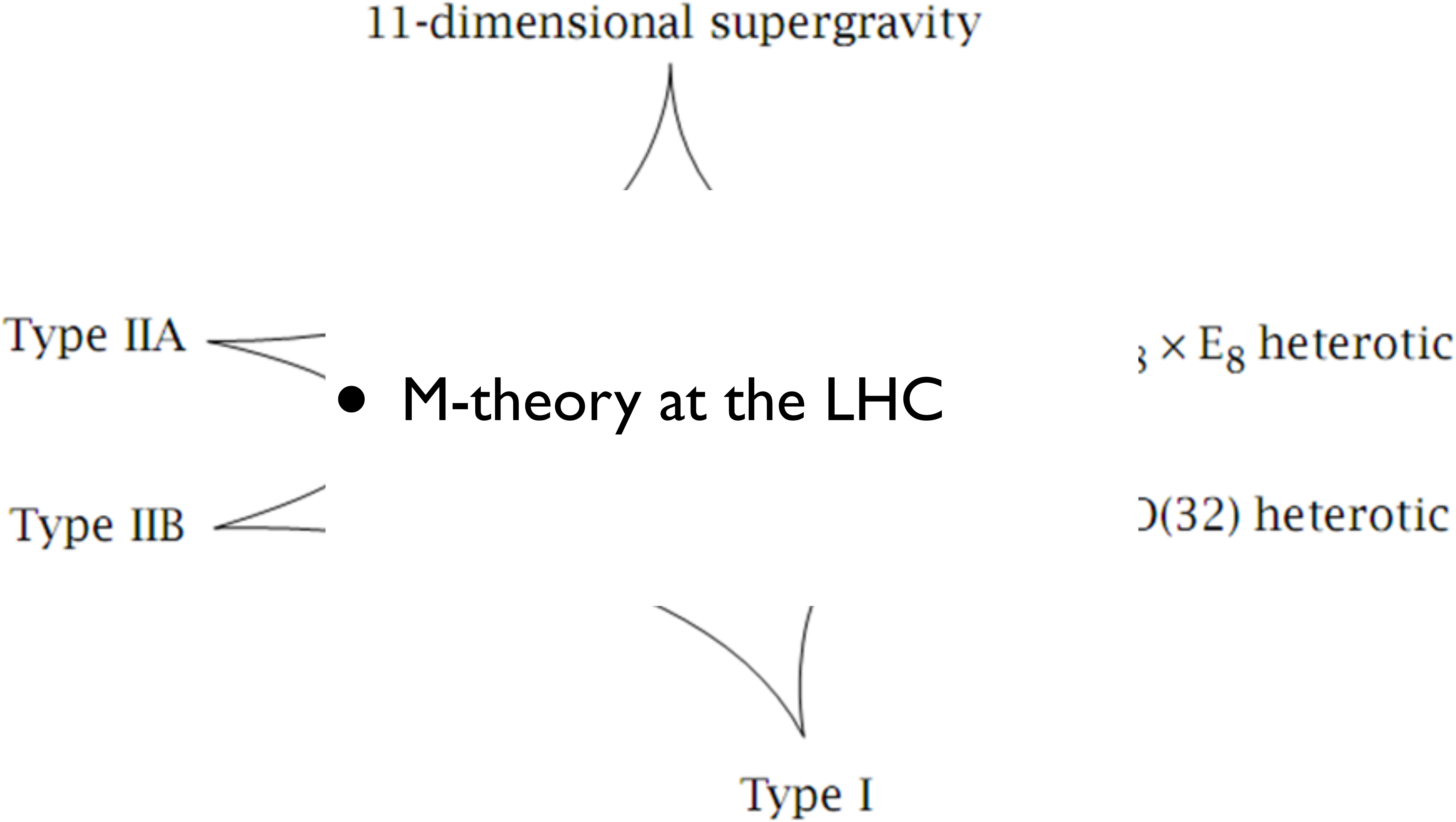
$E_8 \times E_8$ heterotic

Type IIB

SO(32) heterotic

Type I

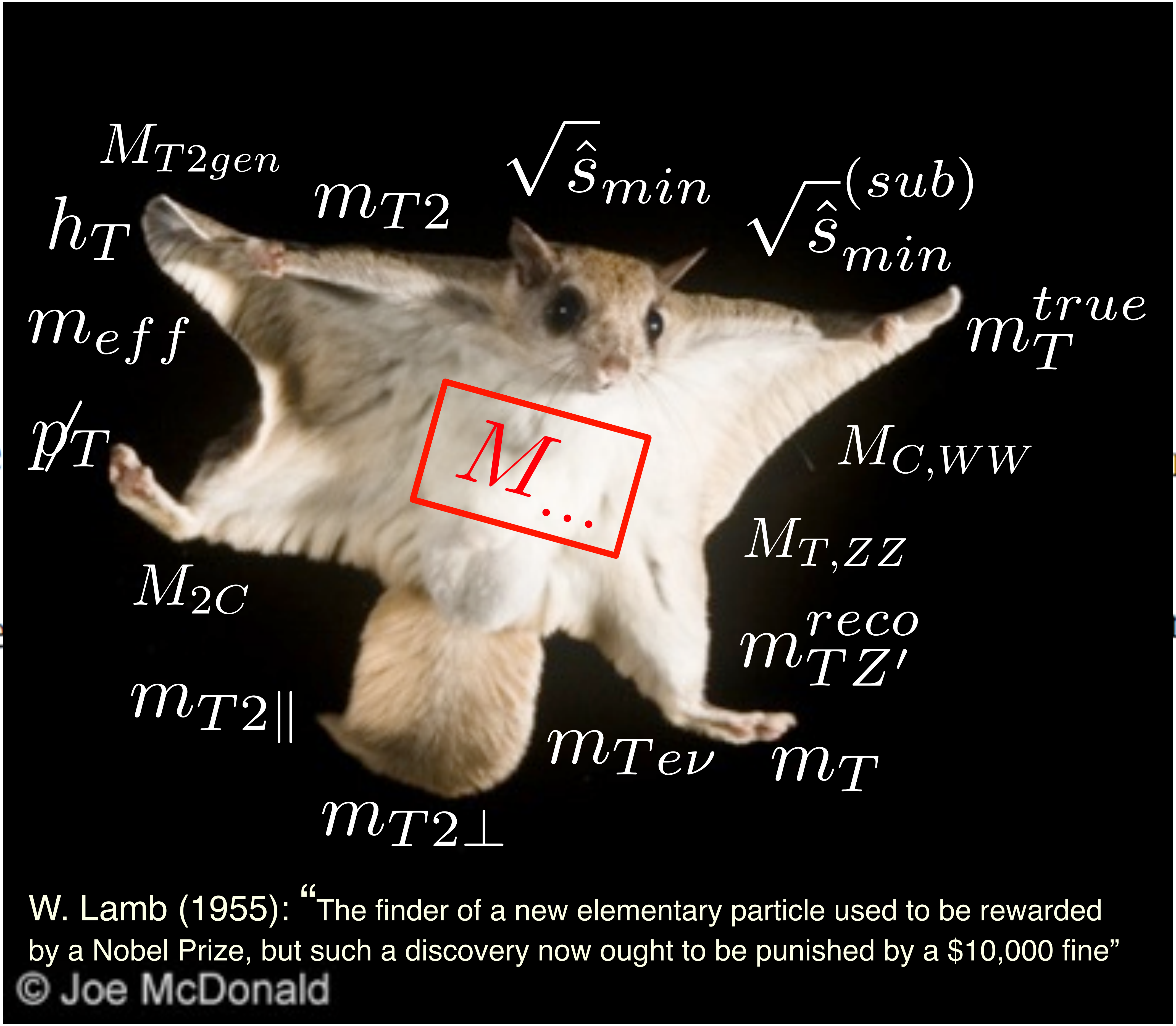

11-dimensional supergravity
Type IIA
Type IIB
M-theory
$E_8 \times E_8$ heterotic
SO(32) heterotic
Type I



- M-theory at the LHC

Type rotic

Type rotic



W. Lamb (1955): “The finder of a new elementary particle used to be rewarded by a Nobel Prize, but such a discovery now ought to be punished by a $10,000 fine”

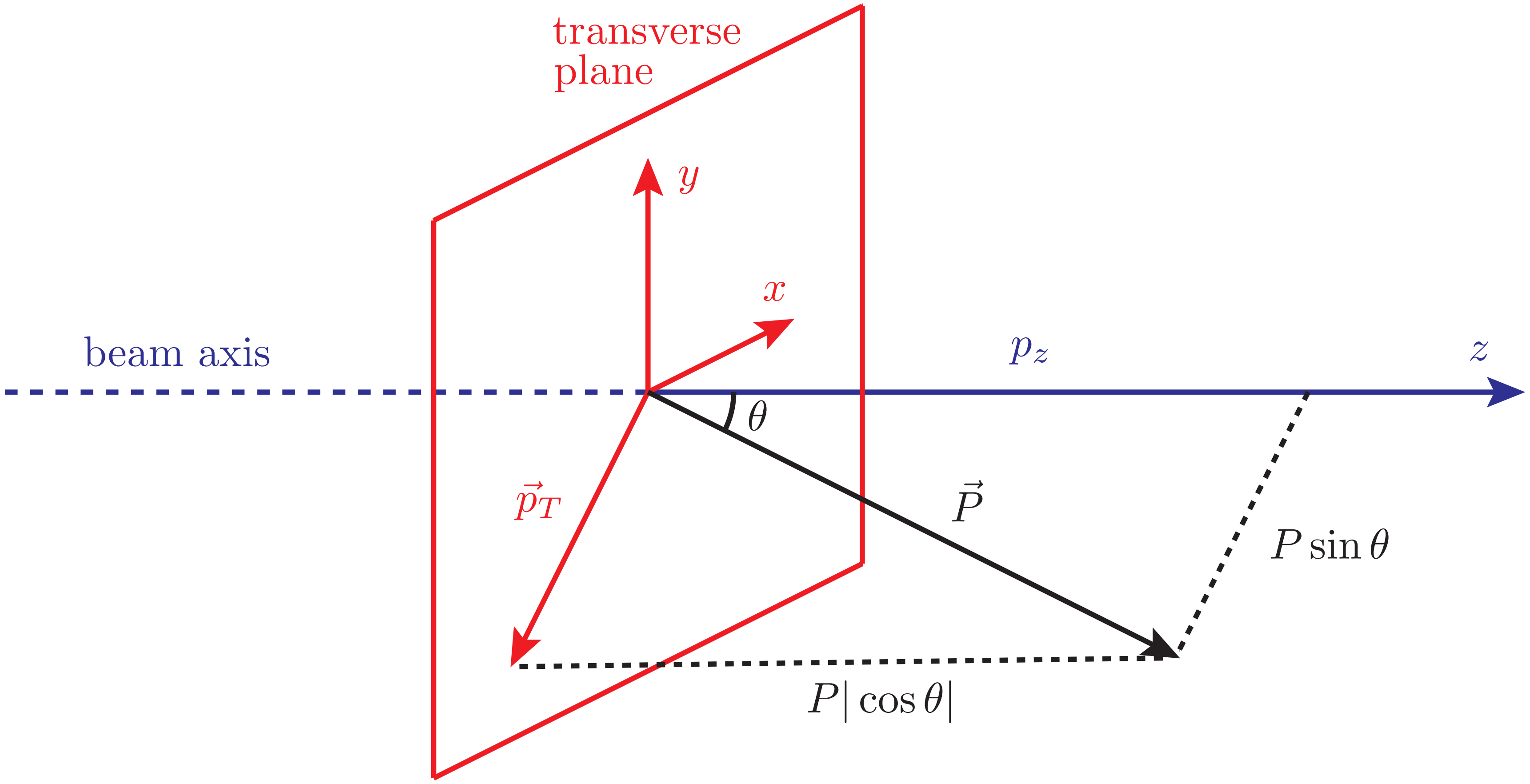
© Joe McDonald

Outline

- Transversification
  - how do we project particle momenta?
- Agglomeration
  - how do we add transverse momenta?
- Interpretation
  - how do we categorize reconstructed objects?
- Generalization
  - how do we define the most general mass-bound variables?
- Specialization
  - how do we recover the existing variables?
  - illustration: dilepton tt-bar and h->WW examples.

Transversification of 3-vectors

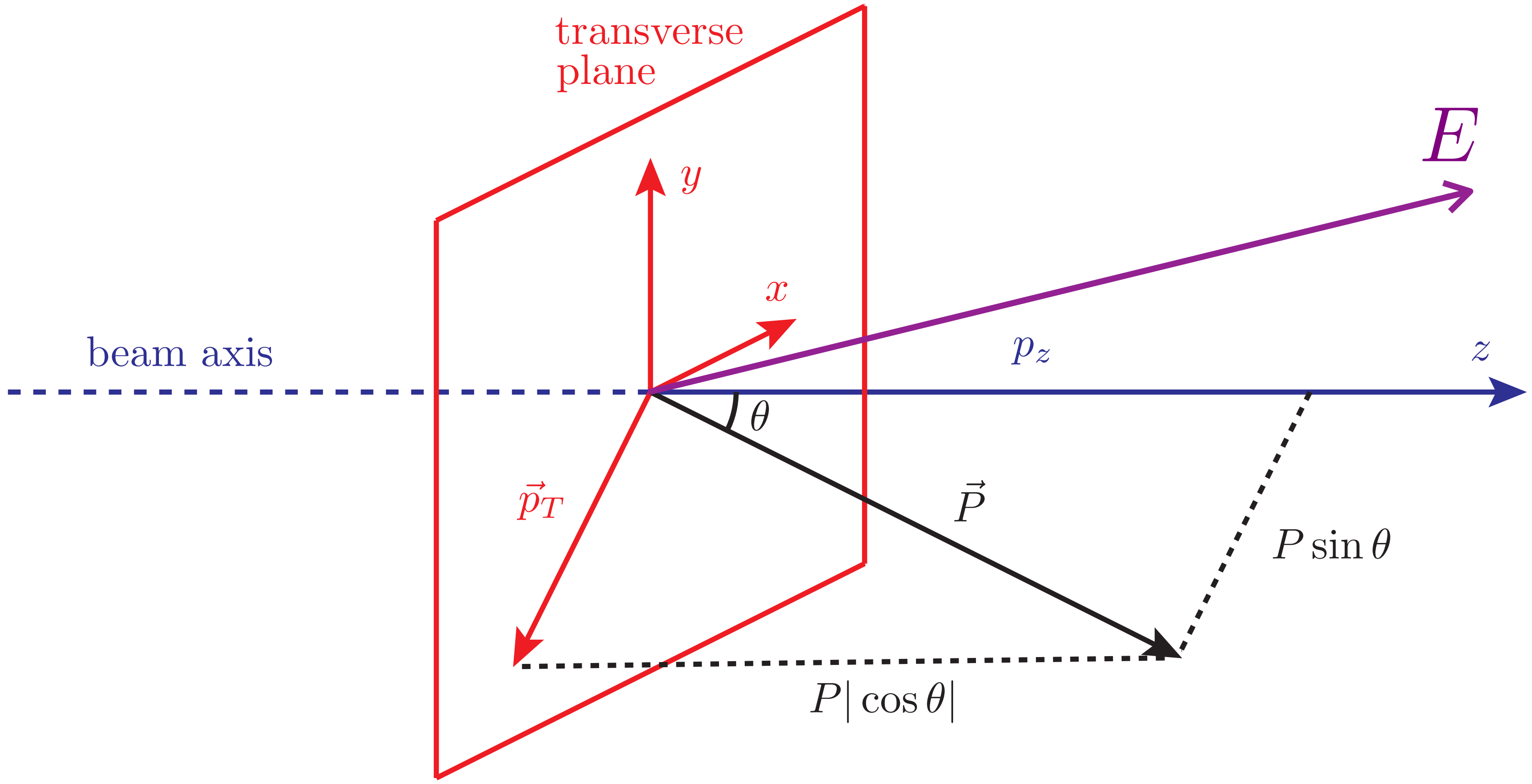
- Warm-up exercise: geometrical projection



$$p_T = P\sin\theta$$

Transversification of 1+3-vectors

- What to do with the energy (time-like) component?



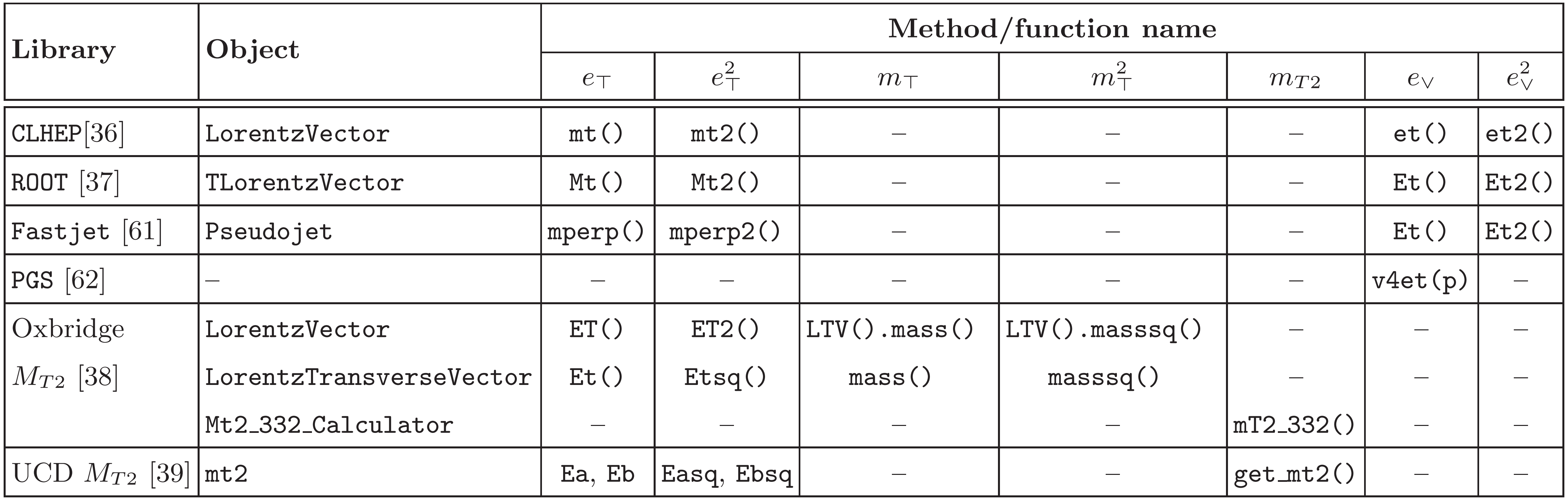
- Well, isn’t it obvious? Not really: there are at least three different options for the “transverse” energy: “T”, “V” and “0”.

Summary of transverse projections

	Transverse projection method		
Quantity	**Mass-preserving '$\top$'**	**Speed-preserving '$\vee$'**	**Massless '$\circ$'**
Original (4)-momentum	$P^\mu = (E, \vec{p}_T, p_z)$		
(1+3)-mass invariant	$M = \sqrt{E^2 - \vec{p}_T^{\,2} - p_z^2}$		
Transverse momentum	$\vec{p}_T \equiv (p_x, p_y)$		
(1+2)-vectors	$p_\top^\alpha \equiv (e_\top, \vec{p}_\top)$	$p_\vee^\alpha \equiv (e_\vee, \vec{p}_\vee)$	$p_\circ^\alpha \equiv (e_\circ, \vec{p}_\circ)$
Transverse momentum under the projection	$\vec{p}_\top \equiv \vec{p}_T$	$\vec{p}_\vee \equiv \vec{p}_T$	$\vec{p}_\circ \equiv \vec{p}_T$
Transverse energy under the projection	$e_\top \equiv \sqrt{M^2 + \vec{p}_T^{\,2}}$	$e_\vee \equiv E\,\lvert\sin\theta\rvert = \lvert\vec{p}_T\rvert / V$	$e_\circ \equiv \lvert\vec{p}_T\rvert$
Transverse mass under the projection	$m_\top^2 = e_\top^2 - \vec{p}_\top^{\,2}$	$m_\vee^2 \equiv e_\vee^2 - \vec{p}_\vee^{\,2}$	$m_\circ^2 \equiv e_\circ^2 - \vec{p}_\circ^{\,2} = 0$
Relationship between transverse quantity and its (1+3) analogue	$m_\top = M$	$m_\vee = M\,\lvert\sin\theta\rvert$	$m_\circ = 0$
	$\frac{1}{v_\top} = \frac{1}{V}\sqrt{1 + (1 - V^2)\frac{p_z^2}{p_T^2}}$	$v_\vee = V$	$v_\circ = 1$
Equivalence classes under $(1+3) \overset{\text{proj}}{\longmapsto} (1+2)$	All P^μ with the same p_x, p_y and M	All P^μ with the same p_x, p_y and V	All P^μ with the same p_x and p_y

A guide to existing computer codes

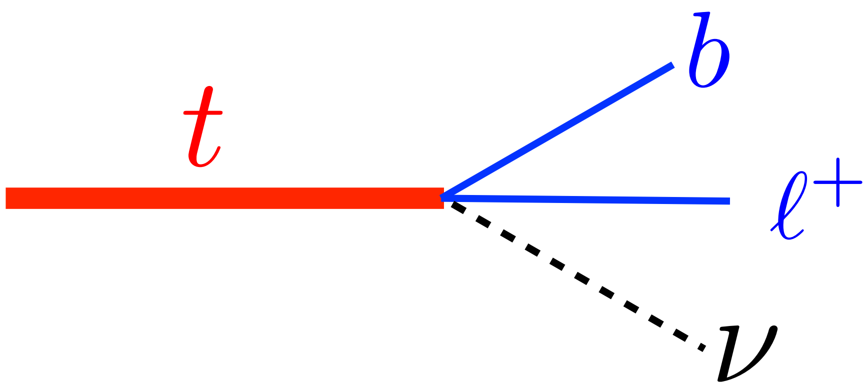
- Both “T” and “V” projections appear to be used in the existing computer libraries and codes

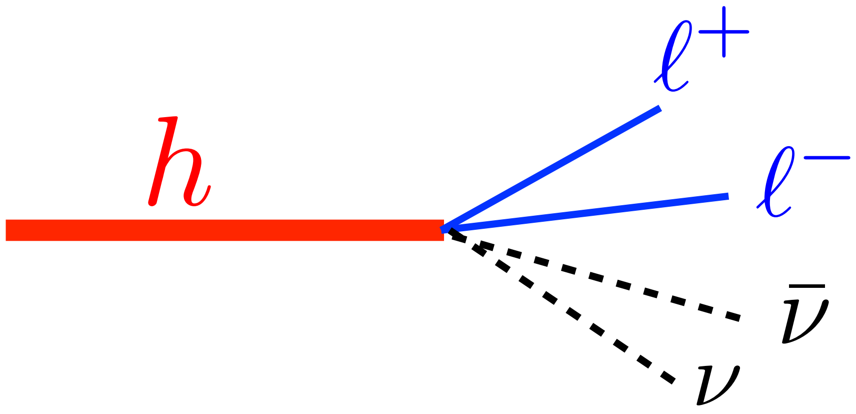


Library	**Object**	**Method/function name**						
		$e_\top$	$e_\top^2$	$m_\top$	$m_\top^2$	m_{T2}	$e_\vee$	$e_\vee^2$
`CLHEP`[36]	`LorentzVector`	`mt()`	`mt2()`	–	–	–	`et()`	`et2()`
`ROOT` [37]	`TLorentzVector`	`Mt()`	`Mt2()`	–	–	–	`Et()`	`Et2()`
`Fastjet` [61]	`Pseudojet`	`mperp()`	`mperp2()`	–	–	–	`Et()`	`Et2()`
`PGS` [62]	–	–	–	–	–	–	`v4et(p)`	–
Oxbridge M_{T2} [38]	`LorentzVector`	`ET()`	`ET2()`	`LTV().mass()`	`LTV().masssq()`	–	–	–
	`LorentzTransverseVector`	`Et()`	`Etsq()`	`mass()`	`masssq()`	–	–	–
	`Mt2_332_Calculator`	–	–	–	–	`mT2_332()`	–	–
UCD M_{T2} [39]	`mt2`	`Ea`, `Eb`	`Easq`, `Ebsq`	–	–	`get_mt2()`	–	–

Agglomeration

- Heavy, promptly, semi-invisibly decaying resonances are reconstructed by agglomerating their daughter particles





$$t \to b\ell^+\nu$$

$$h \to W^+W^- \to \ell^+\ell^-\nu\bar{\nu}$$

- Transverse quantities are constructed by transverse projections
- Which should come first: the projection or the agglomeration? The results are different!

“Early” versus “late” projections

- The order of the operations makes a big difference for the time-like components

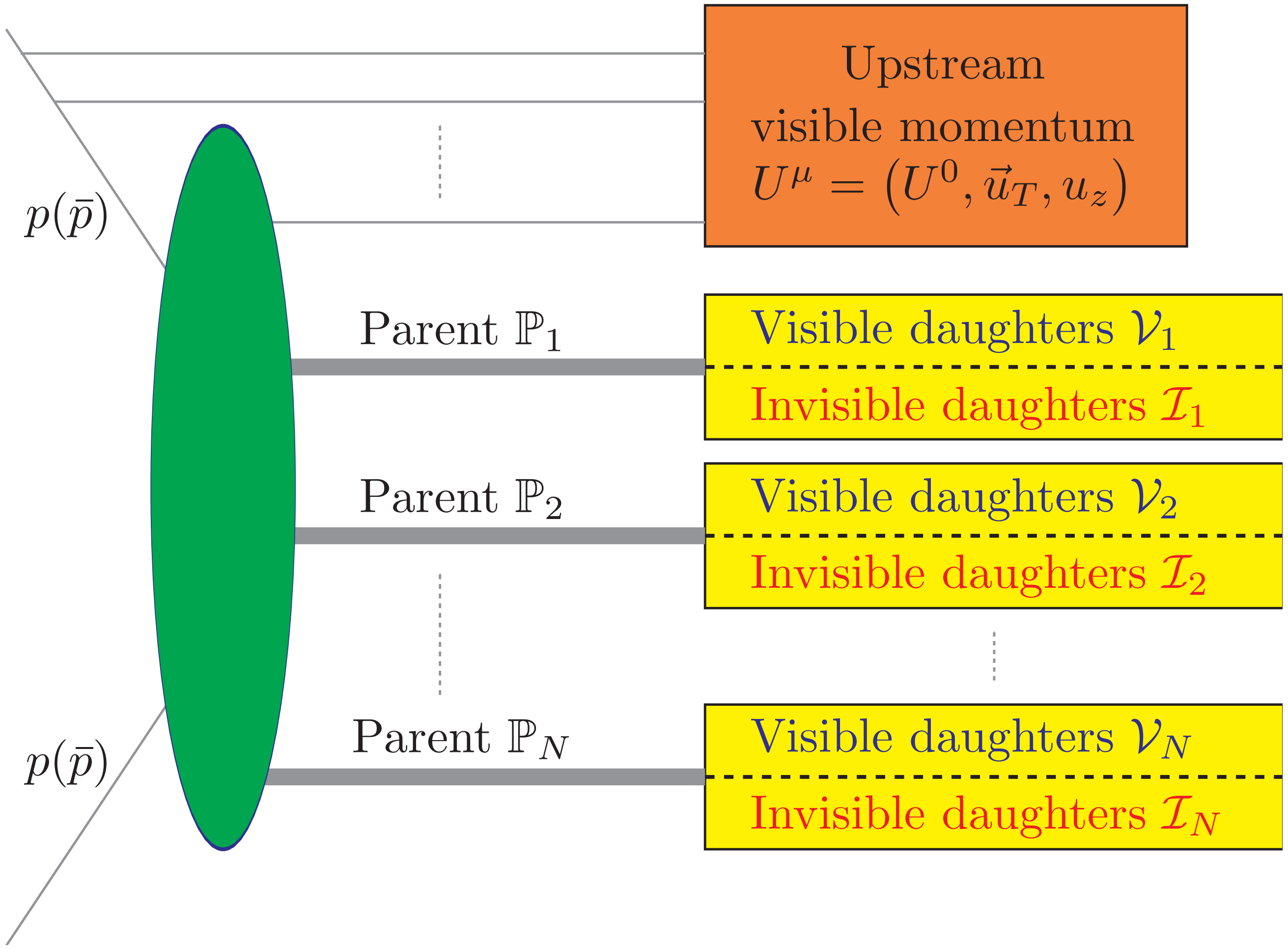
$$\sum_i \vec{p}_{i\top} = \left(\sum_i \vec{P}_i\right)_\top \qquad \sum_i e_{i\top} \neq \left(\sum_i E_i\right)_\top ,$$

$$\sum_i \vec{p}_{i\vee} = \left(\sum_i \vec{P}_i\right)_\vee \qquad \sum_i e_{i\vee} \neq \left(\sum_i E_i\right)_\vee ,$$

$$\sum_i \vec{p}_{i\circ} = \left(\sum_i \vec{P}_i\right)_\circ \qquad \sum_i e_{i\circ} \neq \left(\sum_i E_i\right)_\circ .$$

- Our convention: the order of indices (from left to right) denotes the order of operations, e.g.
  - – add first, project later: $p^\alpha_{aT} \equiv (e_{aT}, \vec{p}_{aT})$
  - – project first, add later: $p^\alpha_{Ta} \equiv (e_{Ta}, \vec{p}_{Ta})$

Interpretation (of an event)



- N "parents". For each:
  - Visible daughters
  - Invisible daughters
- Upstream momentum
- Missing p_T

$$\not{\vec{p}}_T \equiv -\vec{u}_T - \sum_{i=1}^{N_{\mathcal{V}}} \vec{p}_{iT}$$

- Notation for particle momenta:
  - "P" ("p") for visible daughters
  - "Q" ("q") for invisible daughters

How to form mass-bound variables

- Goal: find a lower bound on the mass of the heaviest (next-heaviest, etc.) parent
- There are various possibilities:
  - – 1 unprojected $\mathcal{M}_a \equiv \sqrt{g_{\mu\nu}\,(\mathbf{P}_a^\mu + \mathbf{Q}_a^\mu)(\mathbf{P}_a^\nu + \mathbf{Q}_a^\nu)}$
  - – 3 late-projected $\mathcal{M}_{aT} \equiv \sqrt{g_{\alpha\beta}\,(\mathbf{p}_{aT}^\alpha + \mathbf{q}_{aT}^\alpha)(\mathbf{p}_{aT}^\beta + \mathbf{q}_{aT}^\beta)}$
  - – 3 early-projected $\mathcal{M}_{Ta} \equiv \sqrt{g_{\alpha\beta}\,(\mathbf{p}_{Ta}^\alpha + \mathbf{q}_{Ta}^\alpha)(\mathbf{p}_{Ta}^\beta + \mathbf{q}_{Ta}^\beta)}$
- Then minimize over the momenta of the invisible particles:

$$\begin{aligned} M_N &\equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_a \right] \right] , \\ M_{NT} &\equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{aT} \right] \right] , \\ M_{TN} &\equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{Ta} \right] \right] , \end{aligned}$$

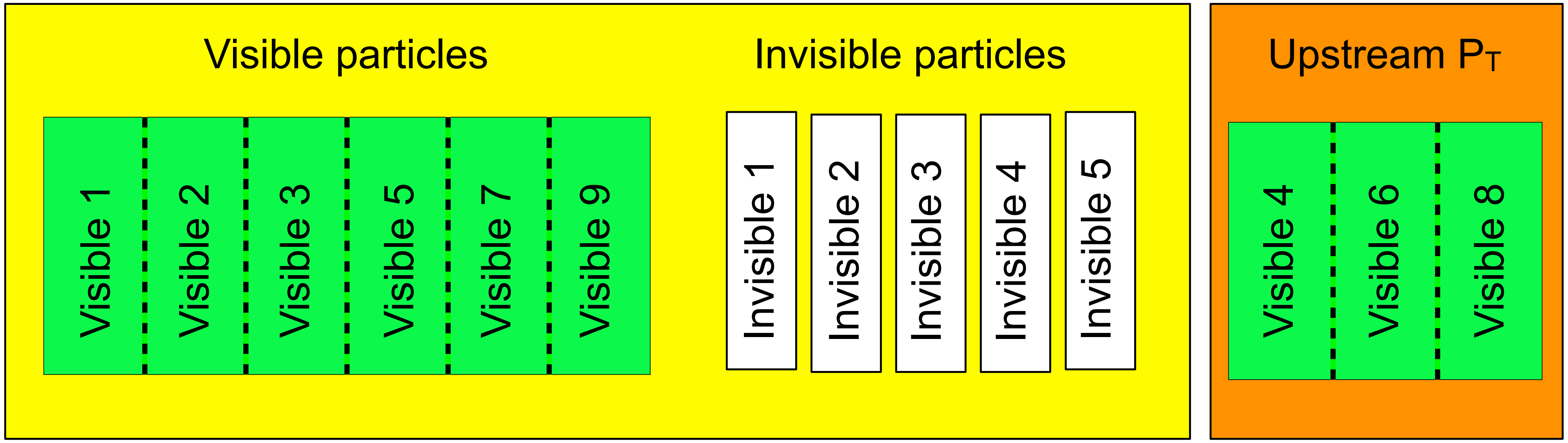
The 7 basic mass bound variables

Type of variables	Operations			Notation
	First	Second	Third	
Unprojected	Partitioning	Minimization	—	M_N ✓
Early partitioned (late projected) M_{NT}	Partitioning	$T = \top$ projection	Minimization	$M_{N\top}$ ✓
	Partitioning	$T = \vee$ projection	Minimization	$M_{N\vee}$
	Partitioning	$T = \circ$ projection	Minimization	$M_{N\circ}$ ✓
Late partitioned (early projected) M_{TN}	$T = \top$ projection	Partitioning	Minimization	$M_{\top N}$ ✓
	$T = \vee$ projection	Partitioning	Minimization	$M_{\vee N}$
	$T = \circ$ projection	Partitioning	Minimization	$M_{\circ N}$ ✓

- Can you recognize which one is the Cambridge M_{T2}?

Example: The unprojected M_1

- This is the minimum total invariant mass of the single-parent subsystem



Visible particles

Visible 1

Visible 2

Visible 3

Visible 5

Visible 7

Visible 9

Invisible particles

Invisible 1

Invisible 2

Invisible 3

Invisible 4

Invisible 5

Upstream P_T

Visible 4

Visible 6

Visible 8

$$M_1^2(\not{\mathbf{M}}_1) \equiv \left(\sqrt{\mathbf{M}_1^2+\mathbf{p}_{1T}^2}+\sqrt{\not{\mathbf{M}}_1^2+\not{p}_T^2}\right)^2 - u_T^2 \equiv \hat{s}_{min}^{(sub)}$$

Total visible mass: $\mathbf{M}_1 \equiv \sqrt{\mathbf{E}_1^2 - \vec{\mathbf{p}}_{1T}^2 - \mathbf{p}_{1z}^2},$

Konar, Kong, Matchev, Park 2010

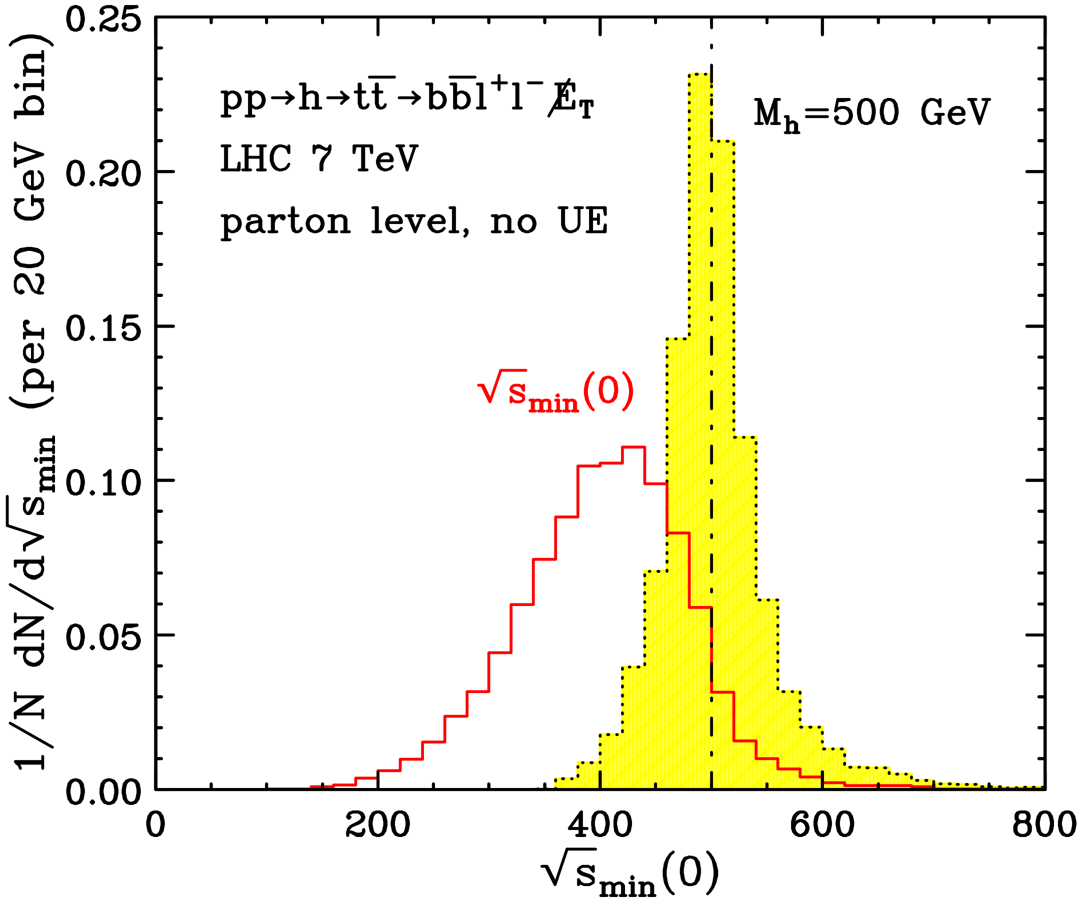
Total invisible mass: $\not{\mathbf{M}}_1 \equiv \sum_{i=1}^{N_{\mathcal{I}}} \tilde{M}_i.$

Applications of $\boxed{\sqrt{s_{min}}}$

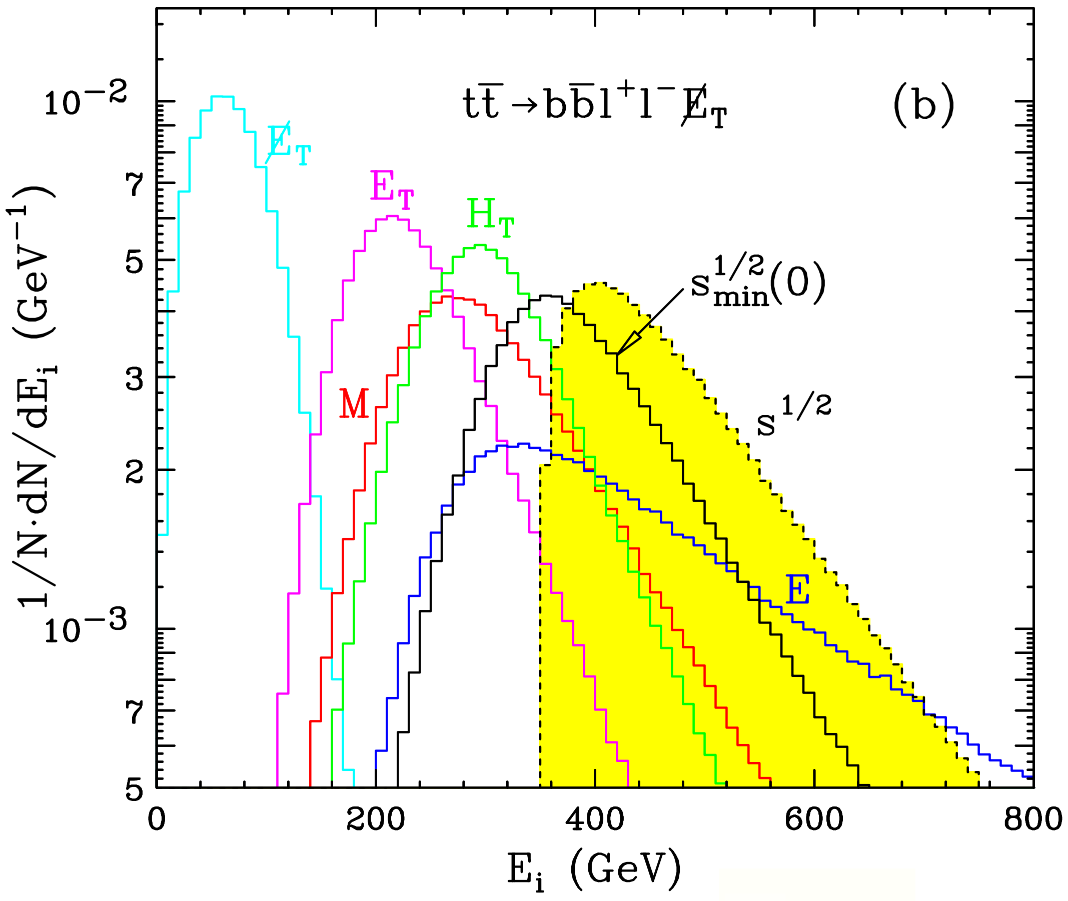
Konar, Kong, Matchev 2008

Konar, Kong, Matchev, Park 2010

- N=1: Single semi-invisibly decaying particle
  - SM Higgs to tt-bar
  - endpoint at the parent mass

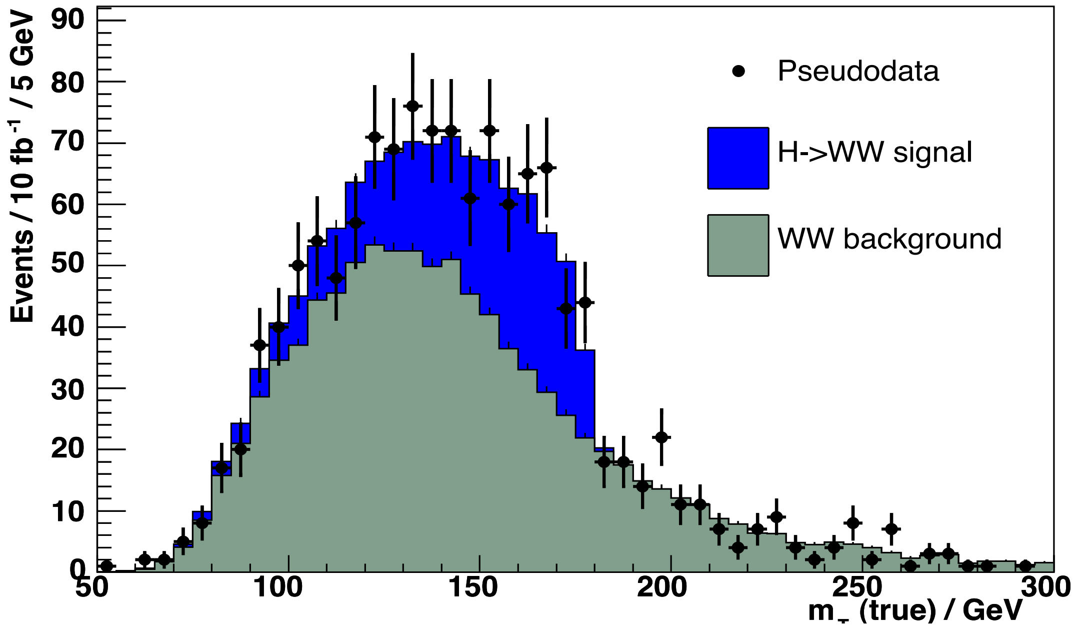


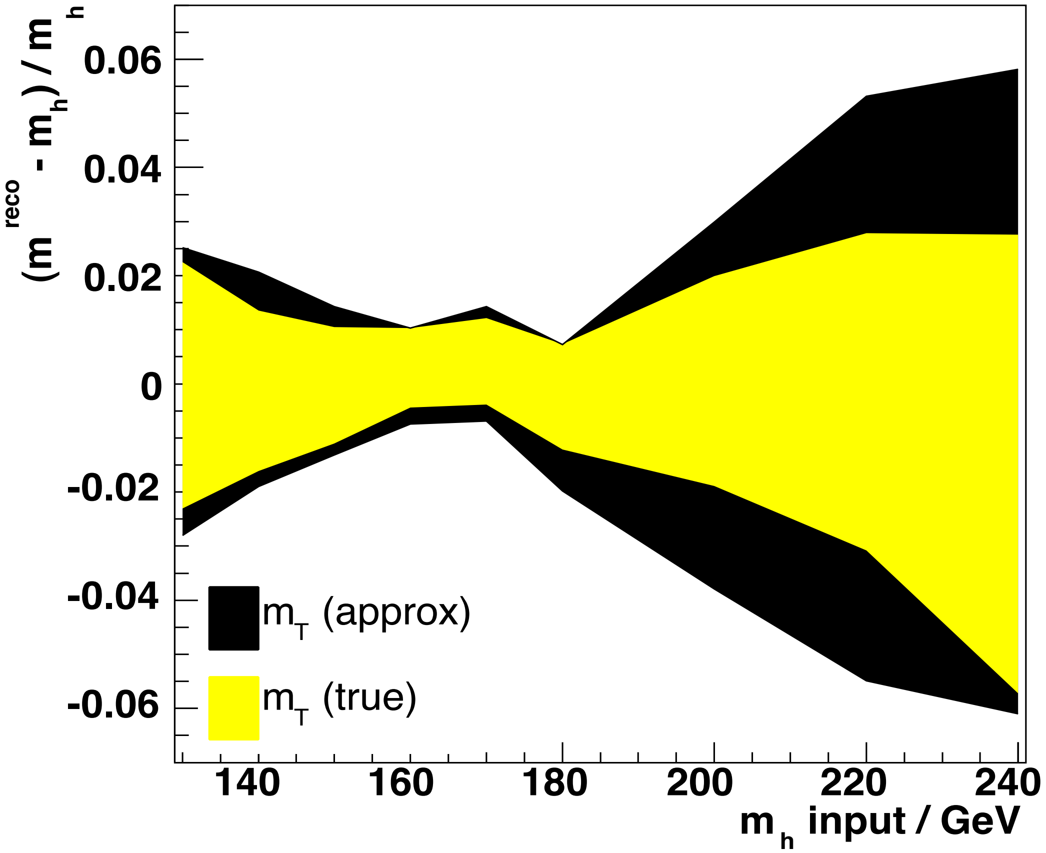
- N=2: A pair of semi-invisibly decaying particles
  - direct tt-bar production
  - peak at the total parent mass



Barr, Gripaios, Lester 2009

$$h \to WW^{(*)} \to \ell^+\ell^-\nu\bar{\nu}$$

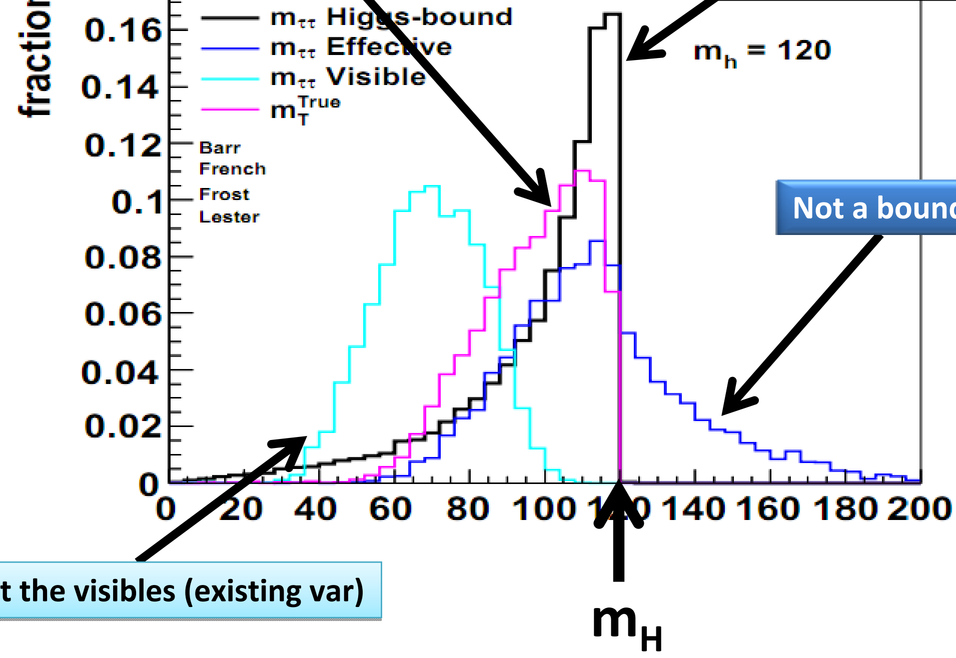




Parent mas bound (no intermediate constraint) = M1T

Result

Including the intermediate constraint (BEST)



Not a bound (existing var)

Just the visibles (existing var)

Dramatic difference to Higgs observability?

Barr, French, Frost, Lester 2011

$$h \to \tau\tau$$

$$m_{\tau\tau}^{\mathrm{Higgs-bound}} = \min_{\{Q_1^\mu, Q_2^\mu | \aleph\}} \sqrt{H^\mu H_\mu}$$

$$H^\mu = P_1^\mu + Q_1^\mu + P_2^\mu + Q_2^\mu$$

$$\begin{aligned} Q_1^\mu Q_{1\mu} &= 0, \\ Q_2^\mu Q_{2\mu} &= 0, \\ (Q_1^\mu + P_1^\mu)(Q_{1\mu} + P_{1\mu}) &= m_\tau^2, \\ (Q_2^\mu + P_2^\mu)(Q_{2\mu} + P_{2\mu}) &= m_\tau^2, \end{aligned}$$

$$\vec{q}_{1\mathrm{T}} + \vec{q}_{2\mathrm{T}} = \not{\vec{p}}_{\mathrm{T}}.$$

History repeats but we learn more and understand better

$$M_T = 2\sqrt{p_T^2(ll) + m^2(ll)},$$

$$M_C = \sqrt{p_T^2(ll) + m^2(ll)} + \not{E}_T$$

Han,Zhang 1998,1999

$$\begin{aligned}\sqrt{s}_{min}^{(sub)}(\not{M}) &= \left\{ \left(\sqrt{E_{(sub)}^2 - P_{z(sub)}^2} + \sqrt{\not{M}^2 + \not{P}_T^2} \right)^2 - P_{T(up)}^2 \right\}^{\frac{1}{2}} \\ &= \left\{ \left(\sqrt{M_{(sub)}^2 + P_{T(sub)}^2} + \sqrt{\not{M}^2 + \not{P}_T^2} \right)^2 - P_{T(up)}^2 \right\}^{\frac{1}{2}} \\ &= \left\{ \left(\sqrt{M_{(sub)}^2 + P_{T(sub)}^2} + \sqrt{\not{M}^2 + \not{P}_T^2} \right)^2 - (\vec{P}_{T(sub)} + \vec{\not{P}}_T)^2 \right\}^{\frac{1}{2}} \\ &= ||p_{T(sub)} + \not{p}_T||\,,\end{aligned}$$

$$\begin{aligned} p_{T(sub)} &\equiv \left(\sqrt{M_{(sub)}^2 + P_{T(sub)}^2}\,,\ \vec{P}_{T(sub)} \right) \\ \not{p}_T &\equiv \left(\sqrt{\not{M}^2 + \not{P}_T^2}\,,\ \vec{\not{P}}_T \right)\,. \end{aligned}$$

Konar, Kong, Matchev, Park, 2008 2010

$$(m_T^{\mathrm{true}})^2 \equiv m_T^2(m_i = 0) = m_v^2 + 2(e_v|\mathbf{p}_i| - \mathbf{p}_v \cdot \mathbf{p}_i)$$

Barr, Gripaios, Lester 2009, 2011

$$\boxed{M_1^2(\not{\mathbf{M}}_1) \equiv \left(\sqrt{\mathbf{M}_1^2 + \mathbf{p}_{1T}^2} + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2} \right)^2 - u_T^2}$$

Barr, Khoo, Konar, Kong, Lester,
Matchev, Park 2011

The late “T”-projected variable M_{NT}

- The order is: agglomerate, “T”-project, then minimize over q_{iT} and q_{iz}. First form each parent mass

$$\mathcal{M}^2_{a\top}(\mathbf{p}^\alpha_{a\top}, \mathbf{q}^\alpha_{a\top}, \tilde{\mu}_a) \equiv (\mathbf{p}_{a\top} + \mathbf{q}_{a\top})^2 \equiv (\mathbf{e}_{a\top} + \tilde{\mathbf{e}}_{a\top})^2 - (\vec{\mathbf{p}}_{aT} + \vec{\mathbf{q}}_{aT})^2$$

- Then minimize the largest one:

$$M_{N\top}(\not{\mathbf{M}}) \equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{a\top}(\mathbf{p}^\alpha_{a\top}, \mathbf{q}^\alpha_{a\top}, \tilde{\mu}_a) \right] \right]$$

- For N=1 the result is

$$M^2_{1\top}(\not{\mathbf{M}}_1) \equiv \left(\sqrt{\mathbf{M}_1^2 + \mathbf{p}_{1T}^2} + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2} \right)^2 - u_T^2 \equiv \hat{s}^{(sub)}_{min}$$

- In general one finds the identity

$$\boxed{M_{N\top} = M_N}$$

The early"T"-projected variable M_{TN}

- The order is: "T"-project, agglomerate, then minimize over q_{iT} (there is no q_{iz} dependence).

$$\mathcal{M}^2_{\top a}(\mathbf{p}^{\alpha}_{\top a}, \mathbf{q}^{\alpha}_{\top a}, \tilde{\mu}_a) \equiv (\mathbf{p}_{\top a} + \mathbf{q}_{\top a})^2 \equiv (\mathbf{e}_{\top a} + \tilde{\mathbf{e}}_{\top a})^2 - (\vec{\mathbf{p}}_{aT} + \vec{\mathbf{q}}_{aT})^2$$

- Then minimize the largest one:

$$M_{\top N}(\not{\mathbf{M}}) \equiv \min_{\sum \vec{q}_{iT} = \not{\vec{p}}_T} \left[\max_a \left[\mathcal{M}_{\top a}(\mathbf{p}^{\alpha}_{\top a}, \mathbf{q}^{\alpha}_{\top a}, \tilde{\mu}_a) \right] \right]$$

- For N=1 the result is

$$M^2_{\top 1}(\not{\mathbf{M}}_1) = \left(\sum_{i=1}^{N_{\mathcal{V}}} \sqrt{M_i^2 + \vec{p}_{iT}^{\,2}} + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2} \right)^2 - u_T^2$$

- For massless visible particles (leptons or jets)

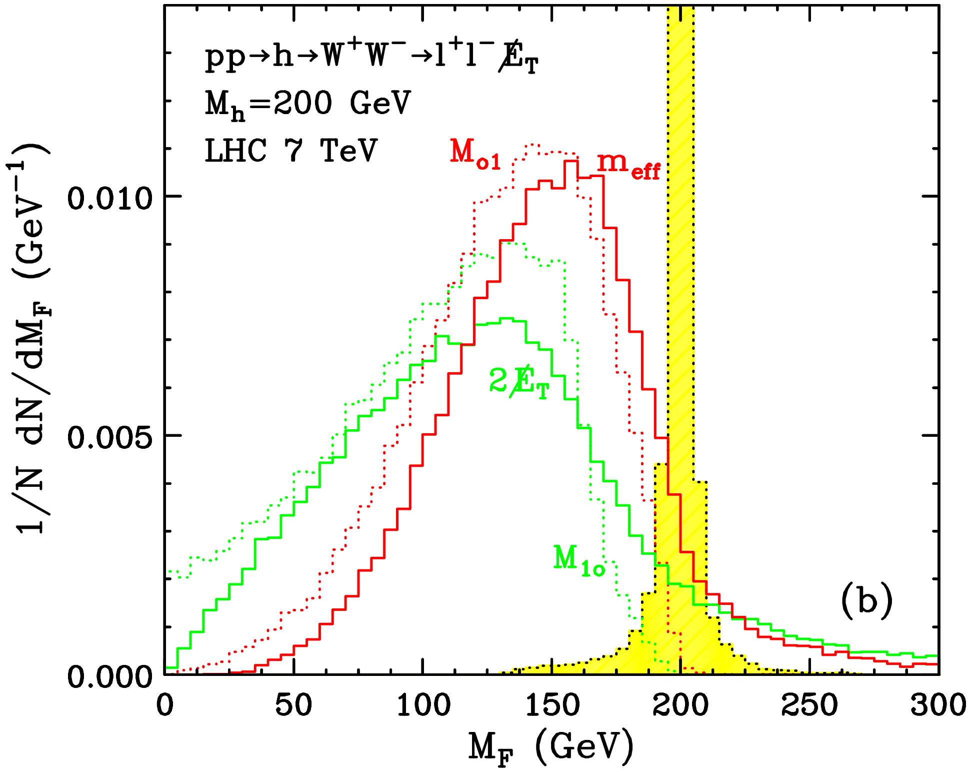
$$\boxed{\lim_{M_i \to 0} M^2_{\top 1}(\not{\mathbf{M}}_1) = \left(h_T + \sqrt{\not{\mathbf{M}}_1^2 + \not{p}_T^2} \right)^2 - u_T^2}$$

Generalized version of Meff

The early “0”-projected variable M_{0N}

$$M_{\circ N} \equiv \min_{\sum \vec{q}_{iT} = \vec{\not p}_T} \left[\max_a \left[\mathcal{M}_{\circ a}(\mathbf{p}^\alpha_{\circ a}, \mathbf{q}^\alpha_{\circ a}) \right] \right]$$

$$\begin{aligned}
M^2_{\circ 1} &= \min_{\sum \vec{q}_{iT} = \vec{\not p}_T} \left[\left(\sum_{i=1}^{N_{\mathcal{V}}} e_{i\circ} + \sum_{i=1}^{N_{\mathcal{I}}} \tilde{e}_{i\circ} \right)^2 - u_T^2 \right] \\
&= \left(\sum_{i=1}^{N_{\mathcal{V}}} e_{i\circ} + \min_{\sum \vec{q}_{iT} = \vec{\not p}_T} \left[\sum_{i=1}^{N_{\mathcal{I}}} \tilde{e}_{i\circ} \right] \right)^2 - u_T^2 \\
&= \left(\sum_{i=1}^{N_{\mathcal{V}}} p_{iT} + \min_{\sum \vec{q}_{iT} = \vec{\not p}_T} \left[\sum_{i=1}^{N_{\mathcal{I}}} q_{iT} \right] \right)^2 - u_T^2 \\
&= \left(\sum_{i=1}^{N_{\mathcal{V}}} p_{iT} + \not p_T \right)^2 - u_T^2 \\
&= \left(h_T + \not p_T \right)^2 - u_T^2, \\
&= m^2_{\text{eff}} - u_T^2.
\end{aligned}$$



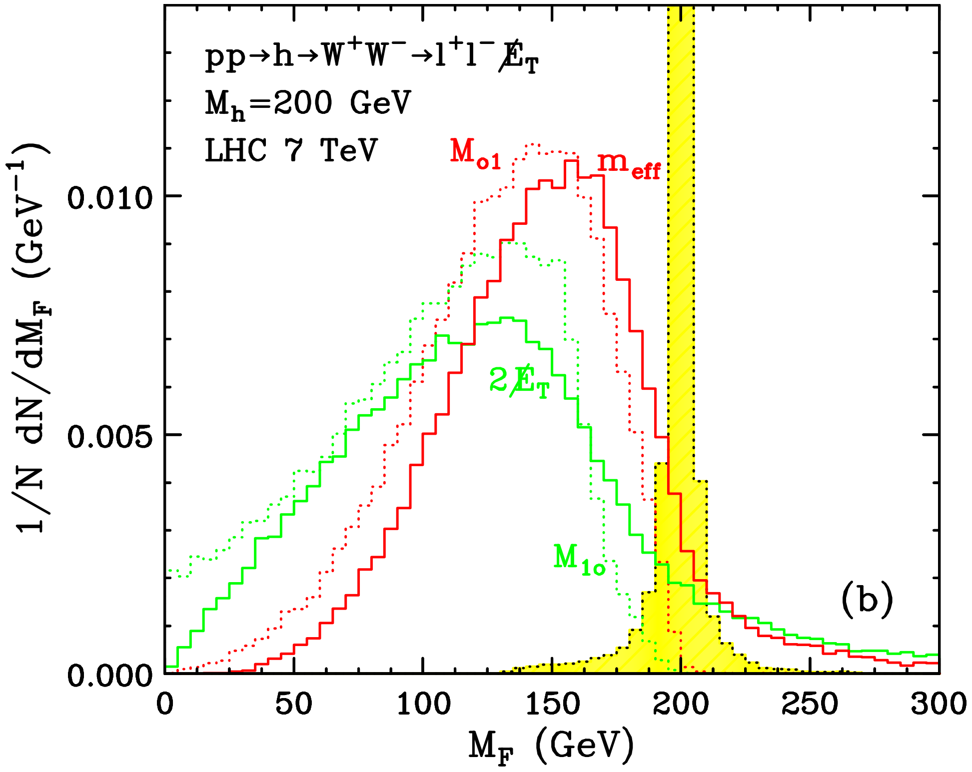
$$h_T \equiv \sum_{i=1}^{N_{\mathcal{V}}} p_{iT}$$

$$m_{\text{eff}} \equiv h_T + \not p_T$$

The late “0”-projected variable M$_{N0}$

$$M_{N\circ} \equiv \min_{\sum \vec{q}_{iT}=\vec{\not{p}}_T} \left[\max_a \left[\mathcal{M}_{a\circ}(\mathbf{p}^{\alpha}_{a\circ}, \mathbf{q}^{\alpha}_{a\circ})\right]\right]$$

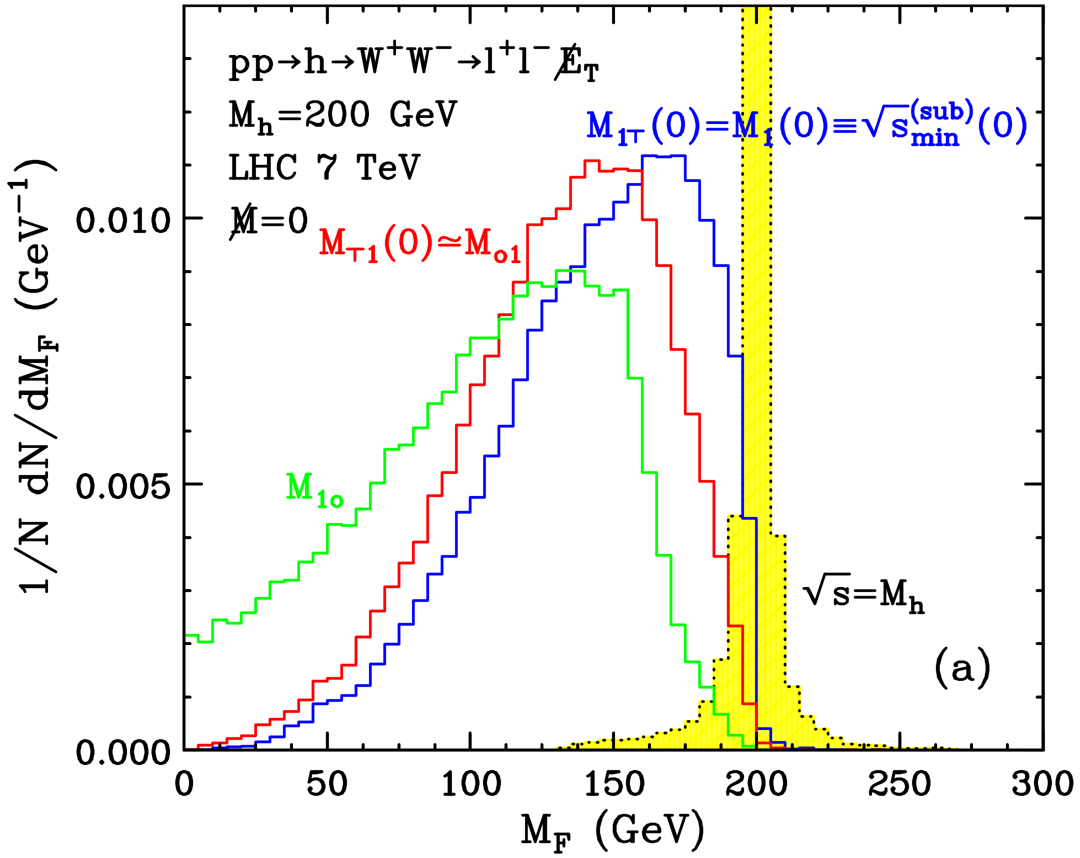
$$\begin{aligned}
M^2_{1\circ} &= \min_{\sum \vec{q}_{iT}=\vec{\not{p}}_T} \left[\left(\mathbf{e}_{1\circ} + \tilde{\mathbf{e}}_{1\circ}\right)^2 - u_T^2\right] \\
&= \left(\mathbf{e}_{1\circ} + \min_{\sum \vec{q}_{iT}=\vec{\not{p}}_T} \left[\tilde{\mathbf{e}}_{1\circ}\right]\right)^2 - u_T^2 \\
&= \left(\mathbf{e}_{1\circ} + \min_{\sum \vec{q}_{iT}=\vec{\not{p}}_T} \left[\left|\sum_{i=1}^{N_{\mathcal{I}}} \vec{q}_{iT}\right|\right]\right)^2 - u_T^2. \\
&= \left(\left|\sum_{i=1}^{N_{\mathcal{V}}} \vec{p}_{iT}\right| + \not{p}_T\right)^2 - u_T^2 \\
&= \left(|\vec{\not{p}}_T + \vec{u}_T| + \not{p}_T\right)^2 - u_T^2 \\
&= 2\left(\vec{\not{p}}_T \cdot (\vec{\not{p}}_T + \vec{u}_T) + \not{p}_T\, |\vec{\not{p}}_T + \vec{u}_T|\right)
\end{aligned}$$

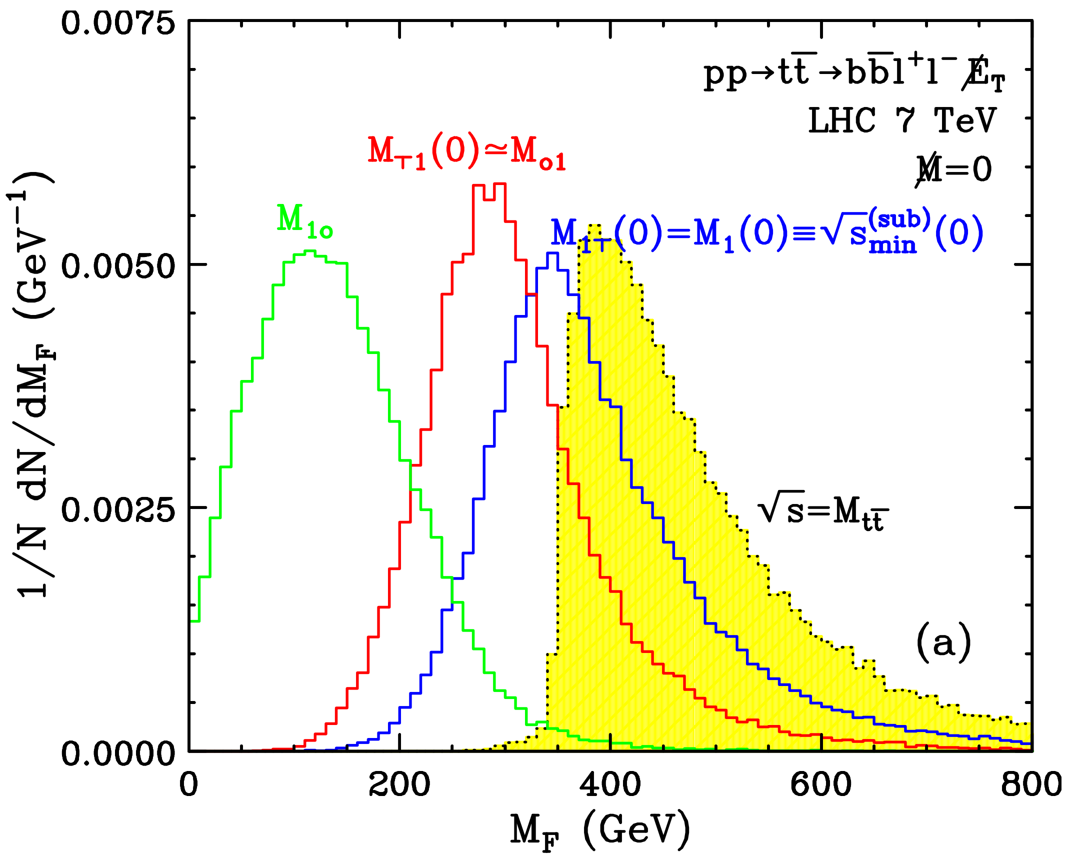


$$\boxed{\lim_{u_T \to 0} M^2_{1\circ} = 4\not{p}_T^2}$$

Which variable is best?

$$M_N = M_{N\top} \geq M_{\top N} \geq M_{\circ N} \geq M_{N\circ}$$



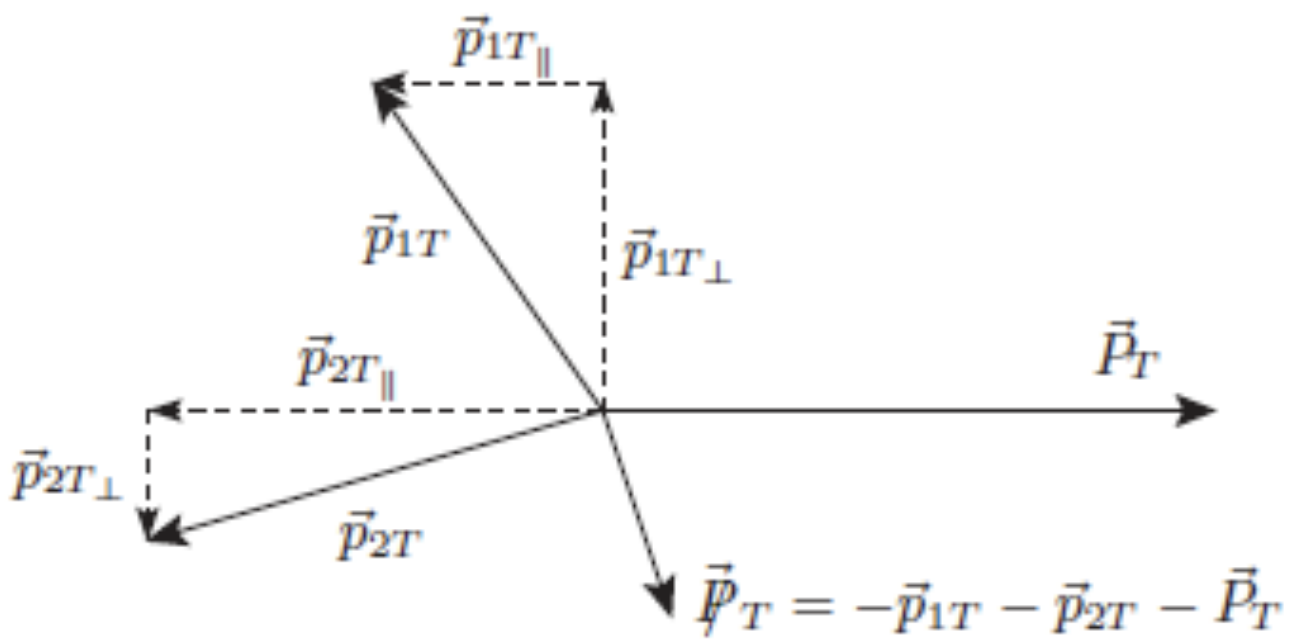


- Late (or no) projection gives a better endpoint structure
- Early projection less sensitive to forward hadronic activity

1D decomposition, 2009

(Konar, Kong, Matchev, Park 2009)

- ISR (UTM) increases the MT2max
- No general analytic expression for MT2



$$e_{iT_\parallel} \equiv \sqrt{m_i^2 + |\vec{p}_{iT_\parallel}|^2}, \quad e_{iT_\perp} \equiv \sqrt{m_i^2 + |\vec{p}_{iT_\perp}|^2}.$$

$$\vec{p}_{iT_\parallel} \equiv \frac{1}{P_T^2}\left(\vec{p}_{iT} \cdot \vec{P}_T\right)\vec{P}_T,$$

$$\vec{p}_{iT_\perp} \equiv \vec{p}_{iT} - \vec{p}_{iT_\parallel} = \frac{1}{P_T^2}\vec{P}_T \times \left(\vec{p}_{iT} \times \vec{P}_T\right),$$

$$M_{T_\parallel}^{(i)} \equiv \sqrt{\tilde{M}_c^2 + 2\left(e_{iT_\parallel}\sqrt{\tilde{M}_c^2 + |\vec{p}_{cT_\parallel}^{(i)}|^2} - \vec{p}_{iT_\parallel}\cdot\vec{p}_{cT_\parallel}^{(i)}\right)},$$

$$M_{T_\perp}^{(i)} \equiv \sqrt{\tilde{M}_c^2 + 2\left(e_{iT_\perp}\sqrt{\tilde{M}_c^2 + |\vec{p}_{cT_\perp}^{(i)}|^2} - \vec{p}_{iT_\perp}\cdot\vec{p}_{cT_\perp}^{(i)}\right)},$$

$$M_{T2_\perp}(\tilde{M}_c, \vec{p}_{iT_\perp}) \equiv \min\left\{\max\left\{M_{T_\perp}^{(1)}, M_{T_\perp}^{(2)}\right\}\right\}. \qquad \textstyle\sum_k(\vec{p}_{cT_\perp}^{(k)} + \vec{p}_{kT_\perp}) = 0:$$

For one massless visible particle, MT2perp becomes simple

$$M_{T2_\perp} = \begin{cases} \tilde{M}_c, & \text{if } \vec{p}_{1T_\perp}\cdot\vec{p}_{2T_\perp} \le 0, \\ \sqrt{A_{T_\perp}} + \sqrt{A_{T_\perp} + \tilde{M}_c^2}, & \text{otherwise,} \end{cases}$$

$$M_{T2_\perp}^{(max)}(\tilde{M}_c) = \mu + \sqrt{\mu^2 + \tilde{M}_c^2}, \qquad \mu \equiv \frac{M_p}{2}\left(1 - \frac{M_c^2}{M_p^2}\right).$$

Define: $$N(\tilde{M}_c) \equiv \sum_{\text{all events}} H\left(M_{T2} - \tilde{M}_p(\tilde{M}_c, 0)\right), \qquad H(x) \equiv \begin{cases} 1, & \text{if } x > 0, \\ 0, & \text{if } x \le 0, \end{cases}$$

$$\tilde{M}_p(\tilde{M}_c, P_T) - \tilde{M}_p(\tilde{M}_c, 0) \ge 0, \quad \Longrightarrow \quad N_{min} \equiv \min\{N(\tilde{M}_c)\} = N(M_c) = 0.$$

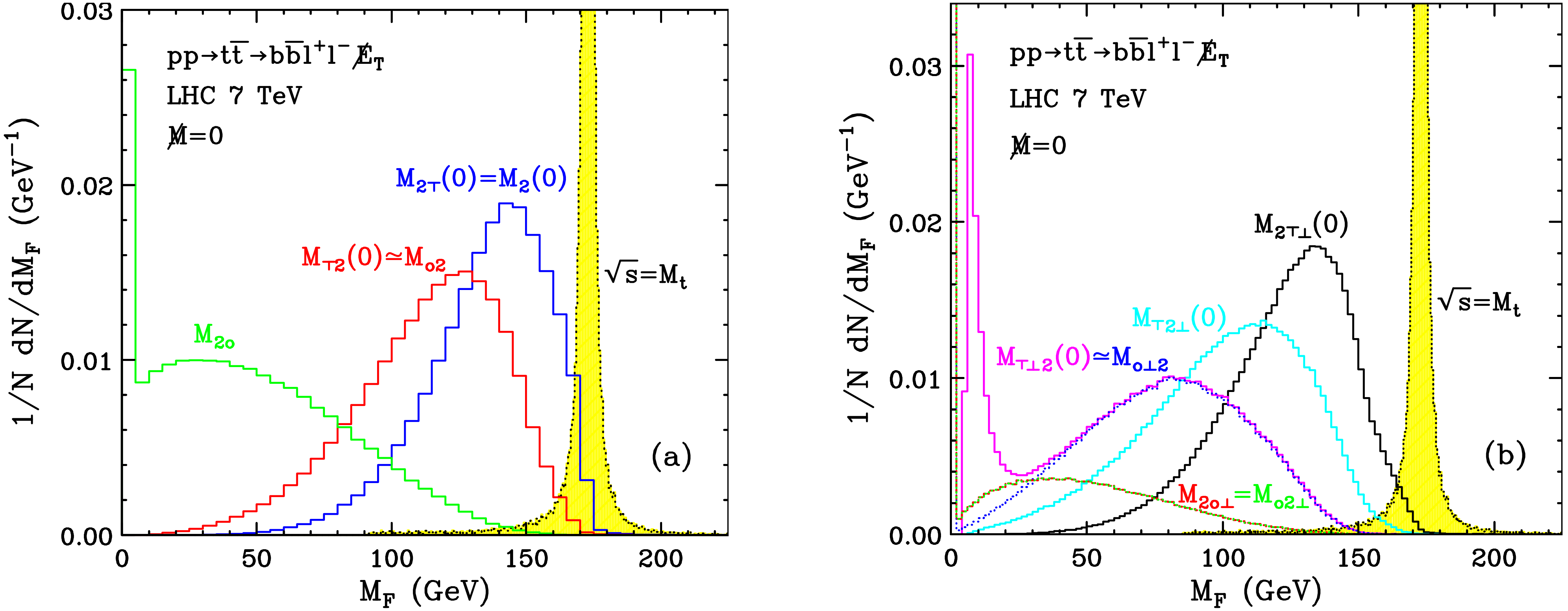
Type of variables	Operations				Notation
	First	Second	Third	Fourth	
Early partitioned doubly projected $M_{NT\perp}$	Partitioning	$T = \top$ projection	$\perp = \top$ projection on $T_\perp$	Minimization	$M_{N\top\perp}$
	Partitioning	$T = \vee$ projection	$\perp = \vee$ projection on $T_\perp$	Minimization	$M_{N\vee\perp}$
	Partitioning	$T = \circ$ projection	$\perp = \circ$ projection on $T_\perp$	Minimization	$M_{N\circ\perp}$
Late partitioned, doubly projected $M_{T\perp N}$	$T = \top$ projection	$\perp = \top$ projection on $T_\perp$	Partitioning	Minimization	$M_{\top\perp N}$
	$T = \vee$ projection	$\perp = \vee$ projection on $T_\perp$	Partitioning	Minimization	$M_{\vee\perp N}$
	$T = \circ$ projection	$\perp = \circ$ projection on $T_\perp$	Partitioning	Minimization	$M_{\circ\perp N}$
In-between partitioned, doubly projected $M_{TN\perp}$	$T = \top$ projection	Partitioning	$\perp = \top$ projection on $T_\perp$	Minimization	$M_{\top N\perp}$
	$T = \vee$ projection	Partitioning	$\perp = \vee$ projection on $T_\perp$	Minimization	$M_{\vee N\perp}$
	$T = \circ$ projection	Partitioning	$\perp = \circ$ projection on $T_\perp$	Minimization	$M_{\circ N\perp}$

TABLE IV. An extended version of Table III, containing the additional variables found by including the option of a $T_\perp$ projection shown in Fig. 9. An analogous set of variables is obtained by considering a $T_\parallel$ projection instead.

Early partition	Hedged partition	Late partition
$M_{N\top\parallel\vee}$ $M_{N\top\parallel\circ}$ $M_{N\top\perp\vee}$, $M_{N\top\perp\circ}$	$M_{\top N\parallel\vee}$ $M_{\top N\parallel\circ}$, $M_{\top N\perp\vee}$, $M_{\top N\perp\circ}$	$M_{\top\parallel\vee N}$ $M_{\top\parallel\circ N}$, $M_{\top\perp\vee N}$, $M_{\top\perp\circ N}$
$M_{N\circ\parallel\top}$ $M_{N\circ\parallel\vee}$ $M_{N\circ\perp\top}$, $M_{N\circ\perp\vee}$	$M_{\circ N\parallel\top}$ $M_{\circ N\parallel\vee}$, $M_{\circ N\perp\top}$, $M_{\circ N\perp\vee}$	$M_{\circ\parallel\top N}$ $M_{\circ\parallel\vee N}$, $M_{\circ\perp\top N}$, $M_{\circ\perp\vee N}$
$M_{N\vee\parallel\circ}$ $M_{N\vee\parallel\top}$ $M_{N\vee\perp\circ}$, $M_{N\vee\perp\top}$	$M_{\vee N\parallel\circ}$ $M_{\vee N\parallel\top}$, $M_{\vee N\perp\circ}$, $M_{\vee N\perp\top}$	$M_{\vee\parallel\circ N}$ $M_{\vee\parallel\top N}$, $M_{\vee\perp\circ N}$, $M_{\vee\perp\top N}$

TABLE V. The 36 hetrogeneously-doubly-projected transverse mass variables for each N. As was the case in Tables III and IV "partition" means the combined operation of partitioning the objects and agglomerating them by summation into composite objects.

Cambridge M_{T2}-type variables



- The “2” in M_{T2} referred to the number of invisibles
- The “2” here refers to the number of parents

A common framework

Existing variable	Mass-bound variable					
	$N = 1$				$N = 2$	
	$M_1(\not\!\mathbf{M}_1) = M_{1\top}(\not\!\mathbf{M}_1)$	$M_{\top 1}(\not\!\mathbf{M}_1)$	$M_{\circ 1}$	$M_{1\circ}$	$M_2(\not\!\mathbf{M}) = M_{2\top}(\not\!\mathbf{M})$	$M_{2\top\perp}(\not\!\mathbf{M})$
$2\not{p}_T = 2\not{E}_T$				$u_T \to 0$		
m_{eff}		$\not\!\mathbf{M}_1 \to 0, u_T \to 0$	$u_T \to 0$			
$\sqrt{\hat{s}}_{\min}^{(\mathrm{sub})}(\not\!\mathbf{M}_1)$	✓					
$\sqrt{\hat{s}}_{\min}(\not\!\mathbf{M}_1)$	$u_T \to 0$					
$m_{Te\nu}(M_e, M_\nu)$	✓	✓	$M_e, M_\nu \to 0$	$M_e, M_\nu \to 0$		
$M_{T,ZZ}(M_Z)$	✓	✓				
$M_{C,WW}$	$\not\!\mathbf{M}_1 \to 0$					
m_T^{true}	$\not\!\mathbf{M}_1 \to 0$					
$m_{TZ'}^{reco}(M_Z)$	$u_T \to 0$	$u_T \to 0$				
$m_{T2}(\not\!\mathbf{M})$					✓	
$m_{T2\perp}(\not\!\mathbf{M})$						✓

All previous variables are just specializations to a specific event topology, massless invisibles or uT=0

Take home lessons

- There are different ways to project on the transverse plane
- Be mindful of the way in which composite particles are agglomerated (before or after T)
- Always think which of the 61 variables is most suited for the particular case at hand
- The early-agglomerated (late-projected) “transverse” variables are “secretly” 1+3 dimensional

$$\boxed{M_{N\top}(\not\!\mathbf{M}) = M_N(\not\!\mathbf{M})}$$

- The dependence on the unknown masses is only through the N summed-mass parameters

$$\boxed{\not\!\mathbf{M}_a \equiv \sum_{i\in\mathcal{I}_a} \tilde{M}_i.}$$

- Proposal to use: $\boxed{m_{\text{eff}}^2 - u_T^2}$ instead of Meff