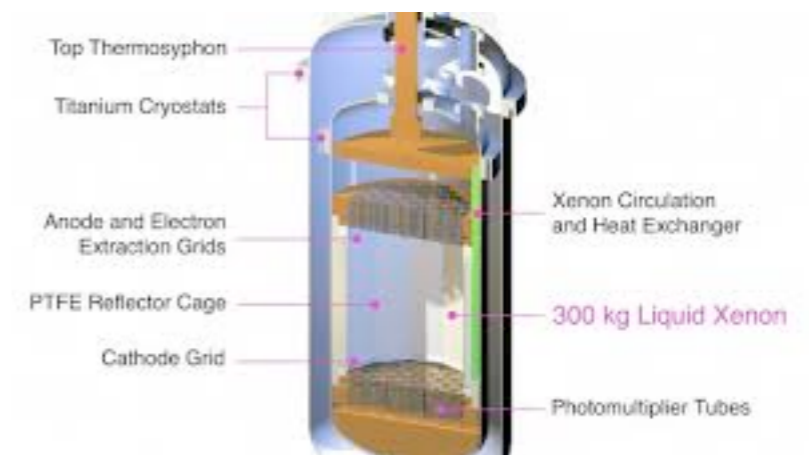
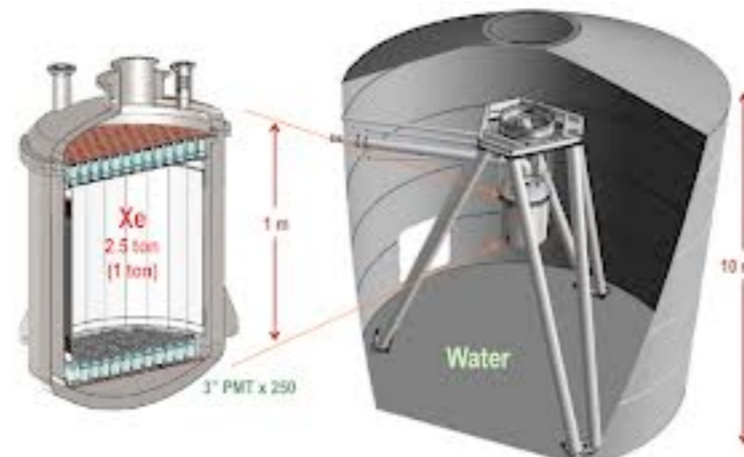
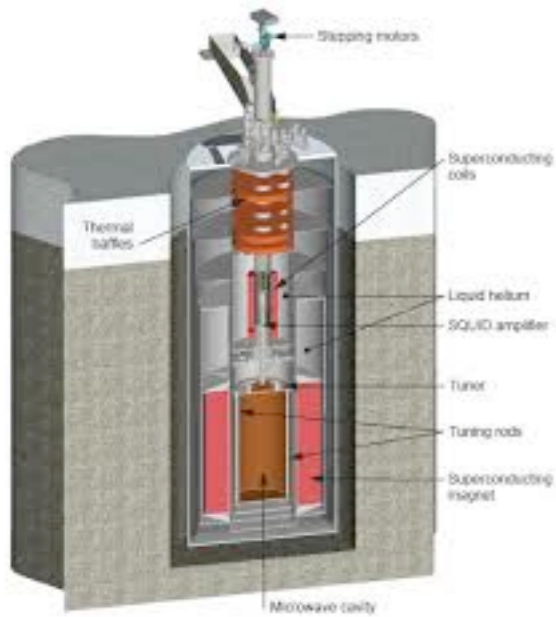


Mixed axion-higgsino CDM from natural SUSY

(Why we need LHC, ILC, wimp and axion detection)

Howard Baer
University of Oklahoma



- $h(125.5 \pm 0.5 \text{ GeV})$ discovered at LHC
- scalars need protective symmetry: SUSY
- $m(h) \sim 125.5 \text{ GeV}$ falls within narrow MSSM expectation
- $m(h)$ requires highly mixed TeV-scale stops
- LHC: no SUSY: $m(\tilde{g}, \tilde{u}) > 1.3 \text{ TeV}$, $m(\tilde{q}, \tilde{d}) > 1.7 \text{ TeV}$, t_1 limits
- impression: then MSSM EW fine-tuned at .1%
- claims: SUSY as expected likely wrong (???)
- this perception arises due to **mis-application of naturalness measures**

A tale of three measures:
“and one ring shall rule them all”

J. R.R. Tolkien

- Simple electroweak fine-tuning Δ_{EW}
- Higgs mass fine-tuning Δ_{HS}
- Traditional EENZ/BG measure Δ_{BG}

We shall see that, if applied properly, then
all three measures agree and imply a rich
program of new physics at ILC:
ILC will be a Higgsino Factory!

First: Naturalness in the Standard Model

SM case: invoke a single Higgs doublet

$$V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$m_h^2 = m_h^2|_{tree} + \delta m_h^2|_{rad}$$

$$m_h^2|_{tree} = 2\mu^2 \quad \delta m_h^2|_{rad} \simeq \frac{3}{4\pi^2} \left(-\lambda_t^2 + \frac{g^2}{4} + \frac{g^2}{8 \cos^2 \theta_W} + \lambda \right) \Lambda^2$$

$m_h^2|_{tree}$ and $\delta m_h^2|_{rad}$ are independent,

If δm_h^2 blows up, can freely adjust (tune) $2\mu^2$ to maintain $m_h = 125.5$ GeV

$$\Delta_{SM} \equiv \delta m_h^2|_{rad} / (m_h^2/2)$$

$$\Delta_{SM} < 1 \Rightarrow \Lambda \sim 1 \text{ TeV}$$

#1: Simplest SUSY measure: Δ_{EW}

Working only at the weak scale, minimize scalar potential: calculate $m(Z)$ or $m(h)$

No large uncorrelated cancellations in $m(Z)$ or $m(h)$

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

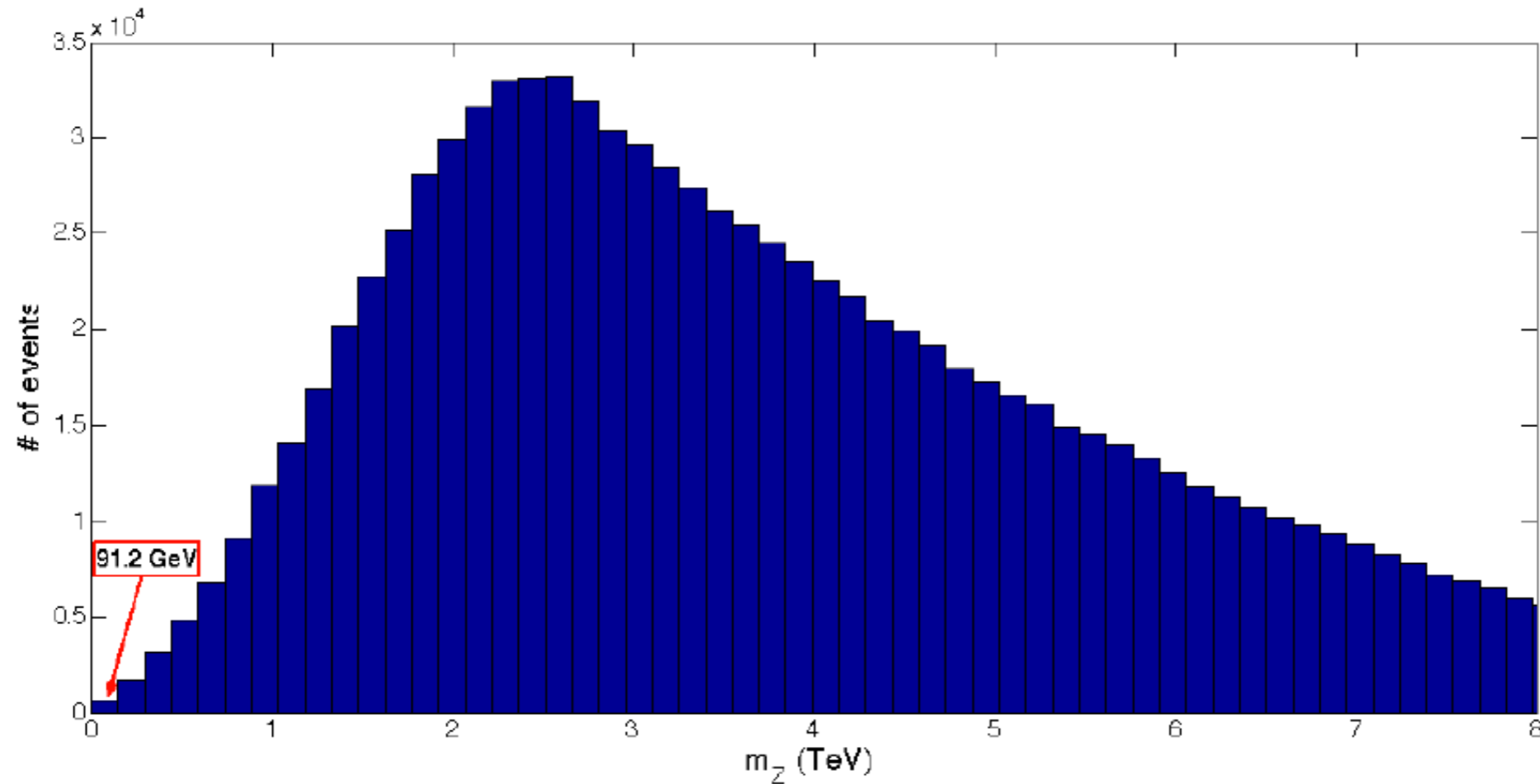
$$\Delta_{EW} \equiv \max_i |C_i| / (m_Z^2/2) \quad \text{with} \quad C_{H_u} = -m_{H_u}^2 \tan^2 \beta / (\tan^2 \beta - 1) \quad \text{etc.}$$

simple, direct, unambiguous interpretation:

- $|\mu| \sim m_Z \sim 100 - 200$ GeV
- $m_{H_u}^2$ should be driven to small negative values such that $-m_{H_u}^2 \sim 100 - 200$ GeV at the weak scale and
- that the radiative corrections are not too large: $\Sigma_u^u \lesssim 100 - 200$ GeV

Large A_t reduces $\Sigma_u^u(\tilde{t}_{1,2})$ whilst lifting m_h to 125.5 GeV

Is Δ_{EW} really a measure of fine-tuning?
 What happens if one doesn't fine-tune $m_{H_u}^2/\mu^2$:



The 20 dimensional pMSSM parameter space then includes

$M_1, M_2, M_3,$
 $m_{Q_1}, m_{U_1}, m_{D_1}, m_{L_1}, m_{E_1},$
 $m_{Q_3}, m_{U_3}, m_{D_3}, m_{L_3}, m_{E_3},$
 $A_t, A_b, A_\tau,$
 $m_{H_u}^2, m_{H_d}^2, \mu, B.$

scan over parameters

Natural value of $m(Z)$ from
 pMSSM is $\sim 2-4$ TeV

#2: Higgs mass or large-log fine-tuning

 Δ_{HS}

$$m_h^2 \simeq \mu^2 + m_{H_u}^2 + \delta m_{H_u}^2|_{rad}$$

$$\frac{dm_{H_u}^2}{dt} = \frac{1}{8\pi^2} \left(-\frac{3}{5}g_1^2 M_1^2 - 3g_2^2 M_2^2 + \frac{3}{10}g_1^2 S + 3f_t^2 X_t \right) \quad X_t = m_{Q_3}^2 + m_{U_3}^2 + m_{H_u}^2 + A_t^2$$

neglect gauge pieces, S , m_{H_u} and running;
then we can integrate from m_{SUSY} to Λ

$$\delta m_{H_u}^2|_{rad} \sim -\frac{3f_t^2}{8\pi^2} (m_{Q_3}^2 + m_{U_3}^2 + A_t^2) \ln(\Lambda^2/m_{SUSY}^2)$$

$$\Delta_{HS} \sim \delta m_h^2 / (m_h^2/2) < 10 \quad \text{then} \quad \begin{aligned} m_{\tilde{t}_{1,2}, \tilde{b}_1} &< 500 \text{ GeV} \\ m_{\tilde{g}} &< 1.5 \text{ TeV} \end{aligned}$$

apparently in violation of LHC constraints!

What's wrong with this argument?

In zeal for simplicity, have neglected that in SUSY

$m_{H_u}^2$ and $\delta m_{H_u}^2|_{rad}$ are not independent

the larger the value of $m_{H_u}^2(\Lambda)$, then the larger is the cancelling correction $\delta m_{H_u}^2|_{rad}$

The dependent terms should be grouped together

$$m_h^2|_{phys} = \mu^2 + (m_{H_u}^2(\Lambda) + \delta m_{H_u}^2)$$

where instead both μ^2 and $(m_{H_u}^2 + \delta m_{H_u}^2)$ should be comparable to $m_h^2|_{phys}$.

After re-grouping: $\Delta_{HS} \simeq \Delta_{EW}$

#3: EENZ/BG traditional measure

Δ_{BG}

Such a re-grouping is properly used
in the EENZ/BG measure:

$$\Delta_{BG} \equiv \max_i [c_i] \quad \text{where} \quad c_i = \left| \frac{\partial \ln m_Z^2}{\partial \ln a_i} \right|$$

Here, the a_i are parameters of the theory

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \mu^2$$



express **weak scale value** in terms of high scale parameters

Express $m(Z)$ in terms of GUT scale parameters:

$$m_Z^2 \simeq -2m_{H_u}^2 - 2\mu^2 \quad (\text{weak scale relation})$$

$$-2\mu^2(m_{SUSY}) = -2.18\mu^2$$

$$\begin{aligned} -2m_{H_u}^2(m_{SUSY}) = & 3.84M_3^2 + 0.32M_3M_2 + 0.047M_1M_3 - 0.42M_2^2 \\ & + 0.011M_2M_1 - 0.012M_1^2 - 0.65M_3A_t - 0.15M_2A_t \\ & - 0.025M_1A_t + 0.22A_t^2 + 0.004m_3A_b \\ & - 1.27m_{H_u}^2 - 0.053m_{H_d}^2 \\ & + 0.73m_{Q_3}^2 + 0.57m_{U_3}^2 + 0.049m_{D_3}^2 - 0.052m_{L_3}^2 + 0.053m_{E_3}^2 \\ & + 0.051m_{Q_2}^2 - 0.11m_{U_2}^2 + 0.051m_{D_2}^2 - 0.052m_{L_2}^2 + 0.053m_{E_2}^2 \\ & + 0.051m_{Q_1}^2 - 0.11m_{U_1}^2 + 0.051m_{D_1}^2 - 0.052m_{L_1}^2 + 0.053m_{E_1}^2, \end{aligned}$$

all GUT scale parameters



Abe, Kobayashi, Omura;
S. P. Martin

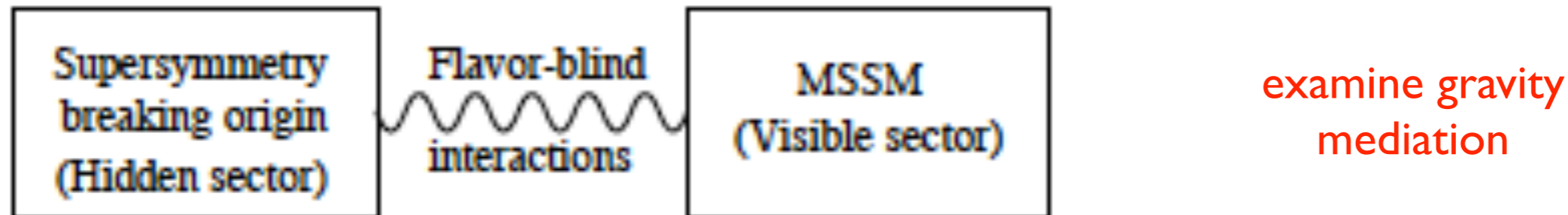
For generic parameter choices, Δ_{BG} is large

But if: $m_{Q_{1,2}} = m_{U_{1,2}} = m_{D_{1,2}} = m_{L_{1,2}} = m_{E_{1,2}} \equiv m_{16}(1,2)$ then $\sim 0.007m_{16}^2(1,2)$

Even better: $m_{H_u}^2 = m_{H_d}^2 = m_{16}^2(3) \equiv m_0^2 \Rightarrow -0.017m_0^2$

For correlated parameters, EWFT collapses in 3rd gen. sector!

To properly apply BG measure, need to identify **independent** soft breaking terms



For any particular SUSY breaking hidden sector, each soft term is some multiple of gravitino mass $m_{3/2}$

$$\begin{aligned} m_{H_u}^2 &= a_{H_u} \cdot m_{3/2}^2, \\ m_{Q_3}^2 &= a_{Q_3} \cdot m_{3/2}^2, \\ A_t &= a_{A_t} \cdot m_{3/2}, \\ M_i &= a_i \cdot m_{3/2}, \\ &\dots \end{aligned}$$

Since we don't know hidden sector, we impose parameters which parameterize our ignorance:

but this doesn't mean each parameter is independent

e.g. dilaton-dominated SUSY breaking: $m_0^2 = m_{3/2}^2$ with $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$

Writing each soft term as a multiple of $m(3/2)$ then we allow for maximal correlations/cancellations:

$$m_Z^2 = -2.18\mu^2 + a \cdot m_{3/2}^2$$

for naturalness, then

$$\mu^2 \sim m_Z^2 \quad \text{and} \quad a \cdot m_{3/2}^2 \sim m_Z^2$$

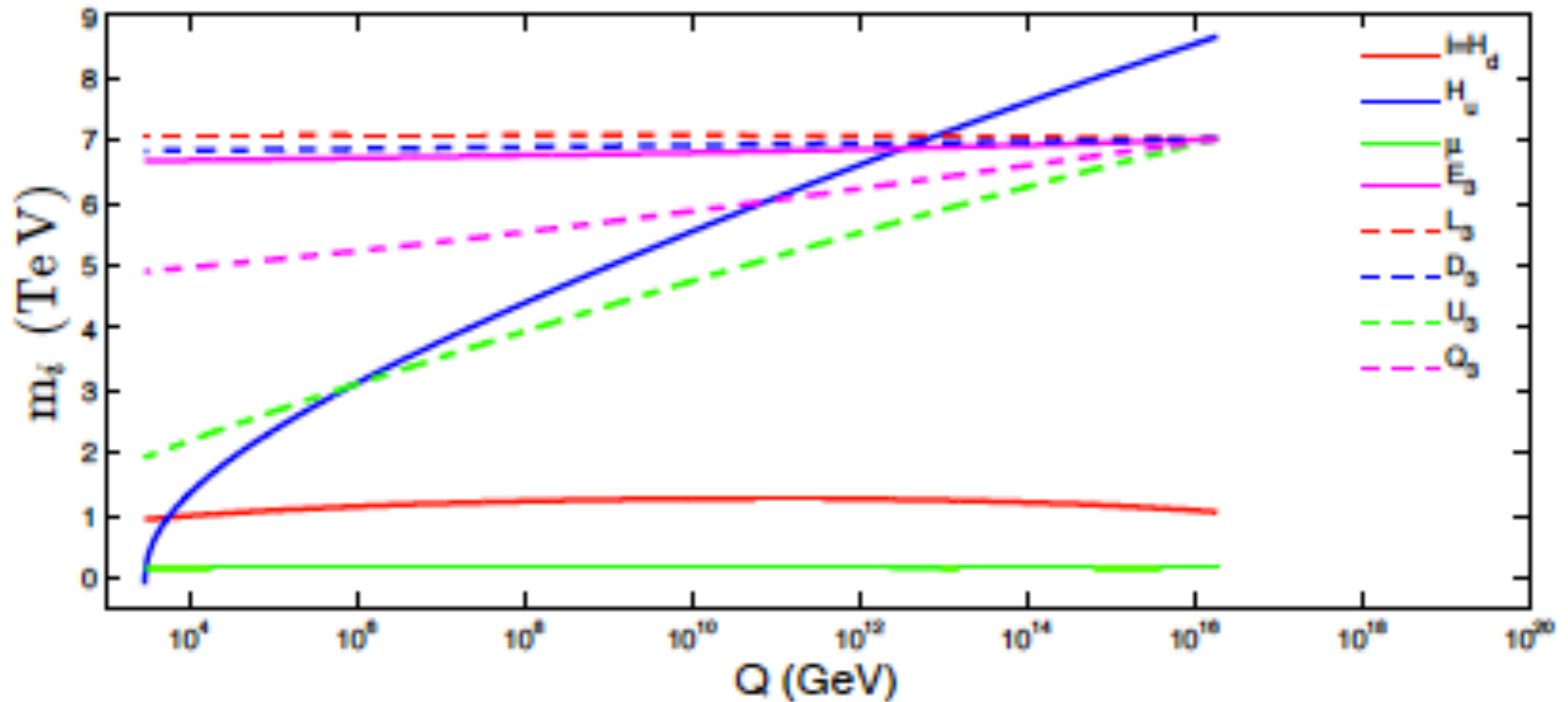
$$m_Z^2 \simeq -2\mu^2(\text{weak}) - 2m_{H_u}^2(\text{weak}) \simeq -2.18\mu^2(\text{GUT}) + a \cdot m_{3/2}^2$$

then

$$-m_{H_u}^2(\text{weak}) \sim a \cdot m_{3/2}^2 \sim m_Z^2$$

$$\lim_{n_{SSB} \rightarrow 1} \Delta_{BG} \rightarrow \Delta_{EW}$$

Applied properly, all three measures agree:
naturalness is unambiguous and highly predictive!

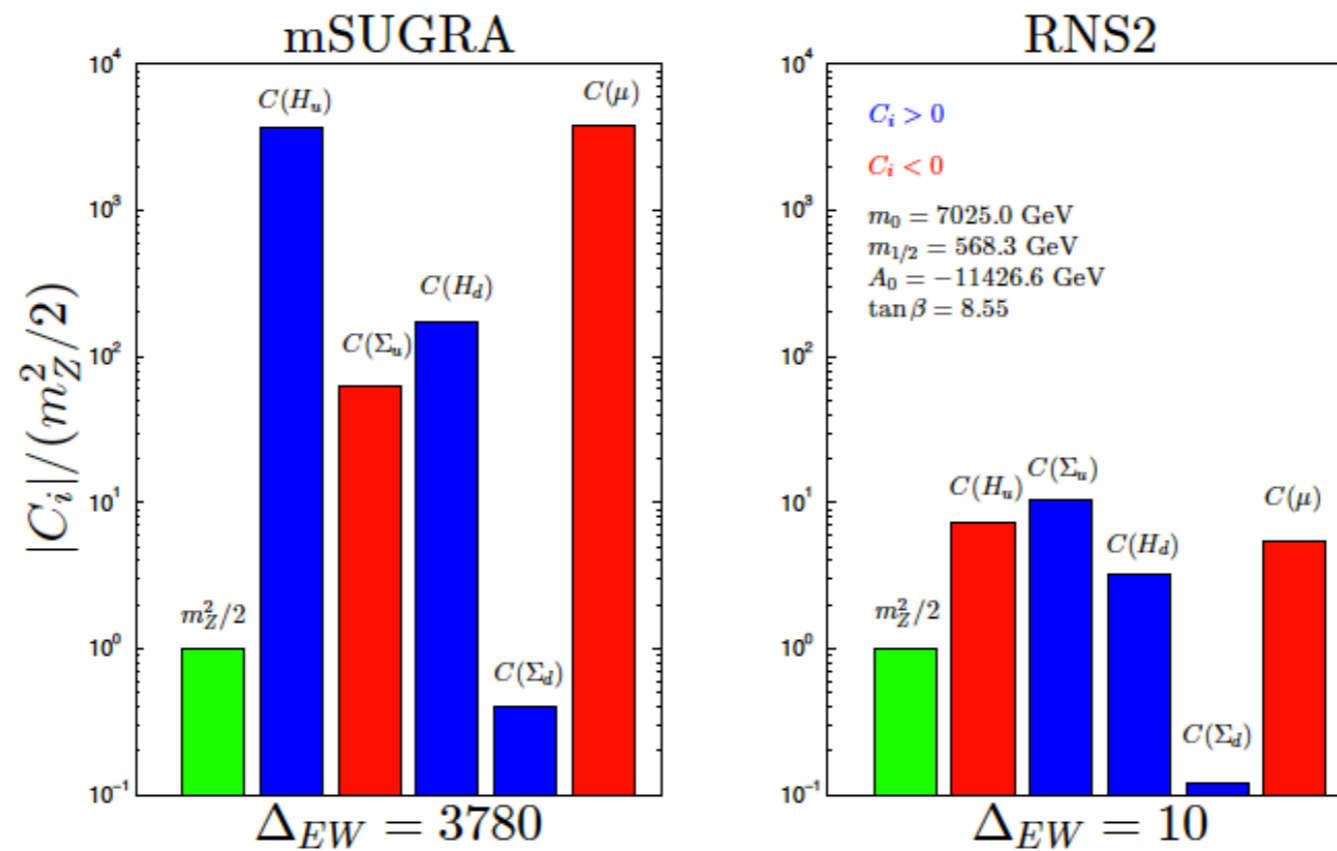


Radiatively-driven natural SUSY, or RNS:

H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, *Phys. Rev. Lett.* **109** (2012) 161802.

H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, *Phys. Rev. D* **87** (2013) 115028 [arXiv:1212.2655 [hep-ph]].

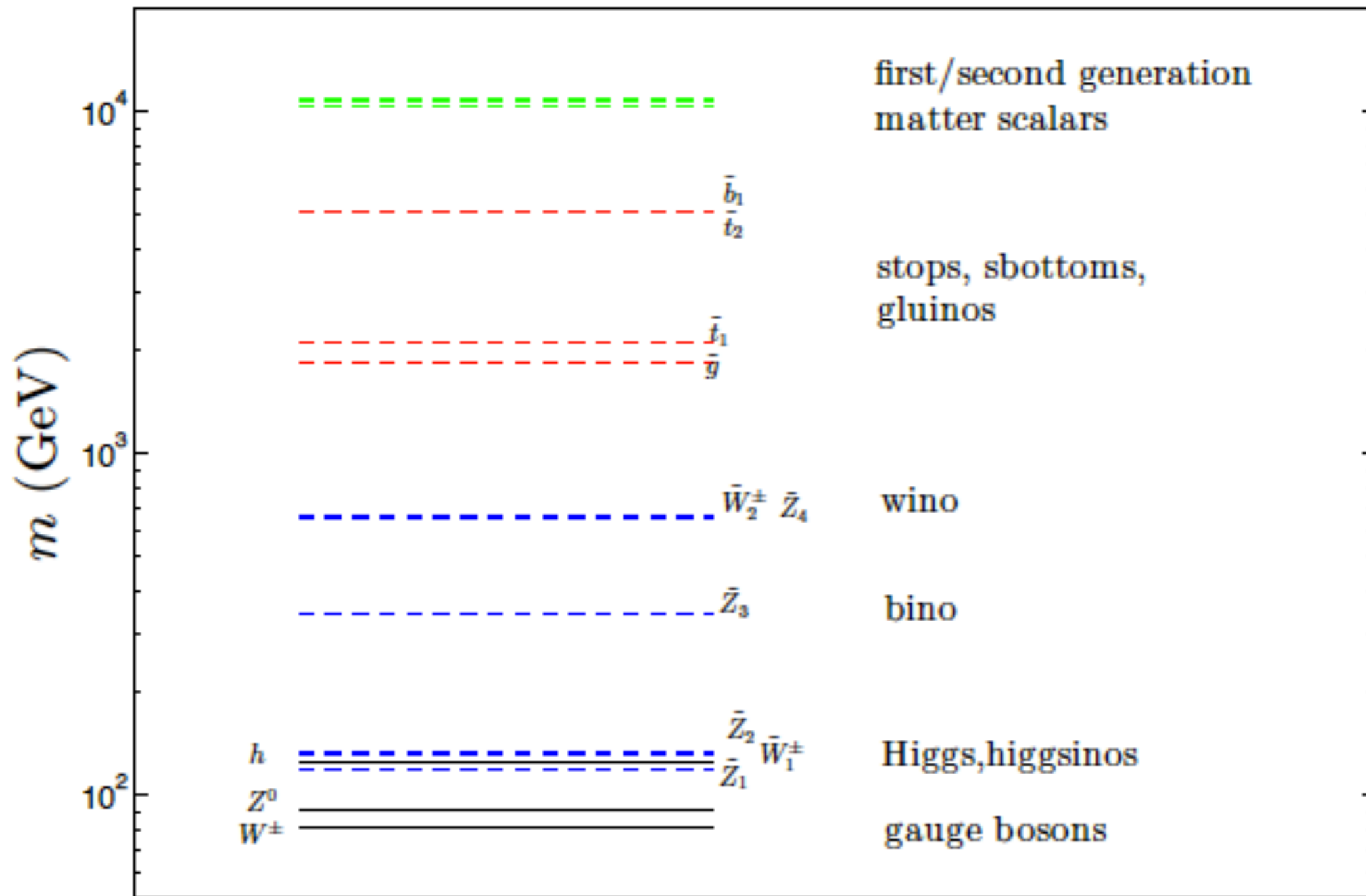
All contributions to $m(Z)$ and $m(h)$ are comparable
to $m(Z)$ and $m(h)$:
model is **natural** in EW sector!



unnatural model

natural model: all
contributions to $m(Z)$
are comparable to $m(Z)$

Typical spectrum for low Δ_{EW} models



There is a Little Hierarchy, but it is **no problem**

$$\mu \ll m_{3/2}$$

SUSY μ problem: μ term is SUSY, not SUSY breaking:
expect $\mu \sim M_{Pl}$ but phenomenology requires $\mu \sim m(Z)$

- NMSSM: $\mu \sim m(3/2)$; beware singlets!
- Giudice–Masiero: μ forbidden by some symmetry:
generate via Higgs coupling to hidden sector
- Kim–Nilles: invoke SUSY version of DFSZ axion
solution to strong CP:

KN: PQ symmetry forbids μ term,
but then it is generated via PQ breaking

Little Hierarchy due to mismatch between
SUSY breaking and PQ breaking scale?

$$\mu \sim \lambda f_a^2 / M_P$$

$$m_{3/2} \sim m_h^2 / M_P$$

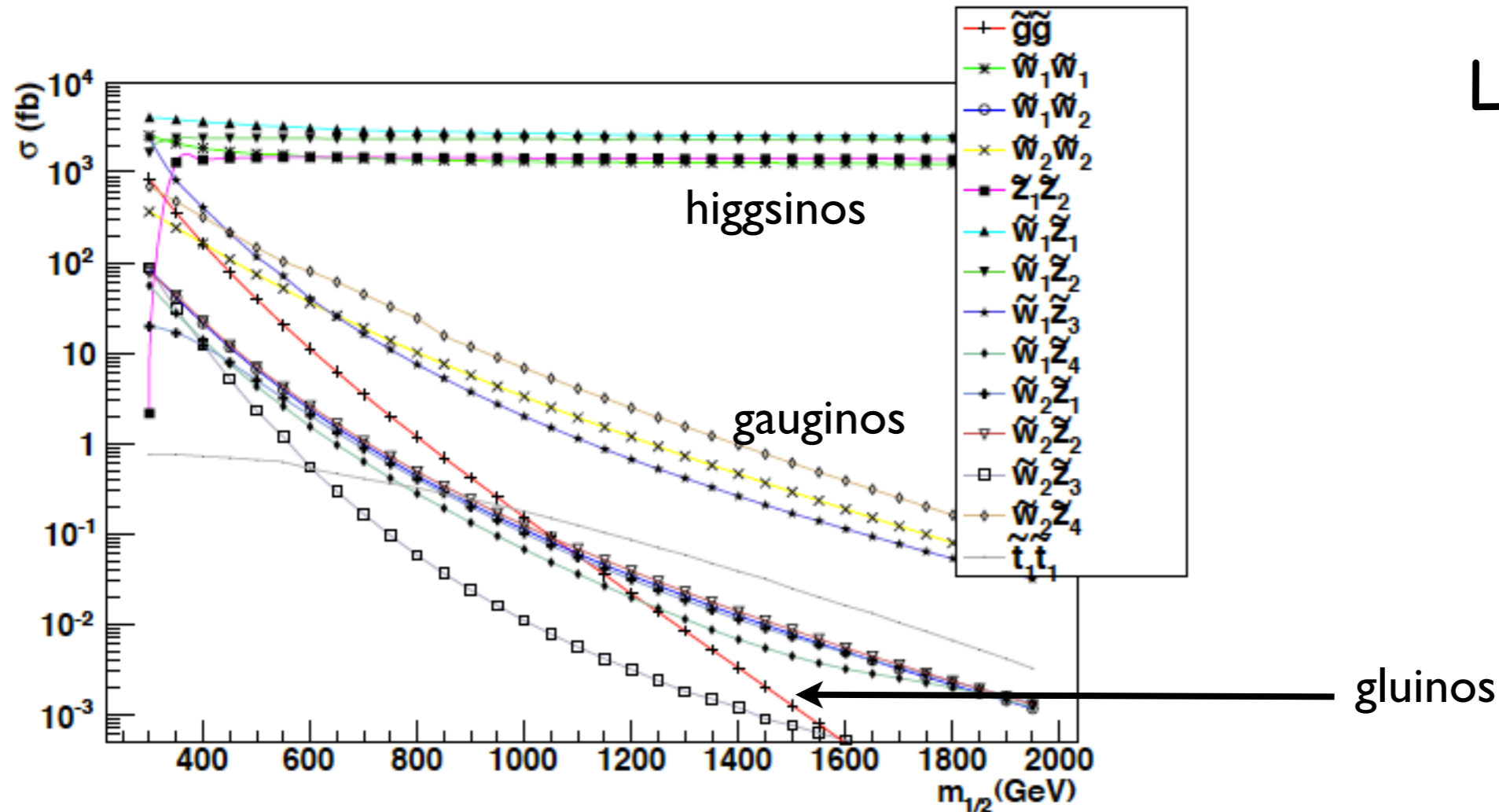
$$f_a \ll m_h$$

Higgs mass tells us where
to look for axion!

$$m_a \sim 6.2 \mu\text{eV} \left(\frac{10^{12} \text{ GeV}}{f_a} \right)$$

Sparticle production along RNS model-line:

LHC14

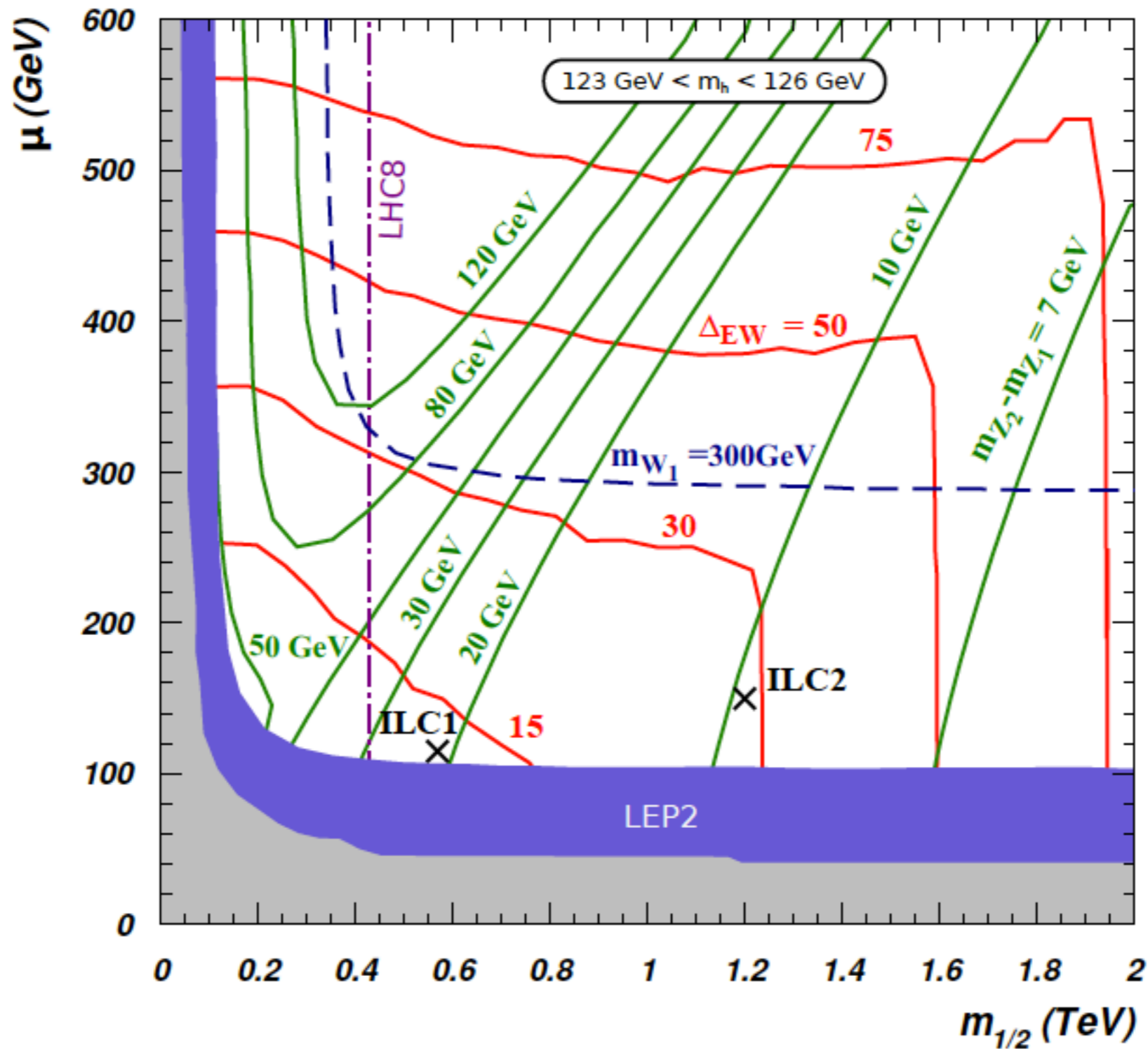


- *higgsino pair production dominant-but only soft visible energy release from higgsino decays
- *largest visible cross section: wino pairs=> SSdB
- *gluino pairs sharply dropping

Radiatively-driven natural supersymmetry at the LHC (with V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata) JHEP1312 (2013) 013.

Landscape of RNS

NUHM2: $m_0=5$ TeV, $\tan\beta=15$, $A_0=-1.6m_0$, $m_A=1$ TeV, $m_t=173.2$ GeV

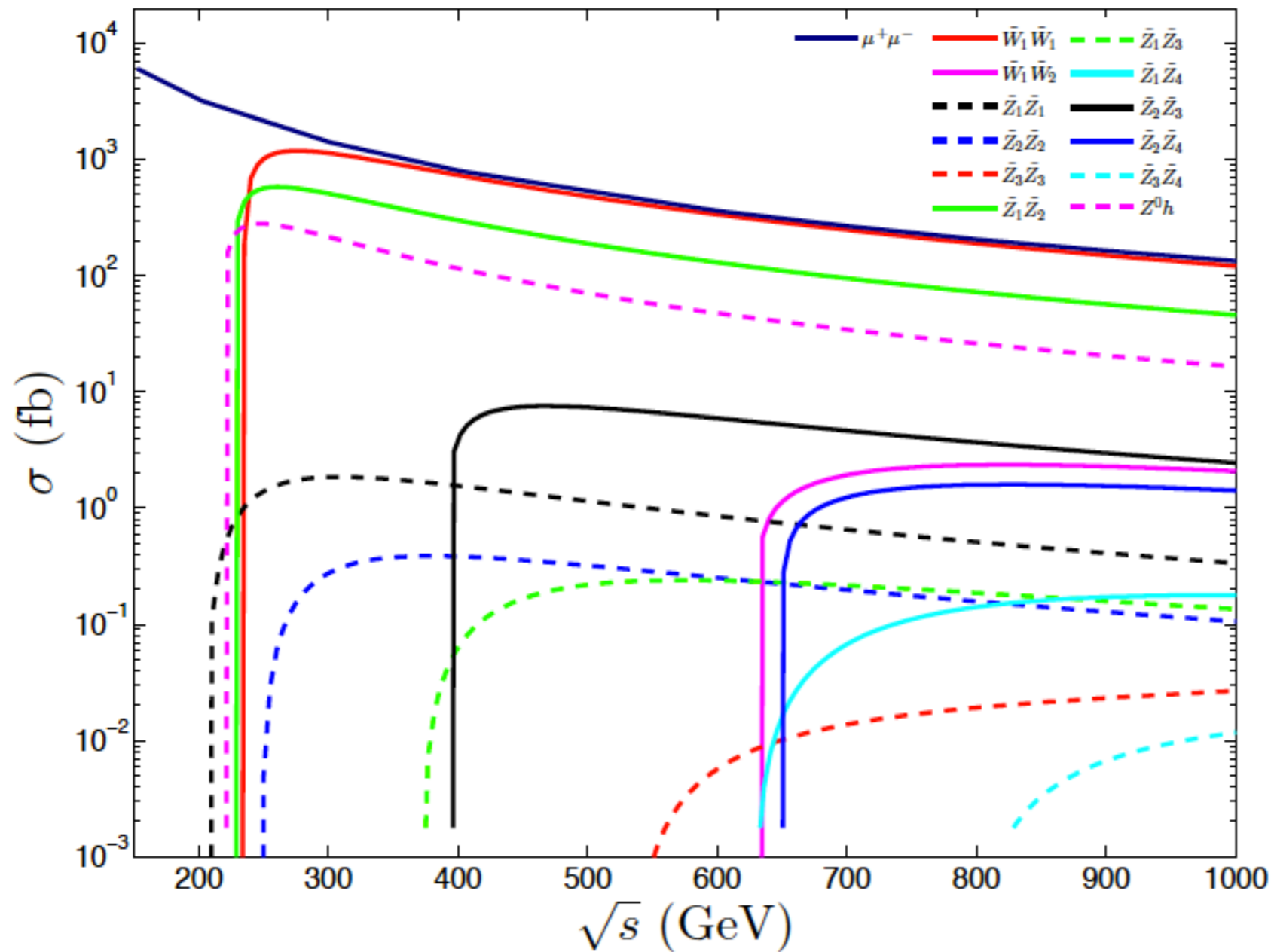


Physics at a Higgsino Factory (with V. Barger, D. Mickelson, A. Mustafayev and Xerxes Tata), arXiv:1404.7510.

Smoking gun signature: light higgsinos at ILC:

ILC is Higgs/higgsino factory!

ILC1: $m_0 = 7025$ GeV, $m_{1/2} = 568.3$ GeV, $A_0 = -11426.6$ GeV, $\tan\beta = 10$, $\mu = 115$ GeV, $m_A = 1000$ GeV



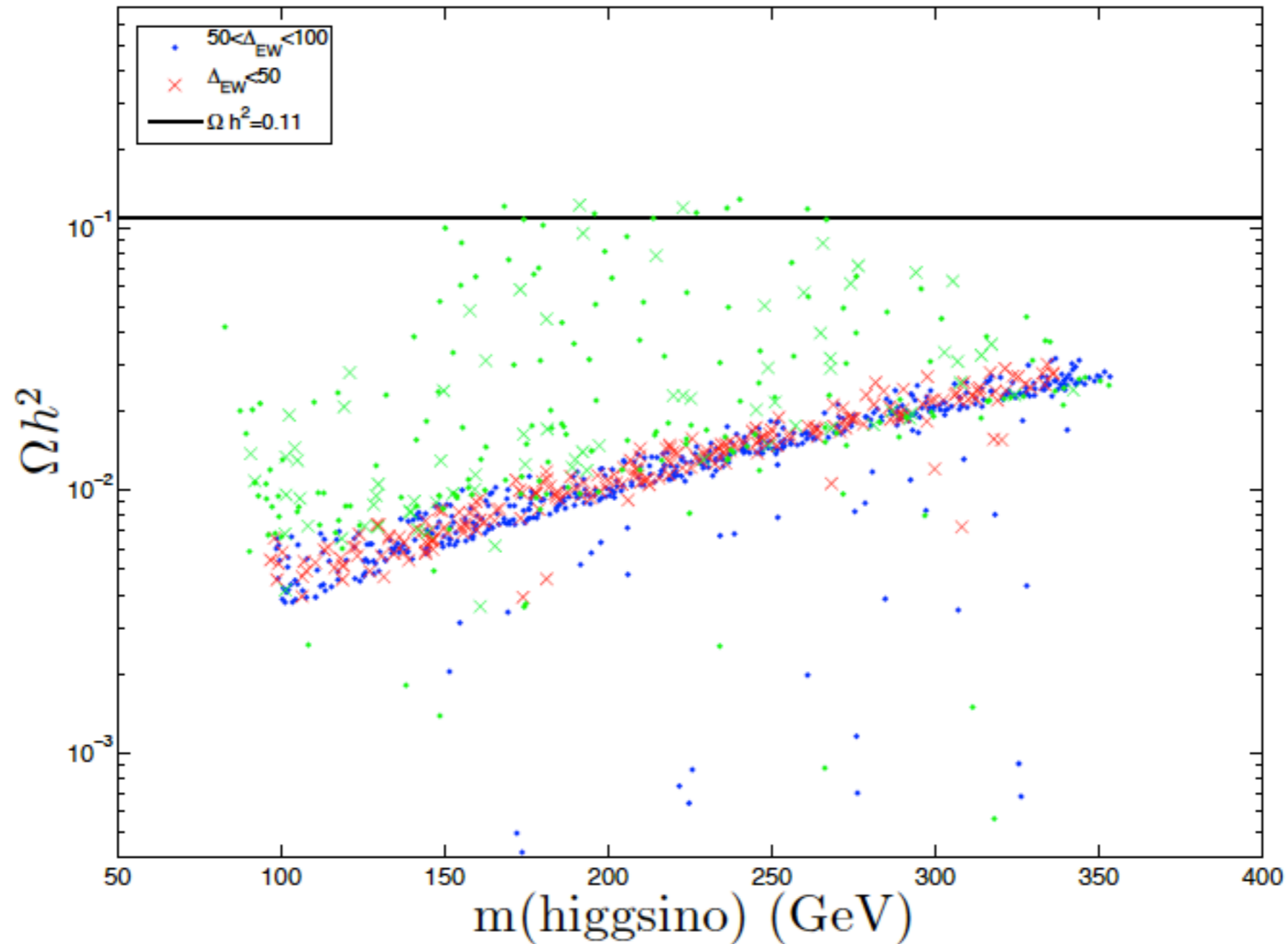
$$\sigma(\text{higgsino}) \gg \sigma(Zh)$$

10–15 GeV higgsino mass
gaps no problem
in clean ILC environment

ILC either sees light higgsinos or natural SUSY dead

Dark matter from RNS: thermally-produced higgsinos can't comprise all CDM!

$\Omega_{\chi}^{TP} h^2$ low by factor 10-15



The QCD fine-tuning problem

- QCD chiral symmetry: expect 4 light pions
- 't Hooft solution: θ -vacuum but then additional term:
- $\bar{\theta} = \theta + \text{arg}(\det \mathcal{M})$
- $\bar{\theta} < 10^{-10}$ (neutron EDM)
- solution: PQ symmetry and axion a

$$\frac{\bar{\theta}}{32\pi^2} F_{A\mu\nu} \tilde{F}_A^{\mu\nu}$$

For DM abundance calculus,
presence of axion changes everything:
spin-0 saxion and spin-1/2 axino with mass \sim TeV

expect mixed axion-higgsino CDM: 2 particles!

As a bonus: axion provides elegant solution to the SUSY μ problem:

In MSSM, Higgs/higgsino mass μ is supersymmetric and not soft breaking: expect $\mu \sim M_P$ but $m(h,Z)$ require $\mu \sim m(\text{weak})$

PQ charges for $H(d)$, $H(u)$ forbid μ term, but $H(d)$, $H(u)$ can couple to axion supermultiplet:

SUSY DFSZ axion model

Kim-Nilles

PQ symmetry breaking:

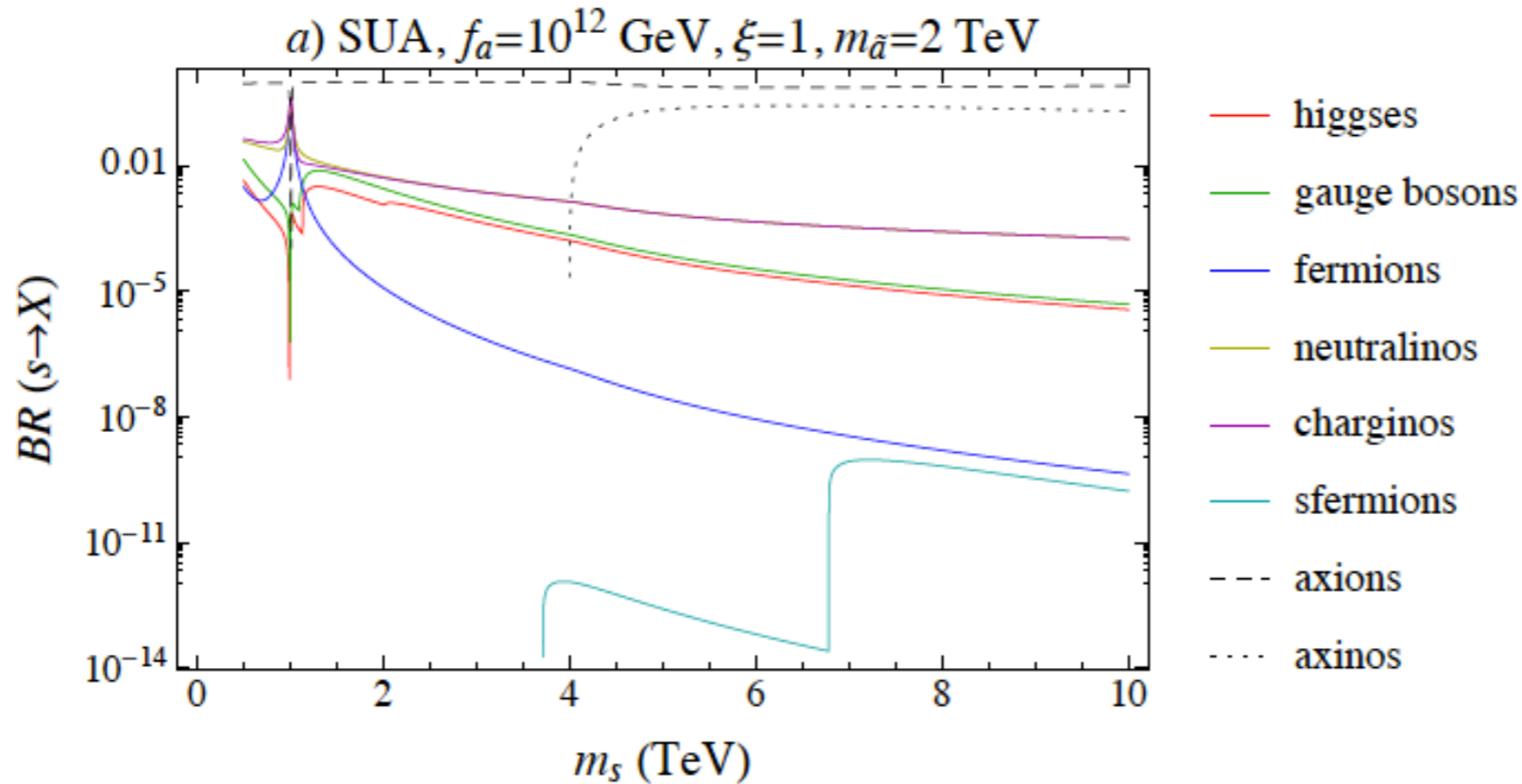
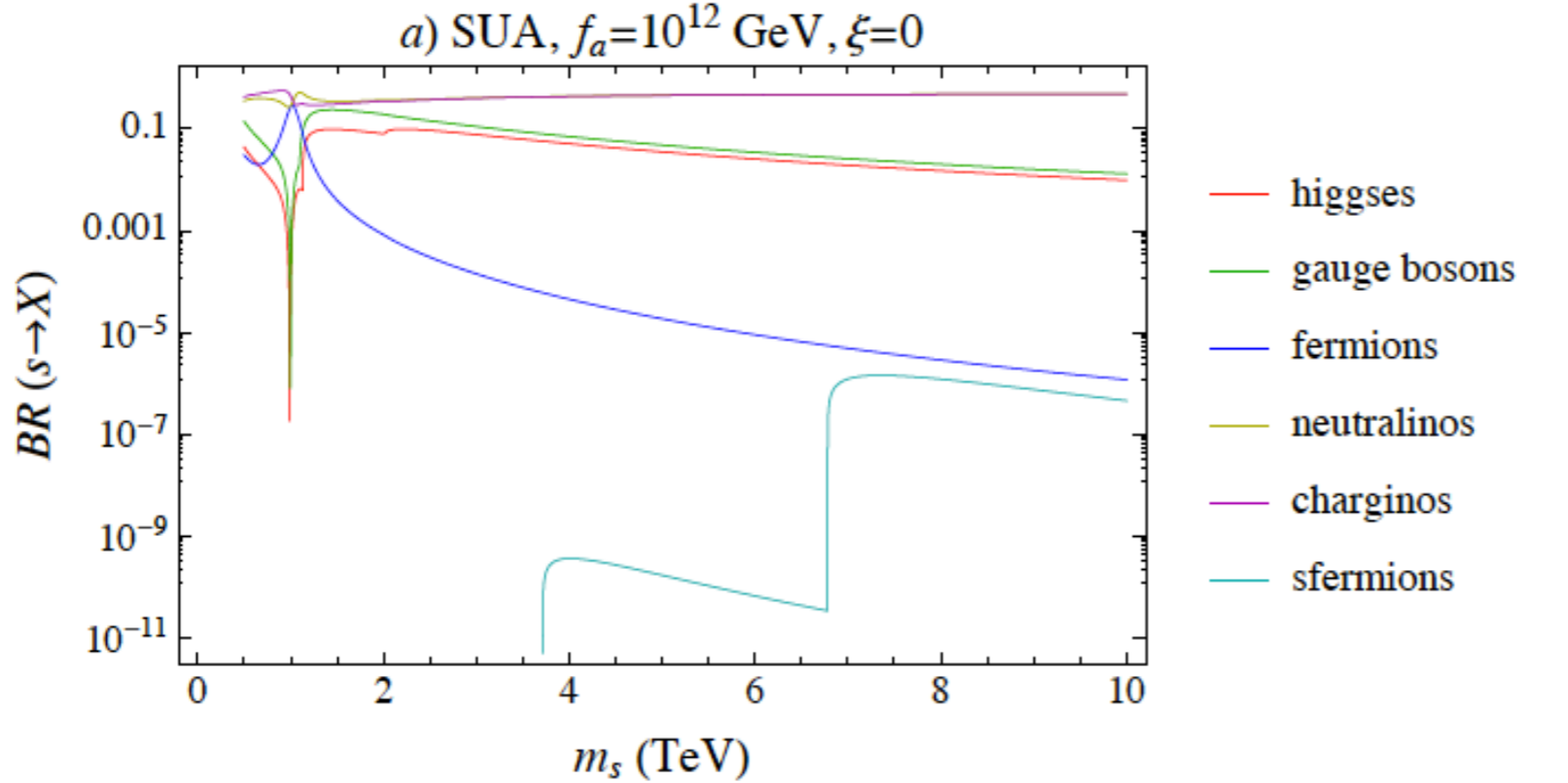
$$\mu \sim f_a^2 / M_P \sim 100 \text{ GeV for } f_a \sim 10^{11} \text{ GeV}$$

Higgs mass tells us where to look for axion and wimp!

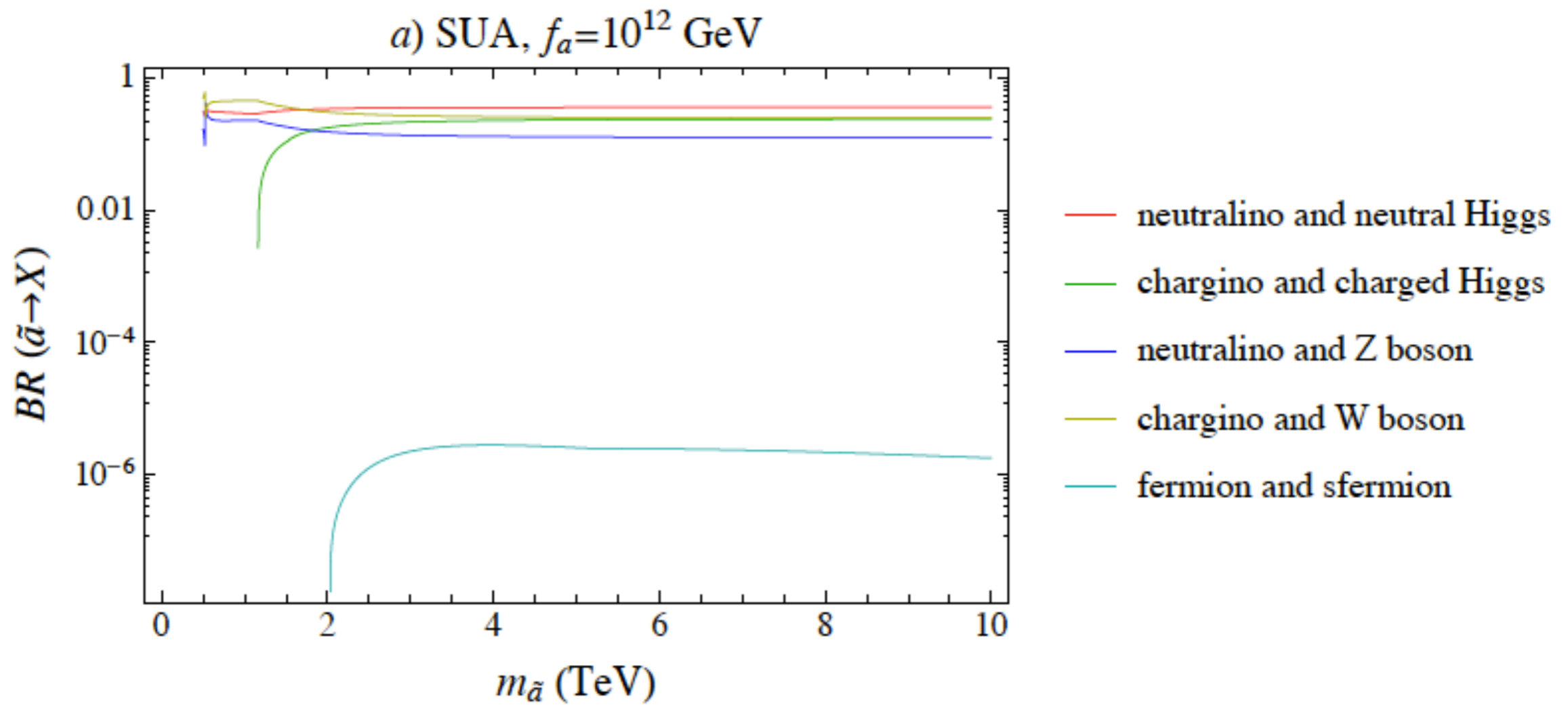
mixed axion-neutralino production in early universe

- neutralinos: thermally produced (TP) or NTP via \tilde{a} , s or \tilde{G} decays
 - re-annihilation at $T_D^{s,\tilde{a}}$
- axions: TP, NTP via $s \rightarrow aa$, bose coherent motion (BCM)
- saxions: TP or via BCM
 - $s \rightarrow gg$: entropy dilution
 - $s \rightarrow SUSY$: augment neutralinos
 - $s \rightarrow aa$: dark radiation ($\Delta N_{eff} < 1.6$)
- axinos: TP
 - $\tilde{a} \rightarrow SUSY$ augments neutralinos
- gravitinos: TP, decay to SUSY

saxion decays in susy dfsz



axino decays in susy dfsz for rns



coupled Boltzmann equations

$$\frac{dn_{\tilde{Z}_1}}{dt} = -3Hn_{\tilde{Z}_1} + \left[\bar{n}_{\tilde{Z}_1}^2 - n_{\tilde{Z}_1}^2 \right] \langle \sigma v \rangle_{\tilde{Z}_1}(T) + \sum_j BR(j \rightarrow \tilde{Z}_1) \Gamma_j m_j \left(n_j - \bar{n}_j \frac{n_{\tilde{Z}_1}}{\bar{n}_{\tilde{Z}_1}} \right) \frac{n_{\tilde{Z}_1}}{\rho_{\tilde{Z}_1}} \quad (3.2)$$

$$\begin{aligned} \frac{dn_{\tilde{G}}}{dt} = & -3Hn_{\tilde{G}} + \left[\bar{n}_{\tilde{G}}^2 - n_{\tilde{G}}^2 \right] \langle \sigma v \rangle_{\tilde{G}} + \sum_j BR(j \rightarrow \tilde{G}) \Gamma_j m_j \left(n_j - \bar{n}_j \frac{n_{\tilde{G}}}{\bar{n}_{\tilde{G}}} \right) \frac{n_j}{\rho_j} \\ & - \sum_j BR(\tilde{G} \rightarrow j) \Gamma_{\tilde{G}} m_{\tilde{G}} \left(n_{\tilde{G}} - \bar{n}_{\tilde{G}} \frac{n_j}{\bar{n}_j} \right) \frac{n_{\tilde{G}}}{\rho_{\tilde{G}}}, \end{aligned} \quad (3.3)$$

$$\frac{dn_s}{dt} = -3Hn_s + \left[\bar{n}_s^2 - n_s^2 \right] \langle \sigma v \rangle_s - \sum_j 2BR(s \rightarrow j) \Gamma_s m_s \left(n_s - \bar{n}_s \left(\frac{n_j}{\bar{n}_s} \right)^2 \right) \frac{n_s}{\rho_s}, \quad (3.4)$$

$$\frac{dn_s^{CO}}{dt} = -3Hn_s^{CO} - \Gamma_s n_s^{CO} / \gamma_s, \quad (3.5)$$

$$\begin{aligned} \frac{dn_{\tilde{a}}}{dt} = & -3Hn_{\tilde{a}} + \left[\bar{n}_{\tilde{a}}^2 - n_{\tilde{a}}^2 \right] \langle \sigma v \rangle_{\tilde{a}} + \sum_j BR(j \rightarrow \tilde{a}) \Gamma_j m_j \left(n_j - \bar{n}_j \frac{n_{\tilde{a}}}{\bar{n}_{\tilde{a}}} \right) \frac{n_j}{\rho_j} \\ & - BR(\tilde{a} \rightarrow j) \Gamma_{\tilde{a}} m_{\tilde{a}} \left(n_{\tilde{a}} - \bar{n}_{\tilde{a}} \frac{n_j}{\bar{n}_j} \right) \frac{n_{\tilde{a}}}{\rho_{\tilde{a}}}, \end{aligned} \quad (3.6)$$

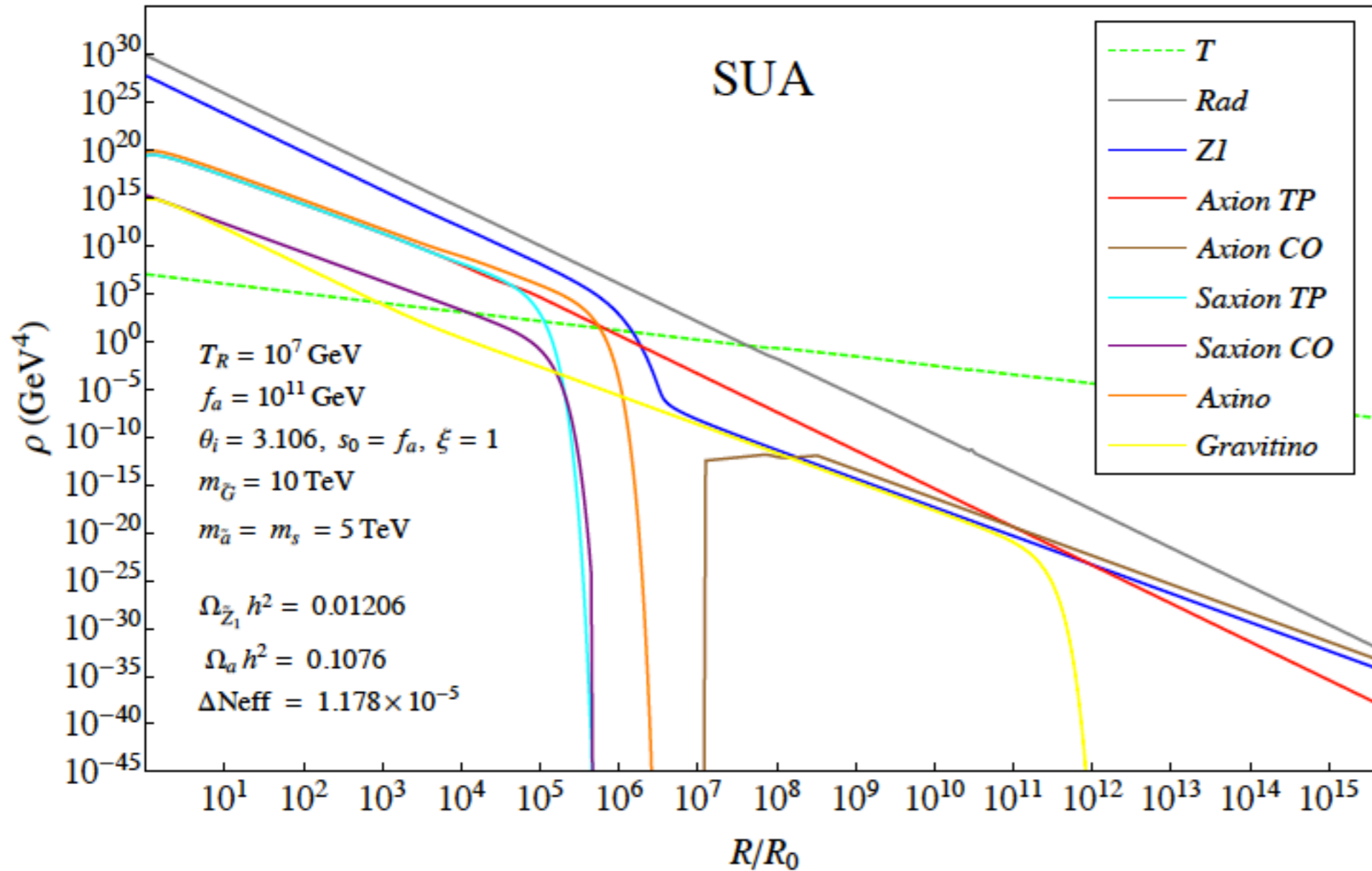
$$\frac{dn_a}{dt} = -3Hn_a + \left[\bar{n}_a^2 - n_a^2 \right] \langle \sigma v \rangle_a + \sum_j 2BR(j \rightarrow a) \Gamma_j m_j \left(n_j - \bar{n}_j \frac{n_a}{\bar{n}_a} \right) \frac{n_j}{\rho_j}, \quad (3.7)$$

$$\frac{dn_a^{CO}}{dt} = -3Hn_a^{CO} + \sum_j BR(j \rightarrow a) \Gamma_j n_j / \gamma_j, \quad (3.8)$$

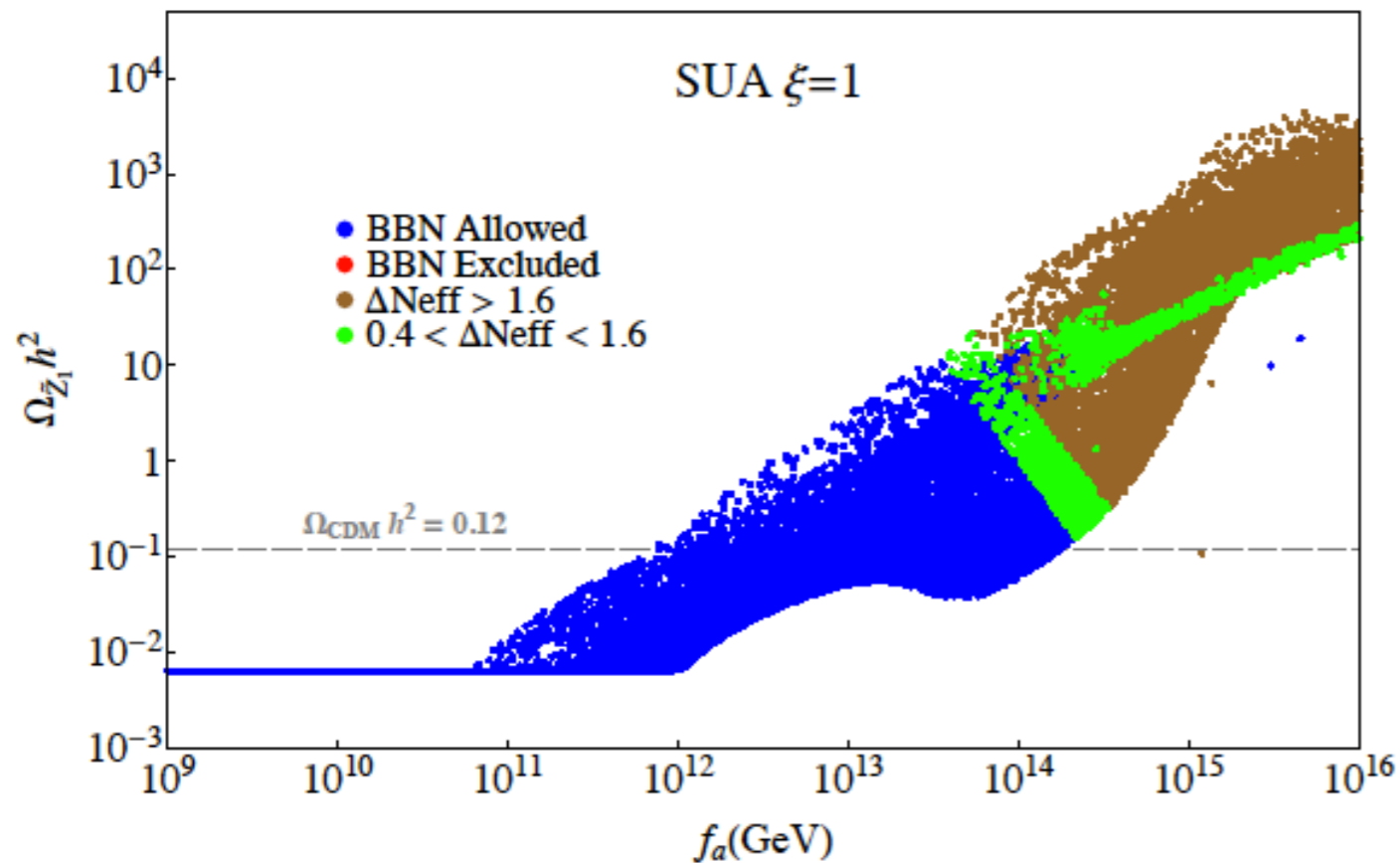
$$\frac{dS}{dt} = \frac{R^3}{T} \sum_i BR(i \rightarrow X) \Gamma_i m_i \left(n_i - \bar{n}_i \sum_{i \rightarrow \dots} B_{ab\dots} \frac{n_a n_b \dots}{\bar{n}_a \bar{n}_b \dots} \right) \quad (3.9)$$

$$H \equiv \dot{R}/R = \sqrt{\rho_T / 3M_P^2}$$

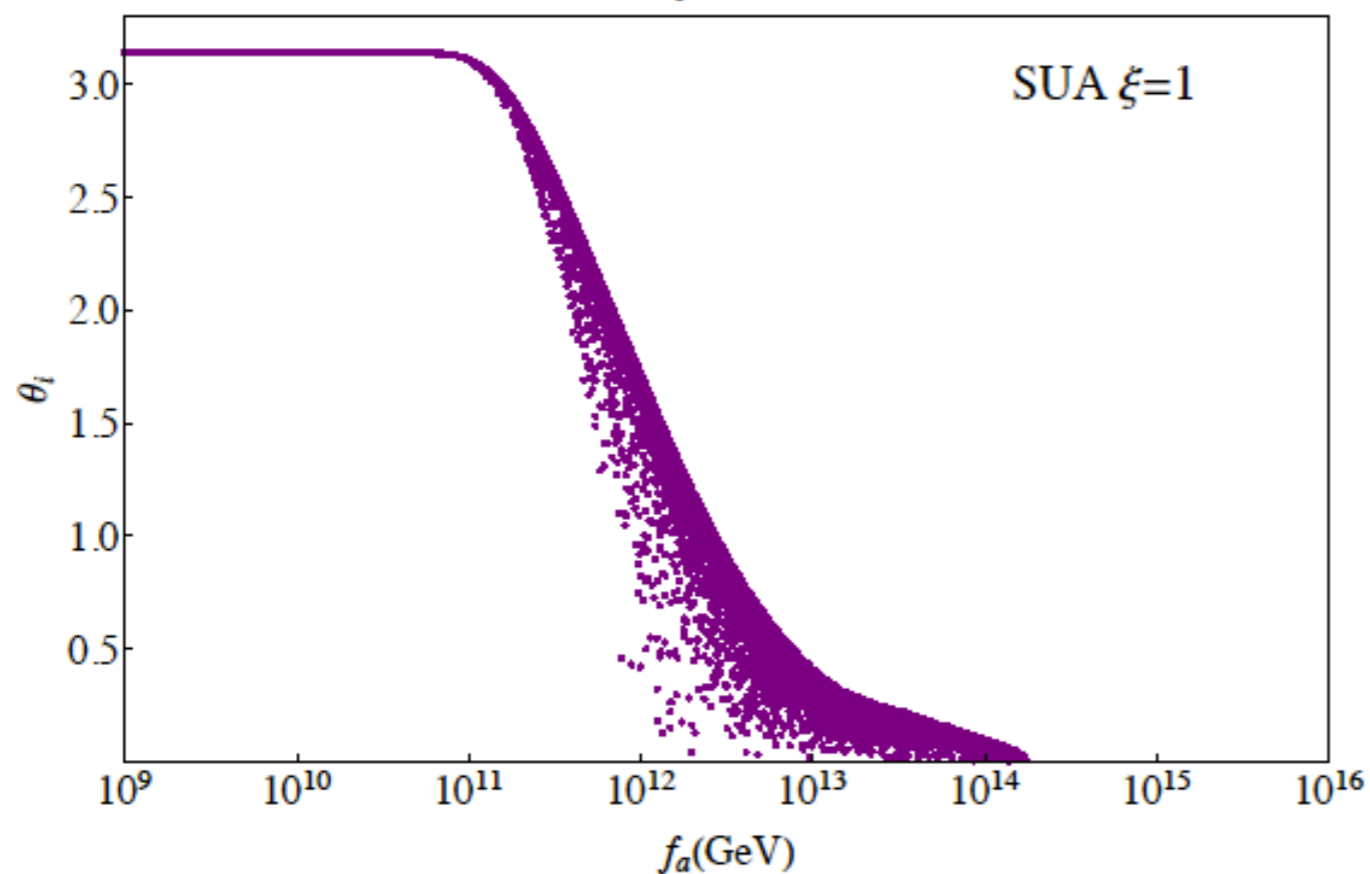
DM production in SUSY DFSZ: solve eight coupled Boltzmann equation



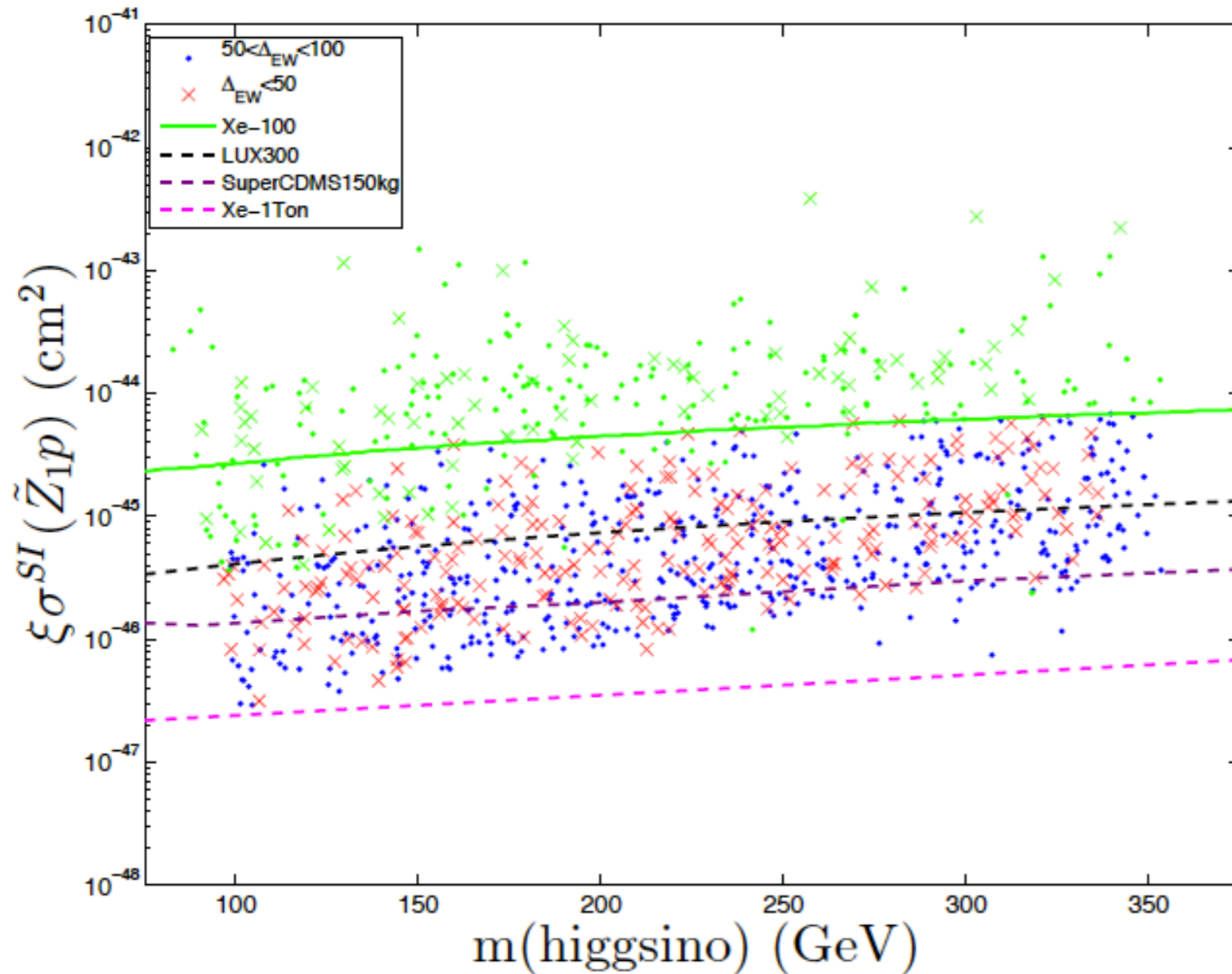
HB, Bae, Chun;
HB, Bae, Lessa, Serce



mainly axion CDM
for $f_a < 10^{11}$ GeV;
for higher f_a , then
get increasing wimp
abundance



Direct higgsino detection rescaled for minimal local abundance



HB, Barger, Mickelson
arXiv:1303.3816

$$\mathcal{L} \ni -X_{11}^h \bar{\tilde{Z}}_1 \tilde{Z}_1 h$$

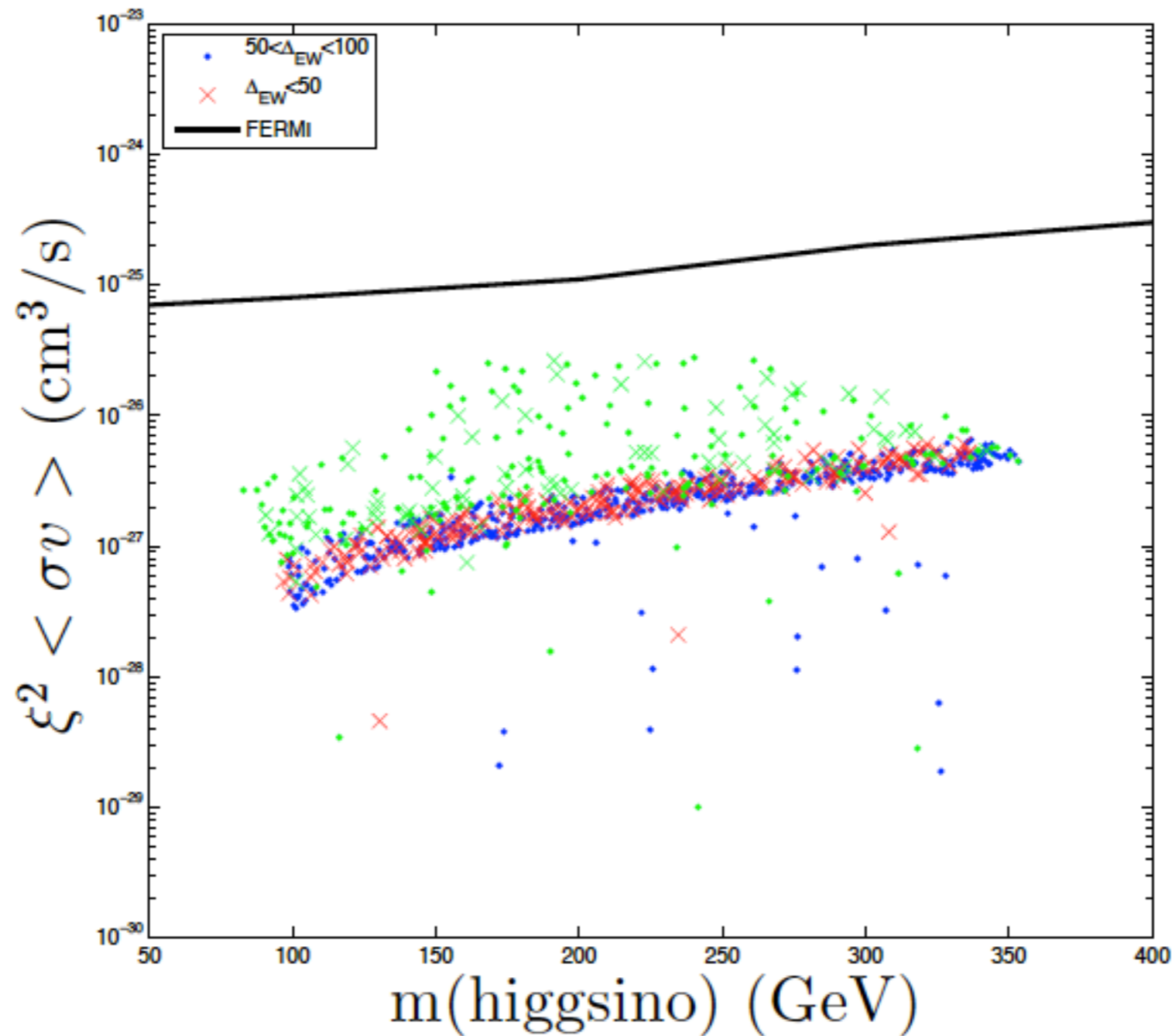
$$X_{11}^h = -\frac{1}{2} (v_2^{(1)} \sin \alpha - v_1^{(1)} \cos \alpha) (g v_3^{(1)} - g' v_4^{(1)})$$

new LUX results

Deployment of Xe-1ton
coming soon!

Can test completely with ton scale detector
or equivalent (subject to minor caveats)

Indirect detection via wimp annihilation suppressed by square of reduced local abundance

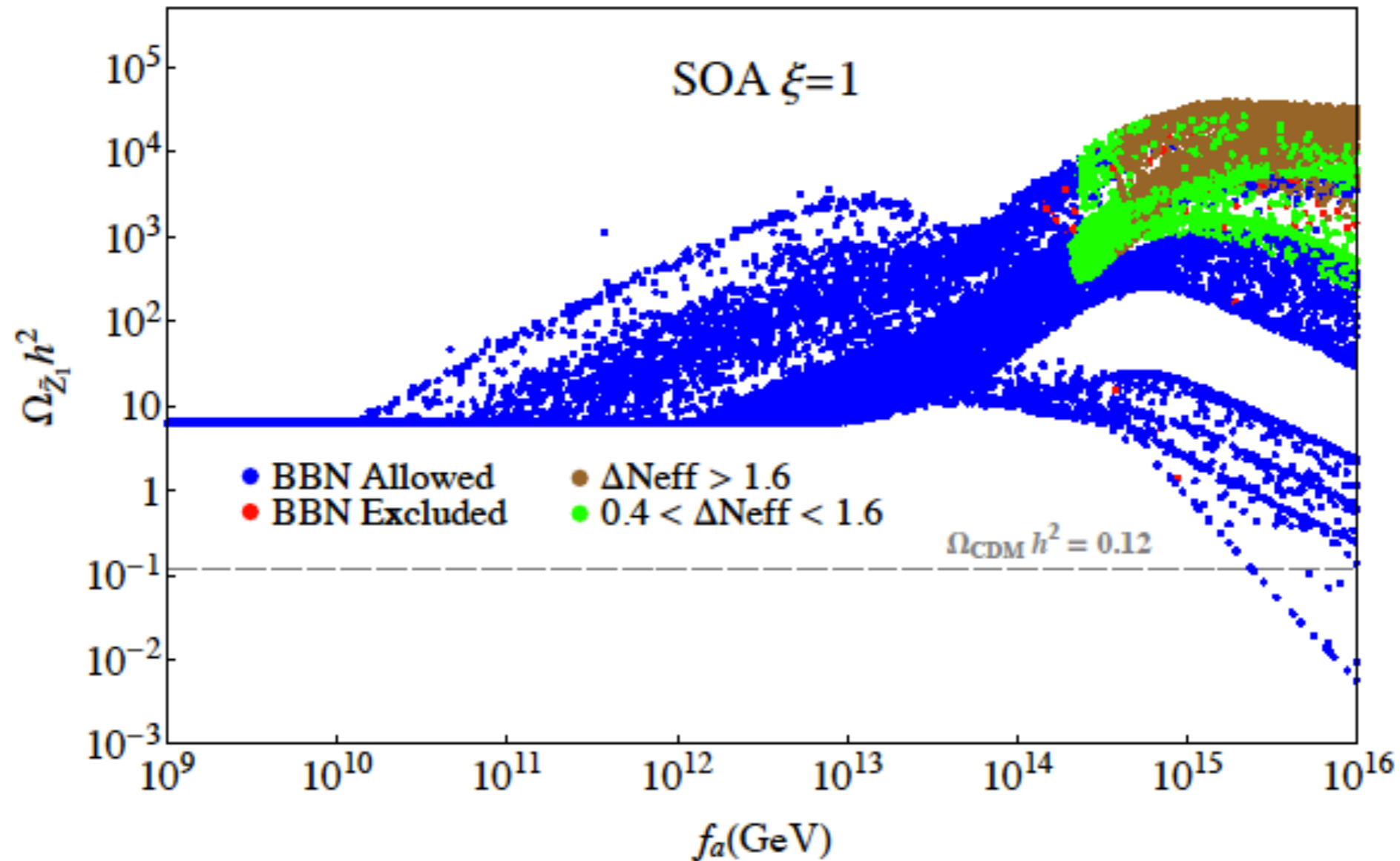


Conclusions: status of SUSY post LHC8

- SUSY EWFT **non-crisis**: EWFT allowed at 10% level in radiatively-driven natural SUSY: SUGRA GUT paradigm is just fine
- naturalness maintained for $\mu \sim 100\text{--}200$ GeV; $t_1 \sim 1\text{--}2$ TeV, $t_2 \sim 2\text{--}4$ TeV, highly mixed; $m(\tilde{g}, \tilde{l}, \tilde{n}) \sim 1\text{--}5$ TeV
- RNS spectra characterized by mainly higgsino-like WIMP: standard relic underabundance
- LHC14 w/ 300 fb^{-1} can see about half of RNS parameter space
- **e^+e^- collider with $\sqrt{s} \sim 500\text{--}600$ GeV needed to find predicted light higgsino states**
- Discovery of and precision measurements of light higgsinos!
- DFSZ invisible axion model: solves strong CP and μ problems while allowing for $\mu \sim m(Z)$
- Expect mainly axion CDM with 5–10% higgsino-like WIMPs over much of p-space
- Ultimately detect both axion and higgsino-like WIMP

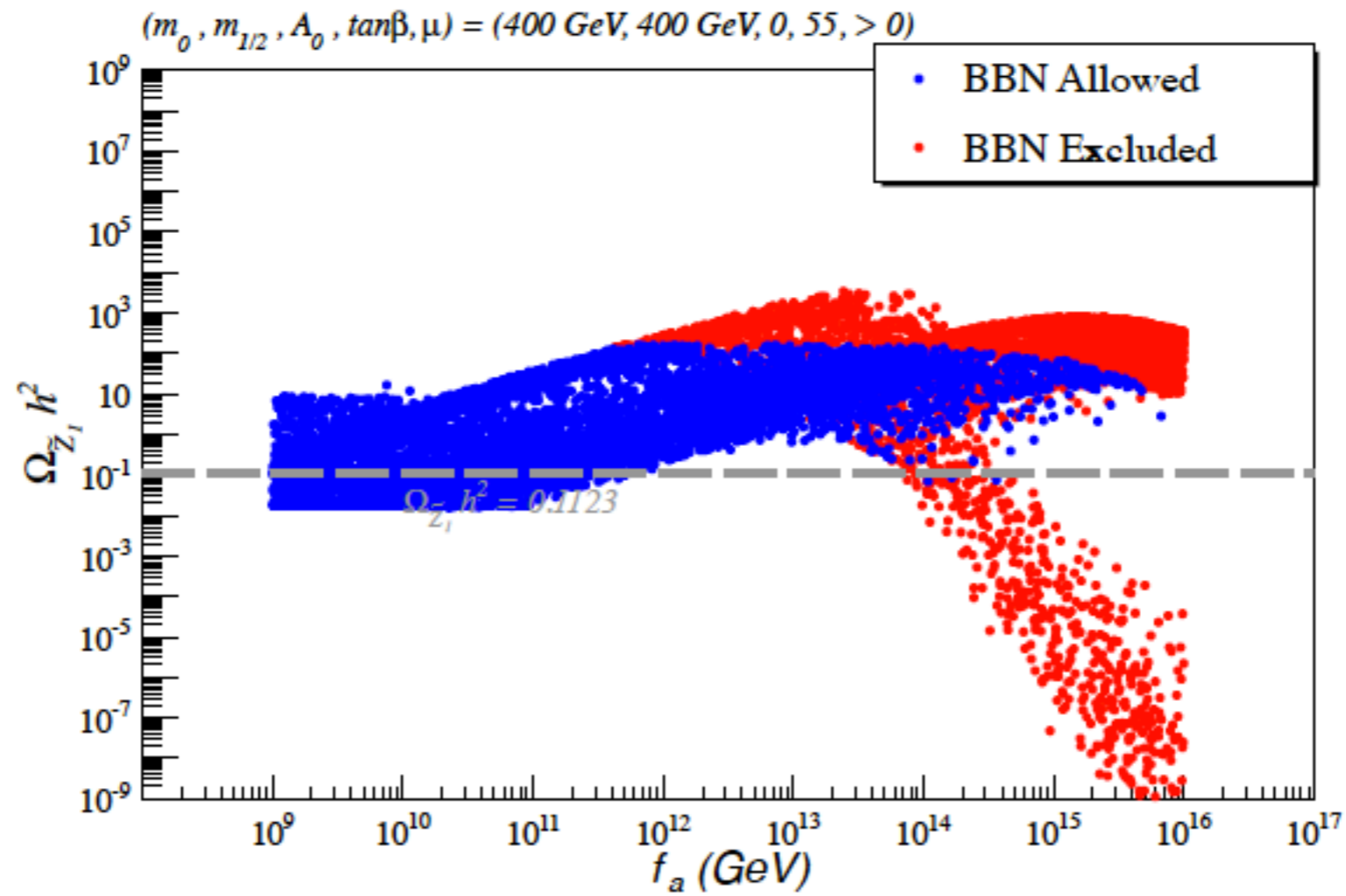
Backup

Case with standard overabundance of bino-like WIMPs



Excluded over most of f_a range except $f_a > 2 \times 10^{15}$ GeV
 where there can be significant entropy dilution so long as $m_s < 2m_{\tilde{Z}_1}$

Mixed axion/higgsino DM in KSVZ with $\xi=0$



mixed axion/higgsino DM in KSVZ with $\xi=1$

