$g-2$ Theory overview

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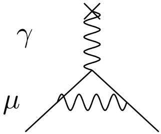
Overview

Introduction

- A lepton has a magnetic moment aligned along its spin

$$\vec{\mu}_\ell = g_\ell \frac{Q_\ell e}{2m_\ell}\vec{s}, \quad g_\ell = 2(1 - a_\ell)$$

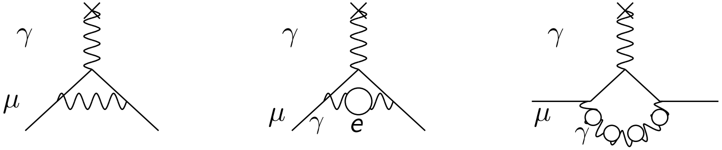
- In the Dirac theory $g_\ell = 2$
- The anomaly $a_\ell = \frac{g_\ell - 2}{2}$ arises from quantum fluctuations



$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{Had.}$$

- Largest contribution comes from the 1ℓ diagram QED, but EW and Hadronic crucial to determine departure from SM
- Arguably the best hint of physics BSM

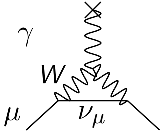
SM: a_μ^{QED}

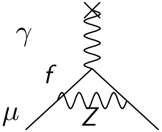


- 1947 → Schwinger: $a_\ell = \frac{\alpha}{\pi} = 0.00116$
- 2012 → Aoyama, Hayakawa, Kihoshita & Nio
  $a_\mu^{5\ell(QED)} = (116584718.951 \pm 0.077 \pm 0.019 \pm 0.009 \pm 0.007) \times 10^{-11}$
  - The main uncertainty comes from α
  - Other uncertainties: from the lepton mass ratios, 4ℓ and 5ℓ

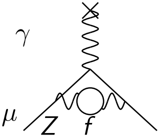
SM: a_μ^{EW}

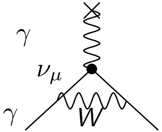
- 1ℓ (LO): [5 diff. groups, Bardeen, Jackiw, Bars, Fujikawa, Sanda, 1972]
  Due to y_μ being small, only W and Z bosons contribute at a measurable level in the LO EW

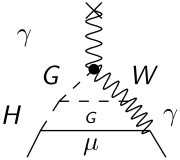




- 2ℓ: Full calculation [A. Czarnecki, Marciano et. al. 1996]
  fermion loops   boson loops (1ℓ corr.)   diag. inv. the Higgs







1. Fermion loops
2. Boson loops
3. Diags. involving the Higgs

- Involve gauge bosons to quarks [a]
- Appear as a correction to 1ℓ diagrams [b]
- Revaluated using the LHC value Higgs [c]

$$a_\mu^{2\ell(EW)} = (153.6 \pm 1.0) \times 10^{-11}$$

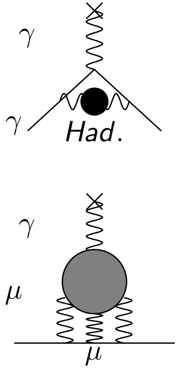
- The error comes from hadronic effects in the 2nd order
- EW diags. with quark triangle loops + unknown 3 ℓ eff.
- Leading logs for the next order are small

[a,b]=A. Czarnecki, B. Krause and W. Marciano, 1996; M. Knecht, S. Peris, M. Perrottet and E. de Rafael 2004

[c]= Stöckinger et. al. 2013

SM: $a_\mu^{Had.}$

- $a_\mu^{Had.:LO}$: Value and error dominated by the LO diagram
- a_μ^{HLbL}: Hadronic light by light contributions are also important



$a_\mu^{Had.}$(LO and NLO)

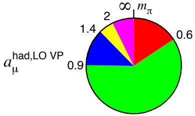
- $a_\mu^{Had.:LO}$: can be calculated from the dispersion relation

$$a_\mu^{Had.:LO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s),$$

$$R \equiv \frac{\sigma_{\rm tot}(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)},$$

  - Kinematic form factor $K(s = m_\pi^2) = 0.4$ to $K(s \to \infty) = 0$
  - The dispersion rel. connects the bare σ for e^+e^- annihilation into hadrons to the hadronic vacuum pol. contritubion to a_μ
  - The E for virtual hadrons is $O(m_\mu c^2) \ll$ QCD perturbative region, but the level of precision required for a_μ needs data up to 2 GeV.

Hagiwara et. al. 2011



- LO at 2011: many different analysis [8], e.g. by Davier & Hagiwara

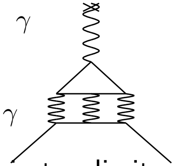
$$a_\mu^{Had.:LO} \to (6923 \pm 42, \qquad 6949 \pm 43) \times 10^{-11}$$

- Previous issues (2011) on data using τ and e^+e^- resolved
- NLO current value [Hagiwara et. al, 2011]

$$a_\mu^{Had.:NLO} = (-98.4 \pm 0.6_{exp} \pm 0.4_{rad}) \times 10^{-11}$$

a_μ^{HLbL}

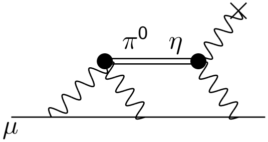
- Must be determined using hadronic models that reproduce QCD properties



- Computations done in two limits: long and short distance

1. Long distance (small p_i):
   - In the "chiral limit" and to the $1/N_c$ expansion calc. analytically

[Knecht, Nyffeler, Perrottet, E. de Rafael, 2002]



2. Short distance (large or mixed p_i): [Melnikov, Vainshtein, 2004]
   - Issues related to OPE of the electromagnetic currents of the virtual photons

But the long and short distance limits are not enough to construct a full model independent evaluation of a_μ^{HLbL}

Most part of the evaluations agree on:

- QCD chiral & large N_c limit
- How to take into account the π^0 conts. modulated by $\pi^0\gamma^*\gamma^*$ form factors

Differ on:

- On short distance limit
- In the shape of the vertex form factors which follow from different models

Agreed value of leading collaborations [arXiv:0901.0306] but room for improvement

$$a_\mu^{HLbL} = (105 \pm 26) \times 10^{-11}$$

& main physics understood (sign settled)

SM overall status

	(arXiv:1311.2198) ($\times 10^{-11}$) UNITS
QED	$116\,584\,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_\alpha$
HVP(LO) [a]	$6\,923 \pm 42$
HVP(LO) [b]	$6\,949 \pm 43$
HVP(HO) [b]	-98.4 ± 0.7
HLbL	105 ± 26
EW	154 ± 1
Total SM [a]	$116\,591\,802 \pm 42_{\rm HLO} \pm 26_{\rm HHO} \pm 2_{\rm other}\ (\pm 49_{\rm tot})$
Total SM [b]	$116\,591\,828 \pm 43_{\rm HLO} \pm 26_{\rm HHO} \pm 2_{\rm other}\ (\pm 50_{\rm tot})$

a. M. Davier, A. Hoecker, B. Malaescu and Z. Zhang, Eur. Phys. J. **C71** (2011) 1515, Erratum-ibid. **C72** (2012) 1874. b. K. Hagiwara, R. Liao, A. D. Martin, D. Nomura and T. Teubner, J. Phys. **G38** (2011) 085003.

- Compare with the combined values of a_μ^+ a_μ^- from E821 and the revised value of $\lambda = \mu_\mu/\mu_p$

$$a_\mu^{E821} = (116592089 \pm 63) \times 10^{-11} \longrightarrow$$

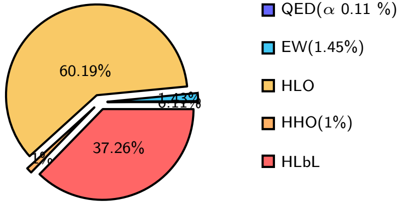
- 3.3 to 3.6 σ difference

$$\Delta a_\mu(E821 - SM) = (287 \pm 80) \times 10^{-11}$$
$$= (261 \pm 80) \times 10^{-11}$$

Future improvements (error estimation)

- a_μ^{HLO}
  - Evaluated from $e^+e^- \to hadrons$:
    - Requires a better combination from different data sets $42 \times 10^{-11} \to 26 \times 10^{-11}$
  - Lattice:
    - Error estimates currently at $\sim 5\%$ $\to$ reduce to $\sim 2\%$ (See Mainz 04.2014 WS)
    - Within a decade should compete with $e^+e^- \to hadrons$
- a_μ^{HLbL} cannot be evaluated by dispersions relations so no big decrease expected but
  - Some experimental data can help to pin down related amplitudes
  - Inputs from improvements on hadronic quantities (form factors, decay constants, etc.)
  - Ultimate goal: reduce from 25% to 10%
  - Calculation on the lattice: even an error $\sim 30\%$ would have an impact

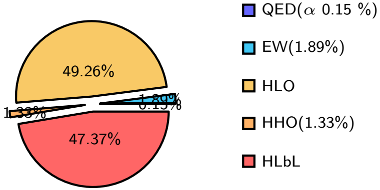
Distribution of errors: Present



- $\delta^{\rm TOT}_{a_\mu^{SM}} = 49 \times 10^{-11} \rightarrow 4 \times 10^{-5}\%$
- $\delta_{\Delta a_\mu(E821-SM)} = \mathbf{80 \times 10^{-11}}$

3.3σ difference

Distribution of errors: Future (5y)

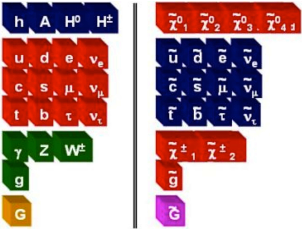


- $\delta^{\rm TOT}_{a_\mu^{SM}} = 36 \times 10^{-11} \rightarrow 3 \times 10^{-5}\%$
- $\delta_{\Delta a_\mu(E821-SM)} = \mathbf{40 \times 10^{-11}}$

7σ difference

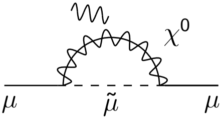
Beyond SM

- Improvement in the coming years will give important constraints on BSM
- Many possibilities
  - Extra-dim, independent effective operators analysis [Freitas et. al. 1402.7065], etc.
  - SUSY: still interesting allowed regions (even 2 ℓ conts. relevant, $O(10)\%$)



SUSY

- Relevant parameters: $m_{\tilde{p}}$, $\tan\beta = \frac{\langle H_u\rangle}{\langle H_d\rangle}$, μ $(\mu H_u H_d)$
- (1ℓ)



$$a_\mu^{\mathrm{SUSY}} = 120 \times 10^{-11} \tan\beta \; \mathrm{sign}(\mu) \left(\frac{100 \text{ GeV}}{\mathrm{m}_{\tilde{\mathrm{p}}}}\right)^2$$

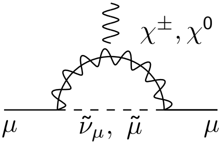
- LHC: $m_{\tilde{q}},\ m_{\tilde{g}} \gtrsim 1$ TeV
- Higgs discovery: $m_H \approx 126$ GeV $\rightarrow m_{\tilde{t}} \gtrsim 1$ TeV
- $a_\mu \rightarrow m_{\tilde{p}} = O(100)$ GeV
- So we consider

$$m_{\tilde{q}},\ m_{\tilde{g}} \gg m_{\tilde{p}} = m_{\tilde{\ell}},\ m_{\tilde{\chi}^\pm},\ m_{\tilde{\chi}^0}$$

Still perfectly possible and 2ℓ effects could be important

Status of loop calculations

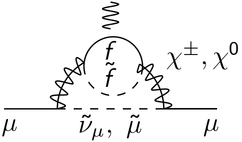
1 ℓ: Complete



$$a_\mu \propto \tan\beta$$

[Fayet 80],
[Kosower et al 83],
[Yuan et al 84],
[Lopez et al 94],
[Moroi 96]

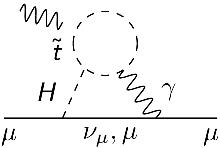
2 ℓ: Partial



$$a_\mu^{2\ell} \propto \log\left(\frac{m_{\tilde{f}}}{m_{SUSY}^{\min}}\right)$$

[Degrassi,Giudice 98] [Marchetti, Mertens, Nierste, DS 08] [Schfer, Stckinger-Kim, v. Weitershausen, DS 10]

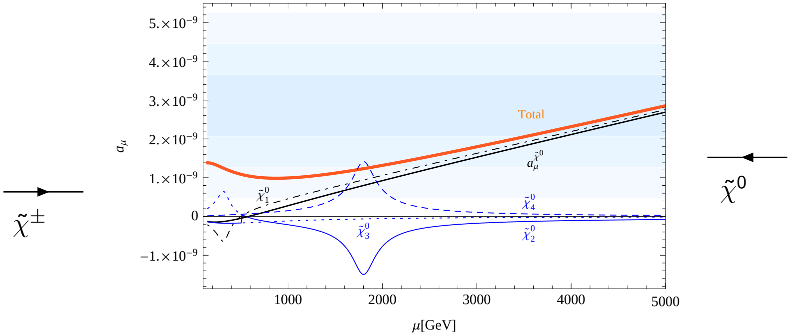
2 ℓ: (1ℓ SM) Complete



$$a_\mu^{2\ell} \propto \tan\beta\, m_{\tilde{t}}$$

[Chen,Geng01][Arhib,Baek 02]
[Heinemeyer,DS,Weiglein 03]
[Heinemeyer,DS,Weiglein 04]

- a_μ clearly distinguishes SUSY models
  - Example [VS et. al. 2014]

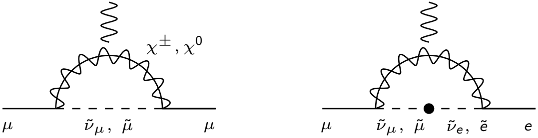


- Due to very different scenarios $(10, 500) \times 10^{-11}$
- Interesting regions compatible with LHC limits
- 2ℓ important (30%) for heavy $\tilde q$ when [Stöckinger et. al. 2013]

$$m_{\tilde Q} \gg m_{\tilde u} \gg m_{\tilde p} \qquad \text{due to large} \quad \log\left(m_{\tilde Q}\right)$$

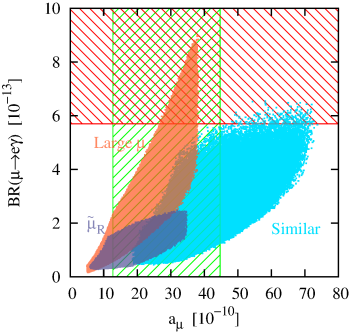
BSM: Correlation to other observables

- Can correlate to other observables [Kersten, Park, Stöckinger, VS, 2014]



- $\mathrm{BR}(\mu \to e\gamma) < 5.7 \times 10^{-13}$

$$\sqrt{\mathrm{BR}(\mu \to e\gamma)} \propto \frac{m^2_{\tilde{L}_{12}}}{m^2_{\tilde{p}}}$$



- Can help to pin down effective models
- Corner supersymmetric parameters

Messages

- **SM:**
  - $\Delta a_\mu(EXP - SM) = (287 \pm 80) \times 10^{-11} \rightarrow > 3\sigma$
  - Intensive work on the improvement of $\alpha_\mu^{Had.}$
  - $\sim 5y$ the error can decrease to $\delta_{\Delta a_\mu(EXP-SM)} = 40 \times 10^{-11}$ (If central value of $\Delta a_\mu(EXP - SM)$ stays the same $\sim 7\sigma$)
- **SUSY:**
  - Contributions possible and compatible with LHC limits + Higgs discovery if

$$m_{\tilde{q}},\ m_{\tilde{g}} \gg m_{\tilde{p}} = m_{\tilde{\ell}},\ m_{\tilde{\chi}^{\pm}},\ m_{\tilde{\chi}^0}$$

  - Typical values $a_\mu^{\mathrm{SUSY}} \in (10, 500) \times 10^{-11}$ can explain the difference between EXP and SM!
  - 2ℓ relevant, especially with the hierarchies possible at LHC
- Waiting for a new measurement