Theory overview of $(\mathbf{g}-\mathbf{2})_\mu$

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Abstract. For more than a decade the SM determination of the anomalous magnetic moment of the muon, $(g-2)_\mu$, has shown a deviation from its experimental value. There is currently a more than 3 standard deviations difference. However, the SM determination could be greatly improved, mainly by the reduction of the uncertainties associated to the calculation of the hadronic contributions to $(g-2)_\mu$. Supersymmetric models are capable to explain the difference between experimental and SM values of $(g-2)_\mu$. In these models, the study of the correlations to other observables, for example BR($\mu \to e\gamma$), can help to select models where constraints from the LHC are compatible and can give definite predictions.

1. SM overview

Current status. A lepton has a magnetic moment, g_ℓ, aligned along its spin: $\vec{\mu}_\ell = g_\ell \frac{Q_\ell e}{2m_\ell}\vec{s}$, where Q_ℓ, is the charge of the lepton, m_ℓ and s its spin. Without quantum corrections $g_\ell = 2$, but with quantum corrections the *anomalous moment*, $a_\ell \equiv (g-2)_\ell/2$, has a non zero value. In the Standard Model (SM), the largest contribution to a_μ comes from the one loop, 1ℓ, QED correction, Fig. 1, but EW and Hadronic corrections are crucial to determine the departure of the SM with the experimental value [1]. The up to date value of $a_\mu^{\rm QED}$ has been computed at 5ℓ [2] and the main uncertainty comes from the fine structure constant α. The other uncertainties come, respectively, from the lepton mass ratios, the 4ℓ and the 5ℓ corrections. This value is presented in Tab. 1. The $a_\mu^{\rm EW}$ contribution has been calculated up to 2ℓ, [3], and is presented in the second row of Tab. 1. Its main uncertainty comes from hadronic considerations at 2ℓ, and unknown 3ℓ effects. The quoted value includes a reevaluation using the latest constraints from the LHC Higgs data, based on previous calculations from [4, 5]. The hadronic contributions

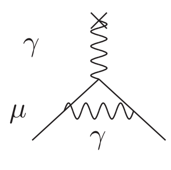

Figure 1. 1ℓ correction to a_μ in QED.

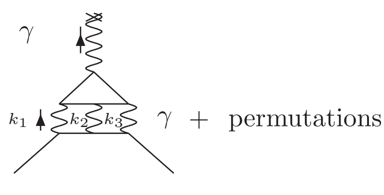

Figure 2. a_μ^{HLbL}: Hadronic Light by Light contributions to a_μ.

are divided in $a_\mu^{HLO,HNLO}$ and Hadronic light by light contributions: a_μ^{HLbL}. The LO Hadronic contribution is calculated from the dispersion relation

$$a_\mu^{HLO} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s)\,, \quad R \equiv \frac{\sigma_{\rm tot}(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} \tag{1}$$

where the kinematic form factor varies from $K(s = m_\pi^2) = 0.4$ to $K(s \to \infty) = 0$. The dispersion relation connects the bare cross section for e^+e^- annihilation into hadrons to the hadronic vacuum polarization contritubution to a_μ. There are presently different analyses, each with different models to calculate the dispersion relation, Eq. (1), based on existing experimental data sets. However, there has not been a consensus on how these values should be put together. The most recent calculations can be found in [6, 7, 8, 9]. Their values are respectively $a_\mu^{HLO} = (6909.6\pm, 46.5, 6812.3 \pm 45.1, 6923 \pm 42, 6949 \pm 43) \times 10^{-11}$. Taking into account an average of these values, we obtain $(6\,898 \pm 44) \times 10^{-11}$. The value of the NLO Hadronic contribution, a_μ^{HNLO}, has been calculated just by one collaboration, [9], and is presented in Tab. 1.

The contribution a_μ^{HLbL}, Fig. 2, must be determined using hadronic models, but currently there is not a single computation valid for all values of the momenta, k_i of the photons, Fig. 2. There are computations done in two limits: long and short distance, which correspond respectively to small momenta k_i, and large or mixed momenta k_i. Most part of the long-distance evaluations agree basically on QCD chiral & large N_c, (c = color) limit and how to take into account the π^0 contributions modulated by $\pi^0\gamma^*\gamma^*$ form factors (see for example [10]). However, up to now, there has been just an independent evaluation of the short distance limit [11]. Despite of this, here is an agreed value of leading collaborations [12], quoted in Tab. 1.

Table 1. Summary of SM results. The results marked with * has been computed here, taking into account references [6, 7, 8, 9]. There is not an agreed official value of the LO contribution, but taking as an average the four references quoted, we obtain $\Delta a_\mu(E821 - SM) = 311 \pm 81$.

	Contribution ($\times 10^{-11}$) UNITS
QED [2]	$116\,584\,718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_\alpha$
EW [3]	154 ± 1
HVP(LO) [*]	$6\,898 \pm 44$
HVP(NLO) [9]	-98.4 ± 0.7
HLbL [12]	105 ± 26
Total SM [*]	$116\,591\,778 \pm 44_{\rm HLO} \pm 26_{\rm HNLO} \pm 2_{\rm other}\ (\pm 51_{\rm tot})$

If we compare the average value a_μ of Tab. 1 to the experimental value from E821, $a_\mu^{E821} = (116592089 \pm 63) \times 10^{-11}$, and the revised value of $\lambda = \mu_\mu/\mu_p$, we obtain $\Delta a_\mu(E821 - SM) = 311 \pm 81$. Since there are different analyses of a_μ^{HLO}, what we can say in general is that there is currently a more than 3σ difference between SM and experimental results.

Future improvements in the theoretical error estimation. The evaluation of a_μ^{HLO} from $e^+e^- \to$ *hadrons* requires a better combination from different experimental data sets used to determine the dispersion relations, Eq. (1), in particular some of the old sets should be revisited. The expected error reduction from the HLO evaluation of [8] is to bring 42×10^{-11} down to 26×10^{-11} [13]. Current lattice estimates for the error are at the $\sim 5\%$ and the plan is to reduce them to $\sim 2\%$, within the next five years and within a decade to be competitive to the theoretical estimates taking into account the experimental information of $e^+e^- \to$ *hadrons* [14]. Since a_μ^{HLbL} cannot be evaluated by dispersions relations, a big decrease is not expected but some experimental data can help to pin down related amplitudes. The ultimate goal is to reduce the uncertainty from 25% to 10%. A calculation on the lattice even with an error $\sim$ 30% would have an impact since it could pin down important difference in the models used to determine short and long distance limits of a_μ^{HLbL} [13, 14].

2. Supersymmetry overview

In the MSSM, at 1ℓ, the relevant parameters to determine the supersymmetric (SUSY) contribution to a_μ are: $m_{\tilde{p}}$, $\tan\beta = \frac{\langle H_u\rangle}{\langle H_d\rangle}$, μ ($\mu H_u H_d$), where $m_{\tilde{p}}$ is the scale of the particles involved in the 1ℓ correction to a_μ, Fig. 3. When all the relevant SUSY masses $m_{\tilde{p}}$ are of the same order, the contribution to a_μ has the form $a_\mu^{\rm SUSY} = 120\times 10^{-11}\ \tan\beta\ {\rm sign}(\mu)\ (100\ {\rm GeV}/{\rm m}_{\tilde{\rm p}})^2$.

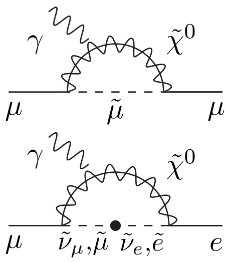

Figure 3. Top: example of a SUSY diagram mediating a_μ. Bottom: The flavor-violating analogous diagram, mediating the transition $\mu \to e\gamma$.

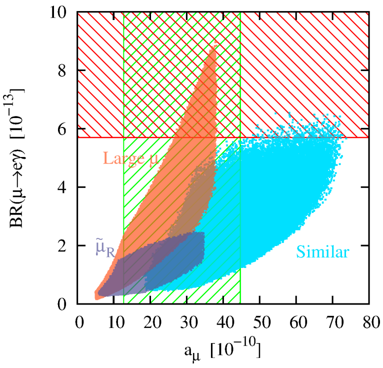

Figure 4. SUSY contribution to a_μ vs BR($\mu \to e\gamma$) for similar SUSY masses (light blue), large μ (orange), and heavy left-handed sleptons ($\tilde{\chi}^0 - \tilde{\mu}_R$, violet). In each case, $\delta_{LL} = \delta_{RR} = 2\times 10^{-5}$, $\tan\beta = 50$.

The LHC searches for SUSY particles, indicate that $m_{\tilde{q}}$, $m_{\tilde{g}} \gtrsim 1$ TeV (the masses of the squarks -in general- and the gluino, respectively). Since a_μ requires $m_{\tilde{p}} = O(100)$ GeV, we should consider that the supersymmetric particles involved in a_μ should be smaller than those for which the LHC has put constraints above the 1 TeV range, hence we need $m_{\tilde{q}}$, $m_{\tilde{g}} \gg m_{\tilde{p}} = m_{\tilde{\ell}}$, $m_{\tilde{\chi}^\pm}$, $m_{\tilde{\chi}^0}$. These last three masses are respectively the masses of the sleptons, the charginos and the neutralinos. With these considerations, a solution to the discrepancy between experimental and SM determinations to a_μ is still possible. Furthermore, when there are some big hierarchies between some sfermions and the sleptons relevant to a_μ, 2ℓ effects could be important [15]. In [16] we have studied cases where there is a hierarchy among the particles entering into the 1ℓ calculation of $a_\mu^{\rm SUSY}$ and hence, the typical behavior of the general 1ℓ corrections does not longer hold. A nice example is the case where the lightest neutralino contribution dominates and we can see an increase of the value of $a_\mu^{\rm SUSY}$ as the value of μ grows. This result holds when the hierarchy is such that $\mu > M_2 > M_1$, where $M_{i=1,2}$ are the masses of the gauginos coupled to the bino and the $SU(2)_L$ gauginos, respectively. In this case, $a_\mu \approx g_1^2 \frac{m_\mu^2}{48\pi^2}\frac{\mu\tan\beta}{M_1} F\left[m^2_{\tilde{\chi}^0_i}/m^2_{\tilde{\ell}_m}\right]$, where in the range of a_μ^{SUSY} that fits the difference $\Delta a_\mu(E821 - SM)$, the function $F[m^2_{\tilde{\chi}^0_i}/m^2_{\tilde{\ell}_m}]$ is $O(1)$ and the value of μ can be up to 5 TeV. This example is a clear departure from the typical behavior at 1ℓ and is relevant for constraints at the LHC, and as we will see next, for correlations to BR($\mu \to e\gamma$). We call this scenario "Large-μ".

3. Correlation to $\mu \to e\gamma$

The SUSY contributions to a_μ and to the decay $\mu \to e\gamma$ are given by very similar Feynman diagrams. There has been already more than a decade (please see references in [16]) of works reporting correlations in specific scenarios, in particular if a_μ is dominated by a single diagram. In [16] we disentangled at which degree the correlations depended on the specific relations on the masses of the MSSM. We have shown how the correlations are weakened by significant cancellations between diagrams in large parts of the MSSM parameter space. Nevertheless, the order of magnitude of BR($\mu \to e\gamma$) for a fixed flavor-violating parameter can often be predicted. We can define flavor violating parameters $(\delta^l_{12})_{XX} \equiv m^2_{\tilde{X}_{12}}/\sqrt{m^2_{\tilde{X}_{11}} m^2_{\tilde{X}_{22}}}$, for $X = L, R$, left and right contributions. What we found is that by fixing this parameter, we can study the correlations between BR($\mu \to e\gamma$) and a_μ in a more systematic way. In Fig. 4 we show a plot of BR($\mu \to e\gamma$) vs. a_μ showing three different regions. In blue (light) we show an scenario where SUSY masses are varied between 300 and 600 GeV, $\tan\beta = 50$ and $(\delta^l_{12})_{LL} = 2 \times 10^{-5}$. The Large-$\mu$ region, which we have described above, is shown in orange (dark) and finally a region labeled by "$\tilde{\mu}_R$" is shown towards the left-bottom corner. The vertical green-shaded area corresponds to the 2σ difference between experimental and SM determinations, using a^{HLO}_μ from [8], and the rest of the values from Tab. 1, $\Delta a_\mu(E821 - SM) = 287 \pm 80$. The red horizontal line corresponds to the upper limit of BR($\mu \to e\gamma$) [17]. We can see that for similar SUSY masses, there is not a correlation, but when we go to a particular hierarchy we start to see a behavior were we can identify a correlation. The use of this correlation is as follows. We can write BR$(\mu \to e\gamma) = \frac{3\pi^2 e^2}{G_F^2 m_\mu^4}\left(|a_{\mu e\gamma L}|^2 + |a_{\mu e\gamma R}|^2\right)$, where $a_{\mu e\gamma X}$, $X = L, R$ are the contributions to the amplitude of the process $\mu \to e\gamma$, with Left and Right chiralities, respectively (see. eq. 5 of [16]). By calculating the values of $a_{\mu e\gamma X}$ and a_μ for the different scenarios, we are able to write $|a_{\mu e\gamma X}| = |a_\mu| f\ m^2_{\tilde{p}}$, where $f = O(1)$ number and now $m^2_{\tilde{p}}$ is the value of the SUSY mass involved in the specific scenario, for example in the Large-μ scenario, the only particles involved will be the particles in the loop of the top figure of Fig. 3. Now, the requirement of a_μ to lie within the difference of the experimental and SM values fixes $m^2_{\tilde{p}}$, the coefficient f changes with the scenario, but the point is that then $|a_{\mu e\gamma X}| \propto (\delta^l_{12})_{XX}$ is constrained if we want to satisfy the bound on BR($\mu \to e\gamma$) and this cannot be evaded by simply raising the value of the parameter $m^2_{\tilde{p}}$. The ultimate application of this kind of analyses would be attained if BR($\mu \to e\gamma$) could eventually be measured: that would give definite indications for the required values of all relevant mass parameters.

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