$\bar{B}_s \to K$ semileptonic decay from an Omnès improved constituent quark model

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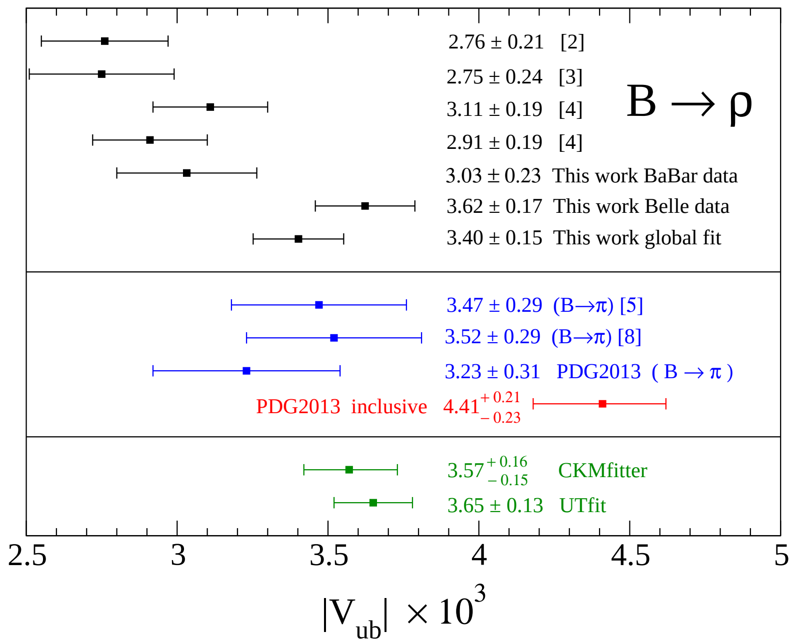
C. Albertus, E.Hernández, C. Hidalgo-Duque and J. Nieves, arXiv:1404:1001

Introduction

- There is a large discrepancy between the $|V_{ub}|$ values obtained from exclusive and inclusive reaction analyses.



$|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$ Inclusive

$|V_{ub}| = (3.23 \pm 0.31) \times 10^{-3}$ Exclusive ($B \to \pi$ dominated)

Taken from C. Albertus et al., PRD in print, arXiv:1406.7782

- Any new determination of $|V_{ub}|$ is relevant.
- In particular the $\bar{B}_s \to K^+ \ell^- \bar{\nu}_\ell$ decay channel is expected to be observed at LHCb and Belle.

Introduction II

We attack the problem as in our previous study of the $B \to \pi$ decay [C. Albertus et al., Phys. Rev. D 72, 033002 (2005)].

- We start with a simple constituent quark model whose valence quark contribution we supplement with a $\bar{B}^*-$pole one. In this way you get a reasonable description of the f^+ dominant form factor at high q^2.
- The quark model does not work well at low q^2 where the kaon recoil is largest.
- To correct the behaviour in that region we use an Omnès functional form for the f^+ form factor and make a combined fit to our quark model results at high q^2 and LCSR results at low q^2.

Form factor decomposition and decay width

$0^- \to 0^-$ transition

$$\langle K^+,\, \vec{p}_K \,|\bar{\Psi}_u(0)\gamma^\mu(1-\gamma_5)\Psi_b(0)|\bar{B}_s,\, \vec{p}_B\,\rangle \quad = \quad \left(P^\mu - q^\mu \frac{M_{B_s}^2 - M_K^2}{q^2}\right) f^+(q^2) + q^\mu \frac{M_{B_s}^2 - M_K^2}{q^2} f^0(q^2)$$

with $P = p_B + p_K$, $q = p_B - p_K$.

For zero lepton masses, the differential decay width reads

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}\,|V_{ub}|^2\,|\vec{p}_K|^3\,|f^+(q^2)|^2$$

Quark model evaluation of the form factors: valence contribution

For a B_s at rest and taking $\vec{q}$ along the $Z+$ direction

$$f^+(q^2) = \frac{1}{2M_{B_s}} \left[V^0(|\vec{q}\,|) + \frac{V^3(|\vec{q}\,|)}{|\vec{q}\,|} \, (E_K - M_{B_s}) \right]$$

$$f^0(q^2) = \frac{1}{2M_{B_s}} \left\{ V^0(|\vec{q}\,|) \, \frac{q^2 + M_{B_s}^2 - M_K^2}{M_{B_s}^2 - M_K^2} + \frac{V^3(|\vec{q}\,|)}{|\vec{q}\,|} \left[E_K \, \frac{q^2 + M_{B_s}^2 - M_K^2}{M_{B_s}^2 - M_K^2} + M_{B_s} \, \frac{q^2 - M_{B_s}^2 + M_K^2}{M_{B_s}^2 - M_K^2} \right] \right\}$$

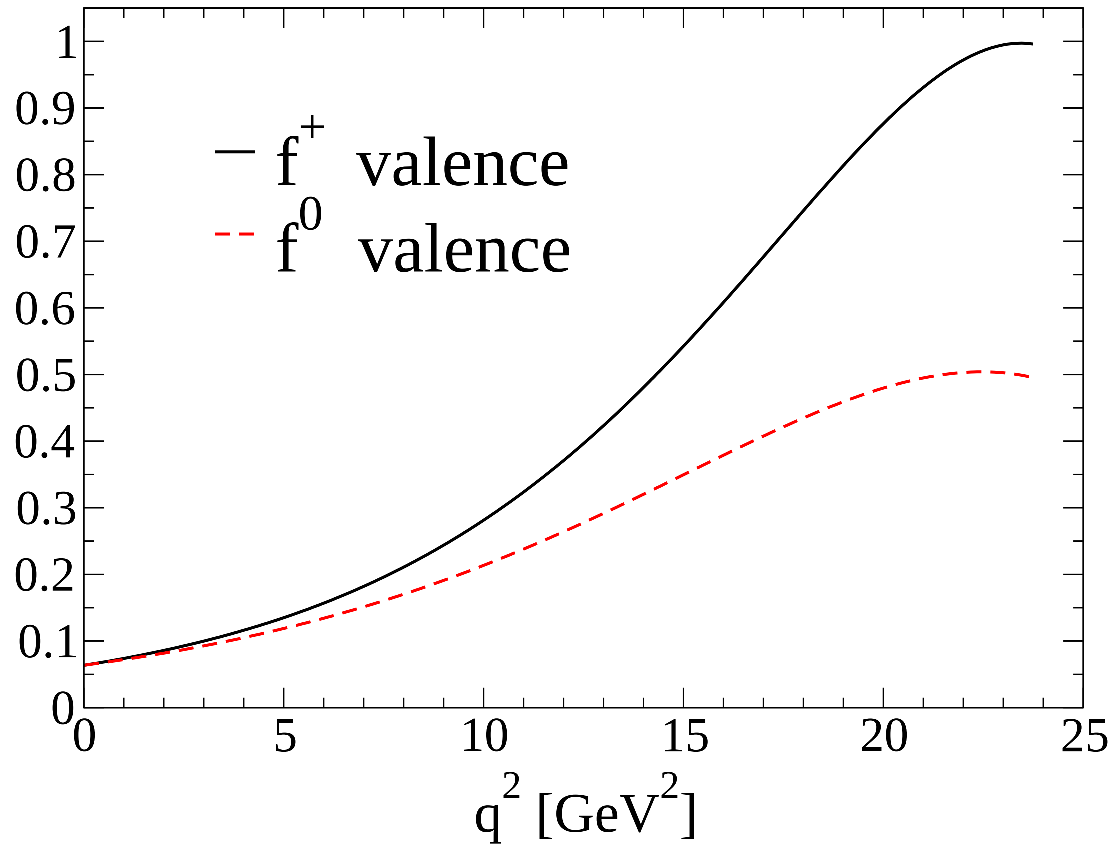
within the quark model the V^0 and V^3 vector matrix elements are given

$$V^0(|\vec{q}\,|) = \sqrt{2M_{B_s} 2E_K} \int d^3p \, \frac{1}{4\pi} \, \Phi_K^*(|\vec{p}\,|) \, \Phi_{B_s}(|\,\vec{p} - \frac{m_s}{m_u + m_s} |\vec{q}\,| \vec{k}\,|) \sqrt{\frac{\widehat{E}_u \widehat{E}_b}{4E_u E_b}} \left(1 + \frac{(-\frac{m_u}{m_u+m_s} \, |\vec{q}\,| \vec{k} - \vec{p}\,) \cdot (\frac{m_s}{m_u+m_s} \, |\vec{q}\,| \vec{k} - \vec{p}\,)}{\widehat{E}_u \widehat{E}_b} \right)$$

$$V^3(|\vec{q}\,|) = \sqrt{2M_{B_s} 2E_K} \int d^3p \, \frac{1}{4\pi} \, \Phi_K^*(|\vec{p}\,|) \, \Phi_{B_s}(|\,\vec{p} - \frac{m_s}{m_u + m_s} |\vec{q}\,| \vec{k}|) \sqrt{\frac{\widehat{E}_u \widehat{E}_b}{4E_u E_b}} \left(\frac{\frac{m_s}{m_u+m_s} \, |\vec{q}\,| - p_z}{\widehat{E}_b} + \frac{-\frac{m_u}{m_u+m_s} \, |\vec{q}\,| - p_z}{\widehat{E}_u} \right)$$

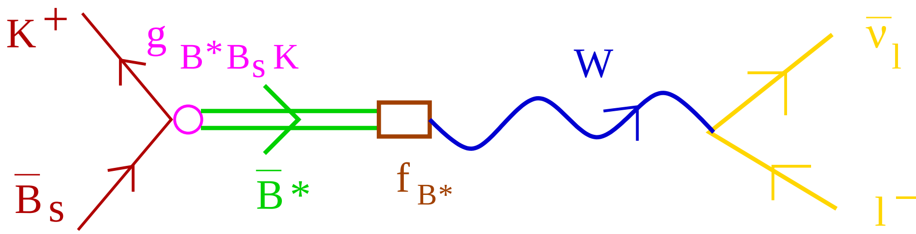
where $\widehat{E}_q = E_q + m_q$. Wave functions are evaluated using the AL1 potential [C. Semay and B. Silvestre-Brac, Z. Phys. C61, 271 (1994)]

Quark model evaluation of the form factors: valence contribution II



Valence contribution to f^+ does not contain the B^*-pole structure at high q^2.
At low q^2 these form factors deviate from the ones calculated in LCSR.

Quark model evaluation of the form factors: B^*-pole contribution



$$g_{B^*B_sK}(q^2)\,\frac{p_K^\mu - q^\mu(p_K\cdot q)/M^2_{B^*}}{M^2_{B^*}-q^2}\sqrt{q^2}f_{B^*}(q^2)$$

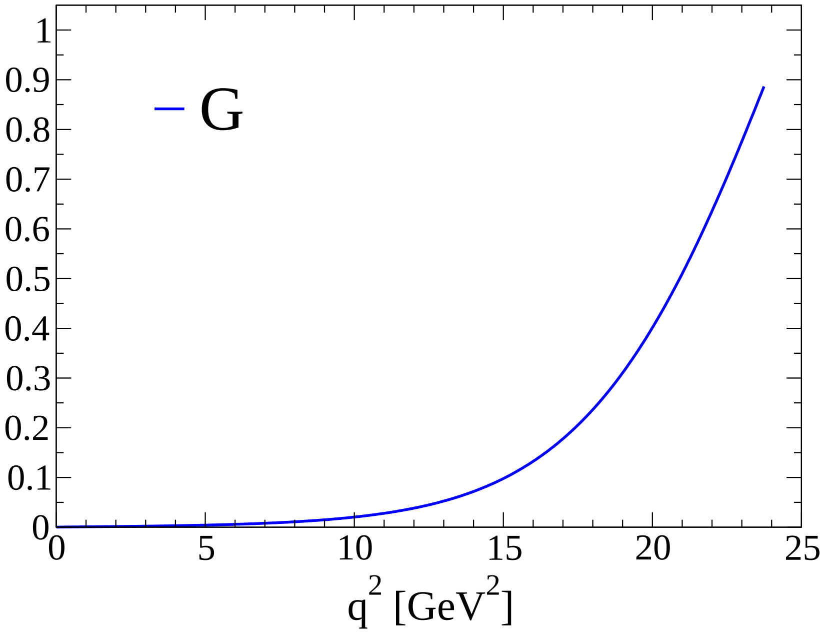
$$f^+_{\mathrm{B}^*-\mathrm{pole}}(q^2) = \frac{g_{B^*B_sK}(q^2)}{2}\sqrt{q^2}f_{B^*}(q^2)\frac{1}{M^2_{B^*}-q^2},$$

$$f^0_{\mathrm{B}^*-\mathrm{pole}}(q^2) = \frac{g_{B^*B_sK}(q^2)}{2}\sqrt{q^2}f_{B^*}(q^2)\frac{M^2_{B_s}-M^2_K-q^2}{(M^2_{B_s}-M^2_K)M^2_{B^*}}.$$

$$f_{B^*}(q^2) = \frac{\sqrt{6}}{(q^2)^{1/4}\,\pi}\int_0^\infty d|\vec{p}|\Phi_{B^*_u}(|\vec{p}|)\,|\vec{p}|^2\sqrt{\frac{\widehat{E}_b\widehat{E}_u}{4E_bE_u}}\left(1+\frac{|\vec{p}|^2}{3\widehat{E}_b\widehat{E}_u}\right) = f_{B^*}\sqrt{\frac{M_{B^*}}{\sqrt{q^2}}}$$

For f_{B^*} we get $f_{B^*} = 151\,MeV$ which is small compared to QCDSR and Lattice determinations thar are around 200 MeV.

Quark model evaluation of the form factors: B^*-pole contribution II



$$g_{B^* B_s K}(q^2) = g_{B^* B_s K}\, G(q^2)$$

and it is determined using PCAC

$$\langle H_s,\ \vec{P}' \,|\, q^\mu\ J_{A\,\mu}(0) \,|\, H^*, \lambda\, \vec{P}\,\rangle = -i\, f_K\ g_{H^* H_s K}(q^2)\ \left[q^\mu\ \varepsilon_\mu^{(\lambda)}(\vec{P}\,)\right]$$

We get $g_{B^* B_s K} = 49.88$ compared to the LCSR prediction $g_{B^* B_s K} = 29$ obtained in Z.-H- Li et al., PRD 64, 057901 (2001).

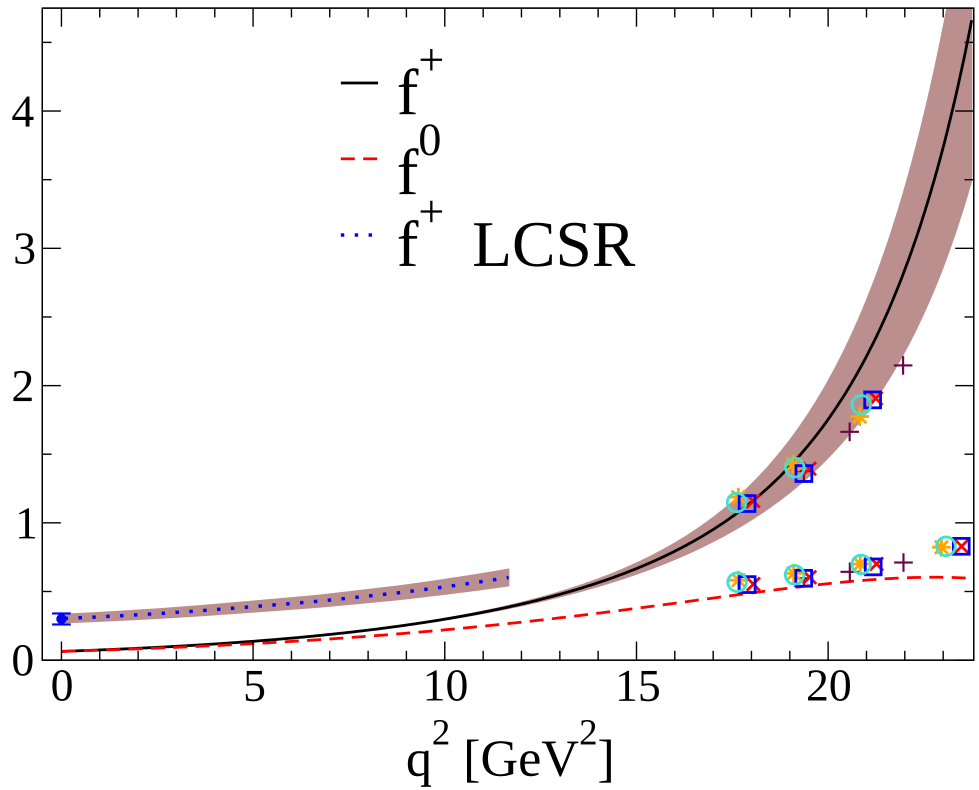
As for the product $g_{B^* B_s K}\ f_{B^*}$ we find $g_{B^* B_s K}\ f_{B^*} = 7.53\,\mathrm{GeV}$

Large compared to the LCSR prediction $g_{B^* B_s K}\ f_{B^*} = 3.57 - 4.19$ GeV by Z.-H- Li et al.

But in agreement with the LQCD plus SU(3) symmetry result
$g_{B^* B_s K}\ f_{B^*} = 7.49 \pm 1.85\,\mathrm{GeV}$
(K.C. Bowler et al., NPB619, 507 (2001). A. Abada et al., JHEP 0402,016 (2004))

We will finally use $g_{B^* B_s K}\ f_{B^*} = 7.49 \pm 2.38\,\mathrm{GeV}$

Quark model evaluation of the form factors: Full calculation



LQCD points from C.M. Bouchard et al., arXiv:1310:3207

LCSR from G. Duplancic and B. Melic, PRD78,054015 (2008)

Omnès representation of the f^+ form factor and fit

Omnès representation is based on analiticity and unitarity. For a sufficiently large number of subtractions one can write

$$f^+(q^2) \approx \frac{1}{M_{B^*}^2 - q^2} \prod_{j=0}^{n} \left[f^+(q_j^2) \left(M_{B^*}^2 - q_j^2 \right) \right]^{\alpha_j(q^2)} \quad , \quad \alpha_j(q^2) = \prod_{j \neq k = 0}^{n} \frac{q^2 - q_K^2}{q_j^2 - q_k^2}$$

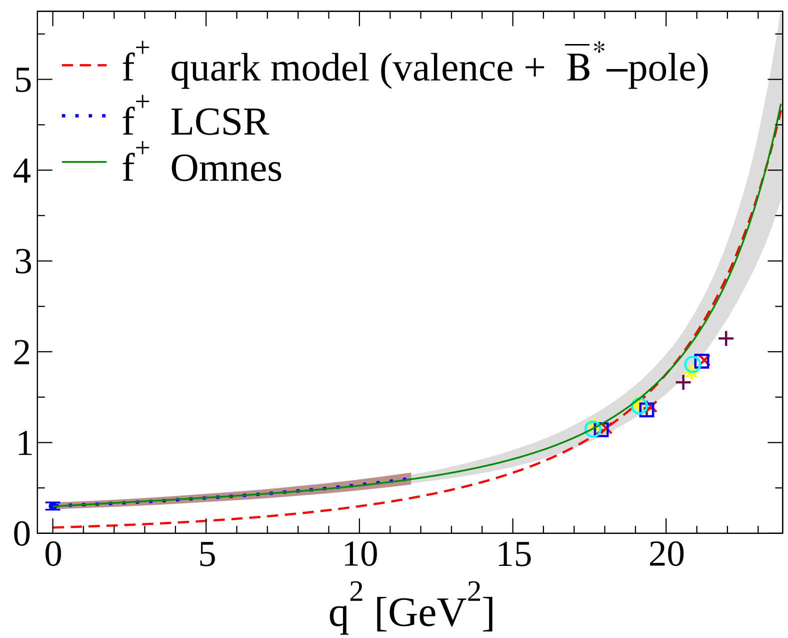
We follow J. M. Flynn and J. Nieves, PRD76,031302 (2007) and take for q_j^2 the four different values 0, $q^2_{\rm max}/3$, $2q^2_{\rm max}/3$ and $q^2_{\rm max}$.

We treat $f^+(q_j^2)$ as free parameters and make a combined χ^2-fit to our quark model results in the high q^2 region and the LCSR predictions by G. Duplancic et al. in the low q^2 region.

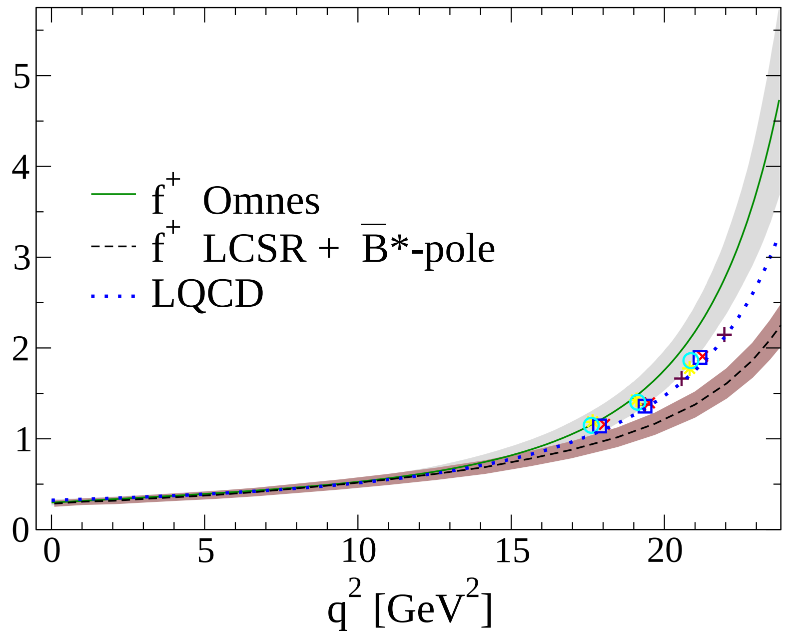
f^+ form factor obtained from the fit

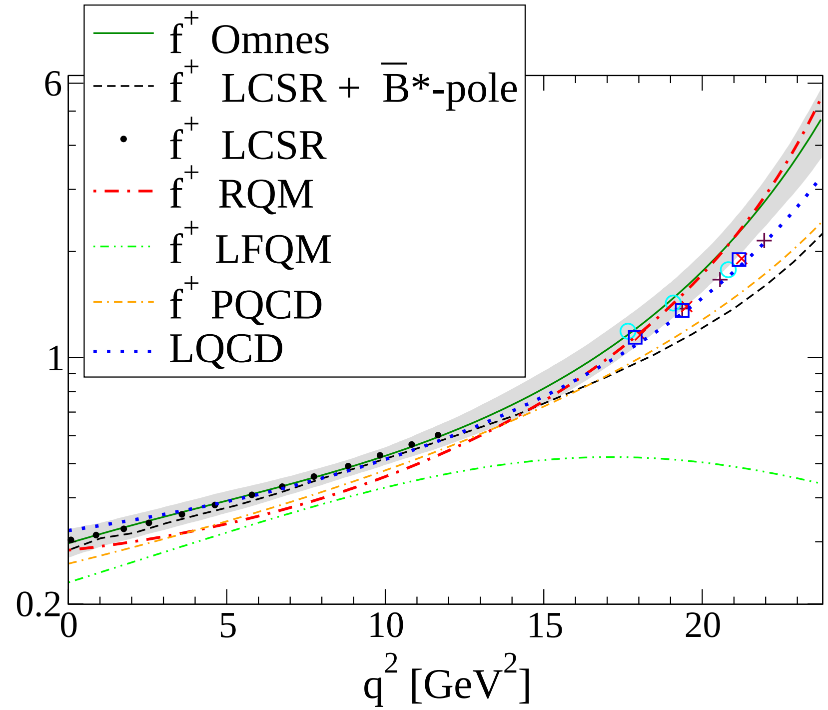
The outcome of the fit is

$$\begin{aligned} f^+(0) &= 0.297 \pm 0.027, \\ f^+(q^2_{\rm max}/3) &= 0.461 \pm 0.025, \\ f^+(2\,q^2_{\rm max}/3) &= 0.902 \pm 0.100, \\ f^+(q^2_{\rm max}) &= 4.738 \pm 0.998 \end{aligned}$$



f^+ form factor obtained from the fit II





LCSR+$\bar{B}^*-$pole Z.-H. Li et al., PRD64, 057901 (2001)

LCSR G. Duplancic and B. Melic, PRD78,054015 (2008)

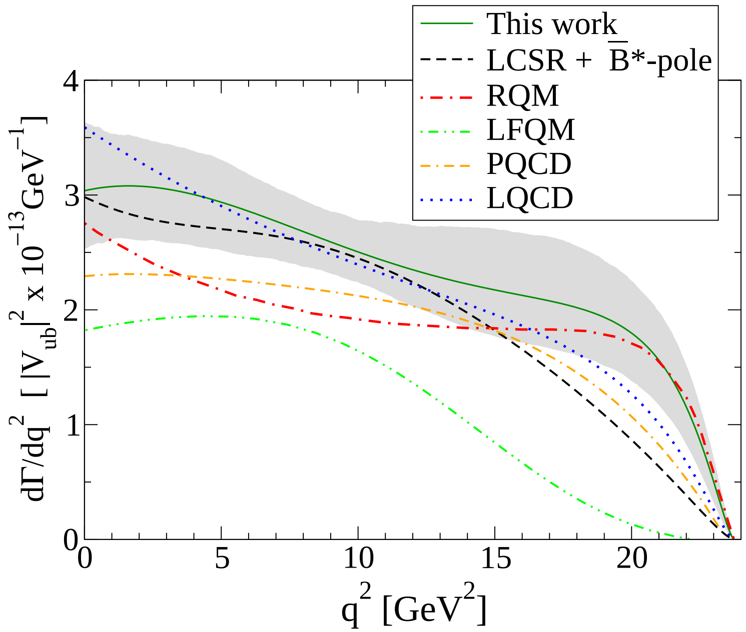
RQM R.N. Faustov and V.O. Galkin, PRD87, 094028 (2013)

LFQM R.C. Verma,J. Phys. G39, 025005 (2012)

PQCD W.-F. Wang and Z.-J. Xiao, PRD86, 114025 (2012).

LQCD C.M. Bouchard et al., arXiv:1406.2279

Decay width



	This work	LCSR+$\bar{B}^*$-pole	RQM	LFQM	PQCD	LQCD
Γ [$\lvert V_{ub}\rvert^2 \times 10^{-9}$ MeV]	$5.47^{+0.54}_{-0.46}$	$4.63^{+0.97}_{-0.88}$	4.50 ± 0.45	2.75 ± 0.24	4.2 ± 2.2	5.1 ± 1.0

A combined result gives

$$\Gamma(\bar{B}_s \to K^+ \ell^- \bar{\nu}_\ell) = (4.94 \pm 0.30)|V_{ub}|^2 \times 10^{-9}\,\text{MeV}$$

Summary

- We have evaluated the $\bar{B}_s \to K^+ l \nu_l$ decay width starting from a constituent quark model.
  - Valence quark contribution gives a poor description of the form factors.
  - The explicit inclusion of a B^*-pole contribution improves significantly the behavior at large q^2.
- We improve the quark model prediction at low q^2 using the Omnès representation of the form factors and fitting them to LCSR results in that region.
- We think our procedure improves previous global f^+ form factor determinations and the final result is comparable to a recent LQCD calculation.
- The form factor thus obtained has been used to evaluate the decay width for which we get $\Gamma(\bar{B}_s \to K^+ \ell^- \bar{\nu}_\ell) = (5.47^{+0.54}_{-0.46})|V_{ub}|^2 \times 10^{-9}\,\mathrm{MeV}$.
- The result for the decay width can be used to obtain $|V_{ub}|$ with a theoretical error of the order of 3%.