$\bar{B}_s \to K$ semileptonic decay from an Omnès improved constituent quark model

C Albertus[1], E Hernández[2], C Hidalgo-Duque[3] and J Nieves[3]

[1] Departamento de Física Atómica, Nuclear y Molecular e Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, Avenida de Fuentenueva s/n, E-18071 Granada, Spain

[2] Departamento de Física Fundamental e IUFFyM, Universidad de Salamanca, Plaza de la Merced s/n, E-37008 Salamanca, Spain

[3] Instituto de Física Corpuscular (IFIC), Centro Mixto CSIC-Universidad de Valencia, Institutos de Investigación de Paterna, Apartado 22085, E-46071 Valencia, Spain

E-mail: albertus@ugr.es,gajatee@usal.es,carloshd@ific.uv.es, jmnieves@ific.uv.es

Abstract. We study the f^+ form factor for the $\bar{B}_s \to K^+\ell^-\bar{\nu}_\ell$ semileptonic decay in a constituent quark model. The valence quark contribution is supplemented with a $\bar{B}^*$-pole term that dominates the high q^2 region. To extend the quark model predictions from its region of applicability near $q^2_{\rm max} = (M_{B_s} - M_K)^2$, we use a multiply-subtracted Omnès dispersion relation. We fit the subtraction constants to a combined input from previous light cone sum rule results in the low q^2 region and the quark model results (valence plus $\bar{B}^*$-pole) in the high q^2 region. From this analysis, we obtain $\Gamma(\bar{B}_s \to K^+\ell^-\bar{\nu}_\ell) = (5.47^{+0.54}_{-0.46})|V_{ub}|^2 \times 10^{-9}$ MeV, which is about 10% and 20% higher than predictions based on Lattice QCD and QCD light cone sum rules respectively.

1. Introduction

Playing a critical role in testing the consistency of the Standard Model of particle physics and, in particular, the description of CP violation, V_{ub} is still the less well known element of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Any new information that can be obtained from experimentally unexplored reactions is thus relevant. This is the case of the $\bar{B}_s \to K^+\ell^-\bar{\nu}_\ell$ semileptonic decay which is expected to be observed at LHCb and Belle and that it could be used to obtain an independent determination of $|V_{ub}|$. In this contribution we present a study of this reaction. All the details and further results to those presented here can be found in Ref. [1].

The hadronic matrix element for the reaction can be parameterized in terms of the $f^+(q^2)$ and $f^0(q^2)$ form factors, of which only $f^+(q^2)$ plays a significant role for the case of a light lepton in the final state ($l = e, \mu$). In fact, for zero lepton masses, the differential decay width is given solely in terms of $f^+(q^2)$ as

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3}\,|V_{ub}|^2\,\frac{\lambda^{3/2}(q^2, M^2_{B_s}, M^2_K)}{M^3_{B_s}}\,|f^+(q^2)|^2 \qquad (1)$$

with G_F the Fermi decay constant and λ the Källen function defined as $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$.

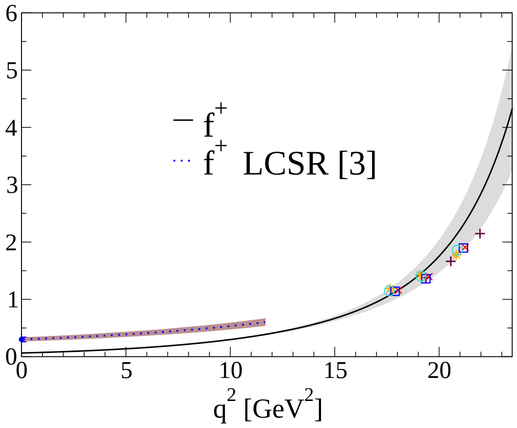

Figure 1. $f^+(q^2)$ form factor evaluated in the quark model (valence plus $\bar{B}^*$-pole contributions). We also show the results obtained in the LCSR calculation of Ref. [3] (dotted-line plus error band) and different lattice data in the high q^2 region reported in Ref. [4].

2. Results and discussion

To obtain the f^+ form factor we shall follow our earlier work in Ref. [2], where similar decays were analyzed, and then we use the quark model to evaluate the valence plus $\bar{B}^*$-pole contributions to the form factors. Calculational details can be found in [1] and references therein. Results are shown in figure 1. Taking into account theoretical uncertainties, shown as a band in the figure, we obtain a reasonable description of the form factor in the high q^2 region, as compared to the preliminary lattice data recently reported in Ref. [4]. For high q^2, the $\bar{B}^*$-pole term dominates but the valence contribution accounts for around 20% of the total. However, there is a large discrepancy in the low q^2 region between the quark model and the light cone sum rule (LCSR) results obtained in Ref. [3]. Since the latter are reliable for low q^2, it is clear that the non-relativistic quark model does not provide a good reproduction of the form factor in that region of q^2 where large relativistic effects are to be expected.

To obtain an $f^+(q^2)$ form factor valid for the whole q^2 region spanned by the decay, we adopt the scheme in Refs. [5, 6, 7], assuming a multiply subtracted Omnès functional ansatz that provides a parameterization of the form factor constrained by unitarity and analyticity properties. We take

$$f^+(q^2) \approx \frac{1}{M_{B^*}^2 - q^2} \prod_{j=0}^{n} \left[f^+(q_j^2)\left(M_{B^*}^2 - q_j^2\right)\right]^{\alpha_j(q^2)} \quad , \quad \alpha_j(q^2) = \prod_{j \neq k = 0}^{n} \frac{q^2 - q_K^2}{q_j^2 - q_k^2} \tag{2}$$

for $q^2 < s_{\rm th} = (M_{B_s} + M_K)^2$ and where $q_0, \cdots q_n^2 \in]-\infty, s_{\rm th}[$ are the $(n+1)$ subtraction points. Note that despite the factor $\frac{1}{M_{B^*}^2 - q^2}$, the functional form is not given by a single pole. The values of $f^+(q_j^2)$ are taken as free parameters and we fix them by making a combined fit to our quark model results in the high q^2 region and to the LCSR results, taken from Ref. [3], in the low q^2 part. As in Ref. [7] we only use four subtraction points corresponding to $q_j^2 = 0,\ q^2_{\rm max}/3,\ 2q^2_{\rm max}/3,\ q^2_{\rm max}$.

Our final result for $f^+(q^2)$ together with its 68% confidence level band is displayed in figure 2. There, we show a comparison with different calculations using LCSR [3], LCSR+$\bar{B}^*$-pole fit [8], relativistic quark model (RQM) [9], light front quark model (LFQM) [10], perturbative QCD (PQCD) [11] and the extrapolation to the physical region done in Ref. [12] of the lattice QCD (LQCD) results obtained in Ref. [4] (also shown). In the LCSR calculation in Ref. [3] the results are only given up to $q^2 = 10\,{\rm GeV}^2$, whereas in Ref. [10] no $\bar{B}^*$-pole contribution is included as can be seen by the behavior of the predicted form factor in the high q^2 region. All other calculations include the $\bar{B}^*$-pole mechanism, but with different strengths. In Ref. [9], where a RQM is used, they obtain a form factor similar to ours for high q^2 values. However, their approach for low and intermediate values of q^2 should not be as appropriate as a LCSR one,

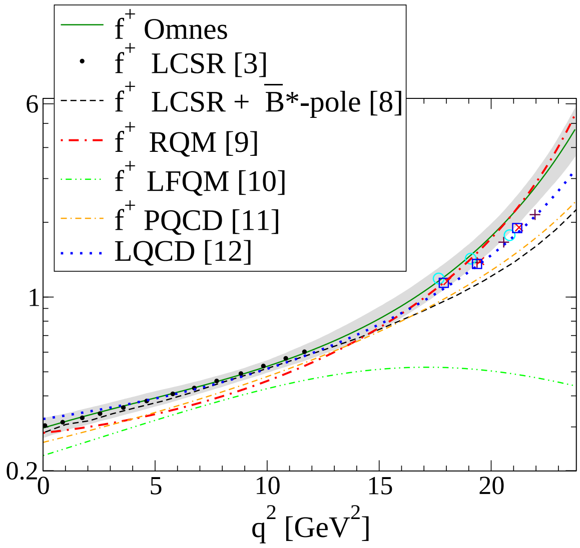

Figure 2. Global comparison of our final result (solid line plus 68% confidence level band) for the f^+ form factor with different calculations using LCSR [3], LCSR+$\bar{B}^*$-pole fit [8], RQM [9], LFQM [10], PQCD [11] and the extrapolation to the physical region done in Ref. [12] of the LQCD data from Ref. [4] which is also shown.

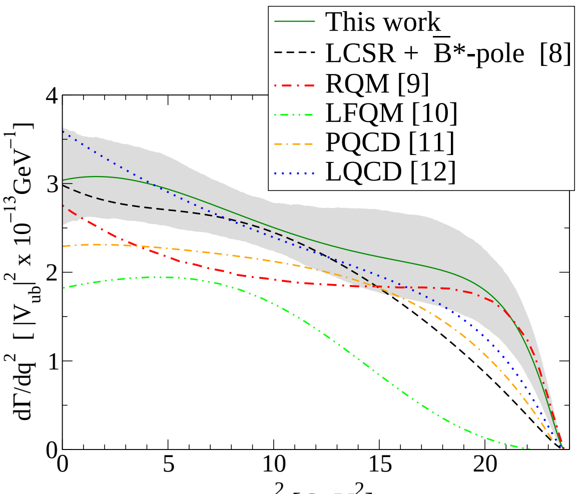

Figure 3. Differential decay width obtained in this work with the Omnès fit (solid line plus 68% confidence level band) and in LCSR+$\bar{B}^*$-pole fit [8], RQM [9], LFQM [10] and PQCD [11] and LQCD [12] approaches.

which we include in our combined analysis. Calculations in Refs. [11] and [8] give similar results at high q^2 but the one in Ref. [11] deviates from LCSR evaluations at small q^2 values. The high q^2 results obtained in LQCD [4, 12] are in between the results obtained in the approaches of Refs. [8, 11] and the quark model ones (both this work and the RQM calculation of Ref. [9]). For very low q^2 however, the central values of the LQCD extrapolation in Ref. [12] lie in the upper part of the LCSR band. Our combined approach should be more adequate in that region of q^2 since we use LCSR data to constraint our form factor.

The differential decay width, together with its 68% confidence level band, is displayed in figure 3. We also show the differential decay width from the calculations in Refs. [8, 9, 10, 11, 12]. For the integrated decay width we obtain

$$\Gamma(\bar{B}_s \to K^+ \ell^- \bar{\nu}_\ell) = (5.47^{+0.54}_{-0.46})|V_{ub}|^2 \times 10^{-9}\,\mathrm{MeV} \tag{3}$$

and a comparison with the results in other approaches is shown in table 1. The calculations in Refs. [8, 9] obtain results that are some 15% smaller than ours. The fact that their results are so similar when compared to each other seems to be a coincidence. As seen in figure 3, their differential decay widths deviate both for small and large q^2 values, but those differences compensate in the integrated width. The result of the PQCD calculation in Ref. [11] is also similar but with a larger uncertainty, around 50%. The LFQM calculation in Ref. [10] gives a much smaller result, in part because no $\bar{B}^*$-pole contribution seems to be included in that approach. The LQCD result in Ref. [12] is the one closest to ours. Its large uncertainty comes from the form factor extrapolation from high q^2, where the lattice points were obtained, to the low q^2 region. Our result is the largest although we are compatible within uncertainties with the predictions of Refs. [8, 9, 11, 12].

Table 1. Decay width in units of $|V_{ub}|^2 \times 10^{-9}$ MeV from several approaches. For the result of Ref. [8] we have propagated a 10% uncertainty in the form factor. Results for Refs. [9, 10, 11] have been adapted from Table IV in Ref. [13].

	This work	LCSR+$\bar{B}^*$-pole [8]	RQM [9]	LFQM [10]	PQCD [11]	LQCD [12]
Γ [$|V_{ub}|^2 \times 10^{-9}$ MeV]	$5.47^{+0.54}_{-0.46}$	$4.63^{+0.97}_{-0.88}$	4.50 ± 0.45	2.75 ± 0.24	4.2 ± 2.2	5.1 ± 1.0

Acknowledgments
This research was supported by the Spanish Ministerio de Economía y Competitividad and European FEDER funds under Contracts Nos. FPA2010-21750-C02-02, FIS2011-28853-C02-02, and the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042), by Generalitat Valenciana under Contract No. PROMETEO/20090090, by Junta de Andalucia under Contract No. FQM-225, by the EU HadronPhysics3 project, Grant Agreement No. 283286, and by the University of Granada start-up Project for Young Researches contract No. PYR-2014-1. C.A. wishes to acknowledge a CPAN postdoctoral contract and C.H.-D. thanks the support of the JAE-CSIC Program.

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