

Towards new frontiers in CP violation in B decays

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Abstract. CP-violating effects in decays of B mesons offer a wide spectrum of probes for testing the phase structure of the quark-flavour sector of the Standard Model. After a brief discussion of the picture emerging from the current LHC data, the focus will be put on two specific topics: hadronic uncertainties from penguin topologies on measurements of the $B_q^0-\bar{B}_q^0$ mixing phases ($q \in \{d, s\}$), and the U -spin-related decays $B_s^0 \rightarrow K^+K^-$ and $B_d^0 \rightarrow \pi^+\pi^-$. Valuable new insights are expected from future studies of CP violation in B decays. For the detection of possible new sources of CP violation, it will be crucial to match the experimental and theoretical precisions and to have a careful look at the underlying assumptions.

1. Introduction

In the Standard Model (SM), the rich phenomenology of quark-flavour physics and CP violation is related to the Cabibbo–Kobayashi–Maskawa (CKM) quark-mixing matrix [1, 2]. Information on the phase structure and elements of this matrix are encoded in weak decays of K , D and B mesons. Since the theory is formulated in terms of quarks while the mesons are bound states of strong interactions, we have to deal with process-dependent, non-perturbative “hadronic” parameters in the calculation of the relevant transition amplitudes. This feature gives rise to the main challenge in studies of CP violation in B decays: hadronic uncertainties.

In the presence of New Physics (NP), typically new sources of flavour and CP violation arise. Analyses of weak meson decays are facing an impressive hierarchy of scales:

$$\underbrace{\Lambda_{\text{NP}} \sim 10^{(0\dots?)} \text{ TeV}}_{\text{short distance physics}} \gg \underbrace{\Lambda_{\text{EW}} \sim 10^{-1} \text{ TeV}}_{\text{short distance physics}} \gg \gg \underbrace{\Lambda_{\text{QCD}} \sim 10^{-4} \text{ TeV}}_{\text{long distance physics}} . \quad (1)$$

In order to deal with this situation, effective field theories offer the suitable theoretical tool. Within this framework, the heavy degrees of freedom (NP particles, top quark, Z and W bosons) are integrated out from appearing explicitly and are described in short-distance loop functions. Perturbative QCD corrections can be calculated in a systematic way, and renormalisation group techniques allow the summation of large logarithms. This machinery was applied to the SM and various popular NP scenarios, such as MSSM, models with universal and warped extra dimensions, little Higgs models, scenarios with extra Z' bosons, etc., as reviewed in [3].

Following these lines, low-energy effective Hamiltonians can be calculated for $\bar{B} \rightarrow \bar{f}$ processes, taking the following general form [4]:

$$\langle \bar{f} | \mathcal{H}_{\text{eff}} | \bar{B} \rangle = \frac{G_{\text{F}}}{\sqrt{2}} \sum_j \lambda_{\text{CKM}}^j \sum_k C_k(\mu) \langle \bar{f} | Q_k^j(\mu) | \bar{B} \rangle, \quad (2)$$

where G_F is Fermi's constant and the λ_{CKM}^j denote combinations of CKM matrix elements. The short-distance contribution to the decay amplitude is described by the Wilson coefficient functions $C_k(\mu)$, which can be calculated in perturbation theory for the SM and its extensions. On the other hand, the hadronic matrix elements $\langle \bar{f} | Q_k^j(\mu) | \bar{B} \rangle$ are non-perturbative quantities, describing the long-distance contributions.

The key players for the exploration of CP violation in B decays are non-leptonic channels. In the previous decade, there were interesting developments for calculations of such processes within QCD: QCD factorisation, the perturbative hard-scattering (PQCD) approach, soft collinear effective theory (SCET) and QCD sum rules; the state of the art is discussed in [5]. There has recently been impressive progress in lattice QCD [6]. However, non-leptonic B decays generally remain a theoretical challenge, which is also indicated by experimental data.

The outstanding feature of analyses of CP violation in B decays is that the calculation of the hadronic matrix elements $\langle \bar{f} | Q_k^j(\mu) | \bar{B} \rangle$ can be circumvented in fortunate cases (for a detailed discussion, see [7]). The corresponding strategies play a key role to “over constrain” the unitarity triangle (UT) of the CKM matrix. Detailed analyses and continuous updates are performed by the CKMfitter [8] and UTfit [9] collaborations. The current picture of the UT shows impressive consistency with the CKM sector of the SM, despite a few tensions.

The previous run of the Large Hadron Collider (LHC) has resulted in the exciting discovery of the Higgs boson. On the other hand, the ATLAS and CMS experiments have so far not seen signals of NP particles, and the SM flavour sector has been confirmed by the LHCb data, apart from a few discrepancies which are unfortunately not yet conclusive. The implications for the general structure of physics beyond the SM are a large characteristic NP scale, i.e. not just $\sim \text{TeV}$, or (and?) symmetries preventing large NP effects in the flavour sector, where models with “Minimal Flavour Violation” are the most prominent example.

Many more interesting results are expected from the next run of the LHC and its upgrade as well as high-precision flavour experiments. In view of the present situation, we have to prepare ourselves to deal with smallish NP effects. It will be crucial for resolving possible signals of NP in the data to have a careful look at the underlying theoretical assumptions and approximations. The challenge will be to match the experimental and theoretical precisions.

2. Penguin effects in benchmark probes of CP violation

Neutral B_q^0 mesons ($q \in \{d, s\}$) show $B_q^0 - \bar{B}_q^0$ mixing [10], which originates from box topologies in the SM but may well receive NP contributions. The CP-violating mixing phases are given by

$$\phi_d = \phi_d^{\text{SM}} + \phi_d^{\text{NP}} = 2\beta + \phi_d^{\text{NP}}, \quad \phi_s = \phi_s^{\text{SM}} + \phi_s^{\text{NP}} = -2\lambda^2\eta + \phi_s^{\text{NP}}, \quad (3)$$

where β is the usual angle of the UT, while $\lambda \equiv |V_{us}| \sim 0.22$ and η are parameters of the Wolfenstein parametrisation of the CKM matrix. The benchmark decays to measure the mixing phases ϕ_q through mixing-induced CP violation are given by $B_d^0 \rightarrow J/\psi K_S$, $B_s^0 \rightarrow J/\psi \phi$ and $B_s^0 \rightarrow J/\psi f_0(980)$. Decays of B_s mesons play the key role at the LHC [11].

These determinations are affected by uncertainties from doubly Cabibbo-suppressed penguin contributions [12–19], which cannot be calculated reliably and are usually neglected. In view of the current situation and the increasing experimental precision, the following questions arise: how important are the penguin contributions and how can they be controlled?

2.1. The $B_{d,s}^0 \rightarrow J/\psi K_S$ system

In the SM, the decay $B_d^0 \rightarrow J/\psi K_S$ originates from a colour-suppressed tree topology and penguin topologies with up, charm and top quarks running in the loops. Using the unitarity of the CKM matrix, the decay amplitude can be written as follows [12]:

$$A(B_d^0 \rightarrow J/\psi K_S) = \left(1 - \lambda^2/2\right) \mathcal{A}' \left[1 + \epsilon a' e^{i\theta'} e^{i\gamma}\right], \quad (4)$$

where \mathcal{A}' and $a'e^{i\theta'}$ are CP-conserving parameters, involving the relevant hadronic matrix elements. Whereas the former quantity is governed by the colour-suppressed tree contribution, the latter measures the ratio of penguin to tree topologies. The key feature of (4) is that the penguin parameter a' enters with the tiny $\epsilon \equiv \lambda^2/(1 - \lambda^2) = 0.05$; γ is the usual UT angle.

CP violation is probed through the time-dependent decay rate asymmetry

$$\frac{\Gamma(B_d^0(t) \rightarrow J/\psi K_S) - \Gamma(\bar{B}_d^0(t) \rightarrow J/\psi K_S)}{\Gamma(B_d^0(t) \rightarrow J/\psi K_S) + \Gamma(\bar{B}_d^0(t) \rightarrow J/\psi K_S)} = C_{J/\psi K_S} \cos(\Delta M_d t) - S_{J/\psi K_S} \sin(\Delta M_d t), \quad (5)$$

where the direct CP asymmetry $C_{J/\psi K_S}$ is proportional to $\epsilon a' \sin \theta' \sin \gamma$. On the other hand, the mixing-induced CP asymmetry can be written in the following form [14]:

$$S(B_d \rightarrow J/\psi K_S) = \sin(\phi_d + \Delta\phi_d), \quad (6)$$

where the hadronic phase shift $\Delta\phi_d$ is proportional to $\epsilon a' \cos \theta' \sin \gamma$ (and is usually neglected).

The decay $B_s^0 \rightarrow J/\psi K_S$ is related to $B_d^0 \rightarrow J/\psi K_S$ through the U -spin symmetry of strong interactions [12]. In the SM, its decay amplitude can be written as

$$A(B_s^0 \rightarrow J/\psi K_S) = -\lambda \mathcal{A} [1 - a e^{i\theta} e^{i\gamma}]. \quad (7)$$

In contrast to (4), a does not enter with ϵ , i.e. is not doubly Cabibbo-suppressed. Consequently, the penguin effects are magnified in $B_s^0 \rightarrow J/\psi K_S$. It is useful to introduce a quantity $H \propto \text{BR}(B_s \rightarrow J/\psi K_S)/\text{BR}(B_d \rightarrow J/\psi K_S)$, which complements the direct and mixing-induced CP asymmetries of the time-dependent CP-violating rate asymmetry of $B_s^0 \rightarrow J/\psi K_S$.

The U -spin symmetry implies $a = a'$ and $\theta = \theta'$, thereby allowing the determination of γ , a and θ from H and the two CP-violating observables of $B_s^0 \rightarrow J/\psi K_S$ [12]. Since 1999, when this strategy was originally proposed, there has been a change of the main focus: a study [16] has shown that the extraction of γ will be feasible at LHCb but not competitive with other methods. As γ will be known by the time CP violation in $B_s^0 \rightarrow J/\psi K_S$ can be detected, the corresponding CP asymmetries allow a clean determination of the penguin parameters a and θ .

The $B_s^0 \rightarrow J/\psi K_S$ channel was observed by CDF [20] and LHCb [21] but its CP asymmetries have not yet been measured. Using currently available data for decays with a CKM structure similar to $B_s^0 \rightarrow J/\psi K_S$, i.e. $B_d^0 \rightarrow J/\psi \pi^0$ and $B^+ \rightarrow J/\psi \pi^+$, and complementing them with $B_d^0 \rightarrow J/\psi K^0$, $B^+ \rightarrow J/\psi K^+$ data, the size of the penguin parameters a and θ can be constrained. An analysis along these lines yields the following preliminary results [22]:

$$a = 0.17_{-0.11}^{+0.13}, \quad \theta = (182.4_{-4.6}^{+4.7})^\circ, \quad \Delta\phi_d = (-0.97_{-0.65}^{+0.72})^\circ. \quad (8)$$

2.2. CP violation in $B_s^0 \rightarrow J/\psi \phi$

The CKM structure of the $B_s^0 \rightarrow J/\psi \phi$ channel is analogous to that of $B_d^0 \rightarrow J/\psi K_S$. However, as the final state is a mixture of CP-even and CP-odd eigenstates $f \in \{0, \parallel, \perp\}$, a time-dependent angular analysis of the $J/\psi[\rightarrow \mu^+ \mu^-] \phi[\rightarrow K^+ K^-]$ decay products has to be performed [23–25]. The impact of the SM penguin contributions is usually neglected. As in the case of $B_d^0 \rightarrow J/\psi K_S$, the expressions for the mixing-induced CP asymmetries are modified as follows [15]:

$$\mathcal{A}_{\text{CP},f}^{\text{mix}} = \sin \phi_s \rightarrow \sin(\phi_s + \Delta\phi_s^f), \quad (9)$$

where the hadronic phase shift $\Delta\phi_s^f$ depends on the final-state configuration f . The current average (neglecting the penguin effects) of the CDF, D0, ATLAS and LHCb data compiled by the Heavy Flavour Averaging Group is given by $\phi_s = (0.0 \pm 4.0)^\circ$ [26], which agrees with the SM value $\phi_s^{\text{SM}} = -(2.11 \pm 0.08)^\circ$ [8] of the B_s^0 - \bar{B}_s^0 mixing phase.

A tool to control the penguin effects is offered by $B_s^0 \rightarrow J/\psi \bar{K}^{*0}$ [15], which was observed by CDF [20] and LHCb [27]. Its branching ratio $(4.4_{-0.4}^{+0.5} \pm 0.8) \times 10^{-5}$ is found in agreement with the prediction $(4.6 \pm 0.4) \times 10^{-5}$ following from $B_d^0 \rightarrow J/\psi \rho^0$, and its polarisation fractions agree well with those of $B_d^0 \rightarrow J/\psi K^{*0}$. The $B_d^0 \rightarrow J/\psi \rho^0$ channel, which shows also mixing-induced CP violation, is another interesting decay to shed light on the hadronic penguin effects [15].

The experimental sensitivity from $B_s^0 \rightarrow J/\psi \phi$ at the LHCb upgrade (50 fb^{-1}) is expected as $\Delta\phi_s|_{\text{exp}} \sim 0.008 = 0.46^\circ$ [28]. This impressive precision will make it mandatory to get a handle on the penguin effects, which may lead to phase shifts at the 1° level (as indicated by (8)).

2.3. CP violation in $B_s^0 \rightarrow J/\psi f_0(980)$

Another interesting probe to study CP violation is provided by $B_s^0 \rightarrow J/\psi f_0(980)$ [29]. In contrast to $B_s^0 \rightarrow J/\psi \phi$, as the $f_0(980)$ is a scalar state with quantum numbers $J^{PC} = 0^{++}$, the final state is present in a p wave and has the CP eigenvalue -1 . Consequently, a time-dependent angular analysis is not needed. On the other hand, the hadronic structure of the $f_0(980)$ is still – after decades – not settled, with a variety of theoretical interpretations ranging from the quark–antiquark picture to tetraquarks. A detailed discussion of the implications of this feature for the extraction of ϕ_s was given in [18] (for $B_{s,d}^0 \rightarrow J/\psi \eta^{(\prime)}$ decays, see [30]), while recent LHCb measurements related to this topic are reported in [31].

2.4. Effective B_s^0 decay lifetimes

The measurement of effective lifetimes of $B_s^0 \rightarrow f$ decays, which are defined as

$$\tau_f \equiv \frac{\int_0^\infty t \langle \Gamma(B_s(t) \rightarrow f) \rangle dt}{\int_0^\infty \langle \Gamma(B_s(t) \rightarrow f) \rangle dt}, \quad (10)$$

offers yet another way to obtain insights into CP violation [32]. Here it is particularly interesting to compare B_s^0 decays into CP-odd final states, such as $B_s^0 \rightarrow J/\psi f_0(980)$, with those into CP-even final states, such as $B_s^0 \rightarrow K^+ K^-$ and $B_s^0 \rightarrow D_s^+ D_s^-$. The measured effective lifetimes can be converted into contours in the ϕ_s – $\Delta\Gamma_s$ plane, where $\Delta\Gamma_s$ is the decay width difference of the B_s -meson system (for an overview of the status of $\Delta\Gamma_s$, see [33]). The lifetime contours are very robust with respect to hadronic uncertainties [32]. For an update with the most recent LHCb data, see [34]. The B_s decay lifetimes result in a picture in agreement with the SM.

2.5. Comments for the LHCb upgrade era

In view of hadronic effects, it is important to give measurements of ϕ_s for the individual decay channels $B_s^0 \rightarrow f$, i.e. $B_s^0 \rightarrow J/\psi \phi$ and $B_s^0 \rightarrow J/\psi f_0(980)$. The pattern of the $(\phi_s)_f$ may provide insights into the hadronic effects: differences in the values of $(\phi_s)_f$ would indicate hadronic effects. On the other hand, should no differences between the individual ϕ_s emerge, there would be evidence for negligible hadronic effects (within the errors) or a universal hadronic phase shift.

The time-dependent analysis of CP violation in $B_s^0 \rightarrow J/\psi K_S$ allows the clean determination of the corresponding penguin parameters (see Subsection 2.1). A sizeable penguin parameter a would indicate a potential problem in the measurement of ϕ_s from the $B_s^0 \rightarrow J/\psi \phi$ and $B_s^0 \rightarrow J/\psi f_0(980)$ channels. On the other hand, smallish penguin effects would give us confidence for the measurement of ϕ_s , although subtleties may arise due to the different final states.

3. CP Violation in $B_s^0 \rightarrow K^+ K^-$ and $B_d^0 \rightarrow \pi^+ \pi^-$

The decays $B_s^0 \rightarrow K^+ K^-$ and $B_d^0 \rightarrow \pi^+ \pi^-$ receive contributions from tree and penguin topologies. In the SM, their decay amplitudes can be written as

$$A(B_s^0 \rightarrow K^+ K^-) \propto C' [e^{i\gamma} + d' e^{i\theta'} / \epsilon], \quad A(B_d^0 \rightarrow \pi^+ \pi^-) \propto C [e^{i\gamma} - d e^{i\theta}], \quad (11)$$

where $d'e^{i\theta'}$, \mathcal{C}' and their unprimed counterparts are CP-conserving strong quantities [35]. The direct and mixing-induced CP asymmetries of the $B_s^0 \rightarrow K^+K^-$ and $B_d^0 \rightarrow \pi^+\pi^-$ decays allow the determination of theoretically clean contours in the γ - d' and γ - d planes, respectively. Since these decays are related to each other through the interchange of all down and strange quarks, the U -spin symmetry of strong interactions implies $d' = d$ and $\theta' = \theta$; the former relation allows the extraction of γ and $d(=d')$ from the contours [35, 36]. Moreover, the strong phases θ and θ' can be determined, allowing an internal consistency check of the U -spin symmetry. Further insights into the hadronisation dynamics are provided by $|\mathcal{C}|$ and $|\mathcal{C}'|$, which can be extracted from the ratio $K \propto \text{BR}(B_s \rightarrow K^+K^-)/\text{BR}(B_d \rightarrow \pi^+\pi^-)$. This strategy is promising for the LHCb physics programme [28]. It will be particularly interesting to compare the resulting value of γ with those following from methods using only tree-diagram-like $B_{(s)}$ -meson decays.

The picture resulting from the current data was explored in detail in [36, 37]; the numerical results given below refer to the update by Rob Knegjens in [38]. An interesting variant of the method was proposed in [39], as discussed in detail by Marco Ciuchini in [40].

Using information on K , CP violation in $B_d^0 \rightarrow \pi^+\pi^-$ and $B_d^0 \rightarrow \pi^\mp K^\pm$, and allowing for U -spin-breaking corrections $\xi \equiv d'/d = 1 \pm 0.15$, $\Delta\theta \equiv \theta' - \theta = \pm 20^\circ$ results in

$$\gamma = (67.7_{-5.0}^{+4.5}|_{\text{input}} +5.0|_{\xi} +0.1|\Delta\theta)^\circ, \quad (12)$$

which agrees with the “tree-level” results $\gamma = (70.0_{-9.0}^{+7.7})^\circ$ [8] and $(69.4 \pm 7.1)^\circ$ [9] within the uncertainties. There are no indications for sizeable non-factorisable $SU(3)$ -breaking corrections in the corresponding data. In the SM, the mixing-induced CP asymmetry is predicted as

$$\mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+K^-)|_{\text{SM}} = -0.220_{-0.054}^{+0.042}, \quad (13)$$

while $\mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+K^-) \approx \mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^\pm K^\pm) = 0.082 \pm 0.04$.

The first LHCb measurement [41] of the CP-violating $B_s \rightarrow K^+K^-$ observables yields

$$\mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+K^-) = -0.30 \pm 0.12 \pm 0.04, \quad \mathcal{A}_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+K^-) = 0.14 \pm 0.11 \pm 0.03, \quad (14)$$

and agrees with the SM predictions given above. In the future, once the experimental precision for the CP asymmetries improves, γ can be extracted exclusively from the γ - $d^{(\prime)}$ contours. The observable K , which is affected by form factors and non-factorisable effects, will then yield insights into hadronic physics. The current data point towards a fortunate situation for the determination of γ which is very robust with respect to U -spin-breaking corrections [36–38].

4. Outlook

The exploration of CP violation in B decays is a very broad field, with many other interesting topics complementing those discussed above. I would like to briefly give two more examples:

- The penguin decay $B_s^0 \rightarrow \phi\phi$ (for a recent theoretical discussion, see [42]). This summer, LHCb announced the first time-dependent angular analysis of CP violation this channel [43]. The result $\phi_s = -0.17 \pm 0.15(\text{stat}) \pm 0.03(\text{syst}) = -(9.7 \pm 8.8)^\circ$ is consistent with the SM at the present level of precision. This channel has a lot of potential for the future.
- A promising decay for Belle II at SuperKEKB is the decay $B_d^0 \rightarrow \pi^0 K_S$. A correlation between the direct and mixing-induced CP asymmetries of this channel can be predicted in the SM, with current data showing an intriguing discrepancy [44]. It will be interesting to monitor the future measurements of the corresponding observables at Belle II.

We are moving towards new frontiers in particle physics. There are still no unambiguous signals for NP at the LHC, and it is impressive – also frustrating – to see how the SM stands more and more stringent tests, both at the high-energy and at the high-precision frontier. Much

more is yet to come with the future running of the LHC and dedicated studies of flavour physics, including CP violation in B decays. However, we have to prepare ourselves to deal with smallish NP effects in the data. In view of the increasing experimental precision, we have to be careful with respect to theoretical assumptions and approximations. The challenge will be the matching of the experimental and theoretical uncertainties in the future high-precision era. Interesting and fruitful years for the further testing of the SM and the search of NP are ahead of us!

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References

- [1] Cabibbo N 1963 *Phys. Rev. Lett.* **10** 531
- [2] Kobayashi M and Maskawa T 1973 *Prog. Theor. Phys.* **49** 652
- [3] Buras A J and Girschbach J 2013 arXiv:1306.3775 [hep-ph]
- [4] Buchalla G, Buras A J and Lautenbacher M E 1996 *Rev. Mod. Phys.* **68** 1125
- [5] Bell G 2014 *these proceedings*
- [6] Jüttner A 2014 *these proceedings*
- [7] Fleischer R 2002 *Phys. Rept.* **370** 537
- [8] Charles J *et al.* [CKMfitter Group] <http://ckmfitter.in2p3.fr>
- [9] Bevan A *et al.* [UTfit Collaboration] <http://www.utfit.org/UTfit/>
- [10] Bobeth C 2014 *these proceedings*
- [11] Borissov G, Fleischer R and Schune M-H 2013 *Ann. Rev. Nucl. Part. Sci.* **63** 205
- [12] Fleischer R 1999 *Eur. Phys. J. C* **10** 299
- [13] Ciuchini M, Pierini M and Silvestrini L 2005 *Phys. Rev. Lett.* **95** 221804
- [14] Faller S, Fleischer R, Jung M and Mannel T 2009 *Phys. Rev. D* **79** 014030
- [15] Faller S, Fleischer R and Mannel T 2009 *Phys. Rev. D* **79** 014005
- [16] De Bruyn K, Fleischer R and Koppenburg P 2010 *Eur. Phys. J. C* **70** 1025
- [17] Ciuchini M, Pierini M and Silvestrini L 2011 arXiv:1102.0392 [hep-ph]
- [18] Fleischer R, Knegjens R and Ricciardi G 2011 *Eur. Phys. J. C* **71** 1832
- [19] Jung M 2012 *Phys. Rev. D* **86** 053008
- [20] Aaltonen T *et al.* [CDF Collaboration] 2011 *Phys. Rev. D* **83** 052012
- [21] Aaij R *et al.* [LHCb Collaboration] 2012 *Phys. Lett. B* **713** 172
- [22] De Bruyn K and Fleischer R 2014 *in preparation*.
- [23] Dighe A S, Duniety I, Lipkin H J and Rosner J L 1996 *Phys. Lett. B* **369** 144
- [24] Dighe A S, Duniety I and Fleischer R 1999 *Eur. Phys. J. C* **6** 647
- [25] Duniety I, Fleischer R and Nierste U 2001 *Phys. Rev. D* **63** 114015
- [26] Amhis Y *et al.* [Heavy Flavour Averaging Group] <http://www.slac.stanford.edu/xorg/hfag/>
- [27] Aaij R *et al.* [LHCb Collaboration] 2012 *Phys. Rev. D* **86** 071102
- [28] Aaij R *et al.* [LHCb Collaboration] 2013 *Eur. Phys. J. C* **73** 2373
- [29] Stone S and Zhang L 2009 *Phys. Rev. D* **79** 074024; arXiv:0909.5442 [hep-ex].
- [30] Fleischer R, Knegjens R and Ricciardi G 2011 *Eur. Phys. J. C* **71** 1798
- [31] Aaij R *et al.* [LHCb Collaboration] 2014 *Phys. Rev. D* **90** 012003
- [32] Fleischer R and Knegjens R 2011 *Eur. Phys. J. C* **71** 1789
- [33] Lenz A 2012 arXiv:1205.1444 [hep-ph].
- [34] Gandini P *talk at Beauty 2014*, 14–18 July 2014, Edinburgh, UK, <http://www.ph.ed.ac.uk/beauty2014>
- [35] Fleischer R 1999 *Phys. Lett. B* **459** 306
- [36] Fleischer R 2007 *Eur. Phys. J. C* **52** 267
- [37] Fleischer R and Knegjens R 2011 *Eur. Phys. J. C* **71** 1532
- [38] Knegjens R 2014 *PhD thesis* Vrije Universiteit Amsterdam
- [39] Ciuchini M, Franco E, Mishima S and Silvestrini L 2012 *JHEP* **1210** 029
- [40] Ciuchini M *talk at Beauty 2014*, 14–18 July 2014, Edinburgh, UK, <http://www.ph.ed.ac.uk/beauty2014>
- [41] Aaij R *et al.* [LHCb Collaboration] 2013 *JHEP* **1310** 183
- [42] Bhattacharya B, Datta A, Duraisamy M and London D 2013 *Phys. Rev. D* **88** 016007
- [43] De Bruyn K 2014 *these proceedings*; Aaij R *et al.* [LHCb Collaboration] 2014 arXiv:1407.2222 [hep-ex]
- [44] Fleischer R, Jäger S, Pirjol D and Zupan J 2008 *Phys. Rev. D* **78** 111501