Properties of doubly and triply heavy baryons

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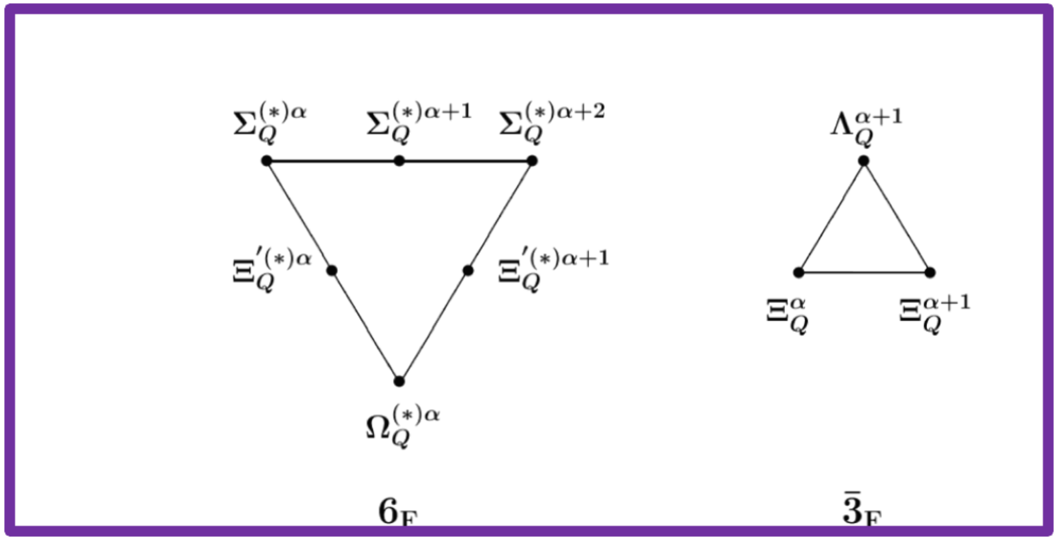
Outline

- Introduction
  - ✓ Heavy baryons
  - ✓ The method
- Masses and residues of the doubly and triply heavy baryons
- Conclusion

Heavy baryons

The quark model predicts heavy baryons containing single, doubly or triply heavy charm or bottom quarks having either spin–1/2 or spin–3/2.

i) Single heavy



So far,

✓baryons with single charm quark have been discovered in the experiments,

✓baryons with single "b" quark and spin 1/2 also have been detected.

✓Amoung baryons with single "b" quark and spin 3/2 only Σ_b^* and Ξ_b^* have been discovered.

S. Chatrchyan *et. al*, CMS Collaboration, Phys. Rev. Lett. 108, 252002 (2012).

ii) Doubly heavy

Spin 1/2	Spin 3/2
Ξ_{bb}	Ξ^*_{cc}
Ω_{bb}	Ω^*_{cc}
Ξ_{bc}	Ξ^*_{bb}
Ω_{bc}	Ξ^*_{bc}
Ξ_{cc}	Ω^*_{bc}
Ω_{cc}	Ω^*_{bb}

Symmetric wavefunctions With respect to exchange of two heavy quarks

Ξ'_{bc}

Ω'_{bc}

Asnti-symmetric

Amoung these states only Ξ^+_{cc} has been discovered by SELEX Collaboration

J. Eigelfried *et. al*, SELEX Collaboration, Nucl. Phys. A 752, 121 (2005).

iii) Triply heavy

Spin 1/2	Spin 3/2
Ω_{bbc}	Ω^*_{bbc}
Ω_{ccb}	Ω^*_{ccb}
	Ω^*_{bbb}
	Ω^*_{ccc}

Have not been discovered yet

Method: QCD sum rule

This is one of the most attractive and applicable non-perturbative phenomenological tools to Hadron physics.

QCD Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_q \bar{\psi}_q (i \not{D} - m_q) \psi_q,$$

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g f^{abc} G^b_\mu G^c_\nu ,$$

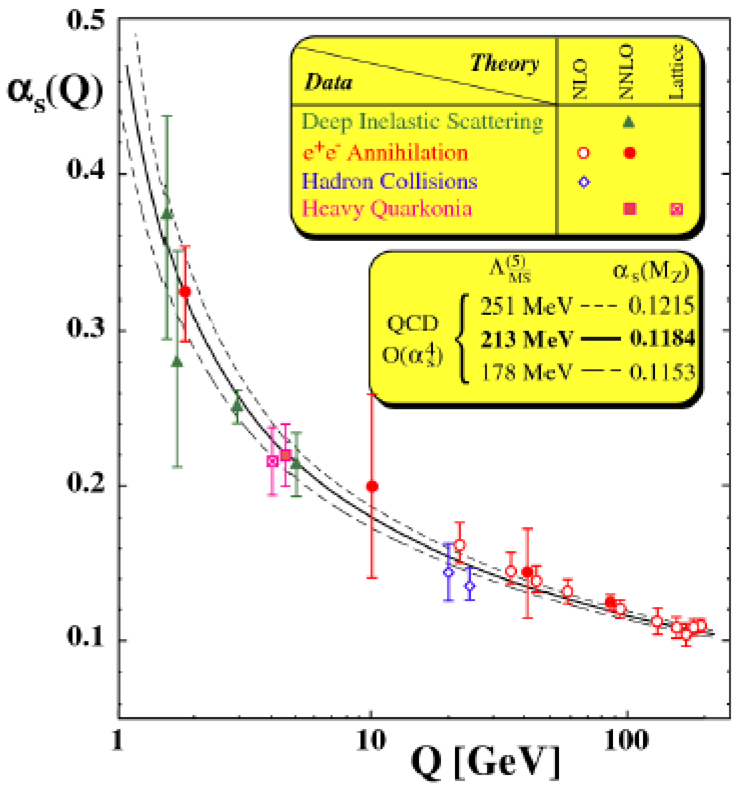
$$\not{D} = D_\mu \gamma^\mu,$$

$$D_\mu = \partial_\mu + i \frac{g}{2} \lambda^a G^a_\mu,$$

In principle, besides the dinamics of quarks and gluons, this lagrangian should be responsible for determination of hadronic properties. Unfortunately, it is valid only in a limited region.

➤In very high energies, due to “asymptotic freedom” we can use this Lagrangian and perturbation theory. However, when energy is decreased the coupling constant between quarks and gluons becomes large and perturbation theory fails.

S. Bethke, J Phys G 26, R27 (2000)



- Hadrons are formed in low energies very far from the "asymptotic freedom" and perturbative region.

→ To investigate their properties, we need some non-perturbative approaches.

Some non-perturbative methods:

- ✓ Different "relativistic" and "non-relativistic" quark models
- ✓ HQET
- ✓ Nambu–Jona-Lasinio model
- ✓ Lattice QCD
- ✓ ....
- ✓ QCD sum rule and its extension: light cone QCDSR.

One of the most Applicable tools to Hadron physics →

- does not include any free parameter
- is based on QCD Lagrangianin
- gives results in a good consistency with existing EXP. data
- its results agree with Lattice predi.

....

QCD sum rules in technique language

✓ In this method, hadrons are represented by their interpolating quark currents.

✓ The main object in this approach is the so called correlation function expressed in terms of these interpolating currents.

- Types of the corr. func.:
  1) two point

$$T = i\int d^4x e^{ipx}\langle 0 \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0\rangle,$$

  we obtain: mass, residue (lep. decay. cons.)
  2) three point

$$T = i\int d^4x d^4y e^{ipx} e^{-ipy}\langle 0 \mid T\{\eta_1(x)\eta^{tr}(0)\bar{\eta}_2(y)\} \mid 0\rangle,$$

  we calculate: form factors used in decay rates, branchin ratio ...

3) light cone:

$$T = i \int d^4 x e^{ipx} \langle \gamma \mid T\{\eta_1(x)\bar{\eta}_2(0)\} \mid 0 \rangle,$$

In this version, the OPE is done in terms of distribution amplitudes of the on-shell particles with different twists. This can be used in all electromagnetic, weak and strong decays.

These correlation functions are calculated in two different ways:

- ✓ Phenomenological or physical side: in terms of hadronic parameters
- ✓ QCD side: in terms of QCD degrees of freedom

The QCD sum rules for physical observables:

Phenomenological = QCD side

Interpolating currents for doubly heavy baryons

✓Spin-1/2

$$\eta^S = \frac{1}{\sqrt{2}}\epsilon_{abc}\Big\{(Q^{aT}Cq^b)\gamma_5 Q'^c + (Q'^{aT}Cq^b)\gamma_5 Q^c + \beta(Q^{aT}C\gamma_5 q^b)Q'^c + \beta(Q'^{aT}C\gamma_5 q^b)Q^c\Big\},$$

$$\eta^A = \frac{1}{\sqrt{6}}\epsilon_{abc}\Big\{2(Q^{aT}CQ'^b)\gamma_5 q^c + (Q^{aT}Cq^b)\gamma_5 Q'^c - (Q'^{aT}Cq^b)\gamma_5 Q^c + 2\beta(Q^{aT}C\gamma_5 Q'^b)q^c$$

$$+ \quad \beta(Q^{aT}C\gamma_5 q^b)Q'^c - \beta(Q'^{aT}C\gamma_5 q^b)Q^c\Big\},$$

T. M. Aliev, K. Azizi, M. Savci, **Nucl.Phys. A895 (2012) 59-70; Phys.Lett. B715 (2012) 149-151**

✓Spin-3/2

$$\eta_\mu = \frac{1}{\sqrt{3}}\epsilon^{abc}\Big\{(q^{aT}C\gamma_\mu Q^b)Q'^c + (q^{aT}C\gamma_\mu Q'^b)Q^c + (Q^{aT}C\gamma_\mu Q'^b)q^c\Big\}$$

T. M. Aliev, K. Azizi, M. Savci, **J.Phys. G40 (2013) 065003**

Interpolating currents for triply heavy baryons

✓Spin-1/2

$$\eta_{QQQ'} = 2\epsilon_{ijk}\left\{\left(Q^{iT}CQ'^{j}\right)\gamma_5 Q^k + \beta\left(Q^{iT}C\gamma_5 Q'^{j}\right)Q^k\right\}$$

Baryon	Q	Q'
Ω_{bbc}	b	c
Ω_{ccb}	c	b

T. M. Aliev, K. Azizi, M. Savci, **JHEP 1304 (2013) 042**

✓Spin-3/2

$$\eta_\mu = \frac{1}{\sqrt{3}}\epsilon^{abc}\left\{2(Q^{aT}C\gamma_\mu Q'^{b})Q^c + (Q^{aT}C\gamma_\mu Q^{b})Q'^{c}\right\} ,$$

Baryon	Q	Q'
Ω^*_{bbc}	b	c
Ω^*_{ccb}	c	b
Ω^*_{bbb}	b	b
Ω^*_{ccc}	c	c

T. M. Aliev, K. Azizi, M. Savci, **J.Phys. G41 (2014) 065003**

Masses and residues of the doubly and triply heavy baryons

Brief introduction to calculations: spin ½ triply heavy

$$\Pi(q) = i \int d^4x e^{iqx} \langle 0 | \mathcal{T}\{\eta_{QQQ'}(x)\bar{\eta}_{QQQ'}(0)\} | 0 \rangle \ ,$$

Hadronic side

$$\Pi(q) = \frac{\langle 0|\eta_{QQQ'}(0)|B(q)\rangle\langle B(q)|\bar{\eta}_{QQQ'}(0)|0\rangle}{q^2 - m_B^2} + \cdots \ ,$$

$$\langle 0|\eta_{QQQ'}(0)|B(q,s)\rangle = \lambda_B u(q,s) \ ,$$

$$\Pi(q) = \frac{\lambda_B^2(\not{q} + m_B)}{q^2 - m_B^2} + \cdots \ ,$$

OPE side

$$\Pi(q) = 4i\epsilon_{ijk}\epsilon_{lmn}\int d^4x e^{iqx}\Big\langle 0\Big|\Big\{-\gamma_5 S_Q^{nj} S_{Q'}^{\prime mi} S_Q^{lk}\gamma_5 + \gamma_5 S_Q^{nk}\gamma_5 Tr\Big[S_Q^{lj} S_{Q'}^{\prime mi}\Big]$$
$$+ \beta\Big(-\gamma_5 S_Q^{nj}\gamma_5 S_{Q'}^{\prime mi} S_Q^{lk} - S_Q^{nj} S_{Q'}^{\prime mi}\gamma_5 S_Q^{lk}\gamma_5 + \gamma_5 S_Q^{nk} Tr\Big[S_Q^{lj}\gamma_5 S_{Q'}^{\prime mi}\Big]$$
$$+ S_Q^{nk}\gamma_5 Tr\Big[S_Q^{lj} S_{Q'}^{\prime mi}\gamma_5\Big]\Big) + \beta^2\Big(-S_Q^{nj}\gamma_5 S_{Q'}^{\prime mi}\gamma_5 S_Q^{lk} + S_Q^{nk} Tr\Big[S_{Q'}^{mi}\gamma_5 S_Q^{\prime lj}\gamma_5\Big]\Big)\Big\}\Big|0\Big\rangle ,$$

where $S' = C S^T C$

$$S_Q(x) = \frac{m_Q^2}{4\pi^2}\frac{K_1(m_Q\sqrt{-x^2})}{\sqrt{-x^2}} - i\frac{m_Q^2 \not{x}}{4\pi^2 x^2} K_2(m_Q\sqrt{-x^2})$$
$$- ig_s\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\int_0^1 du\Big[\frac{\not{k}+m_Q}{2(m_Q^2-k^2)^2}G^{\mu\nu}(ux)\sigma_{\mu\nu} + \frac{u}{m_Q^2-k^2}x_\mu G^{\mu\nu}\gamma_\nu\Big] + \cdots ,$$

where K_1 and K_2 are the modified Bessel functions of the second kind

$$\Pi_i(q) = \int\frac{\rho_i(s)}{s-q^2}ds ,$$

$i = 1$ and 2 correspond to the structures $\not{q}$ and I, respectively.

$$
\begin{aligned}
\rho_1(s) =& \frac{1}{64\pi^4}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}} d\psi d\eta\Bigg\{-3\mu_{QQQ'}\Bigg[-12(-1+\eta)m_Q m_{Q'}(-1+\beta)^2 \\
&+\psi^2\eta(3\mu_{QQQ'}-2s)\Big[5+\beta(2+5\beta)\Big]+\psi\Bigg(2m_Q^2(-1+\beta)^2-12m_Q m_{Q'}(-1+\beta^2) \\
&+(-1+\eta)\eta(3\mu_{QQQ'}-2s)\Big[5+\beta(2+5\beta)\Big]\Bigg)\Bigg]\Bigg\} \\
&+\frac{\langle g_s^2 GG\rangle}{256\pi^4 m_Q m_{Q'}}\int_{\psi_{min}}^{\psi_{max}}\int_{\eta_{min}}^{\eta_{max}} d\psi d\eta\Bigg\{6(-3+4\psi)(-1+\psi+\eta)m_Q^2(-1+\beta^2) \\
&+6(-3+4\eta)(-1+\psi+\eta)m_{Q'}^2(-1+\beta^2)+m_Q m_{Q'}\Bigg[48\psi^2(1+\beta^2)+\psi\Big[-63 \\
&+68\eta-30\beta+8\eta\beta+(-63+68\eta)\beta^2\Big]+2(-1+\eta)\Bigg(-3\Big[3+\beta(2+3\beta)\Big] \\
&+2\eta\Big[5+\beta(2+5\beta)\Big]\Bigg)\Bigg]\Bigg\},
\end{aligned}
$$

$$\mu_{QQQ'} = \frac{m_Q^2}{1-\psi-\eta} + \frac{m_Q^2}{\eta} + \frac{m_{Q'}^2}{\psi} - s \ ,$$

$$\eta_{min} = \frac{1}{2}\left[1-\psi-\sqrt{(1-\psi)\left(1-\psi-\frac{4\psi m_Q^2}{\psi s - m_{Q'}^2}\right)}\ \right] ,$$

$$\eta_{max} = \frac{1}{2}\left[1-\psi+\sqrt{(1-\psi)\left(1-\psi-\frac{4\psi m_Q^2}{\psi s - m_{Q'}^2}\right)}\ \right] ,$$

$$\psi_{min} = \frac{1}{2s}\left[s+m_{Q'}^2-4m_Q^2-\sqrt{(s+m_{Q'}^2-4m_Q^2)^2-4m_{Q'}^2 s}\ \right] ,$$

$$\psi_{max} = \frac{1}{2s}\left[s+m_{Q'}^2-4m_Q^2+\sqrt{(s+m_{Q'}^2-4m_Q^2)^2-4m_{Q'}^2 s}\ \right] .$$

$$\lambda_B^2 e^{-m_B^2/M^2} = \int_{s_{min}}^{s_0} ds \rho_1(s) e^{-s/M^2} ,$$

$$s_{min} = (2m_Q + m_{Q'})^2$$

Borel mass parameter

$$m_B^2 = \frac{\int_{s_{min}}^{s_0} ds\ s\rho_i(s)e^{-s/M^2}}{\int_{s_{min}}^{s_0} ds\ \rho_i(s)e^{-s/M^2}}, \qquad i = 1 \text{ or } 2 ,$$

Continuum threshold

Doubly heavy spin--1/2 baryons :

T.M. Aliev, K. Azizi, M. Savci, Nucl.Phys. A895 (2012) 59

QCDSR, Ioffe

Quark model

Baryon	M^2	$\sqrt{s_0}$	This work	[11]	[12]	[13]	[6]	[19]	Exp [20]
Ξ_{bb}	11.0	10.9	9.96(0.90)	9.78(0.07)	10.17(0.14)	9.94(0.91)	10.202	–	–
Ω_{bb}	11.0	10.9	9.97(0.90)	9.85(0.07)	10.32(0.14)	9.99(0.91)	10.359	–	–
Ξ_{bc}	8.0	7.5	6.72(0.20)	6.75(0.05)	–	6.86	6.933	7.053	–
Ω_{bc}	8.0	7.5	6.75(0.30)	7.02(0.08)	–	6.864	7.088	7.148	–
Ξ_{cc}	5.0	4.6	3.72(0.20)	4.26(0.19)	3.57(0.14)	3.52(0.06)	3.620	3.676	3.5189(0.0009)
Ω_{cc}	5.0	4.6	3.73(0.20)	4.25(0.20)	3.71(0.14)	3.53(0.06)	3.778	3.787	–
Ξ'_{bc}	8.0	7.5	6.79(0.20)	6.95(0.08)	–	–	6.963	7.062	–
Ω'_{bc}	8.0	7.5	6.80(0.30)	7.02(0.08)	–	–	7.116	7.151	–

Table 1: The mass of the doubly heavy spin–1/2 baryons (in units of GeV) at $\beta = \pm 2$.

SELEX Collaboration

M. Mattson et.al, SELEX Collaboration, Phys. Rev. Lett. 89, 112001 (2002).

Baryon	Present work	[11]	[12]
Ξ_{bb}	$0.44(0.08)$	$0.067 \div 0.057$	$0.252(0.064)$
Ω_{bb}	$0.45(0.08)$	–	$0.311(0.077)$
Ξ_{bc}	$0.28(0.05)$	$0.046 \div 0.021$	–
Ω_{bc}	$0.29(0.05)$	–	–
Ξ_{cc}	$0.16(0.03)$	$0.042 \div 0.026$	$0.115(0.027)$
Ω_{cc}	$0.18(0.04)$	–	$0.138(0.030)$
Ξ'_{bc}	$0.30(0.05)$	–	–
Ω'_{bc}	$0.31(0.06)$	–	–

Table 2: The residues of the doubly heavy spin–1/2 baryons (in units of GeV^3).

[11] J. R. Zhang, M. Q. Huang, Phys. Rev. D 78, 094007 (2008).

[12] Z. G. Wang, Eur. Phys. J. A 45, 267 (2010).

QCD sum rules, Ioffe current

Doubly heavy spin--3/2 baryons :

We have no experimental data yet

	Our Work		[10] and [15]	[11]	[6]	[14]
	Structure $\not{q} g_{\mu\nu}$	Structure $g_{\mu\nu}$				
Ξ^*_{cc}	3.69 ± 0.16	3.72 ± 0.18	3.58 ± 0.05	3.90 ± 0.10	3.727	3.61 ± 0.18
Ω^*_{cc}	3.78 ± 0.16	3.78 ± 0.16	3.67 ± 0.05	3.81 ± 0.06	3.872	3.76 ± 0.17
Ξ^*_{bb}	10.4 ± 1.0	10.3 ± 0.2	10.33 ± 1.09	10.35 ± 0.08	10.237	10.22 ± 0.15
Ξ^*_{bc}	7.25 ± 0.20	7.2 ± 0.2	—	8.00 ± 0.26	6.98	—
Ω^*_{bc}	7.3 ± 0.2	7.35 ± 0.25	—	7.54 ± 0.08	7.13	—
Ω^*_{bb}	10.5 ± 0.2	10.4 ± 0.2	10.38 ± 1.10	10.28 ± 0.05	10.389	10.38 ± 0.14

Table 3: The mass spectra of the spin–3/2 doubly heavy baryons in units of GeV.

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[6]: Quark model
[10,11,14,15]: QCD sum rules

	Our Work		[10]	[14]
	Structure $\not{q}g_{\mu\nu}$	Structure $g_{\mu\nu}$		
Ξ^*_{cc}	0.12±0.01	0.12±0.01	0.071 ± 0.017	0.070±0.017
Ω^*_{cc}	0.14±0.02	0.13±0.01	—	0.085±0.019
Ξ^*_{bb}	0.22±0.03	0.21±0.01	0.111 ± 0.040	0.161±0.041
Ξ^*_{bc}	0.15±0.01	0.15±0.01	—	—
Ω^*_{bc}	0.18±0.02	0.17±0.01	—	—
Ω^*_{bb}	0.25±0.03	0.25±0.02	—	0.199±0.048

Table 4: The residues of the spin–3/2 doubly heavy baryons in units of GeV^3.

[10, 14]: QCD sum rules

[10] E. Bagan, M. Chabab, S. Narison, Phys. Lett. B **306**, 350 (1993).

[14] Z. G. Wang, Eur. Phys. J. C **68**, 459 (2010).

Triply heavy spin--1/2 baryons

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	This work ($\not{q}$)	This work (I)	[20]	[12]	[13]	[19]	[14]
Ω_{bbc}	11.73 ± 0.16	11.71 ± 0.16	11.50 ± 0.11	11.139	11.280	10.30 ± 0.10	11.535
Ω_{ccb}	8.50 ± 0.12	8.48 ± 0.12	8.23 ± 0.13	7.984	8.018	7.41 ± 0.13	8.245
$\overline{\Omega}_{bbc}$	10.59 ± 0.14	10.56 ± 0.14	10.47 ± 0.12	-	-	-	-
$\overline{\Omega}_{ccb}$	7.79 ± 0.11	7.74 ± 0.11	7.61 ± 0.13	-	-	-	-

Table 2: The masses of the triply heavy spin–1/2 baryons (in units of GeV). For the baryons with over-line, the $\overline{MS}$ values of the quark masses are used.

	This work ($\not{q}$)	This work (I)	[20]
Ω_{bbc}	0.53 ± 0.17	0.45 ± 0.15	0.68 ± 0.15
Ω_{ccb}	0.38 ± 0.13	0.30 ± 0.10	0.47 ± 0.10
$\overline{\Omega}_{bbc}$	0.85 ± 0.28	0.65 ± 0.22	0.68 ± 0.15
$\overline{\Omega}_{ccb}$	0.56 ± 0.18	0.38 ± 0.13	0.47 ± 0.10

Table 3: The residues of the triply heavy spin–1/2 baryons (in units of GeV^3). For the baryons with over-line, the $\overline{MS}$ values of the quark masses are used.

[12,13,14] QCD Bag model, relativistic three-quark model, quark model, respectively

[19,20] QCD sum rules, Ioffe current

Triply heavy spin--3/2 baryons

	$M^2(GeV^2)$	$\sqrt{s_0}(GeV)$	$m(GeV)$	$\lambda(GeV^3)$
$\Omega_{ccc}(\frac{3}{2}^+)$	$4.5-8.0$	5.6 ± 0.2	4.72 ± 0.12	0.09 ± 0.01
$\Omega_{ccc}(\frac{3}{2}^-)$	$4.5-8.0$	5.8 ± 0.2	4.9 ± 0.1	0.11 ± 0.01
$\Omega_{ccb}(\frac{3}{2}^+)$	$6.0-10.0$	8.8 ± 0.2	8.07 ± 0.10	0.06 ± 0.01
$\Omega_{ccb}(\frac{3}{2}^-)$	$6.0-10.0$	9.0 ± 0.2	8.35 ± 0.10	0.07 ± 0.01
$\Omega_{bbc}(\frac{3}{2}^+)$	$8.0-10.5$	12.0 ± 0.2	11.35 ± 0.15	0.08 ± 0.01
$\Omega_{bbc}(\frac{3}{2}^-)$	$8.0-10.5$	12.2 ± 0.2	11.5 ± 0.2	0.09 ± 0.01
$\Omega_{bbb}(\frac{3}{2}^+)$	$12.0-18.0$	15.3 ± 0.2	14.3 ± 0.2	0.14 ± 0.02
$\Omega_{bbb}(\frac{3}{2}^-)$	$12.0-18.0$	15.5 ± 0.2	14.9 ± 0.2	0.20 ± 0.02

No experimental data yet

$$\langle 0 \left|\eta_\mu\right| B_{(3/2)^+}(q)\rangle = \lambda_{(3/2)^+} u_\mu(q) \ ,$$
$$\langle 0 \left|\eta_\mu\right| B_{(3/2)^-}(q)\rangle = \lambda_{(3/2)^-} \gamma_5 u_\mu(q) \ ,$$

T. M. Aliev, K. Azizi, M. Savci, **J.Phys. G41 (2014) 065003**

Conclusion

✓We have calculated the masses and residues of the doubly and triply heavy baryons predicted by the quark model via QCD sum rules.

✓We have only experimental data on the mass of the Ξ_{cc}^{+} state. Our result is in a good consistency with data.

✓ Our results on the masses are mainly in good consistency with the predictions of other non-perturbative approaches like different quark models, but in the case of residue, somehow we see some differences between our results and existing preditions in the literature.

✓Our results can be checked by future experiments.

Thank You