University of Birmingham — BEACH 2014 — July 2014

Charm and Beauty on the Lattice

Andreas Jüttner
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Motivation

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- but dark matter, CP-violation, … indicate that there must be sth. else
- so far no smoking gun

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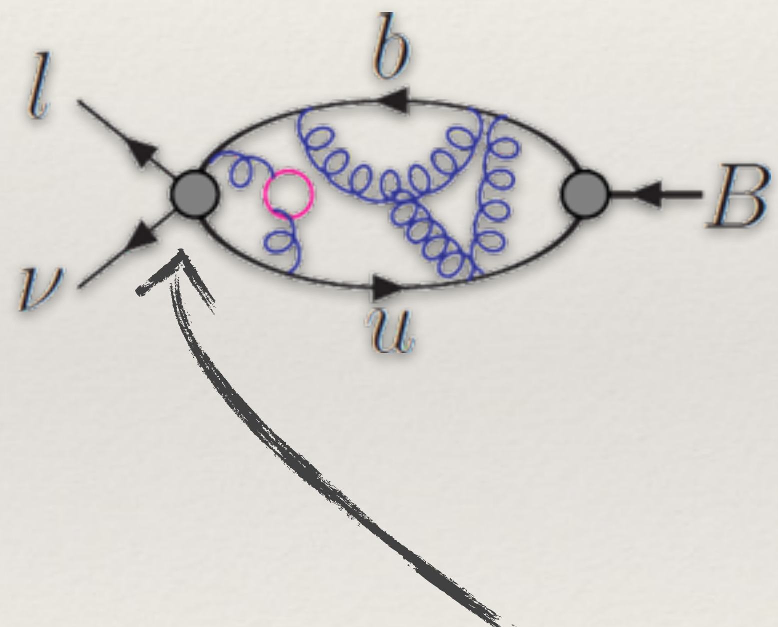
The role of lattice QCD in phenomenology

An example:

$$\Gamma_{\text{exp.}} = V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$$

more specifically e.g tree level leptonic B decay

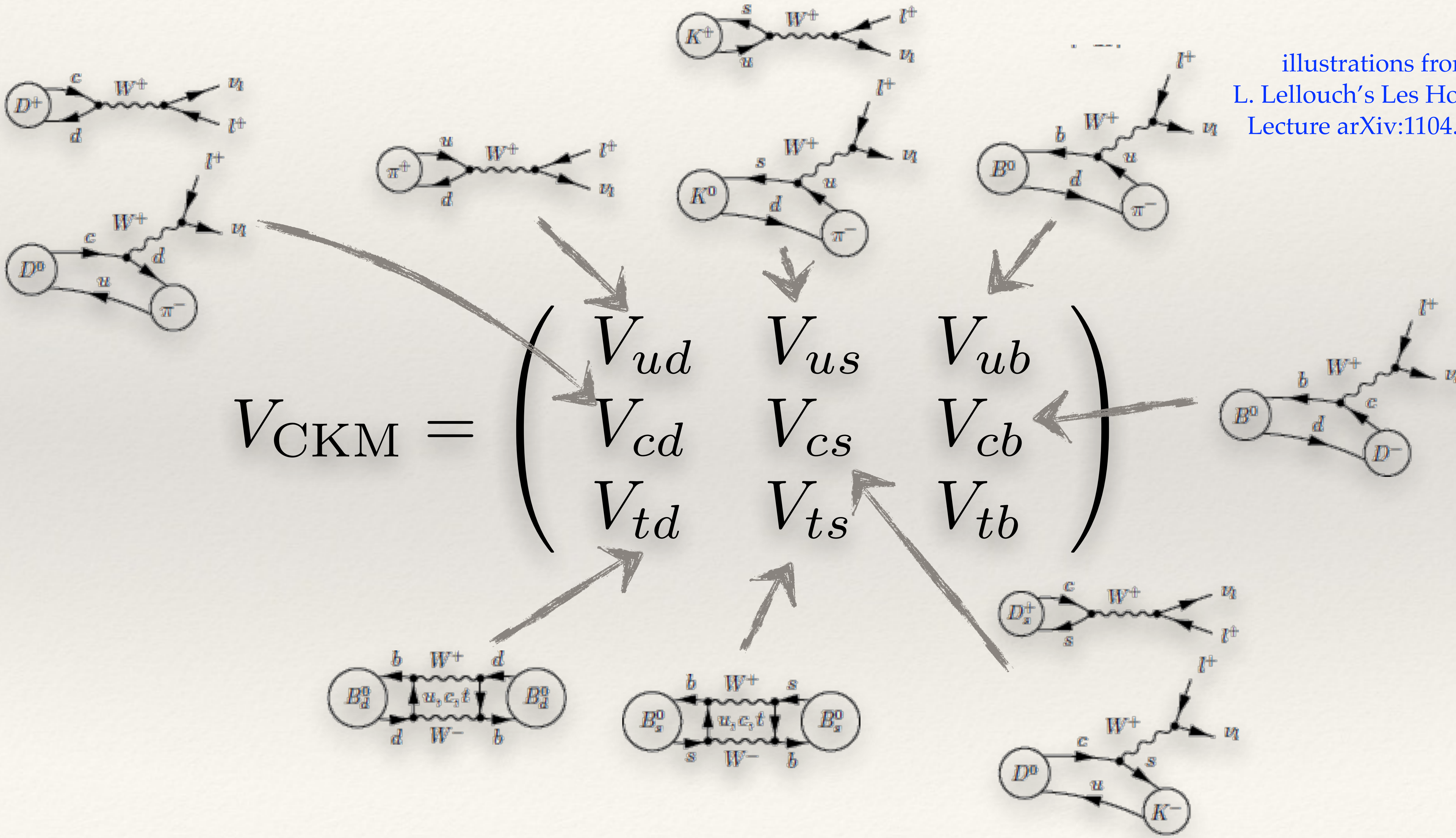


$$\Gamma(B \to l\nu_l) = \frac{m_B}{8\pi} G_F^2 f_B{}^2 |V_{ub}|^2 m_l^2 \left(1 - \frac{m_l^2}{m_B^2}\right)^2$$

Experimental measurement + theory prediction allows for extraction of CKM MEs

Determination of CKM elements

illustrations from
L. Lellouch's Les Houches
Lecture arXiv:1104.5484



$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Lattice QCD

Formulate QCD on Euclidean discretised space-time

- provides gauge-invariant regularisation wt. cut-off $\propto a^{-1}$
- observables in terms of expectation value of discretised path integral

$$\langle 0|O|0\rangle \quad = \quad \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-S_{\mathsf{lat}}[U,\psi,\bar{\psi}]}$$

- Evaluate discretised path integral in finite volume by means of Monte Carlo simulation

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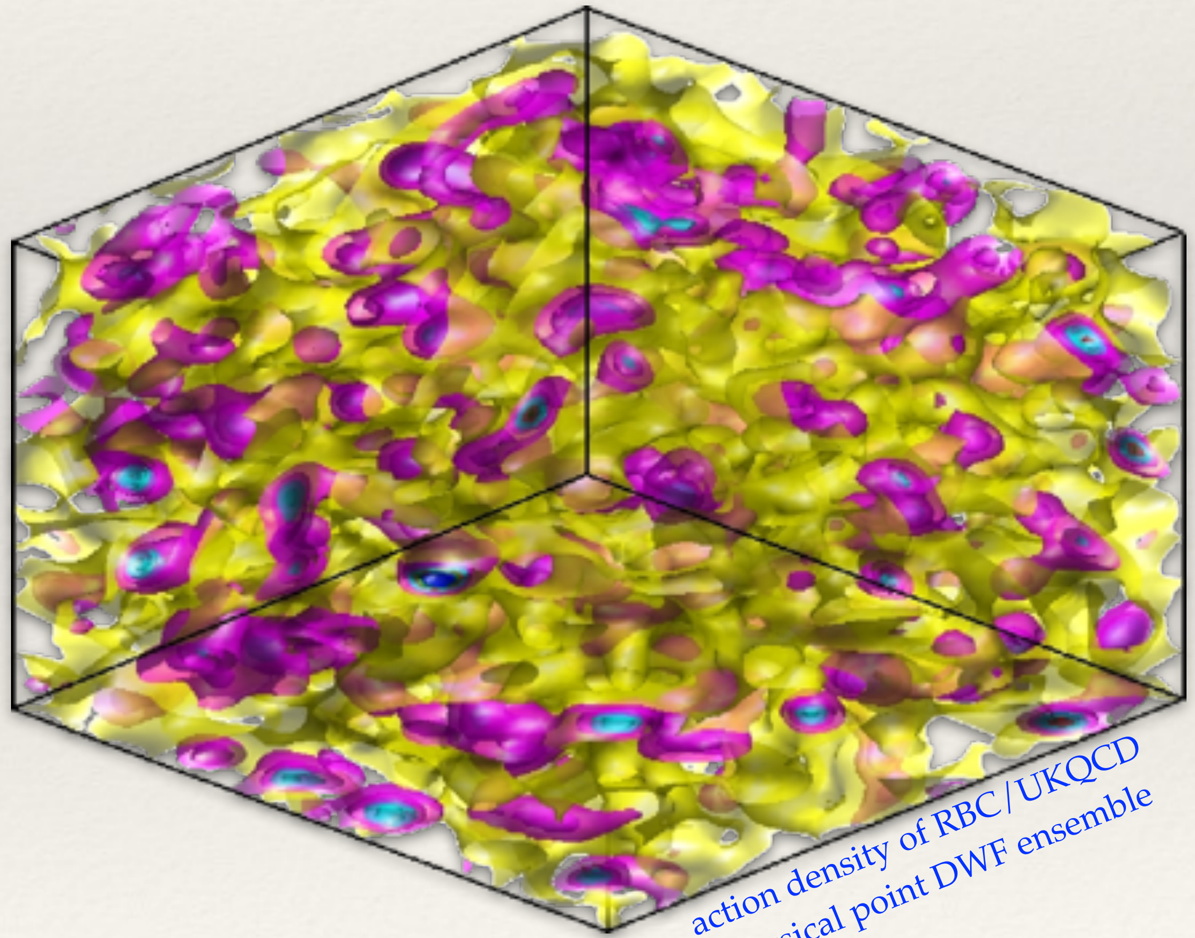
State of the art simulations

What we can do

- mass degenerate up and down quarks at their physical point
- physical strange and charm quarks
  ($\rightarrow N_f = 2,\ 2+1,\ 2+1+1$ QCD)
- bottom needs special treatment
- cut-off $a^{-1} \leq 4\text{GeV}$
- volume $L \leq 6 fm$



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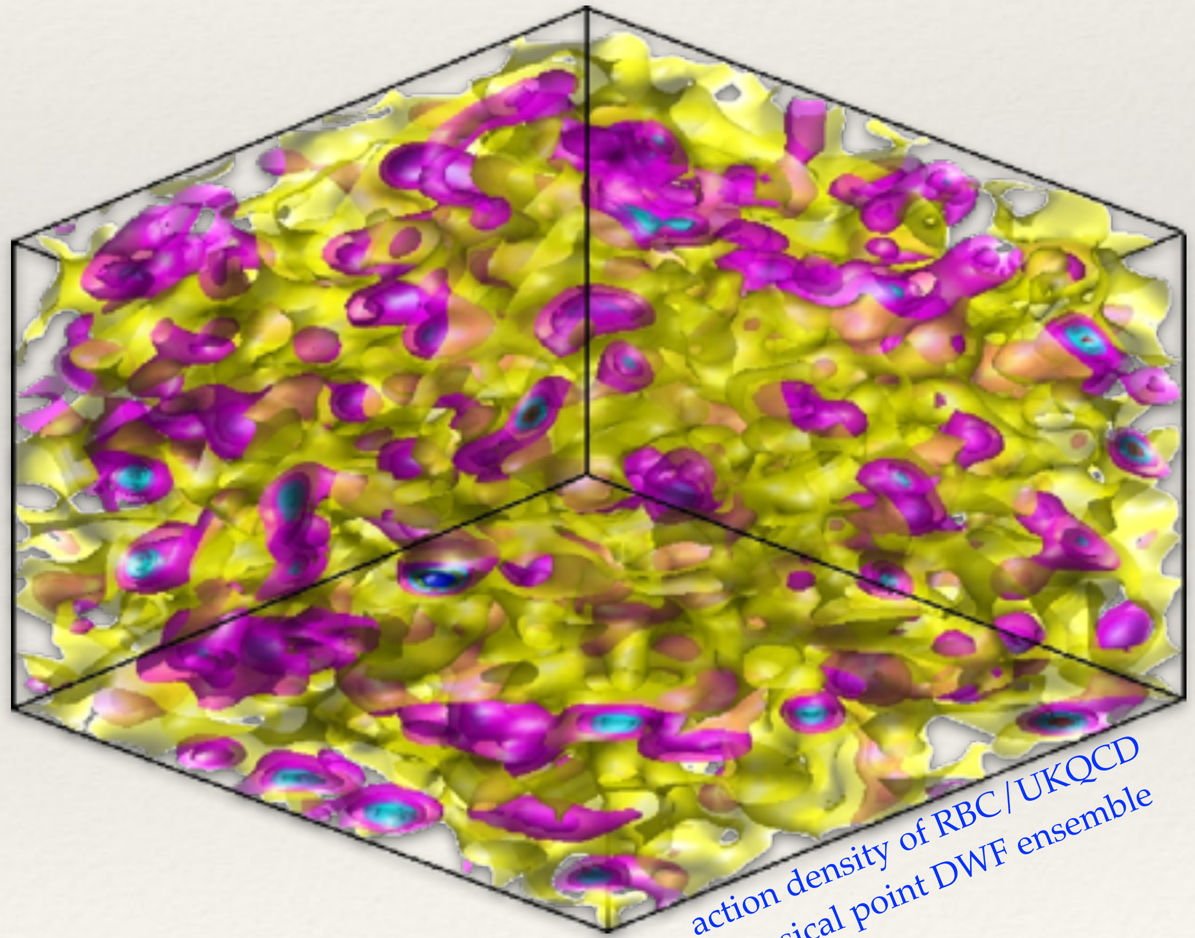
What comes next

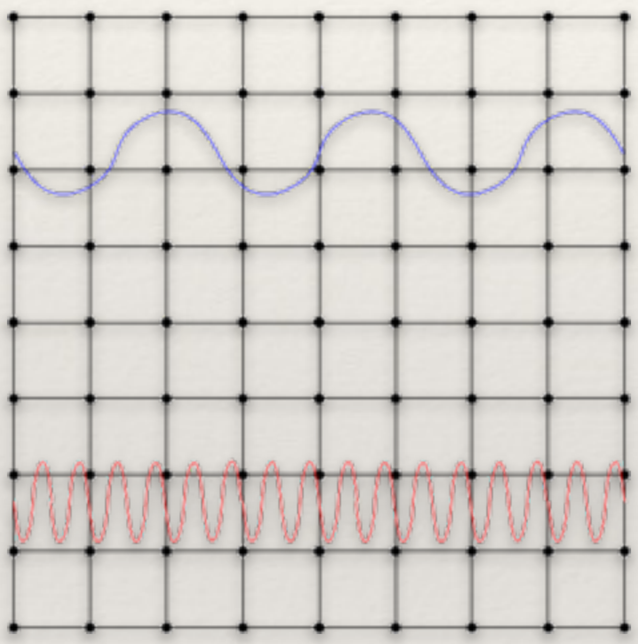
- add isospin breaking
- add electromagnetism
  see e.g. Antonin Portelli's Lattice 2014 plenary



action density of RBC/UKQCD
physical point DWF ensemble


c and b * on the lattice

discretisation effects



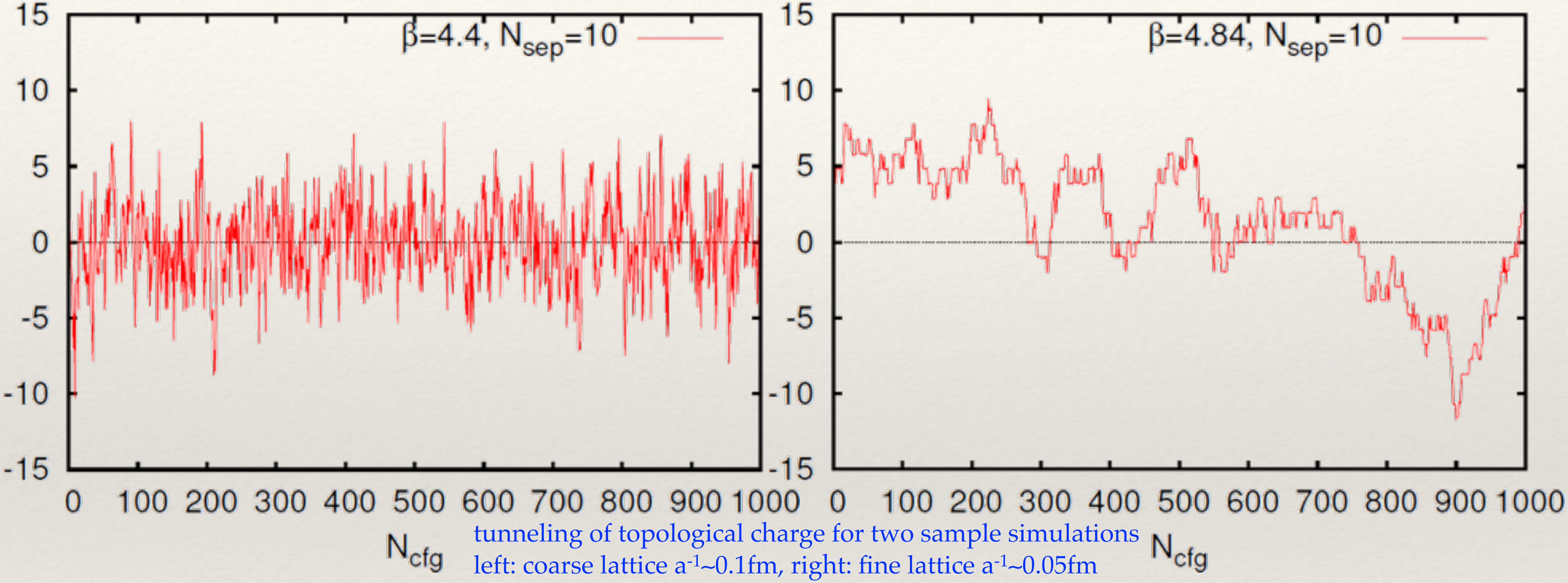
- charm: $am_c<1$ on sufficiently fine lattices
  - fully relativistic quarks
  - fine lattices needed but very expensive (in terms of CPU time)
  - cut-off a^{-1} much larger than 4GeV very hard!

- bottom: $am_b \sim 1$
  → need help from
  - HQET, NRQCD
  - extended Symanzik improvement program
  - extra-/interpolation in $1/m_h$

lattice-c and -b are affected by different systematic effects than the light quarks

* http://www.jicfus.jp/en/promotion/pr/quark-card-dealer/

Critical slowing down



tunneling of topological charge for two sample simulations
left: coarse lattice a^{-1}~0.1fm, right: fine lattice a^{-1}~0.05fm

We have evidence for critical slowing down of algorithms beyond a^{-1}~4GeV

→ needs to be be considered for reliable estimation of stat. errors (ALPHA NP B845 (2011) 93-119)

→ open boundary conditions (Lüscher, Schaefer, JHEP 1107 (2011) 36, McGlynn, Mawhinney arXiv:1406.4551)

Standard, challenging, very challenging processes (with relevance for meson flavour physics)

- **Standard:** single incoming and/or outgoing pseudo-scalar states
  - $\pi, K, D_{(s)}, B_{(s)} \to \mathrm{QCD-vacuum}$
  - $\pi \to \pi, K \to \pi, D \to K, B \to \pi, \ldots$
  - $B_K, (B_D), B_B$

  heavy quark quantities still pose considerable technical challenges and results are less mature

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- **Challenging:** two initial/final hadronic states, one channel
  - e.g. $\pi\pi \to \pi\pi, K\pi \to K\pi, K \to \pi\pi$
  - e.g. $\rho \to \pi\pi$

  good theoretical understanding but numerically/technically challenging

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- **Very challenging - new ideas needed/no clue:**
  - multi-channel final states (hadronic D, B) (e.g. Hansen, Sharpe Phys. Rev. D 86, 016007 (2012))
  - long-distance contributions in e.g. K, D-mixing (Norman's talk)
  - transition MEs with vector final states (e.g. $B \to K^* ll$) (Briceño et al. arXiv:1406.5965)

Standard calculations and results - FLAG

Flavour Lattice Averaging Group

"What's currently the best lattice value for a particular quantity?"

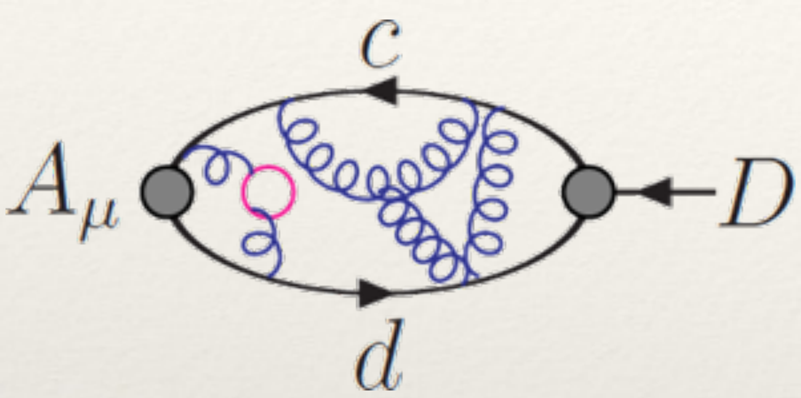
FLAG-1 **(Eur. Phys. J. C71 (2011) 1695)**

FLAG-2 **(http://itpwiki.unibe.ch/flag/, arXiv:1310.8555)**

- quantities:
  - FLAG-1: $m_{u,d}, m_s, f_K/f_\pi, f_+^{K\pi}(0), B_K, SU(2)$ and $SU(3)$ LECs
  - FLAG-2: FLAG-1 + $\alpha_s, f_{D_{(s)}}, f_{B_{(s)}}, B_{B_{(s)}}, B, D$
- summary of results
  - evaluation according to FLAG quality criteria (colour coding)
  - averages of best values where possible
  - detailed summary of properties of individual simulations
  - lattice glossary
- data-deadline 30 November 2013

weak decays

Leptonic decay



$$\mathcal{B}(D_{(s)} \to l\nu_l) = \frac{G_F^2 |V_{cq}|^2 \tau_{D_{(s)}}}{8\pi} f_{D_{(s)}}^2 m_l^2 m_{D_{(s)}} \left(1 - \frac{m_l^2}{m_{D_{(s)}}^2}\right)^2$$

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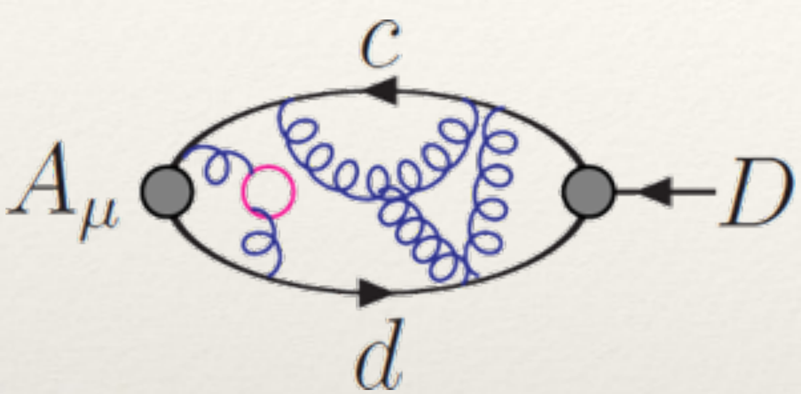

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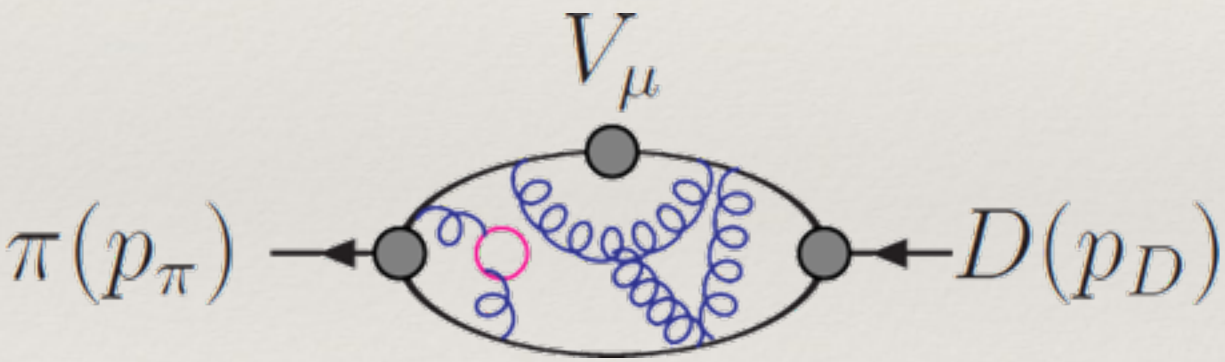
Semileptonic decay



$$\frac{d\Gamma(D \to P\ell\nu)}{dq^2} = \frac{G_F^2 |V_{cx}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (m_D^2 - m_P^2)^2 |f_0(q^2)|^2 \right],$$

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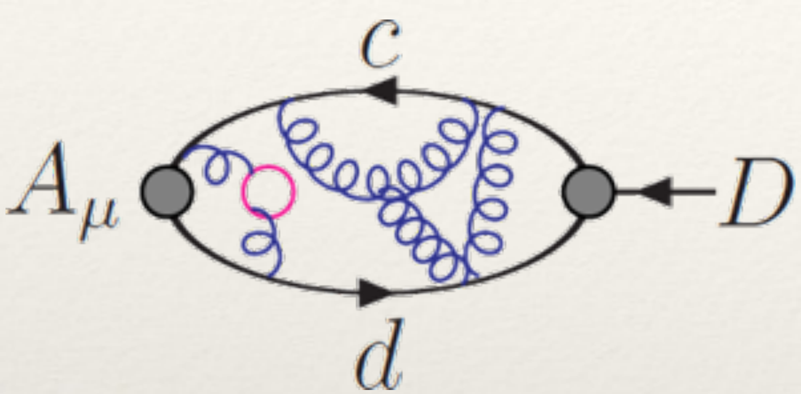

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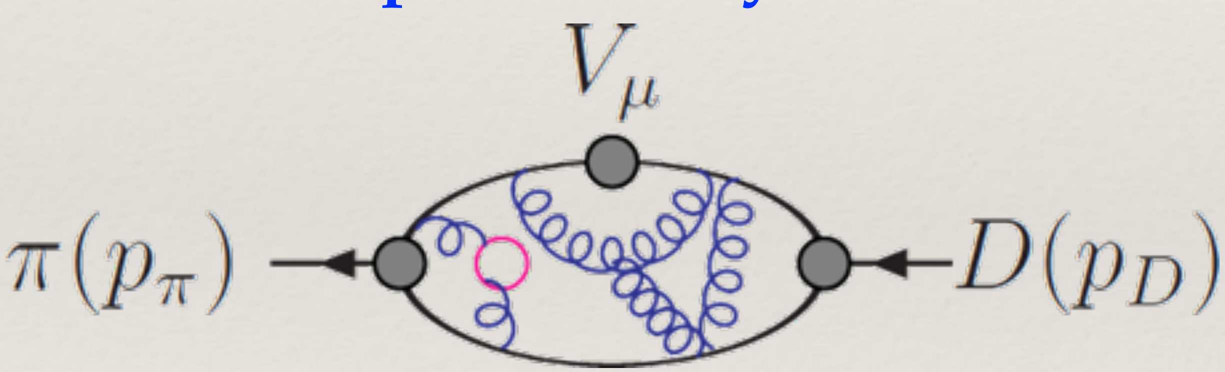

not only tree-level but also rare, e.g.

- $B \to K^{(*)} ll$
- $B_s \to \phi ll$
- $B \to \pi ll$

(Wingate PRL 112, 212003 (2014), PRD 89, 094501 (2014); FNAL/MILC, HPQCD)

or *unusual* channels, e.g.

- $B_s \to K l \nu$ (e.g. HPQCD arXiv:1406.2279)

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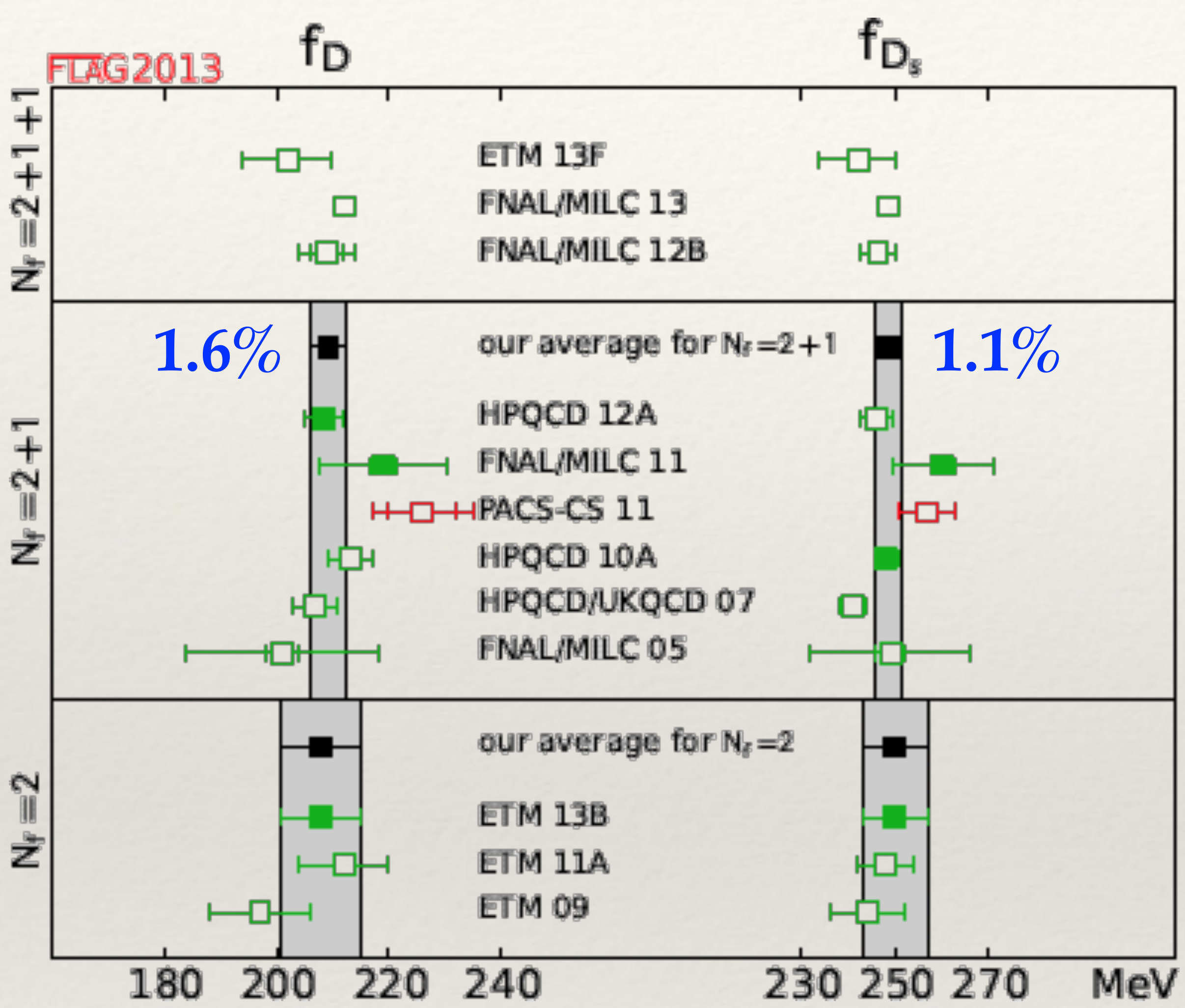
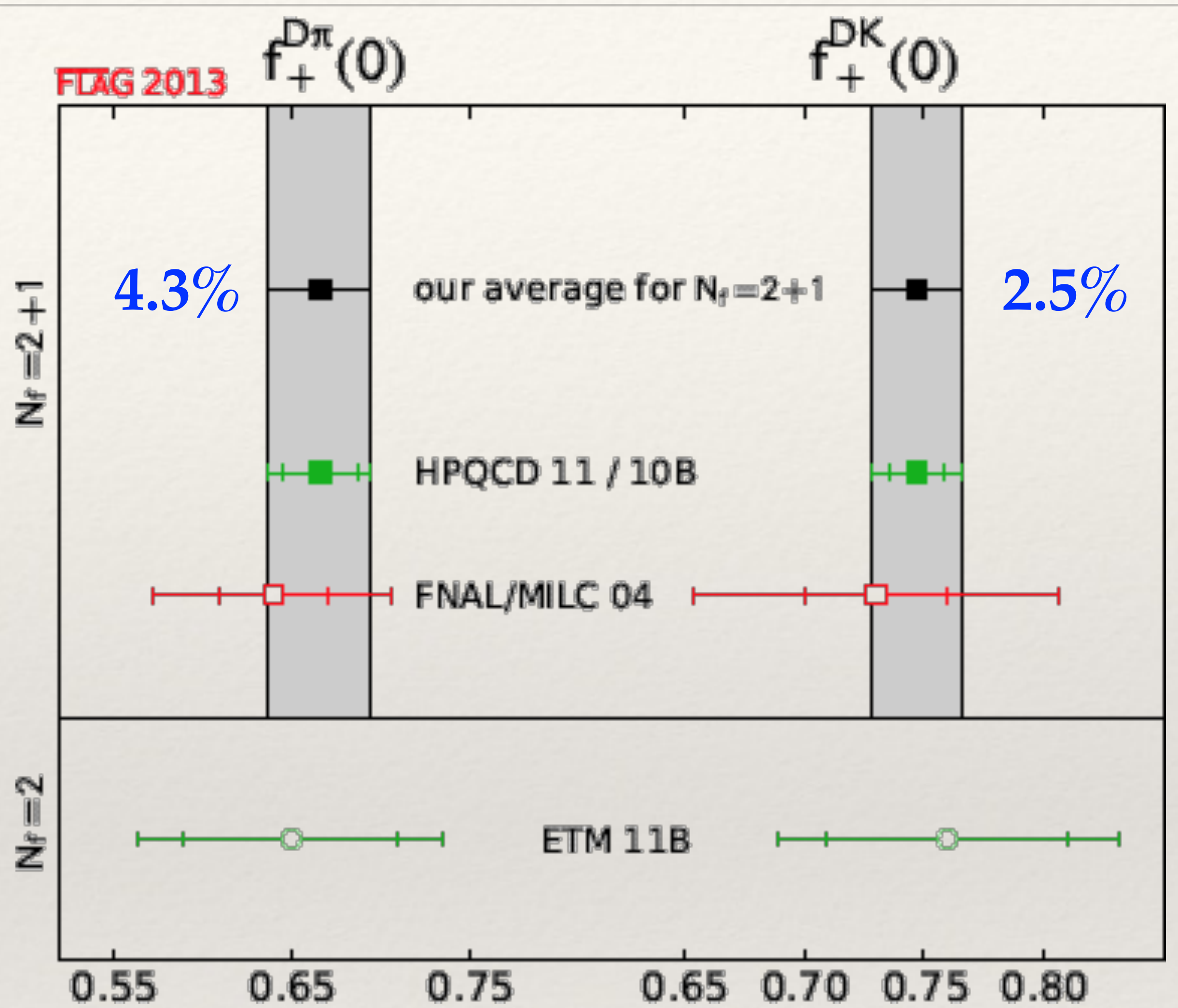
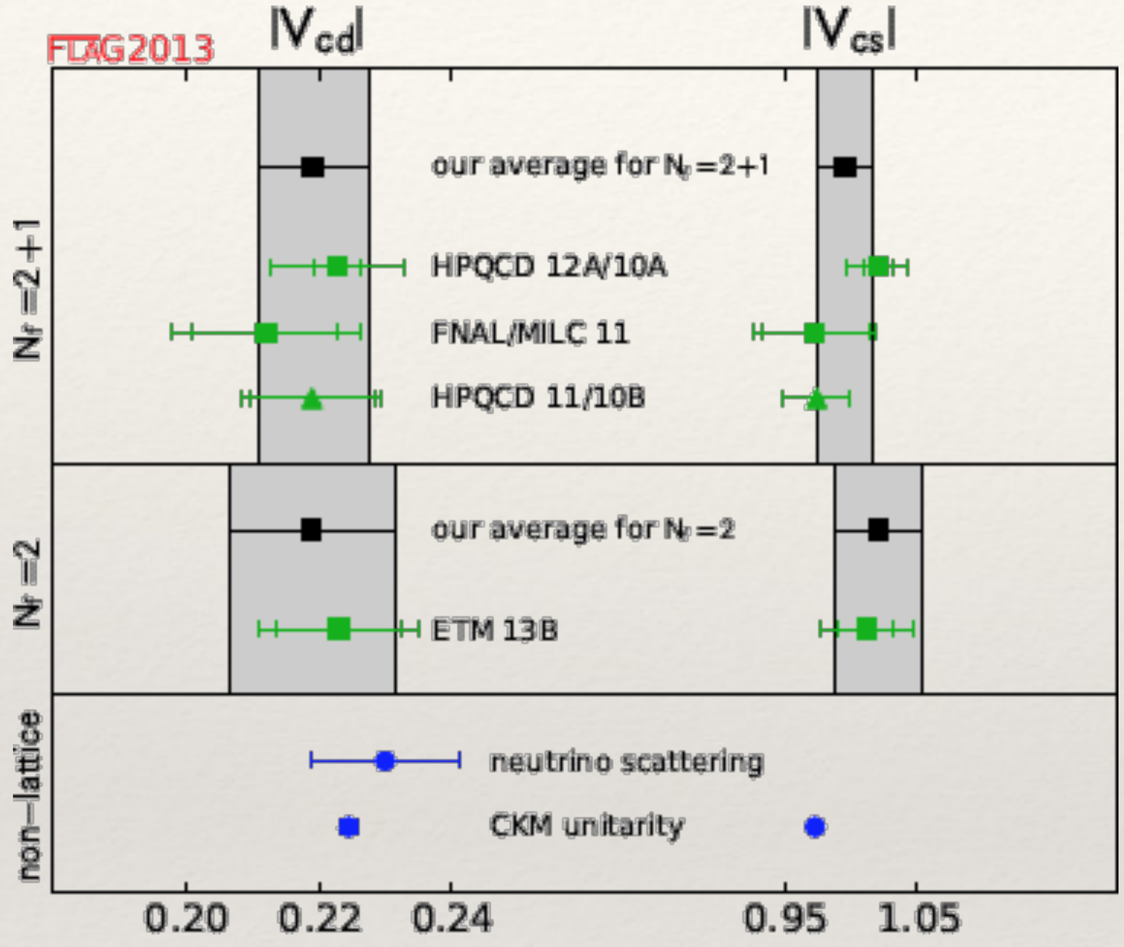
Leptonic $D_{(s)}$ meson decays





- continuum and chiral extrapolation dominant syst. uncertainties
- more activity needed in particular for semi-leptonics (→ Lattice 2014)

Results for $|V_{cd}|$ and $|V_{cs}|$



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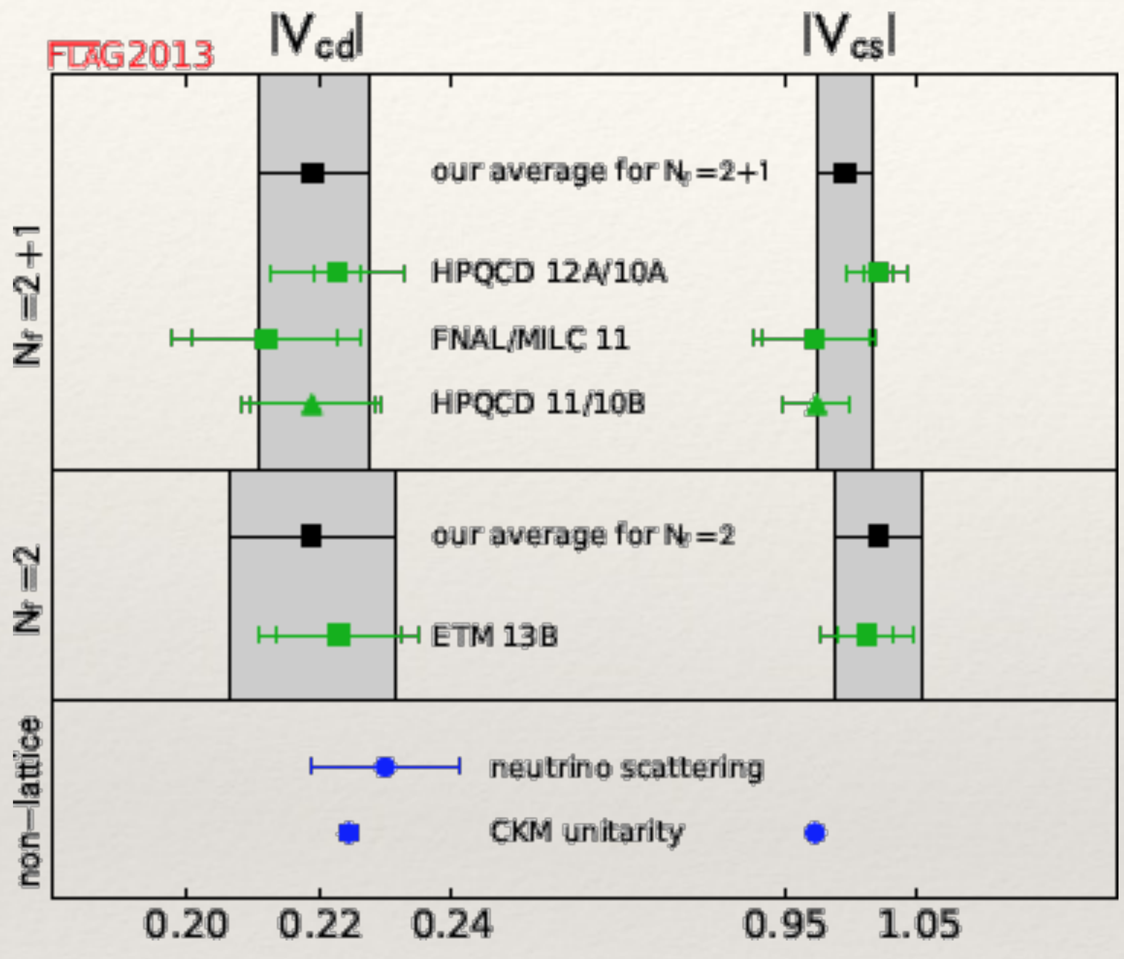
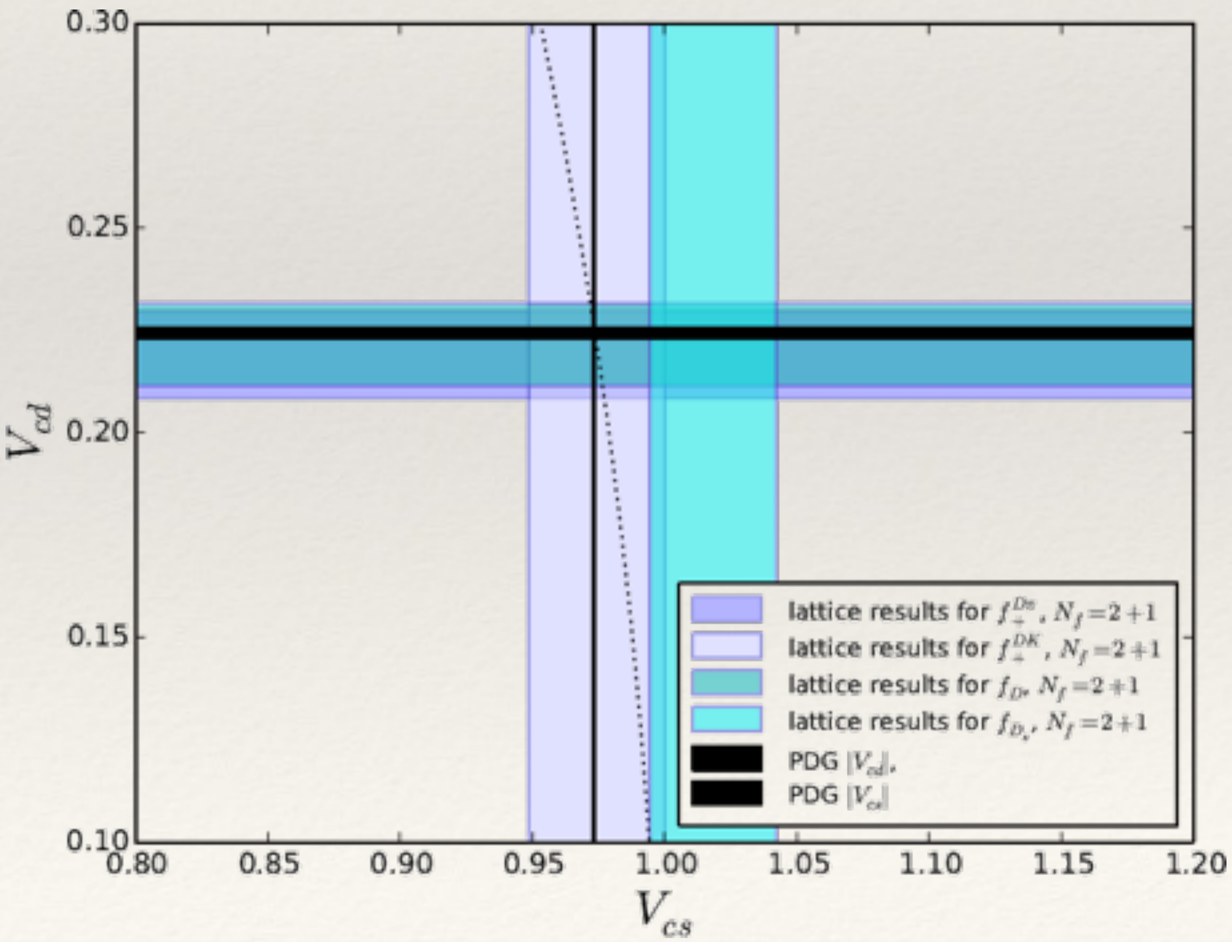
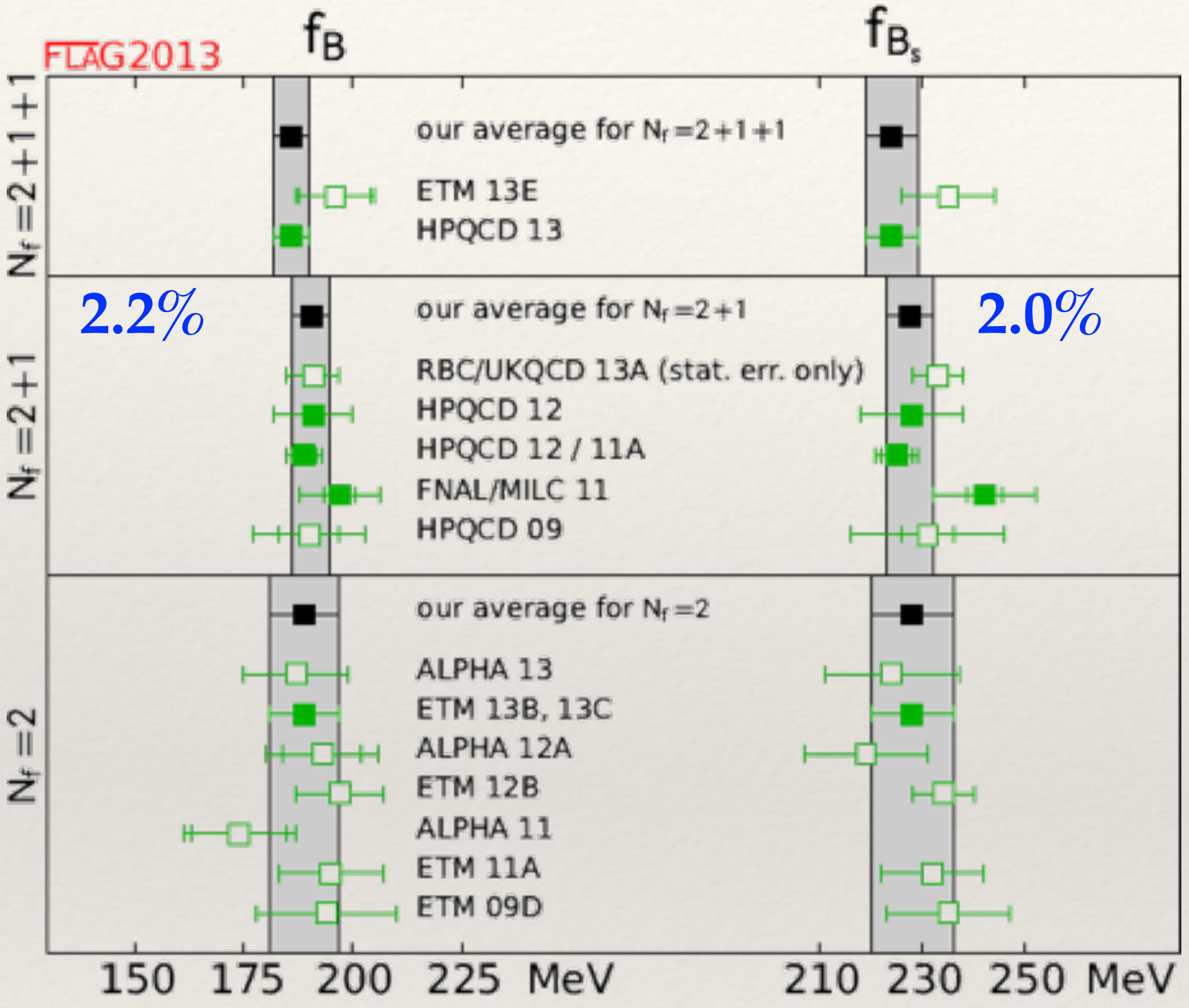


- $|V_{cs}|$ from leptonic decays is slightly larger than from semileptonic decays
- $|V_{cs}|$ from leptonic decays is at tension with CKM-unitarity by 1.9σ (→HPQCD)



Leptonic beauty decays



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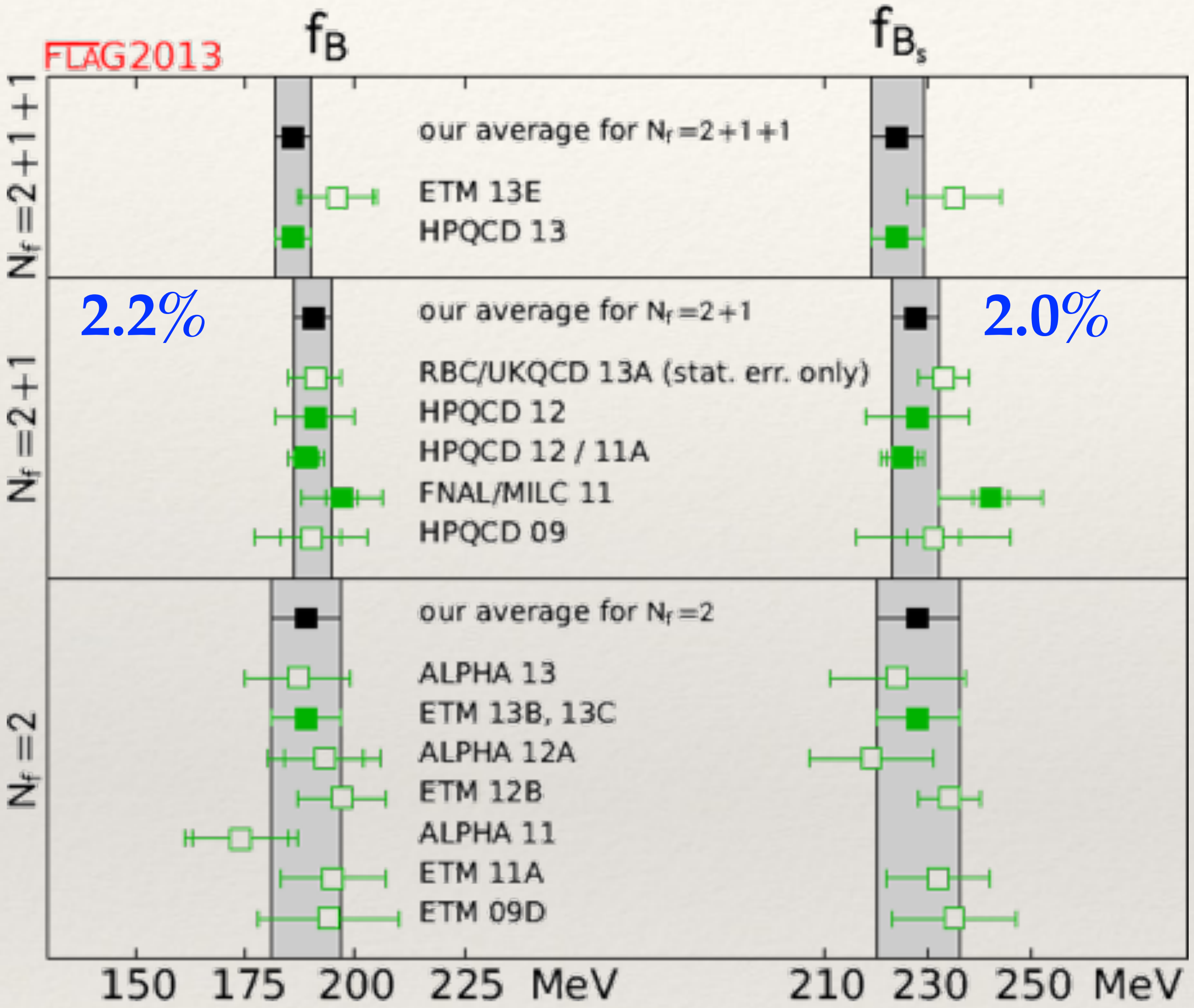
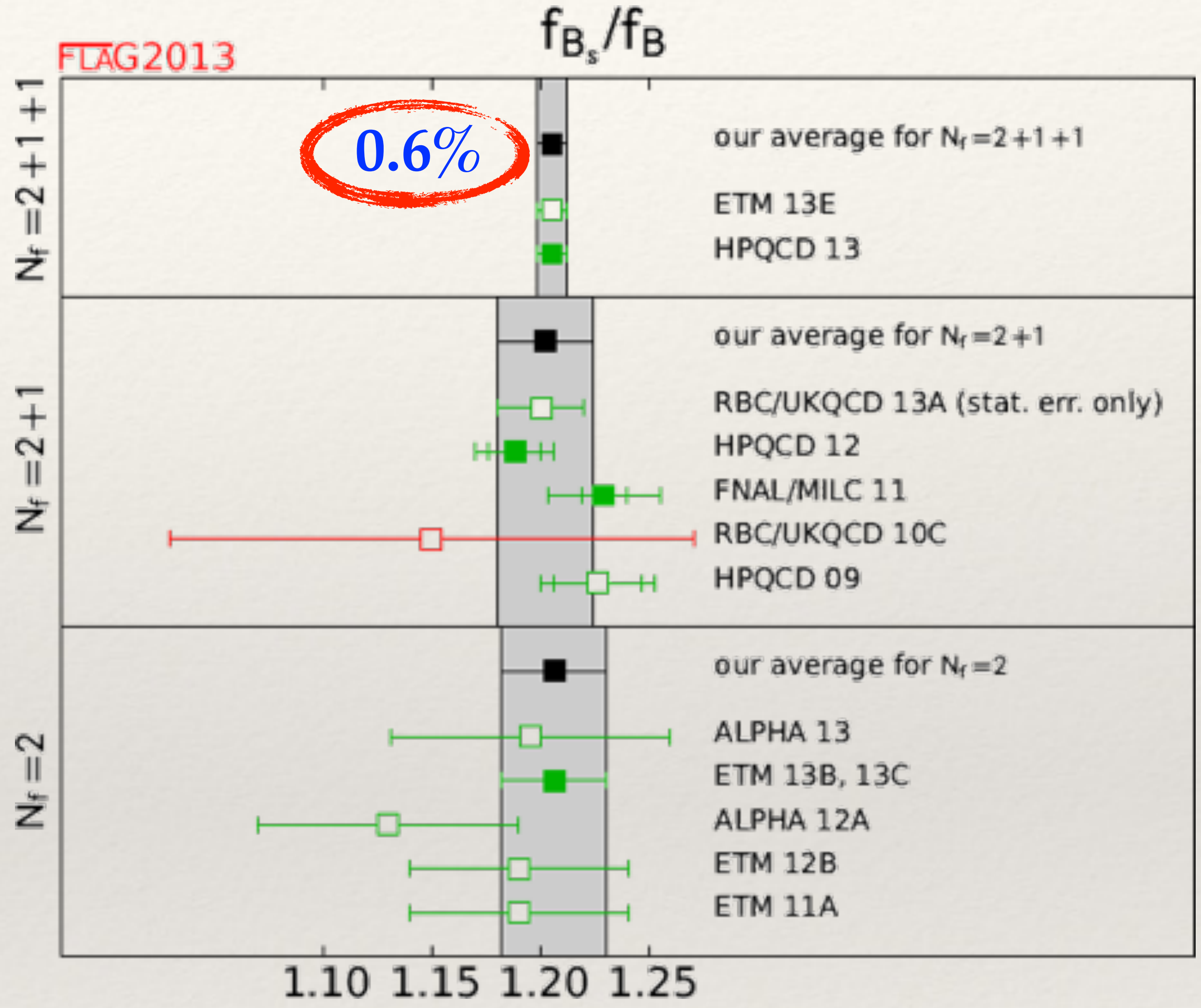
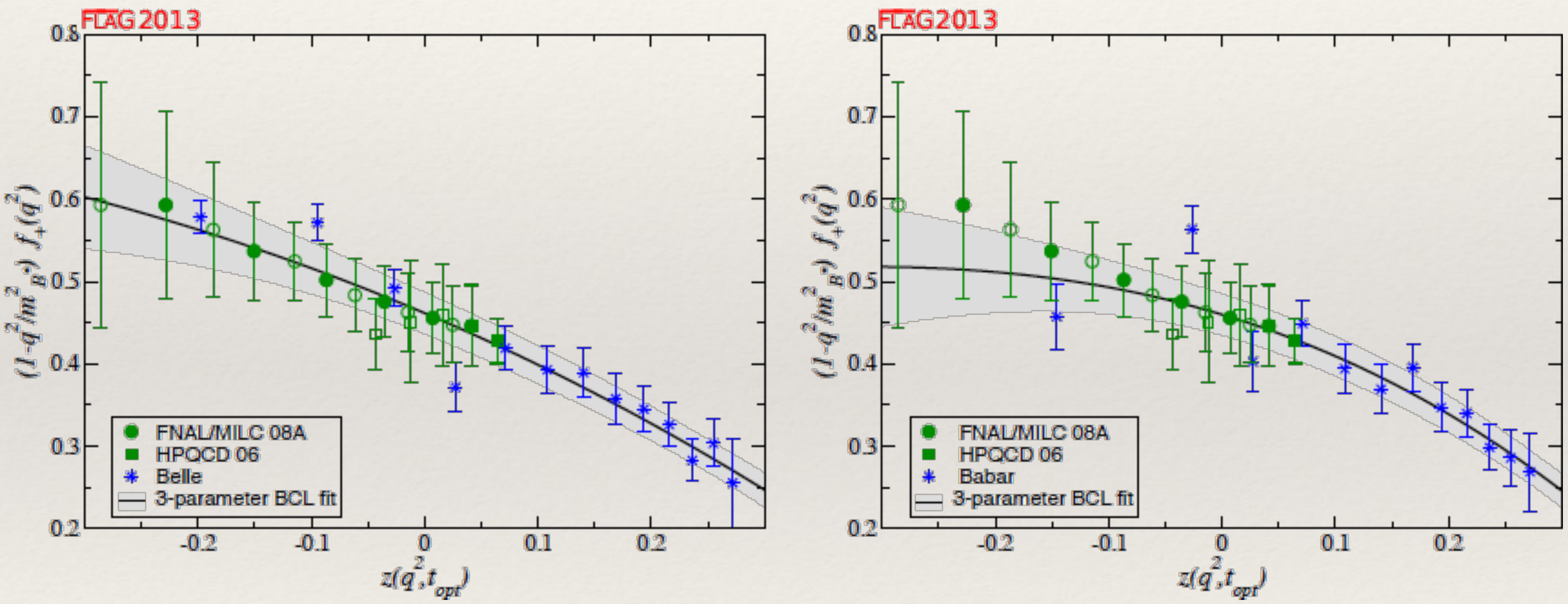


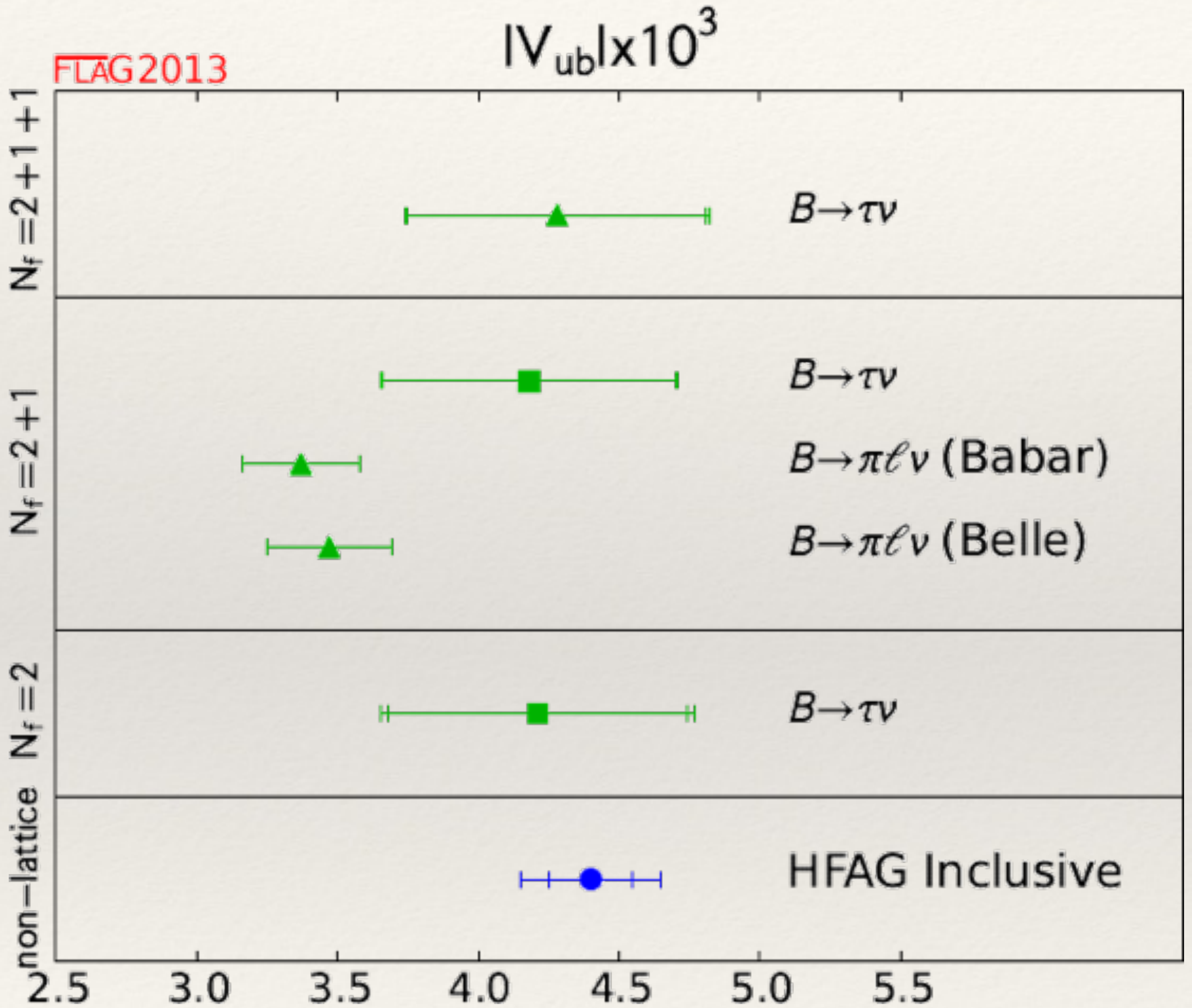


Semileptonic beauty decays

Kinematical reach limited in lattice QCD → extract value of V_{ub} from simultaneous analysis of exp. and lattice data



Results for $|V_{ub}|$



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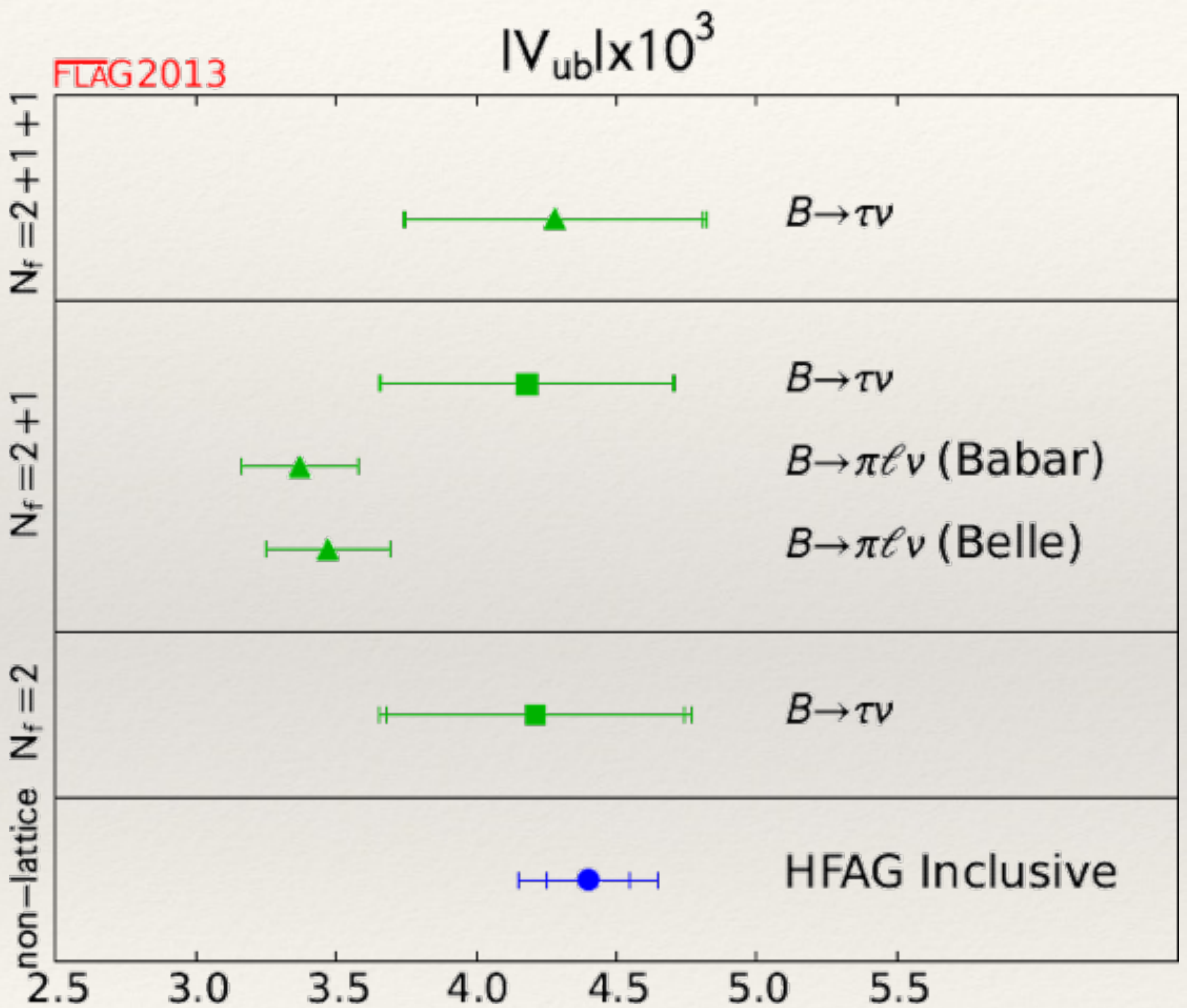

- confirms ~3σ tension between incl. and excl. semilept. decays
- lept. decay lies in between and agrees with both at 1.5σ

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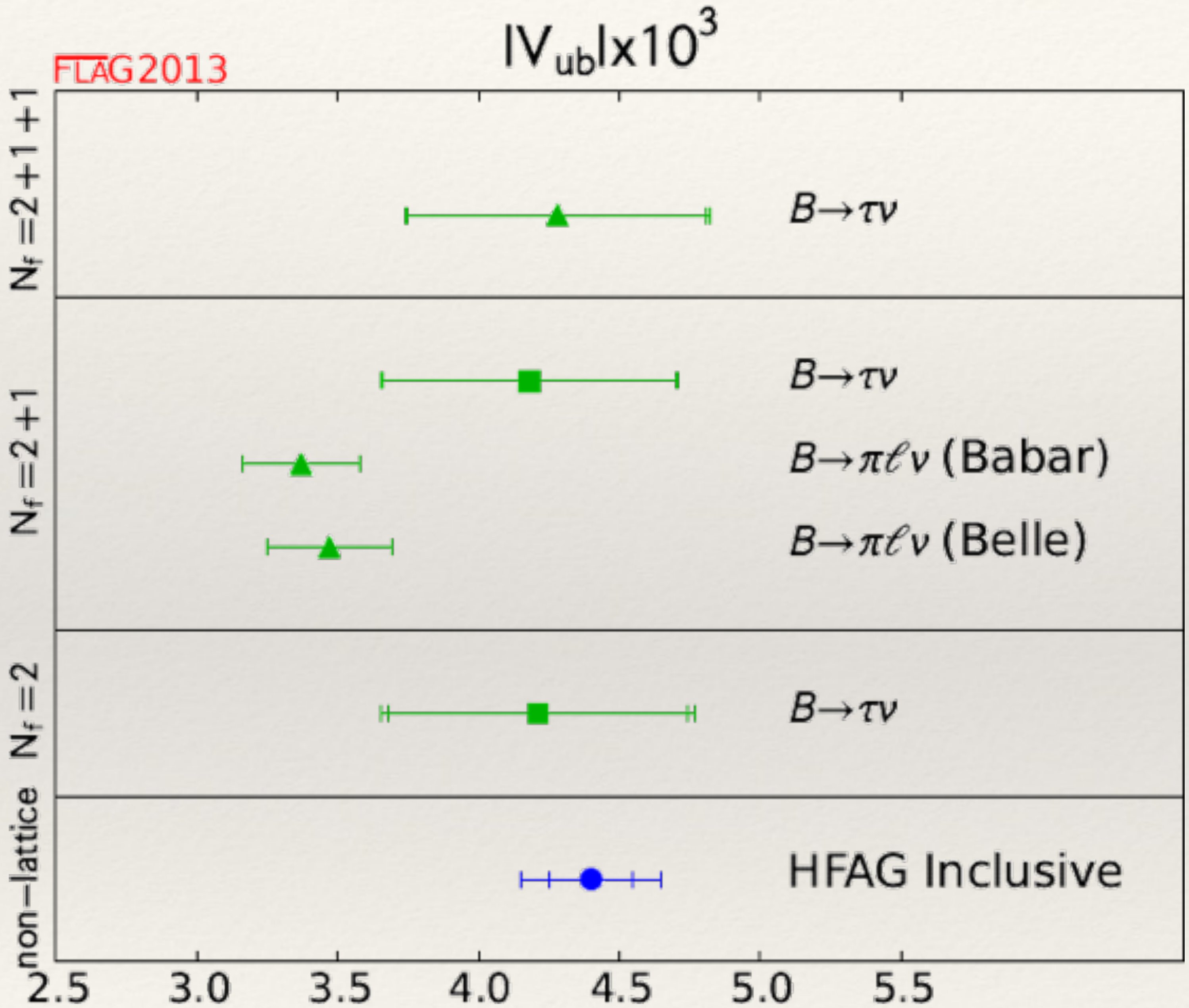
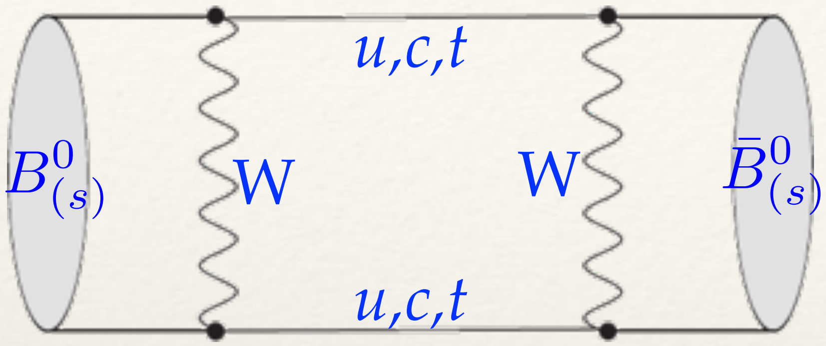
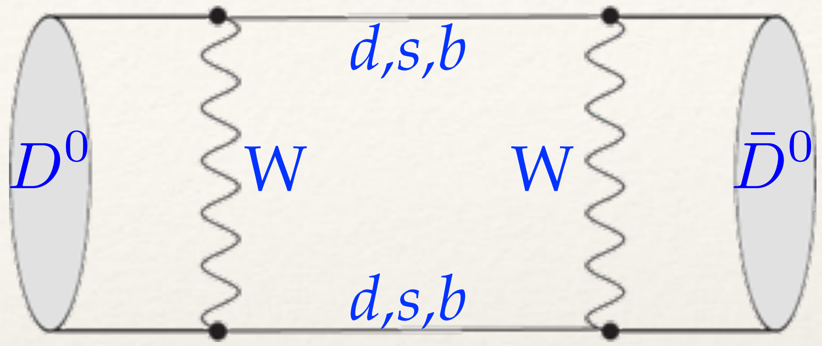


- confirms ~3σ tension between incl. and excl. semilept. decays
- lept. decay lies in between and agrees with both at 1.5σ
- lattice can potentially do better on excl. semileptonics
- Belle II will hopefully improve signal on leptonic channel

SM and BSM mixing (short distance)





- in SM W-boson exchange implies V – A structure, beyond SM other operators possible
- complete set of 4-quark operators:

$$Q_1 = [\bar{c}^a \gamma_\mu (1-\gamma_5) l^a][\bar{c}^b \gamma_\mu (1-\gamma_5) l^b],$$

$$\begin{aligned} Q_2 &= [\bar{c}^a (1-\gamma_5) l^a][\bar{c}^b (1-\gamma_5) l^b], & Q_4 &= [\bar{c}^a (1-\gamma_5) l^a][\bar{c}^b (1+\gamma_5) l^b], \\ Q_3 &= [\bar{c}^a (1-\gamma_5) l^b][\bar{c}^b (1-\gamma_5) l^a], & Q_5 &= [\bar{c}^a (1-\gamma_5) l^b][\bar{c}^b (1+\gamma_5) l^a] \end{aligned}$$

- for B both SM and BSM on the lattice
  for D large distance contributions for SM, so for now only BSM

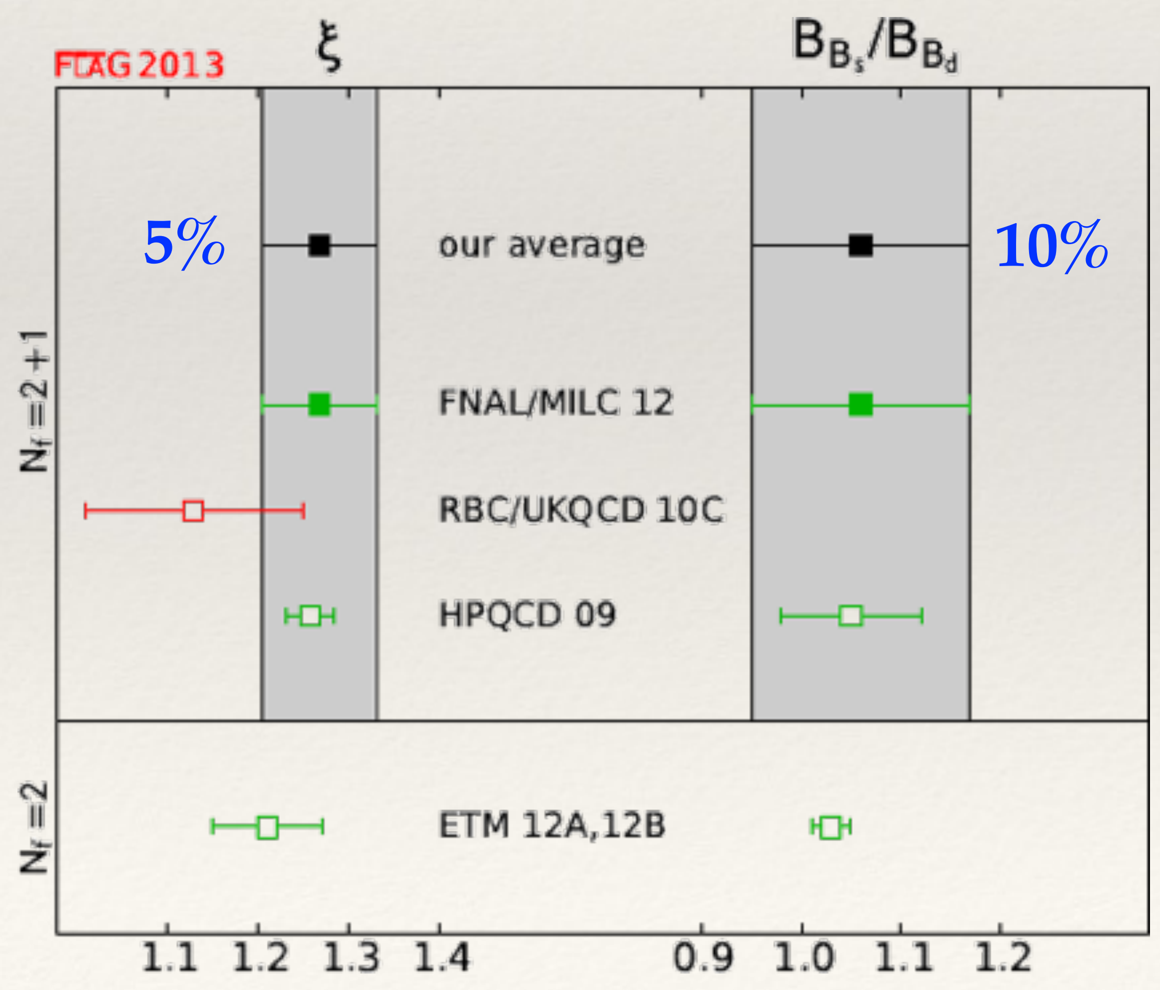
mixing (short distance)

- $D^0 - \bar{D}^0$

ETM Phys.Rev. D90 (2014) 014502

	B_1	B_2	B_3	B_4	B_5
$\overline{\text{MS}}$ (3GeV)	0.75(02)	0.66(02)	0.96(05)	0.91(04)	1.10(05)
RI-MOM (3GeV)	0.74(02)	0.82(03)	1.21(06)	1.09(05)	1.35(06)

- $B_{(s)} - \bar{B}^0_{(s)}$



Summary

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  - quark masses
  - baryons
  - spectroscopy
  - structure
  - …

Summary

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  - baryons
  - spectroscopy
  - structure
  - …
- Some take away messages:
  - we are now simulating physical QCD parameters (→light quarks)
  - there is a large group of quantities which we can pre-/post-dict with an excellent control over systematic effects (→ FLAG)
  - for some of these quantities precision is now such that isospin and EM can no longer be ignored - we are working on it
  - lattice bottom quarks still need help from effective theory (HQET, NRQCD, etc.) and therefore more lattice results with different uses of effective theory desirable

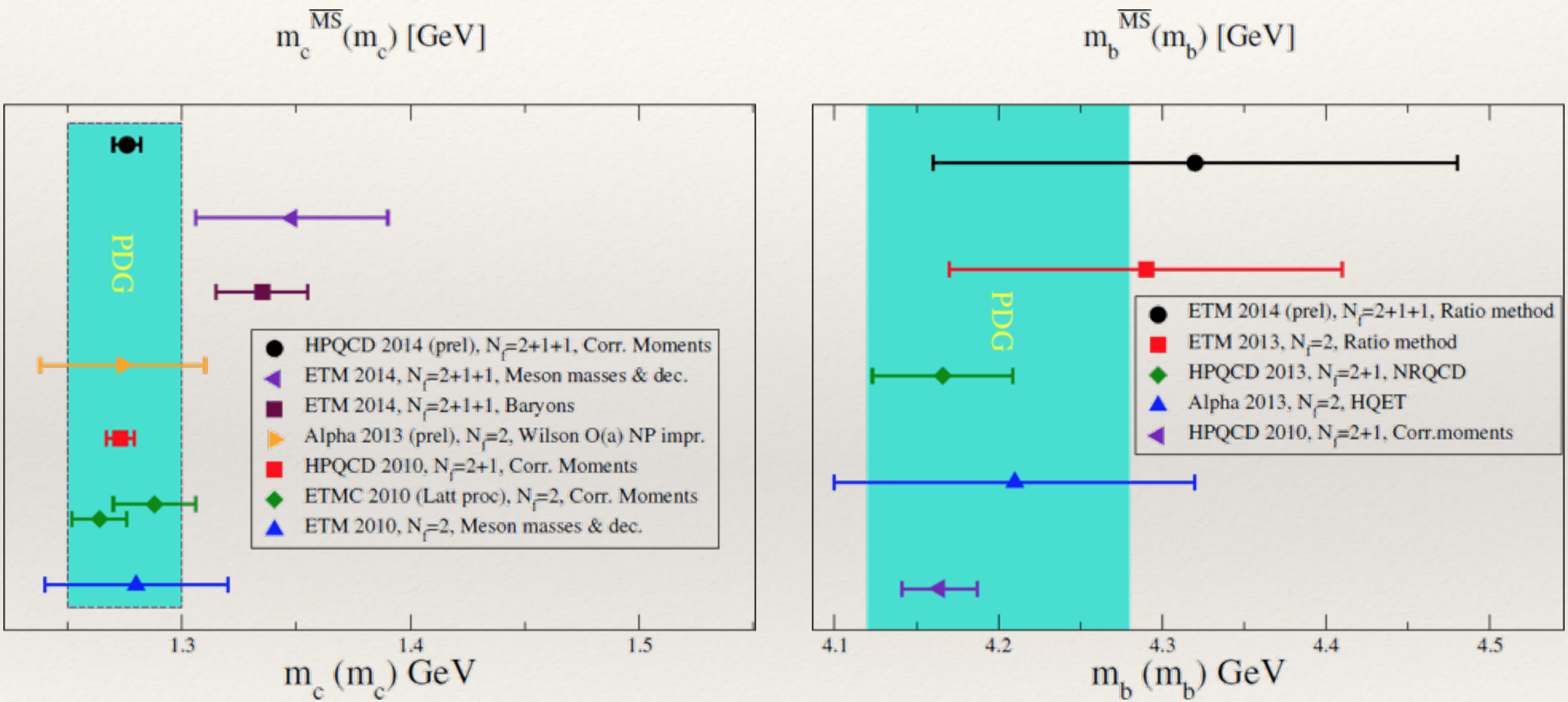
 and  at Lattice 2014

	plenary speaker
c- and b-quark masses	Francesco Sanfilippo
hadron spectroscopy	Sasa Prelovsek
electro-weak matrix elements • leptonic • semi-leptonic (tree, rare) • mixing	Chris Bouchard
hadron structure	Martha Constantinou
isospin breaking	Antonin Portelli

The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) ERC grant agreement No 279757

Supplementary material

Charm and bottom masses



Plots taken from F. Sanfilippo's Lattice 2014 plenary

Results for $|V_{cd}|$ and $|V_{cs}|$

Leptonic decay:

From Experiment Stone and Rosner in PDG:

$$f_D|V_{cd}| = 46.40(1.98)\text{MeV}, \; f_{D_s}|V_{cs}| = 253.1(5.3)\text{MeV}$$

FLAG's analysis:

$$|V_{cd}| = 0.2218(35)(95), \quad |V_{cs}| = 1.018(11)(21), \quad (\text{leptonic decays}, N_f = 2+1)$$

$$|V_{cd}| = 0.2189(83)(94), \quad |V_{cs}| = 1.021(25)(21), \quad (\text{leptonic decays}, N_f = 2)$$

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Semileptonic decay:

From Experiment from HFAG

$$f_+^{D\pi}(0)|V_{cd}| = 0.146(3), f_+^{DK}(0)|V_{cs}| = 0.728(5)$$

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FLAG-average:

$$|V_{cd}| = 0.2191(83), |V_{cs}| = 0.996(21), (\text{our average}, N_f = 2+1)$$

Quarkonia

Main problems:

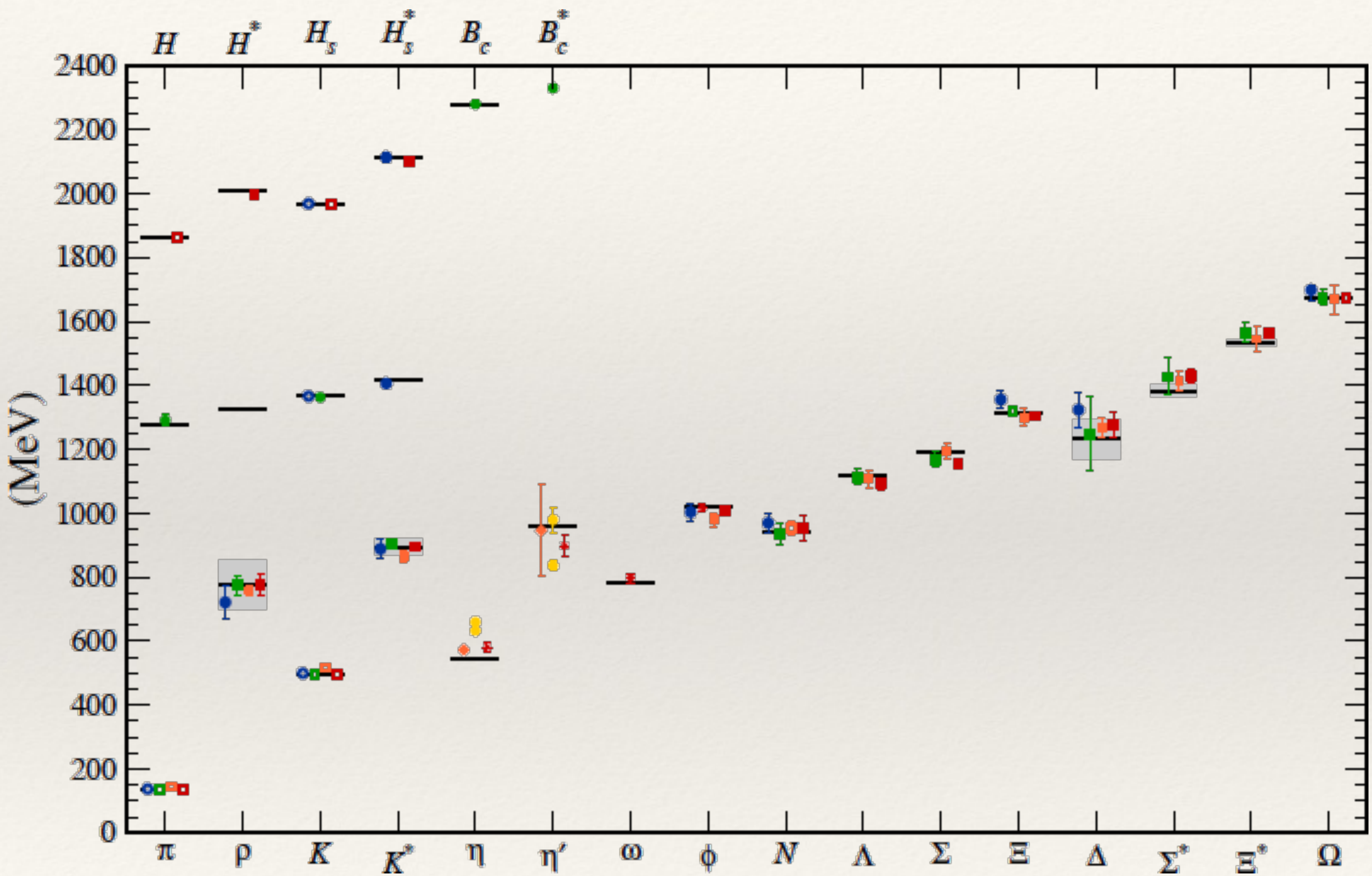
- project on the correct state (large set of bilinear operators)
- get a signal (→GEVP, need large statistics, most existing data for very *heavy pions*)
- deal with plethora of Wick contractions
- scattering in finite volume is hard (Lüscher Nucl.Phys. B354 (1991) 531-578)

What can be done

- precision: below threshold (low-lying charmonium)
- near or above threshold: *single hadron approximation*
- beyond: hard but interesting

Lattice 2014: Hadron spectroscopy Sasa Prelovsek

Some spectra from LQCD vs. experiment



Kronfeld, Ann. Rev. of Nucl. Part. Sci 2012 62

Results for $|V_{ub}|$

- leptonic decays: experimental input for $B\to\tau\nu_\tau$ from Belle and Babar → there is a tension:

	BaBar	Belle
BR(	1.79(48)	0.96(26)
|	3.87(52)(9)	5.28(71)(12)

FLAG combines this to $|V_{ub}|=4.18(52)(9)\times10^{-3}$

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- semileptonic decays:
simultaneous analysis of lattice, Belle and BaBar results (here $N_f=2+1$ lattice input)

	BaBar	Belle
|	3.37(21)	*3.47(22)*

no FLAG average due to unknown exp. correlations

Lattice - systematic uncertainties

In practice need to control a number of sources of systematic uncertainties:

- **discretisation errors** (lattice spacing a)
  effects differ between heavy and light quarks, so currently FLAG's criteria differ from quantity to quantity

- **finite volume errors** (box size L)

- **quark mass extrapolation**
  until very recently mostly unphysical heavy light-quark masses

- **renormalisation, running**

- **heavy quark treatment**

Generally: FLAG considers quantities for which lattice QCD predictions have reached a certain level of maturity

Heavy quark treatment:

✓ RHQ (tl $O(a)$ improved)

NRQCD (tl matched $O(1/m)$
improved through $O(a^2)$)

HQET (including 1/m and leading
cutoff effects at $O(a^2)$)

standard lattice actions
($O(a)$-improved)

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Continuum extrapolation:

★ ≥3 lattice spacings

& $a^2_{\rm max}/a^2_{\rm min} \geq 2$

& $D(a_{\rm min}) \leq 2\%$

& $\delta(a_{\rm min}) \leq 1$

○ two or more lattice spacings

& $a^2_{\rm max}/a^2_{\rm min} \geq 1.4$

& $D(a_{\rm min}) \leq 10\%$

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■ otherwise

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$D(a)$ relative difference between fintest lattice data and continuum limit

$\delta(a)$ deviation of fintest lattice data relative to the statistical and sysetmatic uncertainty of the calculation