

Detecting the long-distance structure of the $X(3872)$

$$[X(3872) \rightarrow D^0 \bar{D}^0 \pi^0]$$

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Isospin violation in the $X(3872)$ [$J^{PC} = 1^{++}$ (LHCb)] decays into $J/\Psi \omega$ and $J/\Psi \rho$

$$\mathcal{B}_X = \frac{\Gamma(X(3872) \rightarrow J/\Psi \overbrace{\pi^+ \pi^-}^{\rho})}{\Gamma(X(3872) \rightarrow J/\Psi \underbrace{\pi^+ \pi^- \pi^0}_{\omega})} = 1.3 \pm 0.5$$

Belle Collab. [PRD84 (2011) 052004]

- $X(3872)$ is not a purely $I = 0$ state **or**
- Transition operator violates isospin

or both...

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The $X(3872)$, with a mass of $m_X = 3871.57 \pm 0.25$ MeV, **is extremely close to the $D^{0*}\bar{D}^0$ threshold** ($m_{D^0} + m_{D^{*0}} = 3871.85 \pm 0.20$ MeV)
 \Rightarrow molecule?

$X(3872) \sim D^0\bar{D}^{*0} \& D^+\bar{D}^{*-}$ bound state ($D\bar{D}^* - hc$ with C-parity=+)

For such a small binding energy, the mass difference (**8 MeV**) between neutral and charged channels plays a role ($m_{D^+} + m_{D^{*-}} = 3879.91 \pm 0.20$ MeV) and the **mass term breaks isospin invariance!**

Isospin violations in X(3872) decays...

Because of the mass differences $D^0 \leftrightarrow D^-$ and $D^{*0} \leftrightarrow D^{*-}$ the kinetic energy (including the mass terms) does not commute with isospin, the X(3872) is an admixture of isospin $I=0$ and $I=1$ (r is the $D\bar{D}^*$ relative distance in the molecule X)

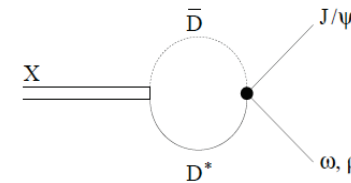
$$\psi_{X(3872)} = \frac{\varphi_{neu}(r) + \varphi_{ch}(r)}{\sqrt{2}} |I=0\rangle + \frac{\varphi_{neu}(r) - \varphi_{ch}(r)}{\sqrt{2}} |I=1\rangle$$

neu: $D^0 \bar{D}^{*0} + cc$ and ch: $D^+ D^{*-} + cc$ (neutral and charged thresholds differ in 8 MeV)

Assuming a contact interaction in the

$I=0$ channel....

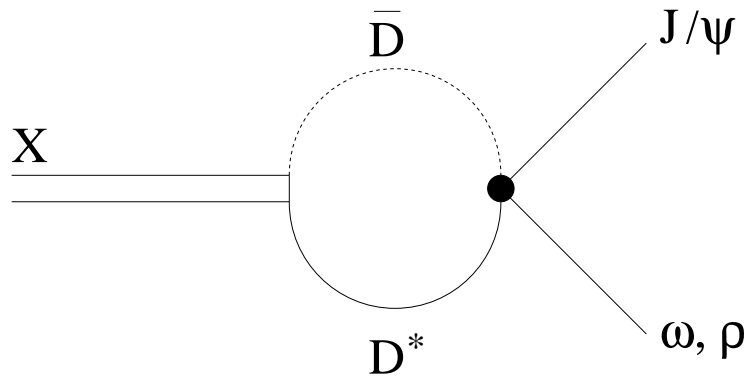
$$\frac{B(X \rightarrow J/\psi \rho)}{B(X \rightarrow J/\psi \omega)} \propto \left(\frac{\varphi_{neu}(0) - \varphi_{ch}(0)}{\varphi_{neu}(0) + \varphi_{ch}(0)} \right)^2 \sim 0.7$$



D. Gammermann, E. Oset Phys. Rev. D80 (2009) 014003; D. Gammermann, J. Nieves, E. Oset and E. Ruiz-Arriola, Phys. Rev. D81 (2010) 014029

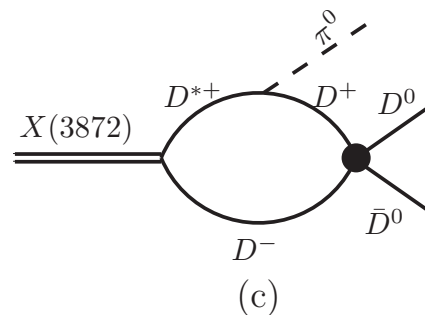
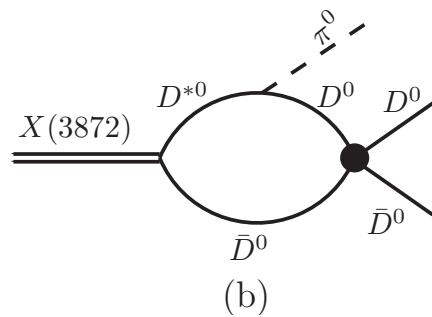
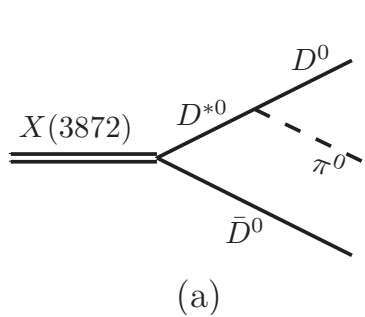
this ratio can be qualitatively understood !

$X(3872) \sim D^0 \bar{D}^{*0} \ \& \ D^+ \bar{D}^{*-}$ bound state ($D\bar{D}^* - hc$ with C-parity=+)



sensitive only to $D\bar{D}^*$ short distances!

$$\int d^3 p \Psi(\vec{p}) \sim \Psi(\vec{0})$$



$$X(3872) \rightarrow D^0 \bar{D}^0 \pi^0$$

$$\sim \Psi(\vec{p}_{D^0})$$

sensitive also to $D\bar{D}^*$
long-distance dynamics!

$$g_0 [X(3872) \bar{D}^0 D^{*0}]$$

$$g_c [X(3872) D^- D^{*+}]$$

$$T[D^0 \bar{D}^0 \rightarrow D^0 \bar{D}^0]$$

$$T[D^+ D^- \rightarrow D^0 \bar{D}^0]$$

HQSS is an appropriate tool to study $c\bar{c}$ XYZ resonances. Though approximate, its predictions arise from QCD in the heavy quark limit and they are more robust than those deduced in other schemes, which starting point is SU(4) flavour symmetry and some arbitrary pattern of its breaking.

HQSS predicts that all types of spin interactions vanish for infinitely massive quarks: the **dynamics is unchanged under arbitrary transformations in the spin of the heavy quark (Q)**. The spin-dependent interactions are proportional to the chromomagnetic moment of the heavy quark, hence are of the order of $1/m_Q$.

To describe the **molecular states of heavy mesons we need an effective Lagrangian that describes the strong interactions of the heavy mesons and antimesons.** We use matrix field $H^{(Q)}$ [$H^{(\bar{Q})}$] to describe the combined isospin doublet of pseudoscalar heavy-meson [antimeson] $P_a^{(Q)} = (P^0, P^+)$ [$P_a^{(\bar{Q})} = (\bar{P}^0, P^-)$] fields and their vector HQSS partners $P_a^{*(Q)}$ [$P_a^{*(\bar{Q})}$]

$$H_a^{(Q)} = \frac{1 + \not{v}}{2} \left(P_{a\mu}^{*(Q)} \gamma^\mu - P_a^{(Q)} \gamma_5 \right), \quad v \cdot P_a^{*(Q)} = 0$$

$$H^{(\bar{Q})a} = \left(P_\mu^{*(\bar{Q})a} \gamma^\mu - P^{(\bar{Q})a} \gamma_5 \right) \frac{1 - \not{v}}{2}, \quad v \cdot P^{*(\bar{Q})a} = 0$$

H^c [$H^{\bar{c}}$] annihilates D [\bar{D}] and D^* [\bar{D}^*] mesons with definite velocity v .

The field $H_a^{(Q)}$ [$H^{(\bar{Q})a}$] transforms as a $(2, \bar{2})$ [$(\bar{2}, 2)$] under the **heavy spin** $\otimes \text{SU}(2)_V$ isospin symmetry

$$H_a^{(Q)} \rightarrow \mathbf{S} \left(H^{(Q)} U^\dagger \right)_a, \quad H^{(\bar{Q})a} \rightarrow \left(U H^{(\bar{Q})} \right)^a \mathbf{S}^\dagger$$

The hermitian conjugate fields are defined by,

$$\bar{H}^{(Q)a} = \gamma^0 H_a^{(Q)\dagger} \gamma^0, \quad \bar{H}_a^{(\bar{Q})} = \gamma^0 \bar{H}^{(\bar{Q})a\dagger} \gamma^0$$

and transform as

$$\bar{H}^{(Q)a} \rightarrow \left(U \bar{H}^{(Q)} \right)^a \mathbf{S}^\dagger, \quad \bar{H}_a^{(\bar{Q})} \rightarrow \mathbf{S} \left(\bar{H}^{(\bar{Q})} U^\dagger \right)^a$$

At LO in the EFT expansion, the **four meson local interaction Lagrangian consistent with HQSS** (T. Alfiky et al., PLB640 238)

$$\begin{aligned}
\mathcal{L}_{4H} &= \mathbf{C}_a \text{Tr} \left[\bar{H}^{(Q)i} H_i^{(Q)} \gamma_\mu \right] \text{Tr} \left[H^{(\bar{Q})i} \bar{H}_i^{(\bar{Q})} \gamma^\mu \right] \\
&+ \mathbf{C}_a^\tau \text{Tr} \left[\bar{H}^{(Q)j} H_i^{(Q)} \gamma_\mu \right] \text{Tr} \left[H^{(\bar{Q})n} \bar{H}_m^{(\bar{Q})} \gamma^\mu \right] \vec{\tau}_j^i \vec{\tau}_n^m \\
&+ \mathbf{C}_b \text{Tr} \left[\bar{H}^{(Q)i} H_i^{(Q)} \gamma_\mu \gamma_5 \right] \text{Tr} \left[H^{(\bar{Q})i} \bar{H}_i^{(\bar{Q})} \gamma^\mu \right] \\
&+ \mathbf{C}_b^\tau \text{Tr} \left[\bar{H}^{(Q)j} H_i^{(Q)} \gamma_\mu \right] \text{Tr} \left[H^{(\bar{Q})n} \bar{H}_m^{(\bar{Q})} \gamma^\mu \gamma_5 \right] \vec{\tau}_j^i \vec{\tau}_n^m
\end{aligned}$$

For each isospin channel (1 or $\vec{\tau} \cdot \vec{\tau}$), we have only two independent constants, which are associated to the two possible spins (0 or 1) of the light quarks in a $H\bar{H}$ system!

Trivial extension to SU(3) light flavor symmetry (still four LEC's) !

$$\begin{aligned}
|D \bar{D}; J = 0, T\rangle &= \frac{\sqrt{3}}{2} |1_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 0, T\rangle + \frac{1}{2} |0_{c\bar{c}}, 0_{\bar{u}\bar{u}}; J = 0, T\rangle \\
|D^* \bar{D}^*; J = 0, T\rangle &= \frac{1}{2} |1_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 0, T\rangle - \frac{\sqrt{3}}{2} |0_{c\bar{c}}, 0_{\bar{u}\bar{u}}; J = 0, T\rangle \\
|D \bar{D}^*; J = 1, T\rangle &= \frac{1}{2} |1_{c\bar{c}}, 0_{\bar{u}\bar{u}}; J = 1, T\rangle - \frac{1}{2} |0_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 1, T\rangle \\
&\quad - \frac{1}{\sqrt{2}} |1_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 1, T\rangle \\
|D^* \bar{D}; J = 1, T\rangle &= \frac{1}{2} |1_{c\bar{c}}, 0_{\bar{u}\bar{u}}; J = 1, T\rangle - \frac{1}{2} |0_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 1, T\rangle \\
&\quad + \frac{1}{\sqrt{2}} |1_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 1, T\rangle \\
|D^* \bar{D}^*; J = 1, T\rangle &= -\frac{1}{\sqrt{2}} |0_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 1, T\rangle - \frac{1}{\sqrt{2}} |1_{c\bar{c}}, 0_{\bar{u}\bar{u}}; J = 1, T\rangle \\
|D^* \bar{D}^*; J = 2, T\rangle &= -|1_{c\bar{c}}, 1_{\bar{u}\bar{u}}; J = 2, T\rangle
\end{aligned}$$

T and $\mathcal{L} = 0_{\bar{u}\bar{u}}, 1_{\bar{u}\bar{u}}$ isospin and total spin of the light dof!

$$\mathbf{HQSS} \Rightarrow \langle S'_{c\bar{c}}, \mathcal{L}'; J, T | H_{\text{QCD}} | S_{c\bar{c}}, \mathcal{L}; J, T \rangle = \delta_{S'_{c\bar{c}} S_{c\bar{c}}} \delta_{\mathcal{L}' \mathcal{L}} \times O_{\mathcal{L}}^T$$

$$\mathcal{B}(J^{PC} = 0^{++}) = \{ |D\bar{D}\rangle, |D^*\bar{D}^*\rangle \}, \quad \mathcal{B}(J^{PC} = 2^{++}) = |D^*\bar{D}^*\rangle$$

$$\mathcal{B}(J^{PC} = 1^{+-}) = \left\{ \frac{1}{\sqrt{2}} (|D\bar{D}^*\rangle + |D^*\bar{D}\rangle), |D^*\bar{D}^*\rangle \right\}$$

$$\mathcal{B}(J^{PC} = 1^{++}) = \left\{ \frac{1}{\sqrt{2}} (|D\bar{D}^*\rangle - |D^*\bar{D}\rangle) \right\}$$

For each isospin channel \mathbf{T} [$O_{\mathcal{L}=0}^T = C_{Ta} - 3C_{Tb}$, $O_{\mathcal{L}=1}^T = C_{Ta} + C_{Tb}$],
with $C_{T(a,b)} = C_{a,b} + (2\vec{T}^2 - 3)C_{a,b}^\tau$ from \mathcal{L}_{4H}

$$V_C(0^{++}) = \begin{pmatrix} C_{Ta} & \sqrt{3}C_{Tb} \\ \sqrt{3}C_{Tb} & C_{Ta} - 2C_{Tb} \end{pmatrix}, \quad V_C(1^{+-}) = \begin{pmatrix} C_{Ta} - C_{Tb} & 2C_{Tb} \\ 2C_{Tb} & C_{Ta} - C_{Tb} \end{pmatrix}$$

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$$\mathbf{V}_C(\mathbf{1}^{++}) = \mathbf{C}_{\mathbf{T}a} + \mathbf{C}_{\mathbf{T}b}, \quad \mathbf{V}_C(\mathbf{2}^{++}) = \mathbf{C}_{\mathbf{T}a} + \mathbf{C}_{\mathbf{T}b}$$

... existence of a heavy quark spin symmetry partner of the $X(3872)$, with $J^{PC} = 2^{++}$, $X(4012)$ around the $D^* \bar{D}^*$ threshold!

$X(3872)$ mass and the branching fraction

$$\mathcal{B}_X = \frac{\Gamma(X(3872) \rightarrow J/\Psi \overbrace{\pi^+ \pi^-}^{\rho})}{\Gamma(X(3872) \rightarrow J/\Psi \underbrace{\pi^+ \pi^- \pi^0}_{\omega})} = 1.3 \pm 0.5$$

fix $(C_{0a} + C_{0b})$ and $(C_{1a} + C_{1b})$, $J^{PC} = 1^{++}$ counter-terms in the $I = 0$ and $I = 1$ sectors (solve a regularized LSE & poles in the FRS).

Heavy Flavor symmetry: $Z_b(10610)$ & $Z_b(10650)$ ($I = 1$, $J^{PC} = 1^{+-}$) have been already discussed by Voloshin (PRD84 031502) and Mehen and Powell (PRD84 114013) in the case of $B^{(*)} \bar{B}^{(*)}$ molecules.

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Heavy Flavor symmetry: $Z_b(10610)$ & $Z_b(10650)$ ($I = 1$, $J^{PC} = 1^{+-}$) $B\bar{B}^*$ and $B^*\bar{B}^*$ molecules $\rightarrow C_{1b}$.

We predict

Solve the LSE in $^{2S+1}L_J$ partial waves (momentum space),

$$\mathbf{T}_{JL'L}^{S'S}(\mathbf{E}; \mathbf{p}', \mathbf{p}) = V_{JL'L}^{S'S}(p', p) + \sum_{L'', S''} \int_0^{+\infty} \frac{dq q^2 4\pi}{(2\pi)^3} \frac{V_{JL'L''}^{S'S''}(p', q) \mathbf{T}_{JL''L}^{S''S}(\mathbf{E}; \mathbf{q}, \mathbf{p})}{E - q^2/2\mu_{12} - M_1 - M_2 + i\epsilon}$$

- UV behavior: a cut-off is needed to regularize the EFT potential
 - $V(p', p) \rightarrow V(p', p) f(\frac{p'}{\Lambda}) f(\frac{p}{\Lambda})$, f.i. $f(x) = e^{-x^2}$ (Gaussian)
- Bound/resonant states: poles in the FRS or SRS of the T -matrix
- Contact interaction $\mathcal{L}_{4H} \Rightarrow \underbrace{\delta_{L0} \delta_{L'0}}_{S\text{-wave}} \delta_{JS} \delta_{J'S'}$ (no mixing of different partial waves).

- RGE:

$$\frac{1}{C_0(\Lambda)} \sim \frac{\mu}{2\pi} \left(\gamma_B - \frac{2}{\pi} \Lambda \right), \quad \gamma_B = \sqrt{-2\mu E_B}$$

LO: \mathcal{L}_{4H} , counter-terms $C's \sim \mathcal{O}(Q^{-1})$ in the EFT expansion (Q is a soft scale) since for shallow bound states

$$\mathcal{O}(V) = \mathcal{O}(VGV), \quad G = \frac{1}{E - H_0} \sim \int \frac{d^3q}{E - M_1 - M_2 - \vec{q}^2/2\mu_{12}} \sim \mathcal{O}(Q)$$

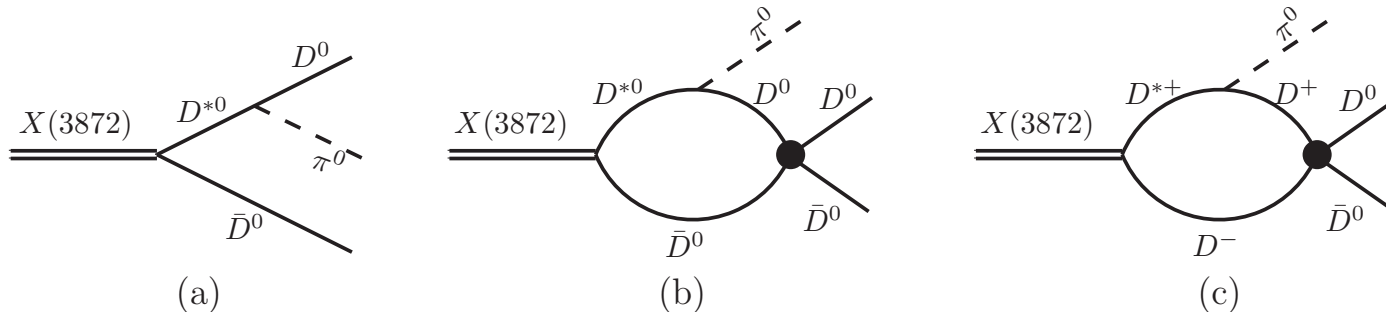
$(Q\bar{Q}'l\bar{l}')$

V_C	$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)	Measurements
C_X	$0(1^{++})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D})$	3875.87	3871.68 (input)	3871.68 ± 0.17 PDG [$X(3872)$]
	$0(2^{++})$	$D^*\bar{D}^*$	4017.3	4012^{+4}_{-5}	?
	$0(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	10604.4	10580^{+9}_{-8}	?
	$0(2^{++})$	$B^*\bar{B}^*$	10650.2	10626^{+8}_{-9}	?
	$0(2^+)$	D^*B^*	7333.7	7322^{+6}_{-7}	?
C_Z	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	10604.4	10602.4 ± 2.0 (input)	10607.2 ± 2.0 Belle [$Z_b(10610)$]
	$1(1^{+-})$	$B^*\bar{B}^*$	10650.2	10648.1 ± 2.1	10652.2 ± 1.5 Belle [$Z_b(10650)$]
	$1(1^{+-})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* + D^*\bar{D})$	3875.87	3871^{+4}_{-12} (V)	$3899.0 \pm 3.6 \pm 4.9$ BESIII [$Z_c(3900)$] $3894.5 \pm 6.6 \pm 4.5$ Belle $3886 \pm 4 \pm 2$ CLEO-c
	$1(1^{+-})$	$D^*\bar{D}^*$	4017.3	4013^{+4}_{-11} (V)	$4026.3 \pm 2.6 \pm 3.7$ BESIII [$Z_c(4020)$]
	$1(1^+)$	D^*B^*	7333.7	$7333.6^{+1}_{-4.2}$ (V)	?

and using also the heavy antiquark-diquark symmetry \Rightarrow triply heavy pentaquarks

$$(QQQl\bar{l}')$$

State	$I(J^P)$	V^{LO}	Thresholds	Mass ($\Lambda = 0.5$ GeV)	Mass ($\Lambda = 1$ GeV)
$\Xi_{cc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	5715	$(M_{\text{th}} - 10)_{-15}^{+10}$	$(M_{\text{th}} - 19)_{-44}^{\dagger}$
$\Xi_{cc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9031	$(M_{\text{th}} - 21)_{-19}^{+16}$	$(M_{\text{th}} - 53)_{-59}^{+45}$
$\Xi_{bb}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12160	$(M_{\text{th}} - 15)_{-11}^{+9}$	$(M_{\text{th}} - 35)_{-31}^{+25}$
$\Xi_{bb}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	15476	$(M_{\text{th}} - 29)_{-13}^{+12}$	$(M_{\text{th}} - 83)_{-40}^{+38}$
$\Xi'_{bc} D^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	8967	$(M_{\text{th}} - 14)_{-13}^{+11}$	$(M_{\text{th}} - 30)_{-40}^{+27}$
$\Xi'_{bc} \bar{B}^*$	$0(\frac{3}{2}^-)$	$C_{0a} + C_{0b}$	12283	$(M_{\text{th}} - 27)_{-16}^{+15}$	$(M_{\text{th}} - 74)_{-51}^{+45}$
$\Xi_{bc}^* D^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	9005	$(M_{\text{th}} - 14)_{-13}^{+11}$	$(M_{\text{th}} - 30)_{-40}^{+27}$
$\Xi_{bc}^* \bar{B}^*$	$0(\frac{5}{2}^-)$	$C_{0a} + C_{0b}$	12321	$(M_{\text{th}} - 27)_{-16}^{+15}$	$(M_{\text{th}} - 74)_{-51}^{+46}$
$\Xi_{bb} \bar{B}$	$1(\frac{1}{2}^-)$	C_{1a}	15406	$(M_{\text{th}} - 0.3)_{-2.5}^{\dagger}$	$(M_{\text{th}} - 12)_{-15}^{+11}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} + \frac{2}{3} C_{1b}$	15452	$(M_{\text{th}} - 0.9)[V]_{\dagger\dagger}^{\text{N/A}}$	$(M_{\text{th}} - 16)_{-17}^{+14}$
$\Xi_{bb} \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{1}{3} C_{1b}$	15452	$(M_{\text{th}} - 1.2)_{-2.9}^{\dagger}$	$(M_{\text{th}} - 10)_{-13}^{+9}$
$\Xi_{bb}^* \bar{B}$	$1(\frac{3}{2}^-)$	C_{1a}	15430	$(M_{\text{th}} - 0.3)_{-2.4}^{\dagger}$	$(M_{\text{th}} - 12)_{-13}^{+11}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{1}{2}^-)$	$C_{1a} - \frac{5}{3} C_{1b}$	15476	$(M_{\text{th}} - 8)_{-7}^{+8}$	$(M_{\text{th}} - 5)_{-8}^{\dagger}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{3}{2}^-)$	$C_{1a} - \frac{2}{3} C_{1b}$	15476	$(M_{\text{th}} - 2.5)_{-3.6}^{\dagger}$	$(M_{\text{th}} - 9)_{-11}^{+9}$
$\Xi_{bb}^* \bar{B}^*$	$1(\frac{5}{2}^-)$	$C_{1a} + C_{1b}$	15476	$(M_{\text{th}} - 4.3)[V]_{+3.3}^{\text{N/A}}$	$(M_{\text{th}} - 18)_{-19}^{+17}$



$$X(3872) \rightarrow D^0 \bar{D}^0 \pi^0$$

$$\sim \Psi(\vec{p}_{D0})$$

**sensitive also to $D\bar{D}^*$
long-distance dynamics!**

$$g_0 [X(3872) \bar{D}^0 D^{*0}]$$

$$g_c [X(3872) D^- D^{*+}]$$

$$T[D^0 \bar{D}^0 \rightarrow D^0 \bar{D}^0]$$

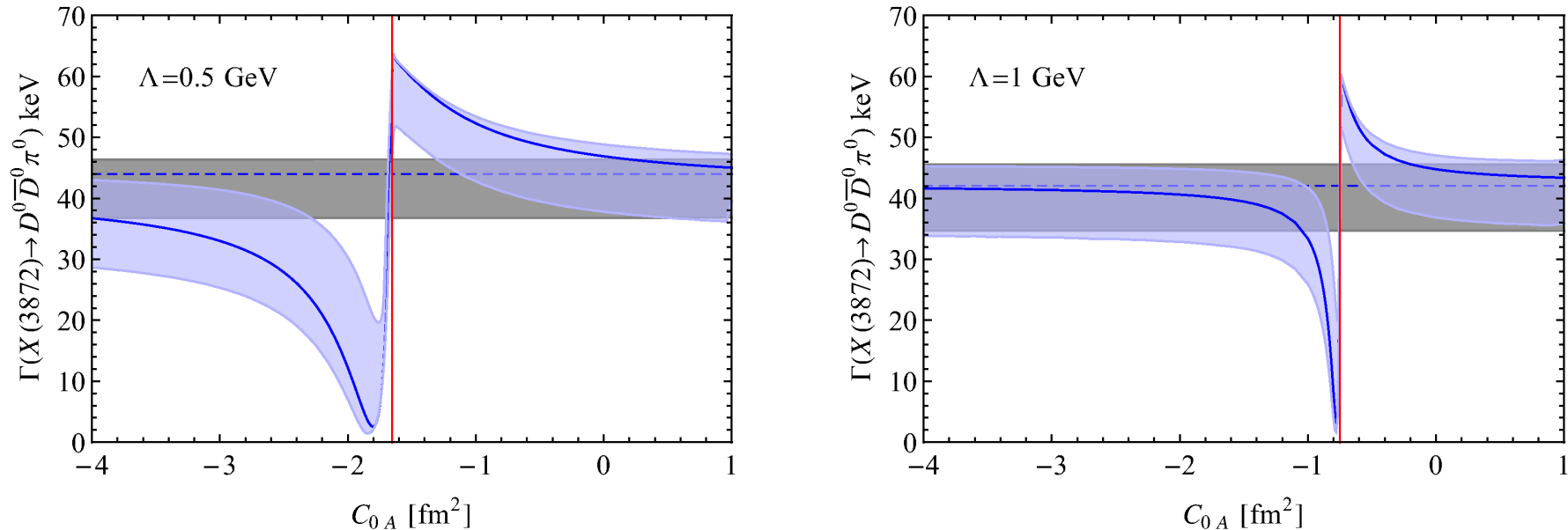
$$T[D^+ D^- \rightarrow D^0 \bar{D}^0]$$

$$T_{\text{tree}} = -2i \frac{g}{f_\pi} \overbrace{g_0^X}^{\text{residue}} \sqrt{M_X M_{D^{*0}} M_{D^0}} \vec{\epsilon}_X \cdot \vec{p}_\pi \left(\frac{1}{p_{12}^2 - M_{D^{*0}}^2} + \frac{1}{p_{13}^2 - M_{D^{*0}}^2} \right),$$

$$d\Gamma = \frac{1}{(2\pi)^3} \frac{1}{32M_X^3} |\overline{T}|^2 dm_{12}^2 dm_{23}^2$$

$$\Gamma(X(3872) \rightarrow D^0 \bar{D}^0 \pi^0)_{\text{tree}} = \underbrace{44.0_{-7.2}^{+2.4}}_{\Lambda=0.5 \text{ GeV}} \left(\underbrace{42.0_{-7.3}^{+3.6}}_{\Lambda=1 \text{ GeV}} \right) \text{ keV}$$

$D^0\bar{D}^0 \rightarrow D^0\bar{D}^0$ and $D^+D^- \rightarrow D^0\bar{D}^0$ FSI contributions...



These contributions depend on the four C_{0a} , C_{0b} , C_{1a} and C_{1b} counter-terms! C_{0b} , C_{1a} and C_{1b} counter-terms are determined by the $X(3872)$, $Z_b(10610)$ and $Z_b(10650)$ resonances. For some values of C_{0a} appears a $D\bar{D}$ bound state close to threshold....

CONCLUSIONS

- We studied the $X(3872) \rightarrow D^0 \bar{D}^0 \pi^0$ decay using an **EFT based on the hadronic molecule assumption for the $X(3872)$** .
- This decay is unique in the sense that it is sensitive to the long-distance structure of the $X(3872)$ as well as the strength of the S -wave interaction between the D and \bar{D} .
- If there was a near threshold pole in the $D\bar{D}$ system, the partial decay width can be very different from the result neglecting the FSI effects. This decay may be used to measure the so far unknown counter-term C_{0a} .
- A future measurement of the $d\Gamma/d|\vec{p}_{D^0}|$ distribution might provide valuable information on the $X(3872)$ wave function at the fixed momentum $\Psi(\vec{p}_{D^0})$
- In addition to **HQSS, HFS and HADS** can be used to predict new heavy meson molecules and triply heavy pentaquarks

BACK UP MATERIAL

EFT framework for the description of heavy meson-antimeson molecules ($T = 0$):

- LO: \mathcal{L}_{4H} (contact range interaction)

$$V_C(0^{++}) = \begin{pmatrix} C_{0a} & \sqrt{3} C_{0b} \\ \sqrt{3} C_{0b} & C_{0a} - 2 C_{0b} \end{pmatrix}, \quad V_C(1^{+-}) = \begin{pmatrix} C_{0a} - C_{0b} & 2 C_{0b} \\ 2 C_{0b} & C_{0a} - C_{0b} \end{pmatrix}$$

$$V_C(1^{++}) = C_{0a} + C_{0b}, \quad V_C(2^{++}) = C_{0a} + C_{0b}$$

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- pion exchanges and particle coupled channel effects to be sub-leading corrections.

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- pion exchanges and particle coupled channel effects to be sub-leading corrections.
- Counter-terms ?

NLO: One Pion Exchange

$H_a^{(Q)}$ [$H^{(\bar{Q})a}$] transforms as a $(2, \bar{2})$ [$(\bar{2}, 2)$] under the heavy spin \otimes **SU(2)_v** **isospin symmetry**

$$H_a^{(Q)} \rightarrow S \left(H^{(Q)} \mathbf{U}^\dagger \right)_a, \quad H^{(\bar{Q})a} \rightarrow \left(\mathbf{U} H^{(\bar{Q})} \right)^a S^\dagger$$

$$\mathcal{L}_{\pi HH} = -\frac{g}{\sqrt{2}f_\pi} \left(\text{Tr} \left[\bar{H}^{(Q)j} H_i^{(Q)} \gamma_\mu \gamma_5 \right] + \text{Tr} \left[H^{(\bar{Q})j} \bar{H}_i^{(\bar{Q})} \gamma^\mu \gamma_5 \right] \right) (\vec{\tau} \partial_\mu \vec{\phi})_j^i + \dots$$

$$\mathbf{V}_{\text{HH}}^{\text{OPE}}(\vec{q}) \sim \frac{g^2}{2f_\pi^2} \frac{(\vec{a} \cdot \vec{q})(\vec{b} \cdot \vec{q})}{\vec{q}^2 + m_\pi^2} \sim \mathcal{O}(Q^0)$$

$g \simeq 0.6$ is the axial coupling between the heavy meson and the pion, $f_\pi \simeq 132 \text{ MeV}$, and \vec{q} the momentum exchanged by the heavy meson and antimeson. Besides, \vec{a} and \vec{b} are the polarization operators.

NLO: One Pion Exchange $\sim \mathcal{O}(Q^0) \Rightarrow$ distinctive feature of the OPE potential can mix different $^{2S+1}L_J$ partial waves.

J^{PC}	$H\bar{H}$	$^{2S+1}L_J$	E ($\Lambda = 0.5$ GeV)	E ($\Lambda = 1$ GeV)	Exp
0^{++}	$D\bar{D}$	1S_0	3708 (3706 \pm 10)	3720 (3712 $^{+13}_{-17}$)	–
1^{++}	$D^*\bar{D}$	3S_1 - 3D_1	Input	Input	3872
1^{+-}	$D^*\bar{D}$	3S_1 - 3D_1	3816 (3814 \pm 17)	3823 (3819 $^{+24}_{-27}$)	–
0^{++}	$D^*\bar{D}^*$	1S_0 - 5D_2	Input	Input	3917
1^{+-}	$D^*\bar{D}^*$	3S_1 - 3D_1	3954 (3953 \pm 17)	3958 (3956 $^{+25}_{-28}$)	3942
2^{++}	$D^*\bar{D}^*$	1D_2 - 5S_2 - 5D_2 - 5G_2	4015 (4012 \pm 3)	4014 (4012 $^{+4}_{-9}$)	–

Predicted masses (in MeV) of the $X(3872)$ HQSS partners when the OPE potential is included. We display results for two different values of the Gaussian cutoff. Now, we find $C_{0a} = -3.46$ fm² and $C_{0b} = 1.98$ fm², and $C_{0a} = -0.98$ fm² and $C_{0b} = 0.69$ fm², for $\Lambda = 0.5$ and 1 GeV, respectively. OPE $\delta(r)$ contribution can be absorbed within the contact range piece $C_{0b} \rightarrow C_{0b} - \frac{g^2}{2f_\pi^2}$.

Binding energies of the states are stable with respect to the iteration of the OPE potential: agreement with the *a priori* EFT expectations! [PRD 86, 056004 (2012)]

NNLO: Particle Coupled Channels Momentum scale (**hard**) associated with the coupled channels is

$$\Lambda_C(0^{++}) = \sqrt{2\mu(2\Delta_Q)} \sim 750 \text{ MeV}, \quad \Lambda_C(1^{+-}) = \sqrt{2\mu\Delta_Q} \sim 520 \text{ MeV}$$

with $\Delta_Q \sim (M_{D^*} - M_D) \Rightarrow G \sim \mathcal{O}(Q^3) \Rightarrow VGV \sim \mathcal{O}(Q^1)$.

- In contrast to the OPE corrections, the **LO counter-term structure stemming from HQSS is not expected to be able to absorb the kind of divergences associated with the coupled channel calculations.**
- One should **add new counter-terms at $\mathcal{O}(Q)$ to soften the regularization scheme dependence** and make the EFT *renormalizable* again. Higher orders will introduce new unknown constants that cannot be fixed at the moment owing to the scarce experimental data available.

J^{PC}	$H\bar{H}$	$E - i\Gamma/2$ ($\Lambda = 0.5$ GeV)	$E - i\Gamma/2$ ($\Lambda = 1$ GeV)	Exp
0^{++}	$D\bar{D}, D^*\bar{D}^*$	3690 (3706 \pm 10)	3694 (3712 $^{+13}_{-17}$)	–
1^{++}	$D^*\bar{D}$	Input	Input	3872
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	3782 (3814 \pm 17)	3782 (3819 $^{+24}_{-27}$)	–
0^{++}	$D\bar{D}, D^*\bar{D}^*$	3939 – $\frac{i}{2}$ 12 (3917)	3937 – $\frac{i}{2}$ 31(3917)	3917 \pm 3 – $\frac{i}{2}$ 28 $^{+10}_{-9}$
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	3984 – $\frac{i}{2}$ 17 (3953 \pm 17)	3982 – $\frac{i}{2}$ 29 (3956 $^{+25}_{-28}$)	3942 \pm 9 – $\frac{i}{2}$ 37 $^{+27}_{-17}$
2^{++}	$D^*\bar{D}^*$	4012 (4012 \pm 3)	4012 (4012 $^{+4}_{-9}$)	–

Masses and widths (in MeV) of the $X(3872)$ HQSS partners when coupled channels effects are included. The contact terms are adjusted to reproduce the $X(3872)$ and $X(3915)$ masses neglecting coupled channel effects. OPE interactions are not included.

$$|\Delta E_B| \simeq |E_B| \left(\frac{\gamma_B}{\Lambda_C} \right)^2 \sim (30 - 40) \text{ MeV}$$

[PRD 86, 056004 (2012)]

NNLO: Particle Coupled Channels [PRD 86, 056004 (2012)]

J^{PC}	$H\bar{H}$	$E - i\Gamma/2$ ($\Lambda = 0.5$ GeV)	$E - i\Gamma/2$ ($\Lambda = 1$ GeV)	Exp
0^{++}	$D\bar{D}, D^*\bar{D}^*$	3658 (3706 \pm 10)	3669 (3712 $^{+13}_{-17}$)	–
1^{++}	$D^*\bar{D}$	Input	Input	3872
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	3730 (3814 \pm 17)	3739 (3819 $^{+24}_{-27}$)	–
0^{++}	$D\bar{D}, D^*\bar{D}^*$	3917 – $\frac{i}{2}$ 23 (3917)	3917 – $\frac{i}{2}$ 50 (3917)	3917 \pm 3 – $\frac{i}{2}$ 28 $^{+10}_{-9}$
1^{+-}	$D^*\bar{D}, D^*\bar{D}^*$	3979 – $\frac{i}{2}$ 24 (3953 \pm 17)	3979 – $\frac{i}{2}$ 39 (3956 $^{+25}_{-28}$)	3942 \pm 9 – $\frac{i}{2}$ 37 $^{+27}_{-17}$
2^{++}	$D^*\bar{D}^*$	4012 (4012 \pm 3)	4012(4012 $^{+4}_{-9}$)	–

Masses and widths (in MeV) of the $X(3872)$ HQSS partners when coupled channels effects are included. The contact terms are adjusted to reproduce the $X(3872)$ and $X(3915)$ masses, while OPE effects are neglected. We find $C_{0a} = -4.16$ fm² and $C_{0b} = 2.21$ fm², and $C_{0a} = -1.14$ fm² and $C_{0b} = 0.35$ fm², for $\Lambda = 0.5$ and 1 GeV, respectively.

- In contrast to the OPE corrections, the **LO counter-term structure stemming from HQSS is not expected to be able to absorb the kind of divergences associated with the coupled channel calculations.**
- One should **add new counter-terms at $\mathcal{O}(Q)$ to soften the regularization scheme dependence** and make the EFT *renormalizable* again (problem: it might occur a transition from a power counting in which Λ_C is a hard scale to a different one in which it is a soft scale). Higher orders will introduce new unknown constants that cannot be fixed at the moment owing to the scarce experimental data available.