Non-lattice QCD for Heavy Flavour Decays

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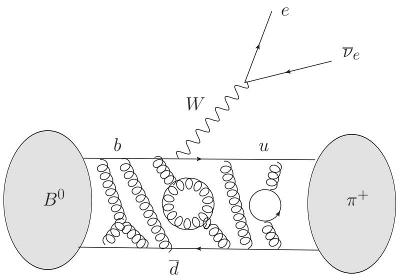
Outline

- Impressive amount of data (LHC, *B*-factories,...) available on exclusive heavy-to-light flavour transitions: $b \to u\ell\nu_\ell$, $b \to s\ell^+\ell^-$, ...
- determination of CKM parameters, coeffs of H_{eff}, constraints on new physics due to non-SM short-distance effects
- all this is not possible without precise knowledge of hadronic matrix elements=
  ={hadronic decay constants, form factors,...}
- continuum QCD (non-lattice) calculations of heavy-light transition matrix elements:
  some results, accuracy, perspectives

Form factors in quark-flavour physics

- a well studied example:
  $B \to \pi\ell\nu_\ell$ (the simplest $b \to u\ell\nu_\ell$ transition)
- hadronic transition matrix element determined by form factors:



$$\langle\pi^+(p)|\bar{u}\gamma_\mu b|B(p+q)\rangle = f^+_{B\pi}(q^2)\Big[...\Big]_\mu + f^0_{B\pi}(q^2)\Big[...\Big]_\mu$$

- a standard source of $|V_{ub}|$ determination [BaBar,Belle]

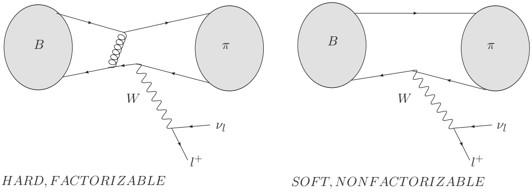
$$\left(\frac{1}{\tau_B}\right)\frac{dBR(\bar{B}^0 \to \pi^+ l^- \nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}p_\pi^3|f^+_{B\pi}(q^2)|^2 + O(m_l^2)$$

$0 < q^2 < (m_B - m_\pi)^2 \sim 26\ \text{GeV}^2$,

- form factors have to be calculated in QCD

QCD dynamics of the form factors

- factorization theorems in $m_b \to \infty$, (originally for $B \to \pi\pi$)
  [M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda (1999)]
- for $B \to \pi$ form factor at large recoil: $(E_\pi \sim m_B/2,\ q^2 \to 0)$
  [M. Beneke, Th. Feldmann (2001)]



$$f_{B\pi}(q^2) \sim \alpha_s(\mu)\int d\omega du\ \phi_B^+(\omega,\mu) T_h(q^2,\omega,u,\mu)\varphi_\pi(u,\mu) + f_{B\pi}^{soft}(q^2)$$

$$\mu = \sqrt{m_b\Lambda}$$

Toolbox of continuum (non-lattice) QCD calculations

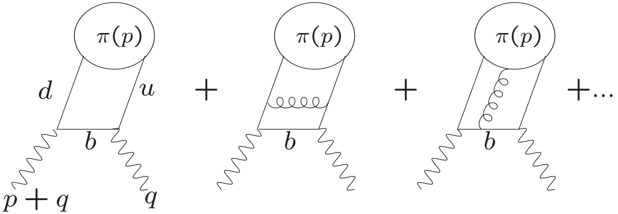
- universal tools: $\Leftarrow$ (also used in the lattice QCD)
  - analyticity & unitarity, hadronic dispersion relations,
  - perturbative expansion in QCD
- dedicated tools:
  - QCD light-cone sum rules (LCSR)
  - two-point (SVZ) QCD sum rules
- LCSR applicable at large hadronic recoil:
  $f_{B\pi}(q^2 \ll m_b^2)$ including "soft" contributions
  lattice QCD: only small recoil (large q^2) region accessible

Light-Cone Sum Rules (LCSR) for $B \to \pi$

- the correlation function an artifitially “designed” amplitude
- external currents with $(p+q)^2, q^2 \ll m_b^2 \Rightarrow b$-quark virtual,
- factorization: $\mu \sim \sqrt{m_b\chi}, \Lambda_{QCD} \ll \chi \ll m_b$

$$F(q^2,(p+q)^2) = \sum_{t=2,3,4,..} \int du\, T^{(t)}(q^2,(p+q)^2,m_b^2,\alpha_s,u,\mu)\,\varphi_\pi^{(t)}(u,\mu)$$

hard scattering amplitudes $\otimes$ pion light-cone DA’s



LO NLO {power corr.}

The OPE result

- current accuracy

$$F(q^2,(p+q)^2) = \left(T_0^{(2)} + (\alpha_s/\pi)T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)}\right) \otimes \varphi_\pi^{(2)}$$
$$+\frac{\mu_\pi}{m_b}\left(T_0^{(3)} + (\alpha_s/\pi)T_1^{(3)}\right) \otimes \varphi_\pi^{(3)}$$
$$+\frac{\delta_\pi^2}{m_b\chi} T^{(4)} \otimes \varphi_\pi^{(4)} + \ldots$$

- LO twist 2,3,4 $q\bar{q}$ and $\bar{q}qG$ terms:

[V.Belyaev, A.K., R.Rückl (1993); V.Braun, V.Belyaev, A.K., R.Rückl (1996)]

-NLO $O(\alpha_s)$ twist 2, (collinear factorization)

[A.K., R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997);]

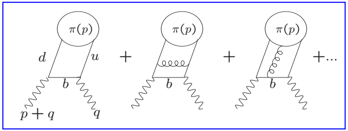
-NLO $O(\alpha_s)$ twist 3 (coll.factorization for asympt. DA)

[P. Ball, R. Zwicky (2001); G.Duplancic,A.K.,B.Melic, Th.Mannel,N.Offen (2007)]

-part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 *[A. Bharucha (2012)]*

Calculating the form factor from LCSR

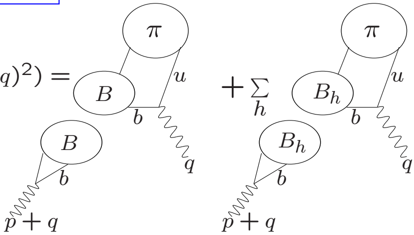
- {correlator OPE } = {sum over intermediate B states}



Unitarity, analyticity,
dispersion relation in $(p+q)^2$:

$\Downarrow$

$F(q^2,(p+q)^2) =$



$f_B f^+_{B\pi}(q^2)$ $\qquad$ $\sum_{B_h} \rightarrow duality\ (s_0^B)$

- varying flavours and J^{PC} yields LCSR's for $B \rightarrow K, \eta$, $D \rightarrow \pi, K$, etc.
- LCSR includes "soft" and "hard" contributions to the form factors
- finite m_b, assessment of $1/m_b$ terms, HQET limit

How accurate are LCSR's

- the "raw" sum rule: OPE = dispersion relation:

$$[F((p+q)^2, q^2)]_{OPE} = \frac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2 - (p+q)^2} + \int\limits_{s_0^B}^{\infty} ds \frac{[\mathrm{Im} F(s, q^2)]_{OPE}}{s - (p+q)^2}$$

$\Uparrow$ $\overline{m}_b$, α_s, $\varphi_\pi^{(t)}(u)$, t=2,3,4;

$\Uparrow$ QCD SR for f_B

$\Uparrow$ quark-hadron duality

- input for OPE: parameters, truncation level, variable scales
- use of 2-point QCD sum rules
- "systematic error" of the quark-hadron duality approximation (suppressed with Borel transformation, controlled by the m_B calculation)
- nailing down the pion DA's (Gegenbauer moments, normalization coeffs.): two-point sum rules, LCSR's for the pion form factors: $\gamma^*\gamma \to \pi$, $\gamma^*\pi \to \pi$ vs exp. (CLEO, BaBar, Belle, Jlab)

$B_{(s)}$ and $D_{(s)}$ decay constants from QCD sum rules

[P.Gelhausen, AK, A.A.Pivovarov, D.Rosenthal, 1305.5432 hep/ph]

Decay constant	Lattice QCD [ref.]	this work
f_B[MeV]	196.9 ± 9.1 [1] 186 ± 4 [2]	207^{+17}_{-9}
f_{B_s}[MeV]	242.0 ± 10.0 [1] 224 ± 5 [2]	242^{+17}_{-12}
f_{B_s}/f_B	1.229 ± 0.026 [1] 1.205 ± 0.007 [2]	$1.17^{+0.04}_{-0.03}$
f_D[MeV]	$212.6(0.4)(^{+1.0}_{-1.2})$[3] 213 ± 4 [2]	201^{+12}_{-13}
f_{D_s}[MeV]	$249.0(0.3)(^{+1.1}_{-1.5})$ [3] 248.0 ± 2.5 [2]	238^{+13}_{-23}
f_{D_s}/f_D	$1.1712(10)(^{+29}_{-32})$ [3] 1.164 ± 0.018 [2]	$1.15^{+0.04}_{-0.05}$

[1], [3]-Fermilab/MILC, [2]-HPQCD

LCSR for $D \to \pi, K$ form factors

AK, Ch. Klein, Th. Mannel, N.Offen (2009)

- important cross-check of the LCSR method
- $b \to c$ in the correlation function (finite quark masses !)

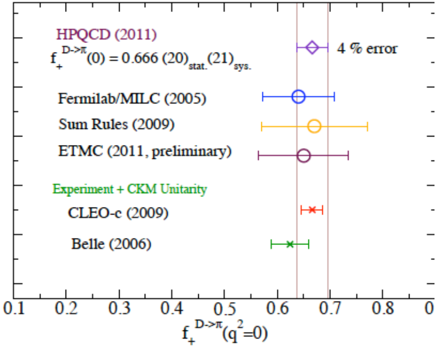


FIG. 6: The $D \to \pi$ form factor $f_+^{D\to\pi}(0)$ from this work an
comparisons with other determinations [12, 13, 23–25].

taken from HPQCD 1109. 1501(2011)

Results for $B \to \pi$ form factor and extraction of $|V_{ub}|$

- $0 < q^2 < 12$ GeV^2 - LCSR *[AK, T.Mannel, N.Offen, Y-M.Wang (2011)]*
  - integrate over the region of validity, taking $q^2_{max} = 12$ GeV^2

$$\Delta\zeta\,(0, q^2_{max}) \equiv \frac{G_F^2}{24\pi^3}\int\limits_0^{q^2_{max}} dq^2 p_\pi^3 |f^+_{B\pi}(q^2)|^2 = \frac{1}{|V_{ub}|^2 \tau_{B^0}} \int\limits_0^{q^2_{max}} dq^2 \frac{d\mathcal{B}(B \to \pi \ell \nu_\ell)}{dq^2}\,,$$

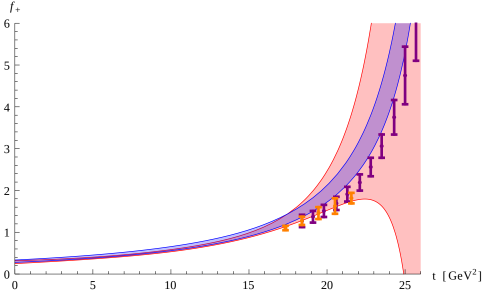
from [Belle Collab 1306.2781 [hep-ex]]

TABLE XII: Values of the CKM matrix element $|V_{ub}|$ based on rates of exclusive $\bar{B} \to X_u \ell^- \bar{\nu}_\ell$ decays and theoretical predictions of form factors within various q^2 ranges. The first uncertainty is statistical, the second is experimental systematic and the third is theoretical. The theoretical uncertainty for the ISGW2 model is not available.

X_u	Theory	q^2 GeV/c^2	$N^{\rm fit}$	$N^{\rm MC}$	$\Delta\mathcal{B}$ 10^{-4}	$\Delta\zeta$ ps^{-1}	$\lvert V_{ub}\rvert$ 10^{-3}
π^0	LCSR [33]	< 12	119.6 ± 16.2	116.5	0.423 ± 0.057	$4.59^{+1.00}_{-0.85}$	$3.35 \pm 0.23 \pm 0.09^{+0.36}_{-0.31}$
	LCSR [34]	< 16	168.2 ± 18.9	153.5	0.588 ± 0.066	$5.44^{+1.43}_{-1.43}$	$3.63 \pm 0.20 \pm 0.10^{+0.60}_{-0.40}$
	HPQCD [35]	> 16	58.6 ± 10.5	57.6	0.196 ± 0.035	$2.02^{+0.55}_{-0.55}$	$3.44 \pm 0.31 \pm 0.09^{+0.59}_{-0.39}$
	FNAL [36]					$2.21^{+0.47}_{-0.42}$	$3.29 \pm 0.30 \pm 0.09^{+0.37}_{-0.30}$
π^+	LCSR [33]	< 12	247.2 ± 18.9	233.1	0.808 ± 0.062	$4.59^{+1.00}_{-0.85}$	$3.40 \pm 0.13 \pm 0.09^{+0.37}_{-0.32}$
	LCSR [34]	< 16	324.2 ± 22.6	305.1	1.057 ± 0.074	$5.44^{+1.43}_{-1.43}$	$3.58 \pm 0.12 \pm 0.09^{+0.59}_{-0.39}$
	HPQCD [35]	> 16	141.3 ± 16.0	116.1	0.445 ± 0.050	$2.02^{+0.55}_{-0.55}$	$3.81 \pm 0.22 \pm 0.10^{+0.66}_{-0.43}$
	FNAL [36]					$2.21^{+0.47}_{-0.42}$	$3.64 \pm 0.21 \pm 0.09^{+0.40}_{-0.33}$

Accessing large q^2 with LCSR input

- $q^2 \to z(q^2, t_0)$, z-series parameterization $\Rightarrow$ extrapolation:
  BCL-version [Bourrely, Caprini, Lellouch, (2008)]
- unitarity bounds for the z-transformed form factor
  [L.Lellouch (1996)], [Th.Mannel,B.Postler(1998)]



PRELIMINARY:

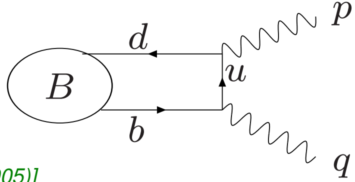
form factor $f^+_{B\pi}(q^2)$: BCL extrapolation (violet) unitarity bounds (magenta), lattice QCD points: HPQCD (orange) and Fermilab-MILC (blue),
correlations shape-normalization

S.Imsong, AK, Th. Mannel, D. van Dyk, work in progress

Other applications of LCSR

- modification of the method: LCSR's with *B*-meson DA's

- a different correlation function:
  B -meson is on-shell,
  π interpolated with a current
  [A.K., T. Mannel, N.Offen,(2005) ,
  F.De Fazio, Th. Feldmann and T. Hurth, (2005)]



- $B \to \gamma_\ell \nu_\ell$ a key process to determine the *B*-meson DA
  recent update in SCET [M.Beneke, J.Rohrwild (2011)],
  soft form factor from LCSR [V.Braun, AK(2013)]

- heavy baryon transition form factors
  see the talk by Yu-Ming Wang at this conference

Using continuum QCD methods for hadronic matrix elements not accessible in the current lattice studies

Hadronic input in $B \to K\ell^+\ell^-$

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}}\frac{\alpha_{em}}{\pi} V_{tb}V_{ts}^* \Bigg[\bar{\ell}\gamma_\mu \ell\, p^\mu \Bigg(C_9 f_{BK}^+(q^2)$$

$$+ \frac{2(m_b + m_s)}{m_B + m_K} C_7^{eff} f_{BK}^T(q^2) + \sum_{i=1,2,\ldots,6,8} C_i\, \mathcal{H}_i^{(BK)}(q^2) \Bigg) + \bar{\ell}\gamma_\mu\gamma_5 \ell\, p^\mu C_{10} f_{BK}^+(q^2) \Bigg]$$

- $B \to K$ form factors: simply $\pi \to K$ in the correlation function, $O(m_s)$ effects included
- additional nonlocal (nonfactorizable) matrix elements:

$$\mathcal{H}_i^{(BK)}(q^2) \sim \langle K(p)| i \int d^4x\, e^{iqx}\, T\{ j^\rho_{em}(x), O_i(0)\} | B(p+q)\rangle$$

$$j^\rho_{em} = \sum_{q=u,d,s,c,b} Q_q \bar{q}\gamma^\rho q\,,$$ O_i -quark-gluon effective operators

- QCD factorization approach *M.Beneke, Th.Feldmann, D.Seidel (2001)*

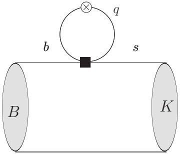
- including soft-gluon nonfactrizable effects and employing hadronic disperison relations to access positive q^2

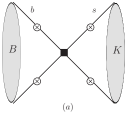
[A.K., Th. Mannel, A. Pivovarov ,Yu-M. Wang, (2010)];

[A.K., Th. Mannel and Yu-M. Wang, (2013)]

The nonlocal matrix elements

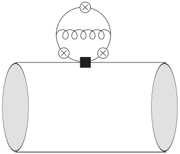
- LO, factorizable and weak annihilation

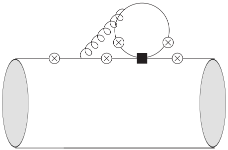


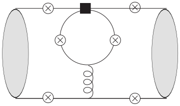


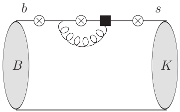
⊗ -virtual photon

- NLO, nonfactorizable ...

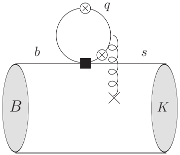


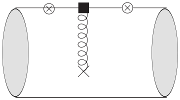






- soft (low virtuality) gluons, nonfactorizable





...

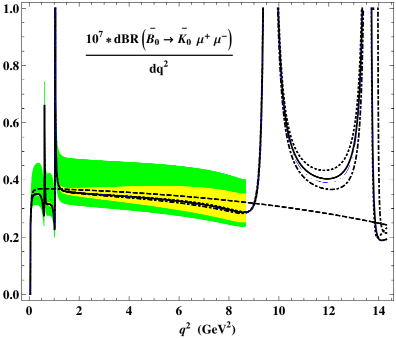
calculated at $q^2 < 0$ and matched to hadronic dispersion relation in the virtual photon channel including $V = \rho, \omega, \phi, ..., J/\psi, \psi(2S)$

$d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$ and bins

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.



$[q^2_{min}, q^2_{max}]$	Belle	CDF	LHCb	LHCb	this work
[0.05, 2.0]	$0.81^{+0.18}_{-0.16} \pm 0.05$	$0.33 \pm 0.10 \pm 0.02$	$0.21^{+0.27}_{-0.23}$	$0.56 \pm 0.05 \pm 0.03$	$0.71^{+0.22}_{-0.08}$
[2.0, 4.3]	$0.46^{+0.14}_{-0.12} \pm 0.03$	$0.77 \pm 0.14 \pm 0.05$	$0.07^{+0.25}_{-0.21}$	$0.57 \pm 0.05 \pm 0.02$	$0.80^{+0.27}_{-0.11}$
[4.3, 8.68]	$1.00^{+0.19}_{-0.08} \pm 0.06$	$1.05 \pm 0.17 \pm 0.07$	1.2 ± 0.3	$1.00 \pm 0.07 \pm 0.04$	$1.39^{+0.53}_{-0.22}$
[1.0, 6.0]	$1.36^{+0.23}_{-0.21} \pm 0.08$	$1.29 \pm 0.18 \pm 0.08$	$0.65^{+0.45}_{-0.35}$	$1.21 \pm 0.09 \pm 0.07$	$1.76^{+0.60}_{-0.23}$

Accessing $B \to V$ form factors

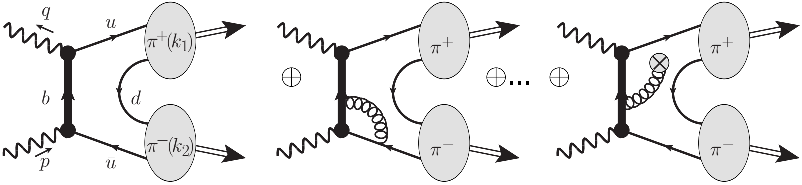
- $B \to \rho \ell \nu_\ell \Rightarrow |V_{ub}|$;
  $B \to K^* \ell^+ \ell^-$ used for angular analysis, search for NP
- LCSR's for $B \to V$ form factors
  [P. Ball, V.Braun (1998), P.Ball, R. Zwicky (2004,...)]
- $\Gamma_V = 0$ approximation ("quenched") $\Rightarrow$ additional uncertainty
- the problem is more general: $\rho(770)$ ($K^*(890)$)
  are strongly coupled to the P-wave of 2π ($K\pi$) state:
  - a simple constraint on the inv. mass of 2π or $K\pi$ may not be precise enough
  - scalar resonances in 2π ($K\pi$): a complete angular analysis is needed

$B \to 2P$ form factors

- disentangling $B \to \pi\pi\ell\nu_\ell$ in terms of $B \to \pi\pi$ form factors, including the partial expansion and resonance contributions;

  S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van Dyk, 1310.6660

- more work in this direction is on the way:
  LCSR's with 2-pion DA's



- the region: $q^2 \ll m_b^2$ (b-quark virtual)
  $k^2 \ll m_b^2$ (2-pion system produced near the LC)
- new nonperturbative input: timelike pion form factors

Conclusions

- LCSR's :
  - predict heavy-light form factors in the large recoil region
  - provide "soft" form factors and power suppressed terms for the QCD factorization theorems
  - accuracy at 10-15 % level; quark-hadron duality - an important issue
  - applications to nonlocal hadronic matrix elements in $b \to s\ell\ell$ exclusive transitions
- experiment can help to improve the accuracy of DA's:
  - $\pi\gamma^*\gamma$, $B \to \gamma\ell\nu_\ell$;
  - radial excitations of D and B mesons
- near future tasks:
  - $B \to 2\pi, K\pi$ form factors
  - nonlocal hadronic matrix elements for $B \to K\pi(K^*)\ell\ell$