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Status of B-mixing in the standard model and beyond

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Abstract. The theoretical methods underlying predictions of mixing quantities ΔM_q and $\Delta\Gamma_q$ are well understood nowadays. Currently the uncertainties are dominated by nonperturbative matrix elements of local $|\Delta B| = 2$ operators of dimension 6 and 7, which require preciser calculations, hopefully with lattice methods in the future. Global fits show good support of the standard model, restricting potential effects of new physics to be small. An improved measurement of $\Delta\Gamma_d$ will be helpful to further analyse the dimuon charge asymmetry as measured by DØ.

1. Introduction

The phenomenon of neutral B_q meson mixing ($q = d, s$) proceeds in the standard model (SM) only at the loop level and provides important constraints in the determination of elements of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, $V_{tb}V_{tq}^*$. Moreover, the loop suppression makes it sensitive to new sources of CP and flavour violation. It affects CP violation in all neutral B_q meson decays in the form of mixing-induced CP asymmetries. In the absence of reliable methods for the calculation of strong (CP-conserving) phases of decay amplitudes, mixing thus provides an important additional access to CP violating parameters.

The time evolution of the two flavour eigenstates B_q and $\overline{B}_q$ is described by

$$i\frac{d}{dt}\begin{pmatrix} |B_q(t)\rangle \\ |\overline{B}_q(t)\rangle \end{pmatrix} = \left(M^q - \frac{\Gamma^q}{2}\right)\begin{pmatrix} |B_q(t)\rangle \\ |\overline{B}_q(t)\rangle \end{pmatrix} . \tag{1}$$

Under the assumption of CPT-invariance, the elements of the 2×2 mass- and decay-matrices, M^q and Γ^q, fulfil the relations $M_{11}^q = M_{22}^q \equiv M_{B_q}$, $M_{12}^q = (M_{21}^q)^*$ and similarly for Γ^q. The diagonalisation yields the light and heavy mass eigenstates $|B_{L,H}\rangle = p|B\rangle \pm q|\overline{B}\rangle$ and their averaged masses, M_{B_q} and decay widths Γ_{B_q}. The according differences, ΔM_q and $\Delta\Gamma_q$, as well as the flavour-specific CP-asymmetries, $a_{\rm sl}^q$

$$\begin{aligned} \Delta M_q &= M_H^q - M_L^q = 2|M_{12}^q| + \ldots \geq 0\,, \\ \Delta\Gamma_q &= \;\Gamma_L^q - \Gamma_H^q \;= 2|\Gamma_{12}^q|\cos(\zeta_q) + \ldots \gtrless 0\,, \\ a_{\rm sl}^q &= \frac{\Gamma(\overline{B}_q \to f) - \Gamma(\overline{B}_q \to \bar{f})}{\Gamma(\overline{B}_q \to f) + \Gamma(\overline{B}_q \to \bar{f})} = \left|\frac{\Gamma_{12}^q}{M_{12}^q}\right| \sin(\zeta_q) + \ldots\,, \end{aligned} \tag{2}$$

Table 1. Compilation of B_q decay constants (left) and bag factors (right) from $N_f = 2+1$ lattice averages (see text for details) and the relative errors they induces in predictions of ΔM_q.

	[MeV]	$\delta(\Delta M_q)$
f_{B_s}	227.7 ± 4.5	4.0%
f_{B_d}	190.5 ± 4.2	4.4%

		$\delta(\Delta M_q)$
$\hat{B}_{B_s}$	1.33 ± 0.06	4.5%
$\hat{B}_{B_d}$	1.27 ± 0.10	7.9%

are determined by the off-diagonal elements M_{12}^q and Γ_{12}^q and their relative phase $\zeta_q = \arg(-M_{12}^q/\Gamma_{12}^q)$. Above the dots stand for terms of $\mathcal{O}(1/8|\Gamma_{12}^q/M_{12}^q|^2 \sin^2\zeta_q)$, which is smaller than 10^{-7} in the SM for both systems $q = d, s$ [1].

The structure of electroweak (EW) interactions in the SM allows for the construction of an effective field theory, in which the off-diagonal element M_{12}^q is given by $|\Delta B| = 2$ operators, whereas Γ_{12}^q by $|\Delta B| = 1$ current-current operators, which again can be matched on local $|\Delta B| = 2$ operators by means of heavy quark expansion (HQE).

2. Standard model

The prediction of M_{12}^q is well understood in the SM,

$$M_{12}^q = \frac{G_F^2 M_{B_q}}{12\pi^2}(V_{tb}V_{tq}^*)^2 m_W^2 S_0(x_t)\hat{\eta} B_{B_q} f_{B_q}^2 \,. \tag{3}$$

Contributions of the EW interaction at short-distance scales of the order of the W-boson mass, m_W, are decoupled and are contained at leading order (LO) in $S_0(x_t = m_t^2/m_W^2)$ [2] depending also on the top-quark mass m_t, at next-to-LO (NLO) QCD $\hat{\eta}$ [3], whereas tiny NLO EW corrections [4] are usually neglected. The accuracy of the determination of CKM elements is strongly limited by the uncertainties of the matrix element of a single local operator $\langle \overline{B}_q|(\bar{b}q)_{V-A}(\bar{b}q)_{V-A}|B_q\rangle = 8/3\, f_{B_q}^2 B_{B_q} M_{B_q}$ in terms of the B_q-meson decay constant, f_{B_q} and the associated bag parameter B_{B_q}. Both are nowadays calculated with the help of lattice QCD techniques — see [5] — and the current precision of latest world averages [6] ($N_f = 2+1$) are summarised in table 1, together with the arising uncertainty on ΔM_q. Since the small uncertainty of the decay constant is currently dominated by a single lattice calculation [7, 8], it would be reassuring to have confirmations from other lattice groups in the future. Beyond the SM, NP scenarios can give rise to additional $|\Delta B| = 2$ operators. The renormalisation group evolution of their Wilson coefficients from m_W to m_b are known up to NLO [9, 10] in QCD, whereas the according bag parameters are currently only available from the single unquenched lattice ($N_f = 2$) calculation [11] — for quenched results see [12] — and some preliminary ($N_f = 2+1$) results are reported in [13].

The calculation of Γ_{12}^q is based on HQE [14] giving rise to a double series in $\lambda = \Lambda_{\rm QCD}/m_b \sim 0.15$ and the strong coupling, α_s,

$$\Gamma_{12}^q = \lambda^3\left(\Gamma_3^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_3^{(1)} + \ldots\right) + \lambda^4\left(\Gamma_4^{(0)} + \ldots\right) + \lambda^5\left(\Gamma_5^{(0)} + \ldots\right) , \tag{4}$$

which is also applied to calculate of the total and partial inclusive decay widths [15]. The individual contributions shown in (4) can be found in [16, 17, 18, 19, 20, 21]. The expansion of $\Delta\Gamma^q$ (and also Γ^q) shows a convergent behaviour and predictions are in satisfactory agreement with measurements — see figure 1 for the B_s system. Especially lifetime ratios can be predicted quite precisely $\tau_{B_s}/\tau_{B_d} \in [0.996, 1.000]$ [1], since they are free of hadronic uncertainties, and agree well with the measured value 0.995 ± 0.006. Concerning $\Delta\Gamma_q$, current measurements [22] and the according SM predictions [1] are

$$\begin{aligned} \Delta\Gamma_d|_{\text{Exp}} &= (0.0059 \pm 0.0079)\,\text{ps}^{-1}\,, & \Delta\Gamma_s|_{\text{Exp}} &= (0.091 \pm 0.008)\,\text{ps}^{-1}\,, \\ \Delta\Gamma_d|_{\text{SM}} &= (0.0029 \pm 0.0007)\,\text{ps}^{-1}\,, & \Delta\Gamma_s|_{\text{SM}} &= (0.087 \pm 0.021)\,\text{ps}^{-1}\,. \end{aligned} \tag{5}$$

A comprehensive summary of theoretical uncertainties [1] shows the that largest sources are due to matrix elements of local $|\Delta B| = 2$ operators of dimension 6 and 7, about 5% and 17%, respectively. Especially bag factors of dim-7 operators are currently known only with low precision from sum rule calculations [23], but no dedicated lattice studies are pursued at the moment. Further, with most recent values of f_{B_q} the according relative uncertainty is the same as for ΔM_q given in table 1. Another large source of uncertainty is the factorisation scale dependence on $\mu_b \sim m_b$ with about 8%.

Whereas experimental uncertainties are below theoretical ones for $\Delta\Gamma_s$, $\Delta\Gamma_d$ is not well known yet. A better measurement of the latter can also help to further investigate the deviation of the dimuon charge asymmetry measured by DØ [24]. Although the interpretation of this measurement within the SM has been refined lately [25]

$$A_{\text{CP}} = C_d\, a_{\text{sl}}^d + C_s\, a_{\text{sl}}^s + C_{\Gamma_d} \frac{\Delta\Gamma_d}{\Gamma_d} + C_{\Gamma_s} \frac{\Delta\Gamma_s}{\Gamma_s}\,, \tag{6}$$

it still points towards large deviations of about 3.6σ from the SM predictions of a_{sl}^q and/or $\Delta\Gamma_d$ [24, 26]. The numerical values of the coefficients C_{d,s,Γ_d} can be extracted from [25, 24] and C_{Γ_s} turns out to be negligible.

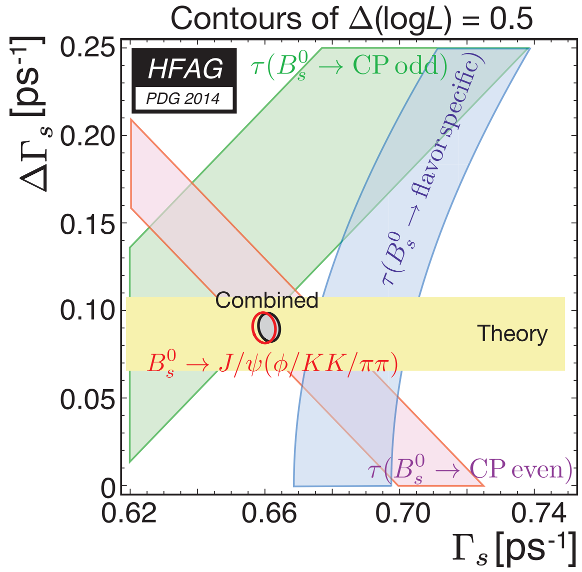

Figure 1. Combined region of $\Delta\Gamma_s$ vs. Γ_s from different channels. The ($B_s \to$ CP-odd) decays allow to measure directly Γ_H^s, whereas ($B_s \to$ CP-even) decays Γ_L^s. Also shown the SM prediction of $\Delta\Gamma_s$ [1]. For comparison, a "prediction" of $\Gamma_s = (0.660 \pm 0.004)$ ps^{-1} might be obtained when using the prediction of τ_{B_s}/τ_{B_d} [1], and the current measured world average of $\tau_{B_d} = (1.519 \pm 0.005)$ ps [22, 26].

3. Beyond the standard model

Effects of NP can be model-independently parametrised with two complex-valued parameters Δ_q and $\tilde{\Delta}_q$ as

$$\begin{aligned} M_{12}^q &= M_{12}^{q,\text{SM}}\, \Delta_q\,, & \Delta_q &= |\Delta_q|\, e^{i\phi_q^\Delta}\,, \\ \Gamma_{12}^q &= \Gamma_{12}^{q,\text{SM}}\, \tilde{\Delta}_q\,, & \tilde{\Delta}_q &= |\tilde{\Delta}_q|\, e^{i\tilde{\phi}_q^\Delta}\,. \end{aligned} \tag{7}$$

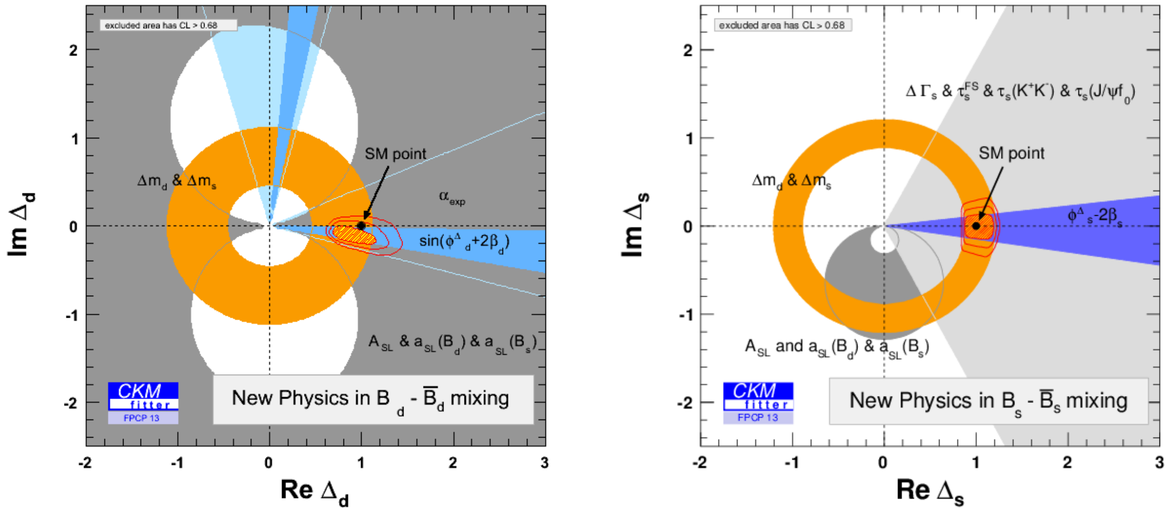

Figure 2. The allowed regions of the NP parameters Δ_d (left) and Δ_s (right), as determined in a global CKM-fit [27].

The global fits of the CKM elements in the framework of the SM take into account also ΔM_q. Beyond the SM, recent constraints on the scenario of NP only in M_{12}^q, i.e. $\Delta_d \neq \Delta_s \neq 0$ and also permitting NP independently in the $K\bar{K}$-system, are shown in figure 2. A 1.5σ deviation is obtained for the 2-dimensional SM hypothesis $\Delta_d = 1$ and 0.0σ for $\Delta_s = 1$. The SM point $\Delta_d = \Delta_s = 1$ is disfavoured by 1σ, compared to the 3.6σ in the year 2010 [28]. The highest pull value of 3.4σ arises for $A_{\rm CP} = C_d\, a_{\rm sl}^d + C_s\, a_{\rm sl}^s$, i.e. setting $C_{\Gamma_d} = 0$. Large deviations from the SM are by now excluded, whereas at 1σ CL deviations of $\mathcal{O}(40\%)$ and $\mathcal{O}(20\%)$ are still allowed

$$\begin{aligned} |\Delta_d| &= 0.81^{+0.27}_{-0.10}\,, & \phi_d^\Delta &= \left(-7.9^{+5.0}_{-2.3}\right)^\circ\,, \\ |\Delta_s| &= 0.97^{+0.20}_{-0.08}\,, & \phi_s^\Delta &= \left(-0.3^{+5.1}_{-5.3}\right)^\circ\,. \end{aligned} \tag{8}$$

An important role in this analysis plays the LHCb measurement of mixing-induced CP asymmetry $S(B \to J/\psi\phi)$ [29, 30], which does not permit an explanation of the DØ result for $A_{\rm CP}$ in terms of NP in M_{12}^s. Instead, from the best fit point of M_{12}^d follows the prediction $a_{\rm sl}^d = (-2.46^{+0.63}_{-0.45}) \cdot 10^{-3}$, which is enhanced by a factor of almost 8 over the SM prediction [1]. Improved determinations of $a_{\rm sl}^d$ at LHCb and Belle II are needed to further elucidate this issue.

In a more general NP scenario, also Γ_{12}^q might be modified. Whereas Γ_{12}^s is determined in the SM by Cabibbo-favoured tree-level exchange $b \to c\bar{c}s$ that gives rise to a large inclusive decay rate $Br(b \to c\bar{c}s) = (23.7 \pm 1.3)\%$, Γ_{12}^d is dominated by $b \to c\bar{c}d$, which leads only to small $Br(b \to c\bar{c}d) = (1.31 \pm 0.07)\%$ [31]. Therefore larger deviations from the SM prediction in Γ_{12}^d are at present less constrained, compared to Γ_{12}^s [32, 33]. Modifications of Γ_{12}^d affect both $\Delta\Gamma_d$ and $a_{\rm sl}^d$, being presently not well measured and thus potentially alleviating current tensions in $A_{\rm CP}$.

Some possible NP scenarios include violations of CKM unitarity [34, 33], large effects in almost unconstrained channels like $b \to \tau\bar{\tau}d$ or modifications of the current-current operators $b \to c\bar{c}d$, $c\bar{u}d$, $u\bar{u}d$ [33]. The latter two scenarios have been investigated in a model-independent fashion based on $|\Delta B| = 1$ dim-6 operators [33]. Concerning $b \to \tau\bar{\tau}d$, model-independent constraints on the complete set of dim-6 operators $O_i \sim [\bar{d}\Gamma_A b][\bar{\tau}\Gamma_B \tau]$ ($\Gamma_A = P_A$, $\gamma_\mu P_A, \sigma_{\mu\nu} P_A$

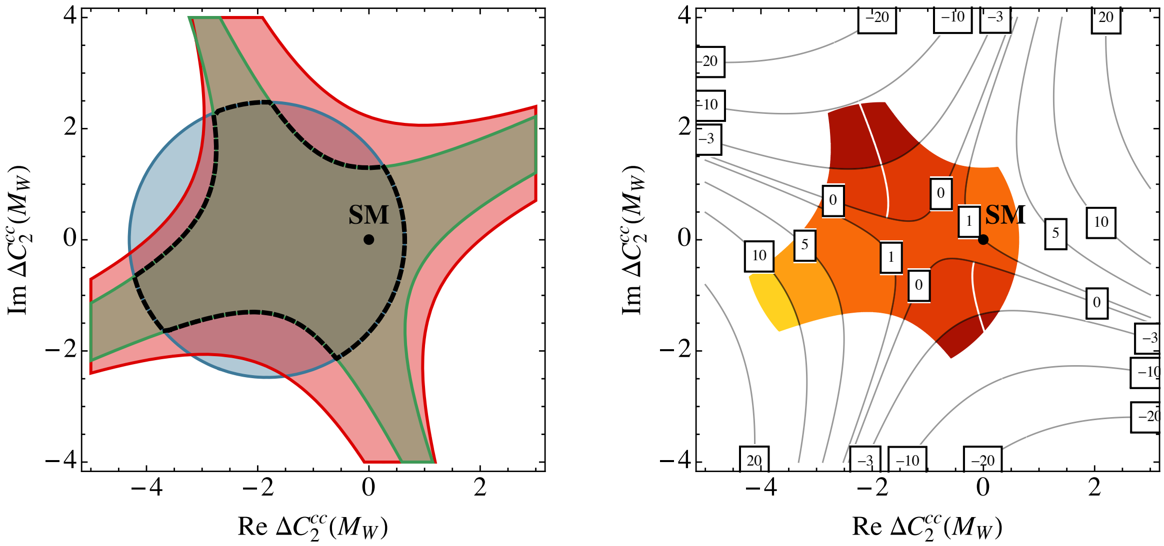

Figure 3. The allowed regions for the NP contributions of the colour-allowed current-current $b \to \tau\bar{\tau}d$ operator [left] from $Br(B \to X_d\gamma)$ (blue, circle-shaped), $a_{\rm sl}^d$ (green, inner star-shaped) and dim-8 contributions to $\sin(2\beta)$ (red, outer star-shaped) [33]. Contours in $\Delta\Gamma_d/\Delta\Gamma_d^{\rm SM}$ [right].

with $P_A = (1\pm\gamma_5)/2$) have been derived on the according Wilson coefficients. In the case of scalar Wilson coefficients direct bounds from $Br(B_d \to \bar{\tau}\tau)$ can not preclude enhancements of about 60% of $\Delta\Gamma_d$. Only indirect constraints from $Br(B \to X_d\gamma)$ and $Br(B^+ \to \pi^+\bar{\tau}\tau)$ are available on vector Wilson coefficients, which can enhance $\Delta\Gamma_d$ by about 270% over the SM prediction. In the case of current-current operators, NP in colour-suppressed $O_1^{pp'} = [\bar{d}^\alpha\gamma^\mu P_L p^\beta][\bar{p}'^\beta\gamma_\mu P_L b^\alpha]$ and colour-allowed $O_2^{pp'} = [\bar{d}\gamma^\mu P_L p][\bar{p}'\gamma_\mu P_L b]$ operators have been assumed, where $pp' = uu, cu, cc$. In the SM the Wilson coefficients $C_i^{pp'} = V_{pd}^* V_{p'b} C_i$ are universal, once factoring out CKM elements. For $b \to (u\bar{u}, c\bar{u})d$, the experimental constraints from $B \to \pi\pi$, $\rho\pi$, $\rho\rho$ and $B \to D^*\pi$, respectively, allow for $\Delta\Gamma_d/\Delta\Gamma_d^{\rm SM} \in [-1.0, 1.4]$, however for $b \to c\bar{c}d$ huge effects of several 100% can not be ruled out, see figure 3.

4. Conclusion

The mixing of neutral B_q mesons ($q = d, s$) plays an important role in CP violating phenomena, tests of the picture of quark mixing in the standard model (SM) and search for new physics (NP). Currently the largest uncertainties in predictions of ΔM_q and $\Delta\Gamma_q$ are due to hadronic parameters: the decay constants f_{B_q} and bag factors of various $|\Delta B| = 2$ operators. There is encouraging progress in lattice predictions concerning ΔM_q in the standard model, dominated by f_{B_q} and a single bag factor. However, NP searches require the knowledge of the bag factors of all $|\Delta B| = 2$ operators of dimension 6, which receives currently much less attention. Moreover, $\Delta\Gamma_q$ also depends strongly on bag factors of $|\Delta B| = 2$ operators of dimension 7 that cause at present the largest uncertainties, but for which so far exist only sum rule estimates.

Recent global CKM fits that account for NP in M_{12}^q only, exclude now larger deviations then $\mathcal{O}(20\%)$ and $\mathcal{O}(40\%)$ in the B_s and B_d system from the SM, respectively. Only the DØ measurement of the like-sign dimuon charge asymmetry has a pull value of about 3σ. This tension could be caused by non-standard contributions to Γ_{12}^d for which at present, large NP contributions can not be excluded. A direct measurement of $\Delta\Gamma_d$, more precise data on the individual semi-leptonic CP asymmetries $a_{\rm sl}^{d,s}$, but also improved experimental determinations of rare and radiative $b \to d$ decays would shed light on this issue and should be pursued with vigour in the future.

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