CHARMLESS HADRONIC B DECAYS: THEORY STATUS

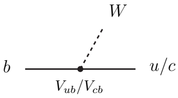
[GUIDO BELL]

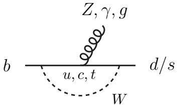
UNIVERSITY OF OXFORD

THE ROYAL SOCIETY

Hadronic B decays

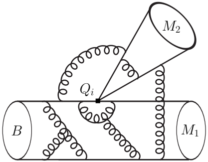
Excellent laboratory to probe the nature of flavour-changing quark transitions





- tree-dominated decays: $B \to \pi\pi$, $B \to \rho\rho$, $B_s \to \pi K$, ...
- penguin-dominated decays: $B \to \pi K$, $B \to \phi K_S$, $B_s \to \phi\phi$, ...

The challenge:



Two strategies:

- flavour symmetries:

  isospin, SU(3), U-spin, V-spin
- heavy-quark expansions:

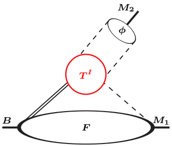
  QCDF, SCET, pQCD

QCD factorisation

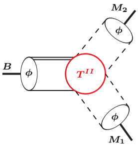
[Beneke, Buchalla, Neubert, Sachrajda 99]

Hadronic matrix elements factorise in heavy quark limit $m_b \gg \Lambda_{QCD}$

$$\langle M_1 M_2 | Q_i | B \rangle \simeq F^{BM_1}(0) \int du \, T_i^I(u) \, \phi_{M_2}(u) + \int d\omega \, du \, dv \, T_i^{II}(\omega, u, v) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$



+



vertex corrections $T^I = 1 + \mathcal{O}(\alpha_s)$ spectator scattering $T^{II} = \mathcal{O}(\alpha_s)$

- valid to all orders in $\alpha_s(m_b)$ and to leading power in Λ_{QCD}/m_b
- strong phases from final-state interactions $\sim \mathcal{O}(\alpha_s), \; \mathcal{O}(1/m_b)$

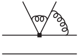
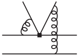
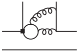
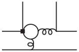
$\Rightarrow$ NNLO is first correction for direct CP asymmetries!

NNLO calculation in QCDF

Two hard-scattering kernels for each operator insertion

$$\langle M_1 M_2 | Q_i | B \rangle \;\simeq\; F^{BM_1}\, T_i^I \otimes \phi_{M_2} \;+\; T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

and two classes of topological amplitudes (trees vs. penguins)

Status	2-loop vertex corrections (T_i^I)	1-loop spectator scattering (T_i^{II})
Trees	 [GB 07, 09] [Beneke, Huber, Li 09]	 [Beneke, Jäger 05] [Kivel 06] [Pilipp 07]
Penguins	 [in progress]	 [Beneke, Jäger 06] [Jain, Rothstein, Stewart 07]

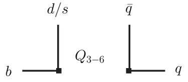
- first NNLO results for tree-dominated observables [GB, Pilipp 09; Beneke, Huber, Li 09]
- no NNLO results for direct CP asymmetries yet
- power-suppressed scalar penguin amplitude known at NLO [BBNS 01]

Missing NNLO ingredient

Various contributions to up/charm QCD penguin amplitudes

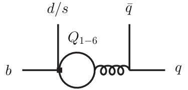
- tree insertions of penguin operators

  2-loop, similar to tree calculation



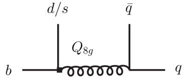
- penguin insertions of current-current and penguin operators

  2-loop, charm quark propagator introduces additional scale



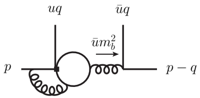
- insertions of magnetic dipole operator

  1-loop, much simpler [Kim, Yoon 11]



$\mathcal{O}(70)$ diagrams at NNLO

- 2 loops, 3 scales (m_b, um_b, m_c), 4 legs
- charm contribution has non-trivial threshold at $\bar{u}m_b^2 \gtrsim 4m_c^2$



2-loop penguin contribution

[GB, Beneke, Huber, Li in progress]

Automated reduction to scalar master integrals (IBP, Laporta)

$\Rightarrow$ $\mathcal{O}(20)$ additional master integrals compared to 2-loop tree calculation

New proposal to choose "optimal" basis of master integrals [Henn 13]

- simple iterated integrations in each order in ε-expansion
- no systematic construction of such a basis exists so far

$\Rightarrow$ found optimal basis for all master integrals

Calculation of master integrals

- analytic results in terms of Goncharov polylogarithms
- two independent numerical implementations for cross checks

  Mellin-Barnes + sector decomposition + Feynman parameters $\Rightarrow$ agreement $\sim 10^{-4}$

Status of calculation

[GB, Beneke, Huber, Li in progress]

Various operator insertions:

- $Q^u_{1,2}$: fully analytic results
- $Q^c_{1,2}$: analytic results for kernels $T^I_i(u)$, numerical results for $\int du\; T^I_i(u)\,\phi_{M_2}(u)$
- $Q_{3\text{-}6}$: 2-loop matrix elements complete, working on UV and IR subtractions . . .

Preliminary result for up penguin amplitude

$$\begin{aligned} a_4^u(\pi\pi) &= -\,[\,0.029\,]_{V_0} \;+\; [\,0.003 - 0.014\,i\,]_{V_1} \;-\; [\,0.003 + 0.007\,i\,]_{V_2} \\ &\quad +\,[\,0.001\,]_{S_1} \;+\; [\,0.001 + 0.002\,i\,]_{S_2} \;+\; [\,0.001\,]_{1/m_b} \\ &= -\,0.025 \;-\; 0.019\,i \end{aligned}$$

- includes 2-loop contribution from $Q^u_{1,2}$ only
- $\sim$ 15% correction to real part, $\sim$ 50% to imaginary part
- $Q^c_{1,2}$ contribution to charm penguin amplitude almost done

Tree amplitudes

Full NNLO results for colour-allowed and colour-suppressed tree amplitudes

$$\begin{aligned}\alpha_1(\pi\pi) &= [\,1.008\,]_{V_0} + [\,0.022 + 0.009\,i\,]_{V_1} + [\,0.024 + 0.026\,i\,]_{V_2}\\ &\quad - [\,0.014\,]_{S_1} - [\,0.016 + 0.012\,i\,]_{S_2} - [\,0.008\,]_{1/m_b}\\ &= 1.015^{+0.020}_{-0.029} + (0.023^{+0.015}_{-0.015})\,i\end{aligned}$$

$$\begin{aligned}\alpha_2(\pi\pi) &= [\,0.224\,]_{V_0} - [\,0.174 + 0.075\,i\,]_{V_1} - [\,0.029 + 0.046\,i\,]_{V_2}\\ &\quad + [\,0.084\,]_{S_1} + [\,0.037 + 0.022\,i\,]_{S_2} + [\,0.052\,]_{1/m_b}\\ &= 0.194^{+0.130}_{-0.095} - (0.099^{+0.057}_{-0.056})\,i\end{aligned}$$

- individual NNLO corrections significant, but cancellations in the sum
- α_1: stable under radiative corrections, precise prediction
- α_2: sizeable hadronic uncertainties, mainly from $\lambda_B^{-1} = \int \frac{d\omega}{\omega}\,\phi_B(\omega)$
- small relative phase between α_1 and α_2

$B \to \pi\pi/\pi\rho/\rho\rho$

[GB, Pilipp 09]

CP-averaged branching ratios in units of 10^{-6}

Mode	QCDF	B	Experiment
$\pi^-\pi^0$	$6.22^{+2.37}_{-2.01}$	5.46	$5.59^{+0.41}_{-0.40}$
$\rho_L^-\rho_L^0$	$21.0^{+8.5}_{-7.3}$	21.3	$22.5^{+1.9}_{-1.9}$
$\pi^-\rho^0$	$9.34^{+4.00}_{-3.23}$	10.4	$8.3^{+1.2}_{-1.3}$
$\pi^0\rho^-$	$15.1^{+5.7}_{-5.0}$	11.9	$10.9^{+1.4}_{-1.5}$
$\pi^+\pi^-$	$8.96^{+3.78}_{-3.32}$	5.21	$5.16^{+0.22}_{-0.22}$
$\pi^0\pi^0$	$0.35^{+0.37}_{-0.21}$	0.63	$1.55^{+0.19}_{-0.19}$
$\pi^+\rho^-$	$22.8^{+9.1}_{-8.0}$	13.2	$15.7^{+1.8}_{-1.8}$
$\pi^-\rho^+$	$11.5^{+5.1}_{-4.3}$	8.41	$7.3^{+1.2}_{-1.2}$
$\pi^\pm\rho^\mp$	$34.3^{+11.5}_{-10.0}$	21.6	$23.0^{+2.3}_{-2.3}$
$\pi^0\rho^0$	$0.52^{+0.76}_{-0.42}$	1.64	$2.0^{+0.5}_{-0.5}$
$\rho_L^+\rho_L^-$	$30.3^{+12.9}_{-11.2}$	22.3	$23.6^{+3.2}_{-3.2}$
$\rho_L^0\rho_L^0$	$0.44^{+0.66}_{-0.37}$	1.33	$0.69^{+0.30}_{-0.30}$

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_B \to \lambda_B/2$ and smaller form factors)

- theory uncertainties highly correlated (form factors, $|V_{ub}|$)
- colour-suppressed modes $\pi^0\pi^0/\pi^0\rho^0/\rho^0\rho^0$ uncertain (λ_B and $1/m_b$)
- overall preference for enhanced colour-suppressed amplitude

[for a similar analysis cf. Beneke, Huber, Li 09]

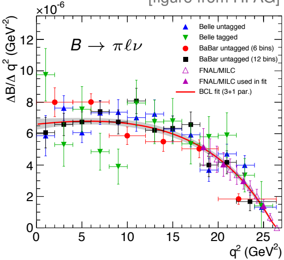
Semileptonic ratios

Can eliminate dependence on form factors and $|V_{ub}|$ via

$$\mathcal{R}_{M_3}(M_1 M_2) = \frac{\Gamma(B \to M_1 M_2)}{d\Gamma(B \to M_3 \ell \nu)/dq^2|_{q^2=0}}$$

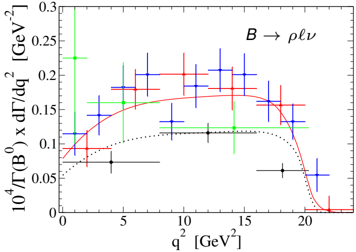
$\Rightarrow$ requires to measure semileptonic decay spectrum and extrapolation to $q^2 = 0$

[figure from HFAG]



$|V_{ub}|F_+^{B\pi}(0)$ from combined data?

[figure from Albertus et al 14]



extraction of $|V_{ub}|A_0^{B\rho}(0)$ possible?

Semileptonic ratios

[GB, Pilipp 09]

Can eliminate dependence on form factors and $|V_{ub}|$ via

$$\mathcal{R}_{M_3}(M_1 M_2) = \frac{\Gamma(B \to M_1 M_2)}{d\Gamma(B \to M_3 \ell\nu)/dq^2|_{q^2=0}}$$

Mode	QCDF	B	Experiment
$\mathcal{R}_\pi(\pi^- \pi^0)$	$0.70^{+0.12}_{-0.08}$	0.95	$0.81^{+0.14}_{-0.14}$
$\mathcal{R}_\rho(\rho_L^- \rho_L^0)$	$1.91^{+0.32}_{-0.23}$	2.38	n.a.
$\mathcal{R}_\rho(\pi^- \rho^0)$	$0.85^{+0.22}_{-0.14}$	1.16	n.a.
$\mathcal{R}_\pi(\pi^0 \rho^-)$	$1.71^{+0.27}_{-0.24}$	2.07	$1.57^{+0.32}_{-0.32}$
$\mathcal{R}_\pi(\pi^+ \pi^-)$	$1.09^{+0.22}_{-0.20}$	0.97	$0.80^{+0.13}_{-0.13}$
$\mathcal{R}_\pi(\pi^+ \rho^-)$	$2.77^{+0.32}_{-0.31}$	2.46	$2.43^{+0.47}_{-0.47}$
$\mathcal{R}_\rho(\pi^- \rho^+)$	$1.12^{+0.20}_{-0.14}$	1.01	n.a.
$\mathcal{R}_\rho(\rho_L^+ \rho_L^-)$	$2.95^{+0.37}_{-0.35}$	2.68	n.a.
$R(\rho_L^- \rho_L^0 / \rho_L^+ \rho_L^-)$	$0.65^{+0.16}_{-0.11}$	0.89	$0.89^{+0.14}_{-0.14}$
$R(\pi^- \pi^0 / \pi^+ \pi^-)$	$0.65^{+0.19}_{-0.14}$	0.98	$1.01^{+0.09}_{-0.09}$

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_B \to \lambda_B/2$ and smaller form factors)

- theory uncertainties largely reduced
- satisfactory description of clean observables
- based on old extraction
  $|V_{ub}| F_+^{B\pi}(0) = (9.1 \pm 0.7) \cdot 10^{-4}$
  [Ball 06]

Tree-dominated B_s decays

[GB, CKM 2010]

CP-averaged branching ratios in units of 10^{-6}

Mode	QCDF	B	Experiment
$\pi^- K^+$	$8.73^{+5.77}_{-4.60}$	4.88	$5.4^{+0.6}_{-0.6}$
$\pi^0 K^0$	$0.50^{+0.71}_{-0.35}$	1.12	n.a.
$\pi^- K^{*+}$	$15.4^{+8.6}_{-7.0}$	11.0	n.a.
$\pi^0 K^{*0}$	$0.39^{+0.58}_{-0.26}$	0.90	n.a.
$\rho^- K^+$	$22.4^{+14.7}_{-11.6}$	12.5	n.a.
$\rho^0 K^0$	$0.73^{+1.28}_{-0.58}$	2.24	n.a.
$\rho_L^- K_L^{*+}$	$40.7^{+22.4}_{-18.3}$	29.1	n.a.
$\rho_L^0 K_L^{*0}$	$0.70^{+1.07}_{-0.54}$	1.87	n.a.
$\rho^- K^+ / \pi^- K^+$	$2.57^{+0.31}_{-0.26}$	2.57	n.a.
$\rho_L^- K_L^{*+} / \pi^- K^{*+}$	$2.64^{+0.31}_{-0.33}$	2.65	n.a.

B: mimics enhanced colour-suppressed amplitude
(with $\lambda_{B_s} \to \lambda_{B_s}/2$ and smaller form factors)

- hadronic parameters less well known (form factors, λ_{B_s})
- simpler pattern of annihilation contributions
- clean ratios of colour-allowed modes can be used to test charming penguins [Zhu 10]

B-meson LCDA

Recent progress in theoretical understanding of $\phi_B(\omega;\mu)$

- expansion in eigenfunctions of 1-loop evolution kernel [GB, Feldmann, Wang, Yip 13]
- formulation in terms of conformal symmetry generators [Braun, Manashov 14]
- consistent implementation of perturbative constraints [Feldmann, Lange, Wang 14]

Most important for phenomenology $\lambda_B^{-1}(\mu) = \int \frac{d\omega}{\omega}\,\phi_B(\omega;\mu)$

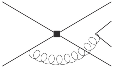
- QCD sum rule estimate $\lambda_B(1\text{GeV}) \simeq (460 \pm 110)$ MeV [Braun, Ivanov, Korchemsky 03]
- but $\pi\pi/\pi\rho/\rho\rho$ branching ratios seem to prefer $\sim$ 200 MeV ?

λ_B can be measured in $B \to \gamma\ell\nu$ decays

- state-of-the-art analyses (NLL, tree-level $1/m_b$) [Beneke, Rohrwild 11; Braun, Khodjamirian 12]
- Babar 09 data $\Rightarrow$ $\lambda_B(1\text{GeV}) > 115$ MeV

Weak annihilation

Power-suppressed annihilation amplitudes introduce model dependence



$$\Rightarrow \qquad X_A = \left(1 + \rho_A \, e^{i\phi_A}\right) \ln \frac{m_B}{\Lambda_h} \qquad \text{IR-cutoff } \; \Lambda_h = 0.5 \text{ GeV}$$

Two-parameter model: $\rho_A \leq 1$ and arbitrary universal soft-rescattering phase ϕ_A

Can gain insights from pure annihilation decays

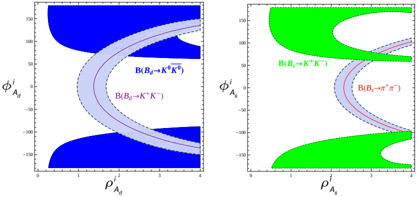
	BBNS (S4)	
$10^6 \, \text{Br}(B_d \to K^+ K^-) = 0.12 \pm 0.05$	0.07	($\Delta D = 1$, exchange topology)
$10^6 \, \text{Br}(B_s \to \pi^+ \pi^-) = 0.73 \pm 0.14$	0.16	($\Delta S = 1$, penguin annihilation)

$\Rightarrow$ challenges universal annihilation model

global $\pi\pi/\pi K/KK$ analysis gives

$\rho^f_A \sim 1.6, \; \rho^i_{A_d} \sim 2.5, \; \rho^i_{A_s} \sim 3.0$

[Wang, Zhu 13]



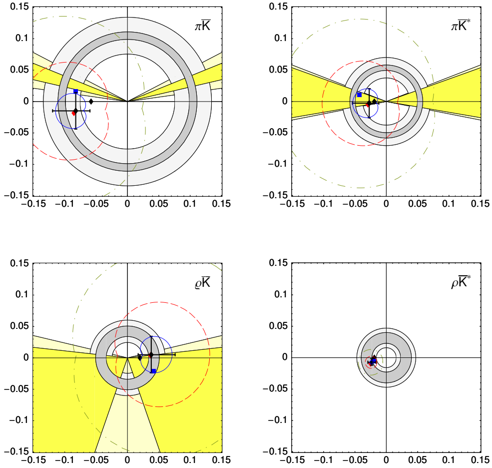
Scalar penguin amplitude

Calculable power correction from insertion of Fierz-transformed penguin operators $\Rightarrow$ $a_6^p(M_1 M_2)$

Distinct final state dependence

- PP $\sim$ $a_4^p + r_\chi \, a_6^p$
- PV $\sim$ a_4^p
- VP $\sim$ $a_4^p - r_\chi \, a_6^p$
- VV $\sim$ a_4^p

$\Rightarrow$ pattern of P/T ratios in good agreement with data



[figures from Beneke, Jäger 06]

Conclusion

- We are about to complete the NNLO calculation in hadronic B decays –
  phenomenlogocial update of all $B \to PP/PV/VV$ observables largely overdue

- Lattice and QCD sum rule calculations provide valuable input for our predictions –
  some hadronic parameters can also be constrained by experimental measurements

- Main limitation of QCDF is poor understanding of power corrections –
  improved modelling based on flavour symmetries possible

Backup slides

Comparison

[BBNS: Beneke, Buchalla, Neubert, Sachrajda 99]
[BPRS: Bauer, Pirjol, Rothstein, Stewart 04]
[pQCD: Keum, Li, Sanda 00]

	BBNS (QCDF)	BPRS (SCET)	pQCD
$\alpha_s(\sqrt{\Lambda m_b})$	perturbative	non-perturbative	perturbative
charm loops	perturbative (small phase)	non-perturbative (large phase from fit to data)	perturbative (small phase)
weak annihilation (power correction)	non-perturbative (model, arbitrary phase)	perturbative (zero bins, small phase)	perturbative (large phase)
perturbative calculation	towards NNLO	NLO	essentially LO
hadronic input	from lattice + QCD sum rules	from QCD sum rules + data, model $\xi_J^{BM}(z)$	from QCD sum rules + data, model $\phi_B(x, b)$

- conceptually QCDF $=$ SCET $\neq$ pQCD

  but phenomenological implementations of BBNS and BPRS differ

- some open problems in BPRS (zero-bin subtractions) and pQCD (glauber gluons)