Charmonium with an Effective Morse Molecular Potential

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OUTLINE OF MY TALK

1) DESPITE THE LONG EFFORTS ALONG MORE THAN 40 YEARS, CHARMONIUM SURPRISES EVERYBODY: THERE ARE NEW STATES NOT WELL DEFINED LINKED TO CHARMONIUM

2) COMPLETELY NEW APPROACH: TO TREAT CHARMONIUM AS A MOLECULE OF c AND $\bar{c}$ AND USE THE MORSE MOLECULAR POTENTIAL AS AN EFFECTIVE POTENTIAL

3) FITTING THE DATA TO THE POTENTIAL

4) DISCUSSION OF RESULTS THAT UNVEILS PART OF THE PUZZLE CONCERNING THE NEW STATES.

5) CONCLUSION

1) DESPITE THE LONG EFFORTS ALONG MORE THAN 40 YEARS, CHARMONIUM SURPRISES EVERYBODY: THERE ARE NEW STATES NOT WELL DEFINED LINKED TO CHARMONIUM

The efforts for the understanding of charmonium began in 1975 with this famous paper

E. Eichten *et al.*, Phys. Rev. Lett. **34**, 369 (1975).

Since 1975 many other papers have been published on charmonium. A set of important papers on this system can be found in the reference http://inspirehep.net/record/129333/references.

The new states not well understood are these states (from PDG)

$\chi(3940)$, $\Psi(4040)$, $\chi(4050)$, $\chi(4140)$, $\Psi(4160)$, $Y(4260)$, $Y(4350)$, $\Psi(4415)$, $\chi(4430)$, and $\chi(4660)$.

2) THE MORSE MOLECULAR POTENTIAL

The Morse molecular potential is used as an effective potential for describing energy levels of diatomic molecules

$$V(r) = D\left(e^{-2\alpha x} - 2e^{-\alpha x}\right)$$

where $-D$ is the minimum of the potential and $x = (r-a)/a$ in which a is the position where $V(r)$ is minimum, and α is a parameter to be found with the fitting.

The potential takes care, effectively, of the attractive QCD force and of the well-known repulsion for very small distances.

For $|x| < 1$ this potential can be expanded about the minimum up to order 3 in x and yields the expression

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda ka^3x^3$$

where $\lambda = \alpha / 2a$.

For this potential the solution of the Schrödinger equation yields the expression for the energy levels

$$E_{nl} = \hbar\omega\left(\upsilon+\frac{1}{2}\right) - A\left(\upsilon+\frac{1}{2}\right)^2 + B_L L(L+1)$$
$$-D_L L^2(L+1)^2 - C_{\upsilon L}\left(\upsilon+\frac{1}{2}\right)L(L+1)+...$$

in which the quantum numbers $\upsilon, L = 0,1,2,3,...$ and where ω and D are related to α, a and m by

$$\omega^2 = \frac{2\alpha^2 D}{ma^2}.$$

In this above expression m is the reduced mass of the constituent quark and antiquark, that is, $m = (1/2)M_c$ in which M_c is the mass of the c quark. The constant B_L is given by $B_L = \hbar^2 / 2ma^2$.

3) THE FITTING

We use the S states $J/\Psi(1S)$ and $\Psi(2S) = \Psi(3686)$, the P states $\chi_c(1P)(\chi_{c0}(1P)$, $\chi_{c1}(1P)$ and $\chi_{c2}(1P))$ and the D state $\Psi(3770)$. In the case of $\chi_c(1P)$ states we take out the spin-orbit interaction contribution. The levels used in the fitting are shown below

Table 1. The states used in the fitting

State (υ, L)	$n^{2S+1}L_J$	Particle	Mass (MeV/c^2)
$(0,0)$	1^3S_1	$J/\Psi(1S)$	3096.916 ± 0.011
$(0,1)$	1^3P	$\chi_c(1P)$	3549.7 ± 37.9
$(1,0)$	2^3S_1	$\Psi(3686)$	3686.09 ± 0.04
$(0,2)$	1^3D_1	$\Psi(3770)$	3778.1 ± 1.2

4. RESULTS AND DISCUSSION

The fitting yields the following values in MeV: $\hbar\omega = 8062.0 \pm 0.1$, $A = 3736.4 \pm 0.1$, $B_L = 282.8 \pm 58.3$, $D_L = 28.3 \pm 5.1$, $D = 4348 \pm 0.5$, $a = (0.28 \pm 0.05)$, $\alpha = 2.15 \pm 0.39$.

This means that there is no charmonium bound state above 4348.8 ± 0.5 MeV.

This is a very important result and shows that the recently found states $\Psi(4415)$, $\chi(4430)$, and $\chi(4660)$ are not bound states of charmonium. On the other hand this result shows that the recently found states $\Psi(4160)$, $\chi(4140)$, $\chi(4050)$, $\Psi(4040)$, $\chi(3940)$, $Y(4260)$, and $Y(4350)$ can be bound states of charmonium.

4.1 The Radii of some Charmonium States

As it was shown above the Morse potential, when expanded about its minimum, yields

$$V(x) = -D + \frac{1}{2}ka^2x^2 - \lambda ka^3x^3$$

For such a potential Robinett obtained the following equation for the average value of position for S states

$$\langle r \rangle_{\upsilon} = a + \frac{3\alpha \hbar\omega}{2m\omega^2 a}\left(\upsilon + \frac{1}{2}\right) = a + \frac{3a\hbar\omega}{4\alpha D}\left(\upsilon + \frac{1}{2}\right)$$

where $\upsilon = 0,1,2,3,....$ We can identify these average values with the radii of charmonium states. We only calculate the radii of the states $J/\Psi(1S)$, $\eta_c(1S)$, $\Psi(3686)$, $\eta_c(2S)$, and $\Psi(3S)$ because the other upper states are far from equilibrium. Using the above values for the constants we obtain the results shown in Table 2 for the radii of three states of charmonium.

Table 2. Radii of some charmonium states.

State (υ, L)	Particle	Radius (fm)
$(0,0)$	$J/\Psi(1S)$, $\eta_c(1S)$	0.35 ± 0.06
$(1,0)$	$\Psi(3686)$, $\eta_c(2S)$	0.49 ± 0.09
$(2,0)$	$\Psi(3S)$	0.63 ± 0.11

5. CONCLUSION

The fitting of some energy levels of charmonium to the Morse potential makes possible the calculation of parameters of the effective molecular potential, prediction of the radii of some states and sheds some light on the character of the states $\chi(3940)$, $\Psi(4040)$, $\chi(4050)$, $\chi(4140)$, $\Psi(4160)$, $Y(4260)$, $Y(4350)$, $\Psi(4415)$, $\chi(4430)$, and $\chi(4660)$.

Therefore, the above results add important information for the understanding of charmonium.

REFERENCES

The long effort for understanding charmonium

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The Morse potential is well discussed in

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THANK YOU!

ANSWER TO POSSIBLE QUESTION ON SPIN CONTRIBUTIONS

As the Hamiltonian (with the molecular potential) does not depend on spin we can fit a set of ^3S or ^3P levels in a sequence because we expect to have the same kind of potential well for spin zero and spin one states, shifted accordingly. In the case of P states we take out the contribution of the spin-orbit interaction.

Differences in mass for some states due to the spin-spin interaction (in MeV)

$J/\Psi(1S) - \eta_c = 1^3S_1 - 1^1S_0 \approx 3096.9 - 2980.3 \approx 116.6$ (116.9 is just 3.765% of 3096.9)

$\Psi(3686) - \eta_c(2S) \approx 2^3S_1 - 2^1S_0 \approx 3686 - 3637 \approx 49$ (49 is just 1.33% of 3686)

It is also interesting to notice that

$\Psi(3686) - J/\Psi(1S) \approx 3686 - 3097 \approx 589$ (Spin 1)

$\eta_c(2S) - \eta_c(1S) \approx 3637 - 2980 \approx 657$

The difference 657-589=68 is only about 2.2% of 3096.9.

Therefore, doing the fitting with spin 1 states or with spin 0 states we should obtain approximately the same parameters for the potential well.

ANSWER TO POSSIBLE QUESTIONS ON CONFINEMENT

As it is well known, confinement is not well understood and there are models that do not consider it inside hadrons. For example, the original MIT bag model treats confinement only at the wall by making the vector current null at it [1]. In the case of the chiral bag model [2], confinement is treated by means of the continuity of the axial vector current at the wall. We do not need to worry about confinement because we only deal with low energy levels.

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ANSWER TO POSSILBE QUESTIONS ON THE STRONG REPULSION AT VERY SMALL DISTANCES

This repulsion attributed to the strong force is well known experimentally and has been mentioned by many authors. For example

S. Aoki *et al.*(HAL QCD collaboration), Baryon-Baryon Interaction from Lattice QCD, 5th Int. Conf. on Quarks and Nuclear Physics (QNP09), Sept 2009, Beijing, Chinese Physics C, **34**, 1229 (2010).

The TOTEM Collaboration of the LHC has seen this repulsion at 7 TeV and 8 TeV.