

Electromagnetic structure of charmed hadrons

G. Erkol¹, K. U. Can², B. Isildak¹, M. Oka², A. Ozpineci³, T. T. Takahashi⁴

¹ Department of Natural and Mathematical Sciences, Faculty of Engineering, Ozyegin University, Nisantep Mah. Orman Sok. No:13, Alemdag 34794 Cekmekoy, Istanbul Turkey

² Department of Physics, H-27, Tokyo Institute of Technology, Meguro, Tokyo 152-8551 Japan

³ Physics Department, Middle East Technical University, 06531 Ankara, Turkey

⁴ Gunma National College of Technology, Maebashi, Gunma 371-8530, Japan

Abstract. We compute the electromagnetic structures of D and D* mesons, the singly charmed Σ_c , Ω_c and the doubly charmed Ξ_{cc} , Ω_{cc} baryons in 2+1 flavor Lattice QCD. We extract the charge radii and the magnetic moments of these hadrons. In general, charmed hadrons are found to be compact as compared to light hadrons. The magnetic moments of the singly charmed baryons are found to be dominantly determined by the light quark and the role of the charm quark is significantly enhanced when it is doubly represented.

1. Introduction

Electromagnetic form factors reveal valuable information about the size and the shape of the hadrons. Determining these form factors is an important step in our understanding of the hadron properties in terms of quark-gluon degrees of freedom. There has been an increasing activity in determining the electromagnetic structure of octet mesons and baryons where the challenge is to understand these quantities directly from QCD, the theory of quarks and gluons. Currently, only the spectrum of the charmed baryons are accessible by experiments and future charm factories like BES-III and PANDA at GSI are expected to probe the charm sector.

In this work, we report on our recent results on the electromagnetic structure of D and D* mesons and the singly charmed Σ_c , Ω_c and the doubly charmed Ξ_{cc} , Ω_{cc} baryons as obtained in 2+1 flavor Lattice QCD [1, 2, 3]. In particular we compute the charge radii and the magnetic moments of these hadrons.

2. Lattice formulation and setup

Here we give the details of our calculations for charmed baryons and refer the reader to Ref. [1] for the charmed mesons. Electromagnetic form factors of baryons can be calculated by considering the baryon matrix elements of the electromagnetic vector current $V_\mu = \sum_q e_q \bar{q}(x) \gamma_\mu q(x)$, where q runs over the quark content of the given baryon:

$$\langle \mathcal{B}(p) | V_\mu | \mathcal{B}(p') \rangle = \bar{u}(p) \left[\gamma_\mu F_{1,\mathcal{B}}(q^2) + i \frac{\sigma_{\mu\nu} q^\nu}{2m_{\mathcal{B}}} F_{2,\mathcal{B}}(q^2) \right] u(p). \quad (1)$$

Here $q_\mu = p'_\mu - p_\mu$ is the transferred four-momentum. Here $u(p)$ denotes the Dirac spinor for the baryon with four-momentum p^μ and mass $m_{\mathcal{B}}$. The Sachs form factors $F_{1,\mathcal{B}}(q^2)$ and $F_{2,\mathcal{B}}(q^2)$

are related to the electric and magnetic form factors by

$$G_{E,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + \frac{q^2}{4m_{\mathcal{B}}^2} F_{2,\mathcal{B}}(q^2), \quad (2)$$

$$G_{M,\mathcal{B}}(q^2) = F_{1,\mathcal{B}}(q^2) + F_{2,\mathcal{B}}(q^2). \quad (3)$$

We use the following ratio

$$R(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma; \mu) = \frac{\langle F^{\mathcal{B}\nu\mu\mathcal{B}'}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma) \rangle}{\langle F^{\mathcal{B}\mathcal{B}}(t_2; \mathbf{p}'; \Gamma_4) \rangle} \left[\frac{\langle F^{\mathcal{B}\mathcal{B}}(t_2 - t_1; \mathbf{p}; \Gamma_4) \rangle \langle F^{\mathcal{B}\mathcal{B}}(t_1; \mathbf{p}'; \Gamma_4) \rangle \langle F^{\mathcal{B}\mathcal{B}}(t_2; \mathbf{p}'; \Gamma_4) \rangle}{\langle F^{\mathcal{B}\mathcal{B}}(t_2 - t_1; \mathbf{p}'; \Gamma_4) \rangle \langle F^{\mathcal{B}\mathcal{B}}(t_1; \mathbf{p}; \Gamma_4) \rangle \langle F^{\mathcal{B}\mathcal{B}}(t_2; \mathbf{p}; \Gamma_4) \rangle} \right]^{1/2},$$

where the baryonic two-point and three-point correlation functions are respectively defined as:

$$\langle F^{\mathcal{B}\mathcal{B}}(t; \mathbf{p}; \Gamma_4) \rangle = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}} \Gamma_4^{\alpha\alpha'} \times \langle 0|T[\eta_{\mathcal{B}}^{\alpha}(x)\bar{\eta}_{\mathcal{B}}^{\alpha'}(0)]|0\rangle, \quad (4)$$

$$\langle F^{\mathcal{B}\nu\mu\mathcal{B}'}(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma) \rangle = -i \sum_{\mathbf{x}_2, \mathbf{x}_1} e^{-i\mathbf{p}\cdot\mathbf{x}_2} e^{i\mathbf{q}\cdot\mathbf{x}_1} \Gamma^{\alpha\alpha'} \langle 0|T[\eta_{\mathcal{B}}^{\alpha}(x_2)V_{\mu}(x_1)\bar{\eta}_{\mathcal{B}'}^{\alpha'}(0)]|0\rangle, \quad (5)$$

with $\Gamma_i = \gamma_i\gamma_5\Gamma_4$ and $\Gamma_4 \equiv (1 + \gamma_4)/2$.

The baryon interpolating fields are chosen as

$$\begin{aligned} \eta_{\Xi_{cc}}(x) &= \epsilon^{ijk} [c^{Ti}(x)C\gamma_5\ell^j(x)]c^k(x), \\ \eta_{\Sigma_c}(x) &= \epsilon^{ijk} [\ell^{Ti}(x)C\gamma_5c^j(x)]\ell^k(x), \\ \eta_{\Omega_{cc}}(x) &= \epsilon^{ijk} [c^{Ti}(x)C\gamma_5s^j(x)]c^k(x), \\ \eta_{\Omega_c}(x) &= \epsilon^{ijk} [s^{Ti}(x)C\gamma_5c^j(x)]s^k(x), \end{aligned} \quad (6)$$

where $\ell = u$ for the doubly charged $\Xi_{cc}^{++}(ccu)/\Sigma_c^{++}(cuu)$ and $\ell = d$ for the singly charged $\Xi_{cc}^+(ccd)/\Sigma_c^+(cdd)$ baryons. Here i, j, k denote the color indices and $C = \gamma_4\gamma_2$. t_1 is the time when the external electromagnetic field interacts with a quark and t_2 is the time when the final baryon state is annihilated. When $t_2 - t_1$ and $t_1 \gg a$, the ratio in Eq. (4) reduces to the desired form

$$R(t_2, t_1; \mathbf{p}', \mathbf{p}; \Gamma; \mu) \xrightarrow[t_2-t_1 \gg a]{t_1 \gg a} \Pi(\mathbf{p}', \mathbf{p}; \Gamma; \mu). \quad (7)$$

We extract the form factors $G_{E,\mathcal{B}}(q^2)$ and $G_{M,\mathcal{B}}(q^2)$ by choosing appropriate combinations of Lorentz direction μ and projection matrices Γ :

$$\Pi(\mathbf{0}, -\mathbf{q}; \Gamma_4; \mu = 4) = \left[\frac{(E_{\mathcal{B}} + m_{\mathcal{B}})}{2E_{\mathcal{B}}} \right]^{1/2} G_{E,\mathcal{B}}(q^2), \quad (8)$$

$$\Pi(\mathbf{0}, -\mathbf{q}; \Gamma_j; \mu = i) = \left[\frac{1}{2E_{\mathcal{B}}(E_{\mathcal{B}} + m_{\mathcal{B}})} \right]^{1/2} \epsilon_{ijk} q_k G_{M,\mathcal{B}}(q^2). \quad (9)$$

Here, $G_{E,\mathcal{B}}(0)$ gives the electric charge of the baryon. Similarly, the magnetic moment can be obtained from the magnetic form factor $G_{M,\mathcal{B}}$ at zero momentum transfer.

We have run our simulations on $32^3 \times 64$ lattices with 2+1 flavors of dynamical quarks and we use the gauge configurations that have been generated by the PACS-CS collaboration [4] with the nonperturbatively $\mathcal{O}(a)$ -improved Wilson quark action and the Iwasaki gauge action. We use the gauge configurations at $\beta = 1.90$ with the clover coefficient $c_{SW} = 1.715$ and they have a lattice spacing of $a = 0.0907(13)$ fm ($a^{-1} = 2.176(31)$ GeV). We consider four different hopping parameters for the sea and the u, d valence quarks, $\kappa_{sea}, \kappa_{val}^{u,d} = 0.13700, 0.13727, 0.13754$ and 0.13770 , which correspond to pion masses of $\sim 700, 570, 410, \text{ and } 300$ MeV. The hopping parameter for the s sea quark is fixed to $\kappa_{sea}^s = 0.1364$.

3. Results and discussion

Our results for the charmed mesons indicate that they are compact as compared to light mesons [1]. As for the charmed baryons we find that the electric charge radii of Ω_{cc}^+ and Ξ_{cc}^+ [2] are about the same size but as compared to that of the proton they are compact. Replacing the light d -quark by an s -quark does not seem to have any effect on the charge radii. Σ_c^{++} baryon has the largest electric charge radius amongst the baryons that we considered. Magnetic charge radii also show a similar pattern as the electric charge radii. The magnetic charge radii of Σ_c^{++} and Σ_c^0 are close to that of the proton's, which is $\langle r_{M,p}^2 \rangle = 0.604 \text{ fm}^2$ [5]. In the case of Ω_c and Ω_{cc} , their electric and magnetic charge radii are somewhat dependent on the sea quarks. However, their magnetic moments are almost independent of the sea-quark effects. A more detailed discussion can be found in Ref. [3].

An analysis of the individual quark contributions [3] shows that the c quark core shifts the centre of mass towards itself thus shrinking the baryon. The charm quark contributions are independent of the quark content of the baryons and the contributions from the u/d - and s -quark are roughly the same. As a result, we find Σ_c^{++} and Ξ_{cc}^{++} , as well as, Ω_{cc}^+ and Ξ_{cc}^+ to have almost the same sizes.

Doubly represented light quarks have the dominant contribution to the magnetic moments and the opposite signs of the light and heavy quark contributions suggest that their spins are generally anti-aligned within the baryon. Σ_c^{++} has the largest magnetic moment of all and the strange baryons Ω_c and Ω_{cc} have somewhat smaller moments. It is interesting to compare these values with the experimental magnetic moment of the proton, which is $\mu_p = 2.793 \mu_N$ [5]. Comparing our magnetic moment results with several other models, although the signs match we see a quantitative disagreement [3].

In conclusion, we have extracted the electromagnetic structures of D and D* mesons, the singly charmed Σ_c , Ω_c and the doubly charmed Ξ_{cc} , Ω_{cc} baryons from 2+1-flavor simulations of QCD on a $32^3 \times 64$ lattice. Our results imply that the charmed baryons are compact with respect to baryons that are composed of only light quarks, *e.g.*, the proton. The existence of the heavy quark shrinks the baryons and doubly charmed baryons are more compact than the singly charmed baryons of the same charge. The size of the baryon is increased when the u/d is replaced by the s quark in a qQQ system. Ω_{cc} has the smallest and Σ_c^{++} and Σ_c^0 baryons have larger and roughly the same magnetic radii. The magnetic moments are dominantly determined by the doubly represented quarks. The signs of the magnetic moments are correctly reproduced on the lattice. However, in general we see an underestimation of the magnetic moments as compared to what has been found with other theoretical methods.

Table 1. The electric and magnetic charge radii in fm^2 , the values of magnetic form factors at $Q^2 = 0$ ($G_{M,B}(0)$), the magnetic moments in nuclear magnetons, for $B \equiv \Sigma_c^{++}$, Σ_c^0 , Ω_{cc} , Ω_c at the chiral point. Charge radii and magnetic moments results are from quadratic and linear fits, respectively.

$\langle r_{E,\Sigma_c^{++}}^2 \rangle [\text{fm}^2]$	$\mu_{\Sigma_c^0} [\mu_N]$	$\mu_{\Sigma_c^{++}} [\mu_N]$	$\langle r_{M,\Sigma_c^0}^2 \rangle [\text{fm}^2]$	$\langle r_{M,\Sigma_c^{++}}^2 \rangle [\text{fm}^2]$
0.240(44)	-0.875(103)	1.499(202)	0.568(130)	0.656(142)
$\langle r_{E,\Omega_{cc}}^2 \rangle [\text{fm}^2]$	$\mu_{\Omega_c} [\mu_N]$	$\mu_{\Omega_{cc}} [\mu_N]$	$\langle r_{M,\Omega_c}^2 \rangle [\text{fm}^2]$	$\langle r_{M,\Omega_{cc}}^2 \rangle [\text{fm}^2]$
0.064(15)	-0.627(43)	0.402(10)	0.343(52)	0.176(24)

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