

Next-to-next-to-leading order QCD and next-to-leading order electroweak corrections to $B_s \rightarrow \mu^+ \mu^-$ within the Standard Model

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Abstract. In this contribution, the recent calculations of next-to-next-to-leading order (NNLO) QCD and next-to-leading order (NLO) electroweak (EW) corrections to the decay $B_s \rightarrow \mu^+ \mu^-$ are reviewed.

1. Introduction

The decay of a B_s meson to two muons is a very promising process to look for deviations from the Standard Model (SM). The reason for this is that, on the one hand, there is both a loop and a helicity suppression which leads to a branching ratio of the order 10^{-9} . On the other hand, there is a quite clean experimental signal and thus, despite its rare nature, it can be measured experimentally. Hence, there was the hope that “New Physics” pops up immediately once a measurement is available. However, in real life this is not the case. In 2013 the CMS and LHCb experiments at the LHC have provided measurements for the averaged time-integrated branching ratios for $B_s \rightarrow \mu^+ \mu^-$ with the preliminary combination [1]

$$\overline{\mathcal{B}}_{s\mu} = (2.9 \pm 0.7) \times 10^{-9}, \quad (1)$$

which is based on Refs. [2] and [3]. This number has to be confronted with the theory prediction [4] given by

$$\overline{\mathcal{B}}_{s\mu} = (3.65 \pm 0.23) \times 10^{-9}, \quad (2)$$

which shows good agreement within the uncertainty. The experimental uncertainty is statistics dominated and will get reduced in the coming years (see discussion in Ref. [5]). On the other hand, the uncertainty of the theory prediction is dominated by the B_s meson decay constant f_{B_s} and CKM matrix elements (see below). The uncertainty which comes from truncation of the perturbative series is sub-leading, thanks to the calculations performed in Refs. [6, 7] as we will briefly discuss in the next sections.

2. NNLO QCD and NLO EW corrections to $B_s \rightarrow \mu^+ \mu^-$

There are several energy scales involved in decays of B mesons: first of all there is the typical scale of the decay process, μ_b , which is of the order of the bottom quark mass. Furthermore,

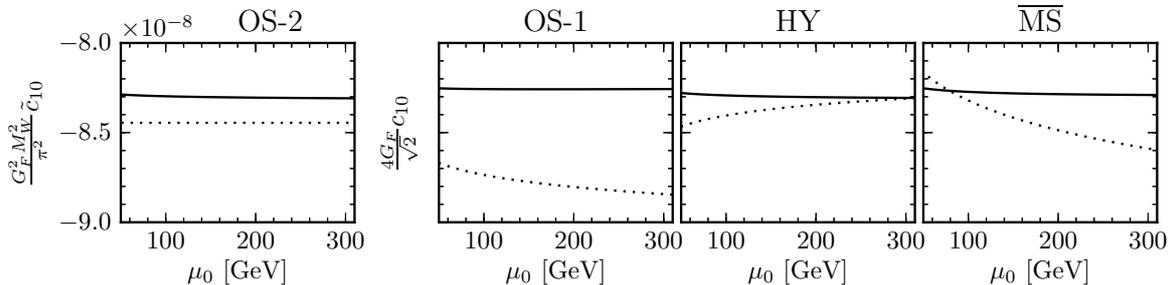


Figure 1. NLO EW corrections adopting different renormalization schemes.

there are masses of the virtual particles in the loops, which in the SM are essentially given by the W boson and the top quark. In the following, the latter scale is denoted by μ_0 . The framework which can be used to perform calculations involving widely separated scales is based on an effective theory where the heavy degrees of freedom are integrated out from the underlying theory. In the case at hand, this leads to an effective Lagrange density with only one relevant effective operator¹ $Q_A = (\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$. In order to arrive at predictions for the decay rate, the corresponding matching coefficient, C_A , has to be computed in a first step at the high scale μ_0 . Afterwards, it has to be evolved to μ_b using renormalization group techniques.

The one-loop calculation of C_A has been performed for the first time in Ref. [8], and NLO QCD corrections have been considered in Refs. [9, 10, 11, 12]. Recently, the three-loop corrections have been computed in Ref. [6]. The calculation can be reduced to vacuum integrals involving the two masses M_W and M_t , which have been solved with the help of expansions for $M_t \gg M_W$ and $M_t \approx M_W$. In this way, only one-scale integrals have to be computed and simple analytic results are obtained, which are of polynomial form with at most logarithmic coefficients. Thus, the numerical evaluation is simple and straightforward. The inclusion of the three-loop QCD corrections to C_A reduces the uncertainties to the branching ratio from scale variation of μ_0 in the interval $M_t/2$ to $2M_t$ from 1.8% to below 0.2% [6].

The complete NLO EW corrections have been computed in Ref. [7]. Before, only the leading M_t^2 terms were known from Ref. [13] and the corresponding uncertainties have been estimated to be of the order of about 7%. In contrast to QCD, there is a non-trivial running of C_A from μ_0 to μ_b after including $\mathcal{O}(\alpha_{em})$ terms which originate from mixing of operators once QED corrections are turned on [14, 15, 16]. In Ref. [7] this effect has been taken into account together with a detailed study of the renormalization scheme dependence. The latter is shown in Fig. 1 where $\tilde{c}_{10} = -2C_A$ is plotted for four different scheme choices (see Ref. [7] for details) at LO and NLO. One observes huge differences at LO (dotted line) which basically disappear at NLO (solid line). After including NLO EW corrections the uncertainty is reduced to below 1%.

3. Prediction for $\overline{\mathcal{B}}(B_s \rightarrow \mu^+\mu^-)$

In Ref. [4] the numbers in Eq. (1) have been confronted with theory predictions including NNLO QCD and NLO EW corrections, as discussed in Section 2. In the following discussion, we concentrate on $B_s \rightarrow \mu^+\mu^-$; the arguments hold analogously also for $B_d \rightarrow \mu^+\mu^-$ and, thus, we also present results for $\overline{\mathcal{B}}_{d\mu}$.

Within the effective-theory framework the branching ratio $\overline{\mathcal{B}}_{s\mu}$ can be cast in the form (see

¹ In beyond-SM theories sizable contributions are also obtained from operators with scalar and pseudo-scalar currents.

		f_{B_q}	CKM	τ_H^q	M_t	α_s	other param.	non-param.	Σ
$\overline{B}_{s\mu}$	$(3.65 \pm 0.23) \times 10^{-9}$	4.0%	4.3%	1.3%	1.6%	0.1%	< 0.1%	1.5%	6.4%
$\overline{B}_{d\mu}$	$(1.06 \pm 0.09) \times 10^{-10}$	4.5%	6.9%	0.5%	1.6%	0.1%	< 0.1%	1.5%	8.5%

Table 1. Central value and relative uncertainties from various sources for $\overline{B}_{s\mu}$ and $\overline{B}_{d\mu}$. In the last column they are added in quadrature.

also Ref. [17])

$$\overline{B}_{s\mu} = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi \Gamma_H^s} \beta_{s\mu} r_{s\mu}^2 |C_A(\mu_b)|^2 + \mathcal{O}(\alpha_{em}), \quad (3)$$

with $N = V_{tb}^* V_{ts} G_F^2 M_W^2 / \pi^2$, $r_{s\mu} = 2m_\mu / M_{B_s}$, $\beta_{s\mu} = \sqrt{1 - r_{s\mu}^2}$ and Γ_H^s denoting the heavier mass-eigenstate total width. M_{B_s} is the B_s -meson mass, and f_{B_s} its decay constant which is defined by the QCD matrix element $\langle 0 | \bar{b} \gamma^\alpha \gamma_5 s | B_s(p) \rangle = i p^\alpha f_{B_s}$. The term “ $\mathcal{O}(\alpha_{em})$ ” originates from the fact that NLO QED corrections to $\overline{B}_{s\mu}$ are not complete as virtual NLO corrections to the matrix element $\langle \mu^+ \mu^- | Q_A | B_s \rangle$ are missing. It has been estimated to be of the order of 0.3% [4] (see also Ref. [18] for more details). It is so small because higher-order QED contributions either remain helicity suppressed or undergo phase-space suppression at the endpoint of the invariant mass distribution.

At that point it is straightforward to evaluate the branching ratio. We refrain from repeating the discussion about the input parameters which can be found in Ref. [4] but want to mention a few important issues in connection to the uncertainty of the branching ratio. A summary is given in Table 1.

- The largest contributions to the uncertainties arise from the decay constants and the CKM matrix elements. The former is based on lattice determinations, and the values for f_{B_s} and f_{B_d} are taken over from a compilation of the Flavour Lattice Averaging Group (FLAG) [19].
- In the case of B_s , we write the CKM factors $|V_{tb}^* V_{ts}|$ as $|V_{cb}| \times |V_{tb}^* V_{ts} / V_{cb}|$, which allows us to use numerical results for the accurately known ratio $|V_{tb}^* V_{ts} / V_{cb}|$. Furthermore, a precise result for $|V_{cb}|$ has recently been obtained in Ref. [20] taking into account both the semileptonic data and the precise quark mass determinations from flavor-conserving processes.
- The parameters M_t (on-shell top quark mass) and α_s enter the matching coefficient $C_A(\mu_b)$ in a non-trivial way. In Ref. [4] formulae for the branching ratio are provided which allow for a convenient change of these parameters.
- The column “other param.” in Table 1 shows that the uncertainties originating from parameters that are not explicitly listed (like the Higgs or gauge boson masses or the Fermi constant) are negligible.
- The contributions to the non-parametric uncertainties, which are estimated to 1.5% both for $\overline{B}_{s\mu}$ and $\overline{B}_{d\mu}$ include

$\mathcal{O}(\alpha_{em})$ in Eq. (3)	0.3%
NNLO QCD μ_0 dependence	0.2%
NLO EW μ_0 dependence	0.2%
NLO EW renormalization scheme dependence	0.6%
Higher-order $M_{B_q}^2 / M_W^2$ power corrections	0.4%
$\overline{\text{MS}}$ –OS top quark mass conversion	0.3%

Combining these uncertainties in quadrature would give 0.9%. Our overall estimate of 1.5% is somewhat more conservative.

- In total relative uncertainties of 6.4% and 8.5% are obtained for $\overline{B}_{s\mu}$ and $\overline{B}_{d\mu}$, respectively.

4. Conclusions

In this contribution it has been discussed that the inclusion of higher order QCD and EW corrections into the theoretical prediction of the decays $B_q \rightarrow \mu^+\mu^-$ ($q = d, s$) significantly reduces the uncertainty, which is now dominated by CKM matrix elements and the decay constant f_{B_q} . Assuming the validity of the SM they can be replaced by the mass difference of the $B_q\overline{B}_q$ system and the bag parameter B_{B_q} which might be a way to further reduce the uncertainties (see, e.g., Refs. [21, 4, 5]).

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