Kaon Physics: Theory Overview

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Outline

V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84 (2012) 399

1. **Theoretical Framework.**

   Short and long-distance physics

2. **Leptonic and Semileptonic Decays.**

   Lepton Universality. CKM determinations

3. **Nonleptonic Decays.**

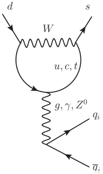
   Octet Enhancement. ε'/ε

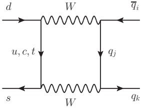
4. **Rare and Radiative Decays.**

   $K \to \pi\nu\bar{\nu}\,,\; K \to \pi\ell^+\ell^-\,,\; K \to \pi\gamma\gamma\;\ldots$

1. Theoretical Framework

- **Sensitivity to Short-Distance Scales:**





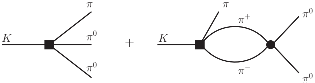
Charm mass prediction

Top quark

GIM cancellation

New Physics ?

- **Long-Distance Physics:**



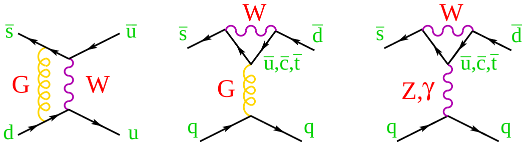
Chiral Dynamics

- **Multi-Scale Problem:**

$\log(M/\mu)$ **(OPE)** , $\log(\mu/m_\pi)$ **(χPT)**

Energy Scale	Fields	Effective Theory
M_W	W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model
	⬇ OPE	
$\lesssim m_c$	$\gamma, g\ ;\ \mu, e, \nu_i$ s, d, u	$\mathcal{L}_{\mathrm{QCD}}^{(n_f=3)}$, $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$
	⬇ $N_C \to \infty$	
M_K	$\gamma\ ;\ \mu, e, \nu_i$ π, K, η	χPT

ΔS = 1 TRANSITIONS



$$\mathcal{L}_{\rm eff}^{\Delta S=1} = -\frac{G_F}{\sqrt{2}}\; V_{ud}\, V_{us}^* \sum_i C_i(\mu)\; Q_i(\mu)$$

$$\begin{aligned}
Q_1 &= (\bar s_\alpha u_\beta)_{\rm V-A}\, (\bar u_\beta d_\alpha)_{\rm V-A} & Q_2 &= (\bar s u)_{\rm V-A} (\bar u d)_{\rm V-A} \\
Q_{3,5} &= (\bar s d)_{\rm V-A} \textstyle\sum_q (\bar q q)_{\rm V\mp A} & Q_4 &= (\bar s_\alpha d_\beta)_{\rm V-A} \textstyle\sum_q (\bar q_\beta q_\alpha)_{\rm V-A} \\
Q_{7,9} &= \tfrac{3}{2} (\bar s d)_{\rm V-A} \textstyle\sum_q e_q (\bar q q)_{\rm V\pm A} & Q_{10} &= \tfrac{3}{2} (\bar s_\alpha d_\beta)_{\rm V-A} \textstyle\sum_q e_q\, (\bar q_\beta q_\alpha)_{\rm V-A} \\
Q_6 &= -8\, \textstyle\sum_q (\bar s_L q_R)(\bar q_R d_L) & Q_8 &= -12\, \textstyle\sum_q e_q\, (\bar s_L q_R)(\bar q_R d_L) \\
Q_{11,12} &= (\bar s d)_{\rm V-A} \textstyle\sum_\ell\, (\bar\ell \ell)_{\rm V,A} & Q_{13} &= (\bar s d)_{\rm V-A} \textstyle\sum_\nu\, (\bar\nu \nu)_{\rm V-A}
\end{aligned}$$

- $q > \mu$: $\quad C_i(\mu) = z_i(\mu) - y_i(\mu)\, (V_{td}\, V_{ts}^* / V_{ud}\, V_{us}^*)$

  $O(\alpha_s^n t^n)\;,\; O(\alpha_s^{n+1} t^n) \qquad [t \equiv \log (M/m)]$ Munich / Rome

- $q < \mu$: $\quad \langle \pi\pi | Q_i(\mu) | K \rangle$? $\qquad$ Physics does not depend on μ

CHIRAL PERTURBATION THEORY (χPT)

- **Expansion in powers of p^2/Λ_χ^2 :** $\mathcal{A} = \sum_n \mathcal{A}^{(n)}$ $(\Lambda_\chi \sim 4\pi F_\pi \sim 1.2\ \mathrm{GeV})$
- **Amplitude structure fixed by chiral symmetry**

$$\mathbf{SU(3)_L \otimes SU(3)_R \;\to\; SU(3)_V}$$

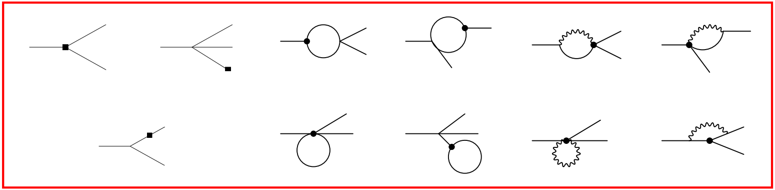
- **Short-distance dynamics encoded in Low-Energy Couplings**
- **$O(p^2)$ χPT:** Goldstone interactions (π, K, η) $\Phi \equiv \frac{1}{\sqrt{2}}\,\vec{\lambda}\,\vec{\varphi}$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \mathrm{Tr}(\lambda L_\mu L^\mu) \,+\, G_{27}\, F^4 \left(L_{\mu 23} L^\mu_{11} + \frac{2}{3}\, L_{\mu 21} L^\mu_{13} \right)$$

$$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^*\, g_R \quad ; \quad L_\mu = -i U^\dagger D_\mu U \quad ; \quad \lambda \equiv \frac{1}{2}\,\lambda_{6-i7} \quad ; \quad U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$$

- **Loop corrections (χPT logarithms) unambiguously predicted**
- **LECs can be determined at $N_C \to \infty$ (matching)**
- **$O(p^2)$ LECs (G_8, G_{27}) can be phenomenologically determined**

$\mathbf{O}\left[\mathbf{p^4}, (\mathbf{m_u} - \mathbf{m_d})\,\mathbf{p^2}, \mathbf{e^2 p^0}, \mathbf{e^2 p^2}\right] \quad \chi\mathbf{PT}$



- **Nonleptonic weak Lagrangian:** $\mathcal{O}(G_F\, p^4)$

$$\mathcal{L}^{(4)}_{\rm weak} \;=\; \sum_i \; G_8\, N_i\, F^2\, O_i^8 \;+\; \sum_i \; G_{27}\, D_i\, F^2\, O_i^{27} \;+\; \mathrm{h.c.}$$

- **Electroweak Lagrangian:** $\mathcal{O}(G_F\, e^2 p^{0,2})$

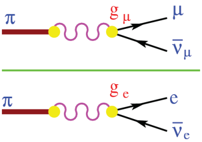
$$\mathcal{L}_{\rm EW} \;=\; e^2 F^6 G_8\, g_{ew}\, \mathrm{Tr}(\lambda U^\dagger \mathcal{Q} U) + e^2 \sum_i \; G_8\, Z_i\, F^4\, O_i^{EW} \;+\; \mathrm{h.c.}$$

- $\mathcal{O}(e^2 p^{0,2})$ **Electromagnetic** $+$ $\mathcal{O}(p^4)$ **Strong:** $Z\,,\, K_i\,,\, L_i$

2. (Semi) Leptonic Decays

Lepton Universality:

$$R_{e/\mu}^{(P)} \equiv \frac{\Gamma(P^- \to e^- \bar{\nu}_e)}{\Gamma(P^- \to \mu^- \bar{\nu}_\mu)}$$



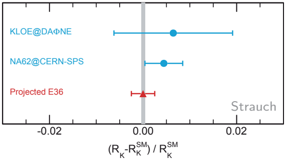
$$R_{e/\mu}^{(\pi)} = (1.2352 \pm 0.0001) \cdot 10^{-4}$$

$$R_{e/\mu}^{(K)} = (2.477 \pm 0.001) \cdot 10^{-5}$$

Cirigliano-Rosell '07

$$R_{e/\mu}^{(\pi)}\Big|_{\rm exp} = (1.230 \pm 0.004) \cdot 10^{-4}$$

$$R_{e/\mu}^{(K)}\Big|_{\rm exp} = (2.488 \pm 0.010) \cdot 10^{-5}$$



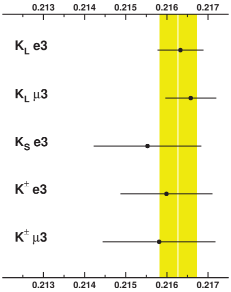
$$\frac{|g_\mu|}{|g_e|} = \begin{cases} 1.0021 \pm 0.0016 & \pi \to \mu/e \\ 0.9978 \pm 0.0020 & K \to \mu/e \\ 1.0010 \pm 0.0025 & K \to \pi\mu/e \\ 1.0018 \pm 0.0014 & \tau \to \mu/e \end{cases}$$

$K \to \pi \, \ell \, \nu_\ell$

$$|\mathbf{V_{us}} \, \mathbf{f_+(0)}| = \mathbf{0.2163 \pm 0.0005}$$

Flavianet Kaon WG, arXiv:1005.2323 [hep-ph]

$$\langle \pi^- | \bar{s} \gamma_\mu u | K^0 \rangle = (p_\pi + p_K)_\mu \; f_+(t) + (p_K - p_\pi)_\mu \; f_-(t)$$



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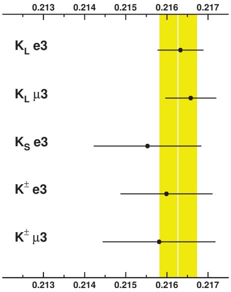

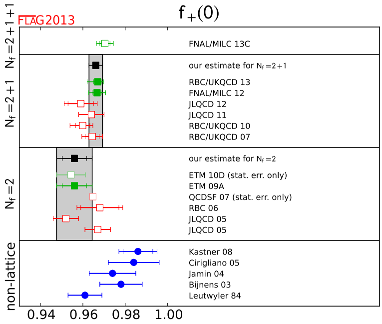

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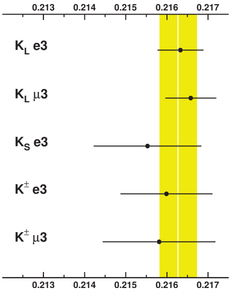

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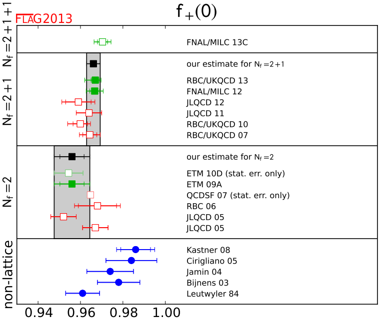


$$f_+(0) = 0.9661\,(32)$$

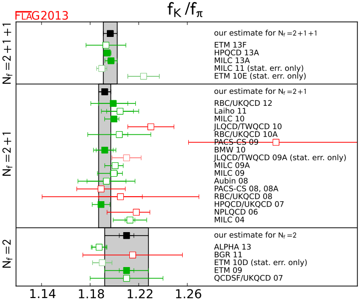
$$\rightarrow \quad |V_{us}| = 0.2239\,(9)$$

$$f_+(0) = 1 + f_2 + f_4 + \cdots$$

Large $\mathcal{O}(p^6)$ χPT correction

$\Gamma(K^+ \to \mu^+\nu_\mu)/\Gamma(\pi^+ \to \mu^+\nu_\mu)$:

$$\frac{|V_{us}\, f_K|}{|V_{ud}\, f_\pi|} = 0.2763 \pm 0.0005 \quad \longrightarrow \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2314 \pm 0.0011$$



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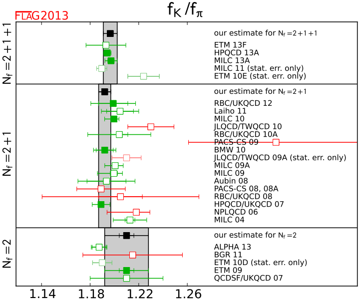

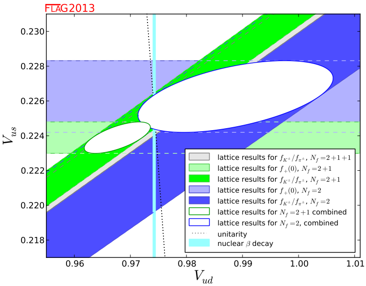

$$|\mathbf{V_{ud}}| = \mathbf{0.97425 \pm 0.00022} \quad , \quad |\mathbf{V_{us}}| = \mathbf{0.2245 \pm 0.0007}$$

$$\mathbf{\Delta}_{\mathrm{CKM}} \equiv |\mathbf{V_{ud}}|^2 + |\mathbf{V_{us}}|^2 + |\mathbf{V_{ub}}|^2 - 1 = -\mathbf{0.0004 \pm 0.0007}$$

3. Nonleptonic Decays

- **Octet Enhancement:** $$\frac{A(K \to \pi\pi)_{I=0}}{A(K \to \pi\pi)_{I=2}} \approx 22$$

  - **Short-distance:** **gluonic corrections, penguins**
  - **Long-distance:** **large χPT corrections (FSI)**
  - **Ongoing Lattice efforts**

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- **Direct CP Violation:** $\eta_{ij} \equiv \frac{A(K_L \to \pi^i \pi^j)}{A(K_S \to \pi^i \pi^j)}$

$$\mathrm{Re}\,(\epsilon'/\epsilon) = \tfrac{1}{3}\left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) = (16.8 \pm 1.4) \cdot 10^{-4}$$

NA31, E731, NA48, KTeV

$$\mathrm{Re}\,(\epsilon'/\epsilon)_{\mathrm{SM}} = \left(19 \pm 2\,{}^{+9}_{-6} \pm 6\right) \cdot 10^{-4}$$

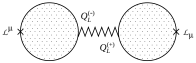
Pallante-Pich-Scimemi

Dynamical understanding of the $\Delta I = 1/2$ rule

AP – E. de Rafael, PL B374 (1996) 186

$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} F^4 \left[a\ \mathrm{Tr}(Q_L^{(-)} L_\mu)\,\mathrm{Tr}(Q_L^{(+)} L^\mu) + b\ \mathrm{Tr}(Q_L^{(-)} L_\mu Q_L^{(+)} L^\mu) + c\ \mathrm{Tr}(Q_L^{(-)} Q_L^{(+)} L_\mu L^\mu) \right]$$

$\mathcal{O}(N_C^2)$



a

$$Q_L^{(+)} = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ; \quad Q_L^{(-)} = Q_L^{(+)\dagger}$$

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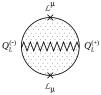
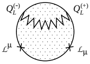


b



c

$$g_8 = \frac{3}{5}(a+b) - b + c$$

$$g_{27} = \frac{3}{5}(a+b)$$

$$a = 1 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \qquad ; \qquad c = \mathrm{Re}C_4 - 16\, L_5\, \mathrm{Re}C_6(\mu^2) \left[\frac{<\bar\psi\psi>}{f_\pi^3}\right]^2 + \mathcal{O}\left(\frac{1}{N_c^2}\right) \simeq 0.3 \pm 0.2$$

$|g_{27}| \simeq 0.29$ ➡ $b \simeq -0.52 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$ ➡ $g_8 \simeq 1.1 + \mathcal{O}\left(\frac{1}{N_c^2}\right)$

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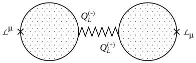

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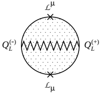
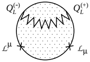


b



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$b < 0$ predicted through explicit calculations AP–E. de Rafael, NP B358 (1991) 311

Confirmed through inclusive QCD analysis M. Jamin–AP, NP B425 (1994) 15

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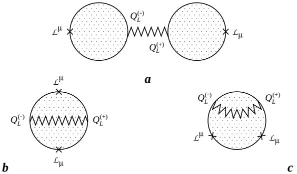

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Confirmed recently by lattice calculations Boyle et al, PRL 110 (2013) 15, 152001

Multi-Scale Problem: Summation of logarithms needed

A large $\log(M_1/M_2)$ compensates a $1/N_C$ suppression

❶ **Short-distance:** $\frac{1}{N_C} \log\left(M_W/\mu\right)$ — Bardeen-Buras-Gerard

$$\Longrightarrow \begin{cases} g_8^\infty = 1.13 \pm 0.05_\mu \pm 0.08_{L_5} \pm 0.05_{m_s} \\ g_{27}^\infty = 0.46 \pm 0.01_\mu \end{cases}$$

Cirigliano et al, Pallante et al

❷ **Long-distance (χPT):** $\frac{1}{N_C} \log\left(\mu/m_\pi\right)$ — Kambor et al, Pallante et al

$$g_8^{\mathrm{LO}} = 5.0 \quad \Longrightarrow \quad g_8^{\mathrm{NLO}} = 3.6$$

$$g_{27}^{\mathrm{LO}} = 0.285 \quad \Longrightarrow \quad g_{27}^{\mathrm{NLO}} = 0.286$$

Cirigliano et al

❸ **Isospin Violation:** $g_{27}^{\mathrm{NLO}} = 0.297$ — Cirigliano et al

Anatomy of ε'/ε calculation

$$\frac{\varepsilon'_K}{\varepsilon_K} \sim \left[\frac{105\ \mathrm{MeV}}{m_s(2\ \mathrm{GeV})}\right]^2 \left\{ B_6^{(1/2)}\,(1-\Omega_{\mathrm{eff}}) - 0.4\ B_8^{(3/2)} \right\}$$

❶ $O(p^4)$ χPT Loops: **Large correction** (FSI) Pallante-Pich-Scimemi

❷ $O(p^4)$ LECs fixed at $N_C \to \infty$: **Small correction**

❸ Isospin Breaking $O\left[(m_u - m_d)\,p^2, e^2 p^2\right]$: **Sizeable corrections**

$$\Omega_{\mathrm{eff}} = 0.06 \pm 0.08$$

Cirigliano-Ecker-Neufeld-Pich

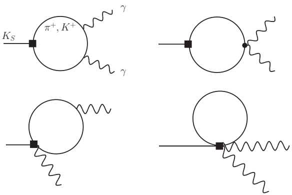
❹ $O(p^4)$ LECs [Re(g_8), Re(g_{27})] and phase-shifts fitted to data

❺ $m_s(2\ \mathrm{GeV}) = 110 \pm 20$ MeV (quark condensate)

4. Rare and Radiative Decays

$K^0 \to \gamma\gamma$

Long-distance dynamics



Finite loop: $\mathrm{Br}_{\mathrm{LO}} = 2.0 \cdot 10^{-6}$

D'Ambrosio-Espriu, Goity

$$\mathrm{Br}(K_S \to \gamma\gamma) = (2.63 \pm 0.17) \cdot 10^{-6}$$

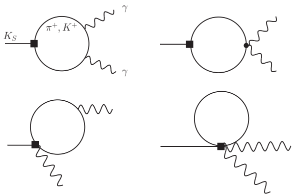
Agreement at $O(p^6)$ (FSI)

$$K_S \to \pi\pi \to \pi^+\pi^- \to \gamma\gamma$$

Kambor-Holstein, Buchalla et al

4. Rare and Radiative Decays

$K^0 \to \gamma\gamma$ **Long-distance dynamics**



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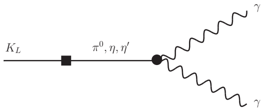
Agreement at $O(p^6)$ (FSI)

$K_S \to \pi\pi \to \pi^+\pi^- \to \gamma\gamma$

Kambor-Holstein, Buchalla et al



$\mathrm{Br}(K_L \to \gamma\gamma) = (5.47 \pm 0.04) \cdot 10^{-4}$

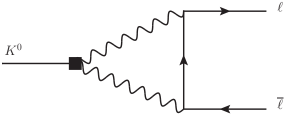
WZW Anomaly

$\mathbf{T}_{\mathrm{LO}} = \mathbf{0}$ [$\mathcal{O}(p^4)$, GMO cancel.]

$\mathcal{O}(p^6)$: SU(3) breaking, η–η' mixing

Well understood

$K^0 \to \ell^+\ell^-$



$K_S \to \ell^+\ell^-$

Long-distance dynamics

Finite 2-loop amplitude: Ecker-Pich

$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{LO}} = 2.1 \cdot 10^{-14}$$

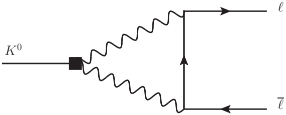
$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{LO}} = 5.1 \cdot 10^{-12}$$

$$\mathrm{Br}(K_S \to e^+e^-)_{\mathrm{exp}} < 9 \cdot 10^{-9}$$

$$\mathrm{Br}(K_S \to \mu^+\mu^-)_{\mathrm{exp}} < 9 \cdot 10^{-9}$$ LHCb

(90% CL)

$K^0 \to \ell^+\ell^-$



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(90% CL)

$K_L \to \ell^+\ell^-$

$$\mathrm{Br}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$$

$$\mathrm{Br}(K_L \to e^+e^-) = (9\,^{+6}_{-4}) \cdot 10^{-12}$$

Saturated by absorptive contrib.

Local countertem ⟷ SD

LD extracted from $\pi^0, \eta \to \ell^+\ell^-$

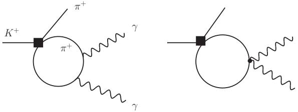
Gomez-Dumm, Pich

Fitted SD contrib. agrees with SM

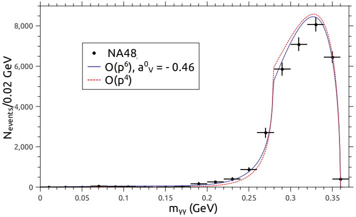
Longitudinal Polarization: Ecker-Pich

$$|\mathcal{P}_L| = (2.6 \pm 0.4) \cdot 10^{-3}$$

$K \to \pi\gamma\gamma$



$\mathrm{Br}(K_L \to \pi^0\gamma\gamma) = (1.27 \pm 0.03)\cdot 10^{-6}$



Finite 1-loop amplitude $[\mathcal{O}(p^4)]$:

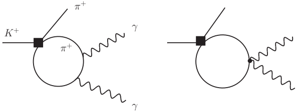
$$\mathrm{Br}(K_L \to \pi^0\gamma\gamma)_{\mathrm{LO}} = 6.8 \cdot 10^{-7}$$

Ecker-Pich-de Rafael, Cappiello-D'Ambrosio, Sehgal

$\mathcal{O}(p^6)$ unitarity corrections needed

Cohen et al, Cappiello et al, D'Ambrosio-Portolés

$K \to \pi\gamma\gamma$



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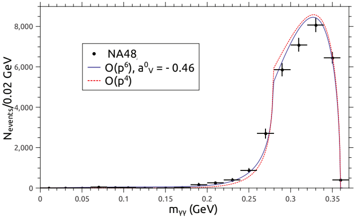

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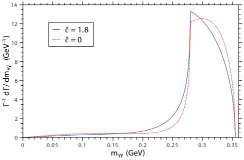
Ecker-Pich-de Rafael, Cappiello-D'Ambrosio, Sehgal

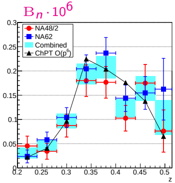
$\mathcal{O}(p^6)$ unitarity corrections needed

Cohen et al, Cappiello et al, D'Ambrosio-Portolés

$$\mathrm{Br}(K^+ \to \pi^+\gamma\gamma) = 1.003\,(56) \cdot 10^{-6}$$

NA48/2–NA62





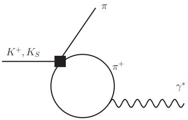
Local $\mathcal{O}(p^4)$ LEC: Ecker-Pich-de Rafael

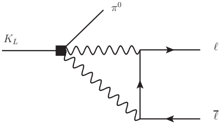
$$\hat{c} = \begin{cases} 1.72 \pm 0.21 & \mathcal{O}(p^4) \\ 1.86 \pm 0.25 & \mathcal{O}(p^6) \end{cases}$$

Small higher-order corrections

D'Ambrosio-Portolés

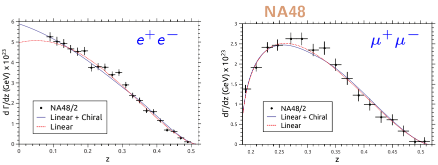
$K \to \pi \ell^+ \ell^-$





$$\mathrm{Br}(K^\pm \to \pi^\pm e^+ e^-) = 3.00\,(9) \cdot 10^{-7}$$

$$\mathrm{Br}(K^\pm \to \pi^\pm \mu^+ \mu^-) = 9.62\,(25) \cdot 10^{-8}$$

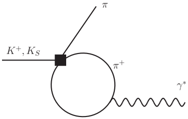


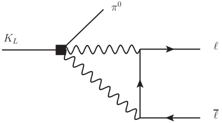
Local $\mathcal{O}(p^4)$ LECs Ecker-Pich-de Rafael

Electromagn. transition form factor

$\mathcal{O}(p^6)$ corrections D'Ambrosio et al

$K \to \pi\, \ell^+ \ell^-$





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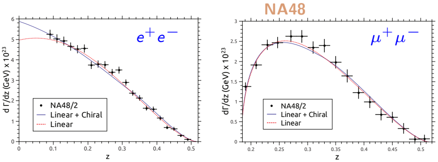


Local $\mathcal{O}(p^4)$ LECs Ecker-Pich-de Rafael

Electromagn. transition form factor

$\mathcal{O}(p^6)$ corrections D'Ambrosio et al

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$$

$$\mathrm{Br}(K_L \to \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10}$$

(90% CL), KTeV

3 contributions: Ecker-Pich-de Rafael

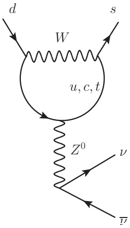
- **Direct $\not{CP}$**
- **Indirect $\not{CP}$**
- **CP conserving** (2γ)

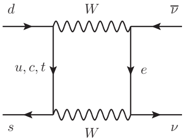
$\not{CP}$ **dominates for e^+e^-:**

$$\mathrm{Br}(K_L \to \pi^0 e^+ e^-) = 3.1\ (0.9) \cdot 10^{-11}$$

Buchalla et al

$K \to \pi \nu \bar{\nu}$





$$T \sim F\left(V_{is}^{*} V_{id}, \frac{m_i^2}{M_W^2}\right) \left(\bar{\nu}_L \gamma_\mu \nu_L\right) \langle \pi | \bar{s}_L \gamma^\mu d_L | K \rangle$$

Negligible long-distance contribution

$$\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = (7.8 \pm 0.8) \cdot 10^{-11} \sim A^4 \left[\eta^2 + (1.4 - \rho)^2\right]$$

$$\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) = (2.4 \pm 0.4) \cdot 10^{-11} \sim A^4 \eta^2$$

Buras et al

Brod et al

$\mathcal{A}(K_L \to \pi^0 \nu\bar{\nu}) \neq 0$ ➡ **Direct** $\not{C}\not{P}$

BNL-E949: few events! ➡ $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu}) = \left(1.73\,^{+1.15}_{-1.05}\right) \cdot 10^{-10}$

KEK-E391a: $\mathrm{Br}(K_L \to \pi^0 \nu\bar{\nu}) < 2.6 \cdot 10^{-8}$ (90% CL)

Ongoing Experiments: NA62, K0TO

Summary

Kaons continue providing important physics information:

- **Interesting interplay of short and long-distances**
- **Sensitive to heavy mass scales. New Physics?**
- **Superb probe of flavour dynamics and $\not{C}\!\!\not{P}$**
- **Excellent testing ground of χPT dynamics**

Theoretical challenge: precise control of QCD effects

Increased sensitivities at ongoing experiments ($K \to \pi\nu\bar{\nu}$)

Future data could bring interesting surprises