Theoretical overview of kaon decays

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Abstract. Kaon decays are an important testing ground of the electroweak flavour theory. They can provide new signals of CP violation and, perhaps, a window into physics beyond the Standard Model. At the same time, they exhibit an interesting interplay of long-distance QCD effects in flavour-changing transitions. A brief overview is presented, focusing on a few selected topics of particular interest. A more detailed and comprehensive review can be found in Ref. [1].
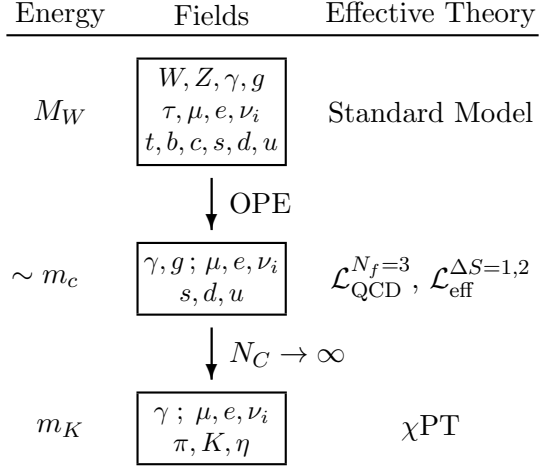
1. Effective Field Theory

Kaon physics has been at the origin of many fundamental ingredients of the Standard Model (SM), such as flavour quantum numbers, parity violation, meson-antimeson mixing, quark mixing, CP violation and the GIM mechanism [1]. Rare kaon decays provide sensitivity to short-distance scales (c, t, $W^\pm$, Z) and offer the exciting possibility of unravelling new physics beyond the SM. Searching for forbidden lepton-flavour-violating processes ($K_L \to e^\pm \mu^\mp$, $K_L \to e^\pm e^\pm \mu^\mp \mu^\mp$, $K^+ \to \pi^+ \mu^\pm e^\mp \ldots$) beyond the 10^{-10} level, one is actually exploring energy scales above the 10 TeV region. The study of allowed decay modes provides, at the same time, very interesting tests of the SM itself, improving our understanding of the interplay among electromagnetic, weak and strong interactions. In addition, new signals of CP violation, which would help to elucidate the source of CP-violating phenomena, can be looked for.

Owing to the presence of very different mass scales ($m_\pi < m_K \ll M_W$), the QCD corrections are amplified by large logarithms. The short-distance logarithmic corrections can be summed up, using the operator product expansion (OPE) and the renormalization-group, all the way down from M_W to scales $\mu < m_c$ [2]. One gets in this way an effective Lagrangian, defined in the three-flavour theory, which is a sum of local four-fermion operators Q_i, constructed with the light degrees of freedom ($u, d, s; e, \mu, \nu_\ell$), modulated by Wilson coefficients $C_i(\mu)$ which are functions of the heavy (Z, W, t, b, c, τ) masses:

$$\mathcal{L}_{\text{eff}}^{\Delta S=1} \;=\; -\frac{G_F}{\sqrt{2}}\, V_{ud} V_{us}^* \sum_i C_i(\mu)\, Q_i\,, \qquad\qquad C_i(\mu) \;=\; z_i(\mu) - y_i(\mu)\, \frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*}\,. \qquad (1)$$

The CP-violating decay amplitudes are proportional to the components $y_i(\mu)$. The overall renormalization scale μ separates the short- ($M > \mu$) and long-distance ($m < \mu$) contributions, which are contained in $C_i(\mu)$ and Q_i, respectively. The Wilson coefficients are fully known at the next-to-leading order (NLO) [3, 4]; this includes all corrections of $\mathcal{O}(\alpha_s^n t^n)$ and $\mathcal{O}(\alpha_s^{n+1} t^n)$, where $t \equiv \log{(M_1/M_2)}$ refers to the logarithm of any ratio of heavy mass scales ($M_{1,2} \geq \mu$). In order to calculate the kaon decay amplitudes, we also need to know the nonperturbative matrix elements of the operators Q_i between the initial and final states.

Energy	Fields	Effective Theory

Figure 1. Evolution from M_W to m_K.
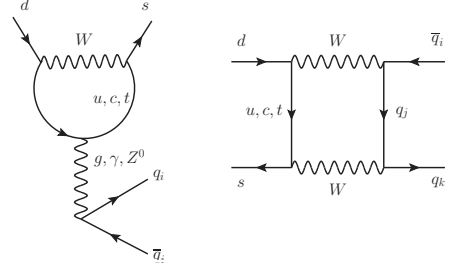
Figure 2. Short-distance diagrams.
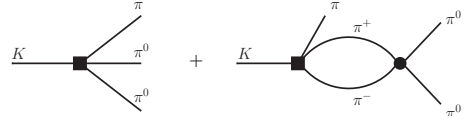
Figure 3. Long-distance diagrams.

At low energies, one can use symmetry considerations to define another effective field theory in terms of the QCD Goldstone bosons (π, K, η). Chiral Perturbation Theory (χPT) [5,6] describes the pseudoscalar-octet dynamics through a perturbative expansion in powers of momenta and quark masses over the chiral symmetry-breaking scale $\Lambda_\chi \sim 1$ GeV. Chiral symmetry fixes the allowed operators, while all short-distance information is encoded in their low-energy couplings (LECs) [7,8]. At LO the most general effective Lagrangian, with the same $SU(3)_L \otimes SU(3)_R$ transformation properties as the short-distance Lagrangian (1), contains three terms [1]:

$$\mathcal{L}_2^{\Delta S=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ g_8 \langle \lambda L_\mu L^\mu \rangle + g_{27} \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + e^2 g_8 g_{\text{ew}} F^6 \langle \lambda U^\dagger Q U \rangle \right\}, \quad (2)$$

where $U \equiv \exp(i\vec{\lambda}\vec{\phi}/F)$ parameterizes the Goldstone fields, $L_\mu = iF^2 U^\dagger D_\mu U$ represents the octet of $V - A$ currents, $\lambda \equiv (\lambda_6 - i\lambda_7)/2$ projects onto the $\bar{s} \to \bar{d}$ transition, $Q = \frac{1}{3} \text{diag}(2, -1, -1)$ is the quark charge matrix and $\langle \rangle$ denotes a 3-dimensional flavour trace. The LECs g_8 and g_{27} measure the strength of the two parts of $\mathcal{L}_{\text{eff}}^{\Delta S=1}$ transforming as $(8_L, 1_R)$ and $(27_L, 1_R)$, respectively, under chiral rotations, while g_{ew} accounts for the electromagnetic penguin operators.

The χPT framework determines the most general form of the K decay amplitudes, compatible with chiral symmetry, in terms of the LECs multiplying the relevant chiral operators. These LECs, which encode the short-distance dynamics, can be determined phenomenologically and/or calculated in the limit of a large number of QCD colours N_C. Chiral loops generate non-polynomial contributions, with logarithms and threshold factors as required by unitarity. Fig. 1 shows schematically the procedure used to evolve down from M_W to m_K. While the OPE resums the short-distance logarithmic corrections $\log(M/\mu)$, the χPT loops take care of the large infrared logarithms $\log(\mu/m_\pi)$ associated with unitarity corrections (final-state interactions).

2. Leptonic and Semileptonic Decays

In (semi)leptonic decays strong interactions only appear through the hadronic matrix elements of the left-handed current, which can be precisely studied within χPT and with lattice simulations.

The ratio $\Gamma[K^- \to e^- \bar{\nu}_e(\gamma)]/\Gamma[K^- \to \mu^- \bar{\nu}_\mu(\gamma)]$ has been calculated [9,10] and measured [11,12] with high accuracy, allowing for a precise test of charged-current lepton universality: $|g_\mu/g_e| = 0.9978\,(20)$. Similar precisions have been achieved in $K \to \pi \ell \nu_\ell$ [13], $\pi \to \ell \nu_\ell$ and $\tau \to \nu_\tau \ell \nu_\ell$ [14] decays. The ratio $\Gamma[K^- \to \mu^- \bar{\nu}_\mu(\gamma)]/\Gamma[\pi^- \to \mu^- \bar{\nu}_\mu(\gamma)]$ provides information

on the quark mixing matrix [13, 15]. With a careful treatment ot electromagnetic and isospin-violating corrections, one extracts $|V_{us}/V_{ud}||F_K/F_\pi| = 0.2763 \pm 0.0005$ [16]. Taking for the ratio of decay constants the ($N_f = 2 + 1 + 1$) lattice average $F_K/F_\pi = 1.194 \pm 0.005$ [17], this gives

$$|V_{us}/V_{ud}| \; = \; 0.2310 \pm 0.0011 \,. \tag{3}$$

Including electromagnetic and isospin-breaking corrections [18, 19], the most recent $K_{\ell 3}$ data leads to $|V_{us} \, f_+(0)| = 0.2163 \pm 0.0005$ [13], with $f_+(0) = 1 + \mathcal{O}[(m_s - m_u)^2]$ the $K^0 \to \pi^- \ell^+ \nu_\ell$ vector form factor. The exact value of $f_+(0)$ has been thoroughly investigated since the first precise estimate by Leutwyler and Roos, $f_+(0) = 0.961 \pm 0.008$ [20]. While analytical calculations based on χPT obtain higher values [21, 22], owing to the large ($\sim +0.01$) 2-loop corrections [23], lattice results [17] used to agree with the Leutwyler–Roos estimate. The most recent lattice analyses [24, 25] give, however, larger values in better agreement with the χPT expectations. The FLAG $N_f = 2 + 1$ average $f_+(0) = 0.9661\,(32)$ [17] implies $|V_{us}| = 0.2239\,(9)$, while a recent $N_f = 2 + 1 + 1$ determination [24], $f_+(0) = 0.9704\,(32)$, results in

$$|V_{us}| \; = \; 0.2229 \pm 0.0005_{\mathrm{exp}} \pm 0.0007_{\mathrm{th}} \,. \tag{4}$$

Together with $|V_{ud}| = 0.97425 \pm 0.00022$ and the tiny $|V_{ub}|$ contribution [26], Eqs. (3) and (4) imply an stringent test of the unitarity of the quark mixing matrix:

$$\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 \; = \; -0.0007 \pm 0.0007 \,. \tag{5}$$

3. Nonleptonic Decays

The measured $A(K \to \pi\pi)_I$ decay amplitudes show a strong enhancement of the octet $\Delta I = \frac{1}{2}$ transition amplitude into a 2π final state with isospin $I = 0$: $|A_0/A_2| \approx 22$. In the χPT framework this manifests as a huge difference between the LECs in Eq. (2). A LO fit to the data gives $|g_8| \simeq 5.0$ and $|g_{27}| \simeq 0.285$. Part of the enhancement originates in the strong rescattering of the final pions, which at one loop increases A_0 by roughly 35%. Taking the χPT 1-loop contributions into account, one finds a sizeably smaller octet coupling (the central values change slightly to 3.61 and 0.297, respectively, if isospin violation is included) [1, 27, 28]:

$$|g_8| \; = \; 3.62 \pm 0.28 \,, \qquad\qquad |g_{27}| \; = \; 0.286 \pm 0.28 \,. \tag{6}$$

In the absence of QCD, the SM (W exchange) prediction $g_8 = g_{27} = \frac{3}{5}$ would disagree with (6). The short-distance QCD corrections show the needed qualitative trend to understand the data. The matching of the effective descriptions $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1}$ (short-distance) and $\mathcal{L}_2^{\Delta S=1}$ (χPT) can be done in the large-N_C limit; including the large short-distance logarithmic corrections $\sim \frac{1}{N_C} \log{(M_W/\mu)}$, this gives the results $g_8^\infty = 1.13 \pm 0.18$ and $g_{27}^\infty = 0.46 \pm 0.01$ [1], which show the relevance of missing NLO corrections in $1/N_C$.

A dynamical understanding of the $\Delta I = \frac{1}{2}$ enhancement was achieved long time ago [29], in the limit of a large number of QCD colours. With one virtual $W^\pm$ field emitted and reabsorbed, and to LO in the chiral expansion (two L_μ insertions at most), there are three possible chiral invariant configurations which give rise to the effective Lagrangian

$$\mathcal{L}_{\mathrm{eff}} \; = \; -\frac{G_F}{\sqrt{2}} \left\{ a \, \langle Q_L^{(-)} L_\mu \rangle \, \langle Q_L^{(+)} L^\mu \rangle \, + \, b \, \langle Q_L^{(-)} L_\mu Q_L^{(+)} L^\mu \rangle \, + \, c \, \langle Q_L^{(-)} Q_L^{(+)} L_\mu L^\mu \rangle \right\} \,. \tag{7}$$

The operators $Q_L^{(+)} = Q_L^{(-)\dagger}$ represent the emission and absorption of the virtual $W^\pm$ field; thus, $[Q_L^{(+)}]_{ij} = \delta_{i1}\delta_{j2}V_{ud} + \delta_{i1}\delta_{j3}V_{us}$. The underlying functional integral over quark and gluon fields
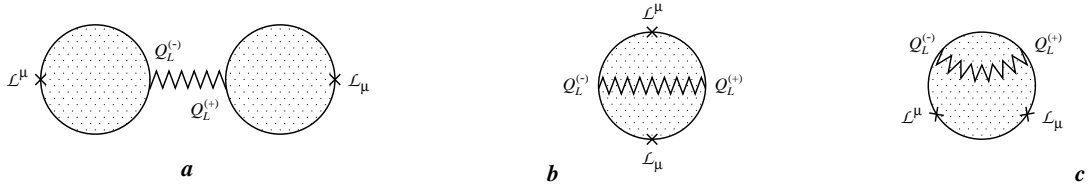
Figure 4. $K \to \pi\pi$ topologies. The solid lines represent quark fields propagating in a gluon background simulated by the dotted lines. The $W^\pm$ is exchanged between two $Q_L^{(\pm)}$ sources [29].

which gives rise to the effective couplings in (7) is represented diagrammatically in Fig. 4. When further restricted to $\Delta S = 1$ transitions, the effective Lagrangian (7) reduces to (2) with [29]

$$g_8 = \frac{3}{5}(a+b) - b + c, \qquad\qquad g_{27} = \frac{3}{5}(a+b). \qquad (8)$$

The topology which leads to the a-type coupling is $\mathcal{O}(N_C^2)$, while those generating b and c are $\mathcal{O}(N_C)$. In the large-N_C limit the coupling a can be calculated because the four-quark operators factorize into QCD currents with well-known χPT realizations. This factorization is only broken by (at least two) gluonic exchanges which are of NNLO in the $1/N_C$ expansion. The c-type configuration corresponds to the so-called penguin-like diagrams which can also be calculated at LO in terms of known phenomenological parameters. Taking into account the factor $F^2 \sim \mathcal{O}(N_C)$, included in the definition of the currents L_μ, one easily finds [29]:

$$a = 1 + \mathcal{O}(1/N_C^2), \qquad\qquad b = \mathcal{O}(1/N_C),$$
$$c = \mathrm{Re}\, C_4 - 16 L_5 \,\mathrm{Re}\, C_6(\mu) \left(\langle 0|\bar\psi\psi|0\rangle(\mu)/F^3 \right)^2 + \mathcal{O}(1/N_C^2) \simeq 0.3 \pm 0.2, \qquad (9)$$

where L_5 is an $\mathcal{O}(p^4)$ coupling of the strong χPT Lagrangian. To $\mathcal{O}(1/N_C)$ the scale dependence in $C_6(\mu)$ cancels with the one in $\langle 0|\bar\psi\psi|0\rangle(\mu)$, while C_4 is scale-independent.

The non-leading topology b cannot be evaluated in a model-independent way. The important observation made in Ref. [29] is that it contributes with opposite signs to g_8 and g_{27}. Taking Eq. (9) into account, the measured value of A_2, i.e. $|g_{27}| \approx 0.29$, implies [29]:

$$b \approx -0.52 + \mathcal{O}(1/N_C^2), \qquad\qquad g_8 \approx 1.1 + \mathcal{O}(1/N_C^2). \qquad (10)$$

Thus, the measured ratio $|g_8/g_{27}|$ requires a large and negative value of b, generating a significant cancellation in g_{27} and a sizeable enhancement of g_8. This is precisely what was previously predicted through model-dependent calculations [30–33] and confirmed through a rigorous inclusive NLO analysis of the two-point correlators associated with the short-distance Lagrangians $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1,2}$ [29, 30, 34]. Recently, the predicted cancellation in g_{27} has been observed by a lattice calculation of A_2 which finds $b/a \approx -0.7$ [35]; the corresponding enhancement of g_8 is also seen in A_0, although the present lattice results are still obtained at unphysical kinematics.

A similar cancellation is observed in lattice simulations of the $\Delta S = 2$ transition amplitude [36]. In the chiral limit, the so-called B_K parameter which regulates the K^0–$\bar{K}^0$ hadronic matrix element is given by $B_K = \frac{3}{4}(a+b)$ [29], exhibiting the same behaviour as g_{27}. This result agrees with explicit model calculations [30–33, 37] and QCD sum-rule determinations [38, 39].

4. Direct CP Violation: ε'/ε

The measured CP-violating ratio [40–43] $[\eta_{ab} \equiv A(K_L \to \pi^a\pi^b)/A(K_S \to \pi^a\pi^b)]$

$$\mathrm{Re}\,(\varepsilon'/\varepsilon) = \frac{1}{3}\left(1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right| \right) = (16.8 \pm 2.0) \times 10^{-4} \qquad (11)$$

demonstrates the existence of direct CP violation in the $K \to 2\pi$ decay amplitudes. When CP violation is turned on, the amplitudes A_I acquire imaginary parts. To first order in CP violation,

$$\varepsilon' = -\frac{i}{\sqrt{2}} e^{i(\chi_2-\chi_0)} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} \right], \tag{12}$$

where the strong phases χ_I can be identified with the S-wave $\pi\pi$ scattering phase shifts at $\sqrt{s} = m_K$, up to isospin-breaking effects [27,28]. The phase $\phi_{\varepsilon'} = \chi_2 - \chi_0 + \pi/2 = (42.5 \pm 0.9)°$ is very close to the so-called superweak phase, $\phi_\varepsilon \approx \tan^{-1}\left[2(m_{K_L} - m_{K_S})/(\Gamma_{K_S} - \Gamma_{K_L})\right] = (43.51 \pm 0.05)°$, implying that $\cos(\phi_{\varepsilon'} - \phi_\varepsilon) \approx 1$. The CP-conserving amplitudes $\mathrm{Re}A_I$ can be set to their experimentally determined values, avoiding in this way the large uncertainties associated with the hadronic matrix elements of the four-quark operators in $\mathcal{L}_{\mathrm{eff}}^{\Delta S=1}$. Thus, one only needs a first-principle calculation of the CP-odd amplitudes $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$; the first one is completely dominated by the strong penguin operator Q_6, while the leading contribution to the second one comes from the electromagnetic penguin Q_8. Fortunately, those are precisely the only operators that are well approximated through a large-N_C estimate of LECs, because their anomalous dimensions are leading in $1/N_C$. Owing to the large ratio $\mathrm{Re}A_0/\mathrm{Re}A_2$, isospin violation plays also an important role in ε'/ε [28]. The one-loop χPT enhancement of the isoscalar amplitude [44,45] destroys an accidental LO cancellation of the two terms in (12) [46–48], bringing the SM prediction of ε'/ε in good agreement with the experimental measurement in Eq. (11) [1,44,45]:

$$\mathrm{Re}\left(\epsilon'/\epsilon\right) = \left(19 \pm 2_\mu \, {}^{+9}_{-6}{}_{m_s} \pm 6_{1/N_C} \right) \times 10^{-4}. \tag{13}$$

5. Rare and Radiative Decays

Kaon decays mediated by flavour-changing neutral currents are suppressed in the SM and their main interest, other than their own understanding, relies on the possible observation of new physics effects. Most of these processes ($K \to \gamma^{(*)}\gamma^{(*)}$, $K \to \ell^+\ell^-$, $K \to \pi\gamma^{(*)}\gamma^{(*)}$, $K \to \pi\ell^+\ell^- \dots$) are dominated by long-distance contributions which can be analyzed with χPT techniques [49–62]. However, there are also processes governed by short-distance amplitudes, such as $K \to \pi\nu\bar{\nu}$ [46, 63, 64]. The decays $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \nu\bar{\nu}$ provide very interesting measures of CP violation. The first one receives contributions from both direct and indirect CP violation which dominate the decay amplitude, the CP-conserving contribution being suppressed by an additional power of α [52]. The decay $K_L \to \pi^0 \nu\bar{\nu}$ violates CP and it is completely dominated by the direct-CP contribution. A detailed overview of rare K decays with a complete reference list can be found in Refs. [1,65].

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