Semileptonic B decays at Belle

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on behalf of the Belle Collaboration

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Outline

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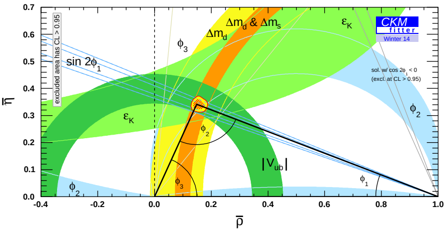
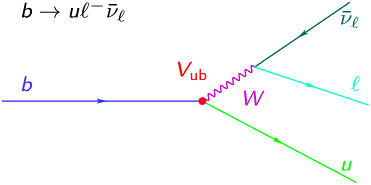
- $B^0 \to D^{*-}\ell\bar{\nu}_\ell$ at Belle
- Preliminary $B \to D\ell\bar{\nu}_\ell$ results with hadronic tag at Belle
- $|V_{\rm cb}|$ determination
- $B^+ \to D_s^{(*)-}K^+\ell\bar{\nu}_\ell$ at Belle
- B_s^0 semileptonic decays
- Semileptonic decays with hadronic tag
    - $B^- \to p\bar{p}\ell^-\bar{\nu}_\ell$ decays
    - $\bar{B} \to \pi\ell^-\bar{\nu}_\ell$ decays
    - $\bar{B} \to \rho\ell^-\bar{\nu}_\ell$ decays
    - $B^- \to \omega\ell^-\bar{\nu}_\ell$ decays
- $|V_{\rm ub}|$ determination
- Conclusion

To test the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and to search for new physics BSM it is important to precisely measure values of the matrix elements. The values of $|V_{ub}|$ and $|V_{cb}|$ can be determined from $b \to u$ and $b \to c$ tree-level transitions in which semileptonic B-meson decays have relatively clean theoretical interpretation and QCD uncertainties are under control. Pure leptonic decays are even more clean theoretically but more difficult from experimental side.





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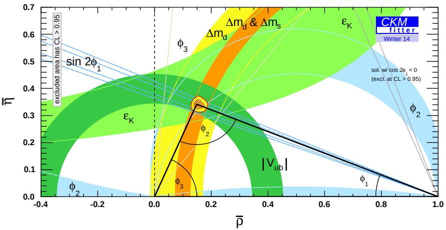

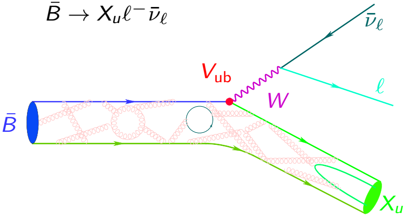

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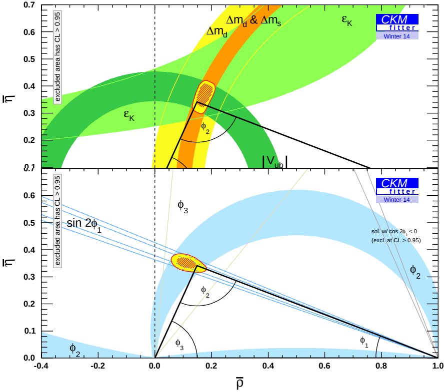
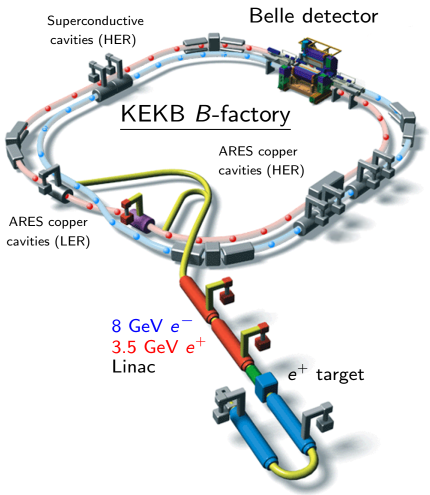
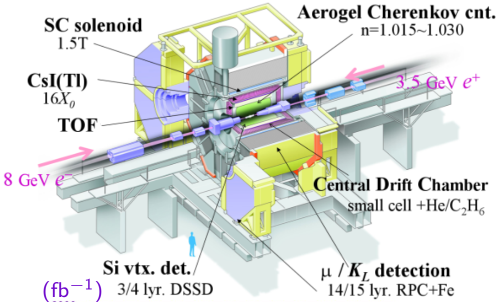
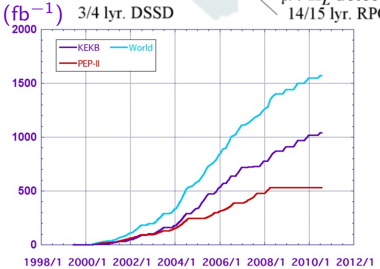


Sides of the UT are still less accurate than angles $\Rightarrow$ we need more precise measurements of semileptonic decays especially charmless for $|V_{ub}|$ element.

Belle detector and KEKB accelerator (1999-2010)







Energy region	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$	$\Upsilon(4S)$	$\Upsilon(5S)$	Off reson./scan	Total
$\mathcal{L}$, fb^{-1}	6	25	3	711	121	100	> 1000

$|V_{cb}|$ from exclusive B decays

Matrix element of $B \to X_q \ell \bar{\nu}_\ell$ decay at the first order

$$\mathcal{M}(B \to X_q \ell \bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{qb} L^\mu H_\mu, \;\; L^\mu = \bar{u}_\ell \gamma^\mu (1 - \gamma^5) v_\nu$$

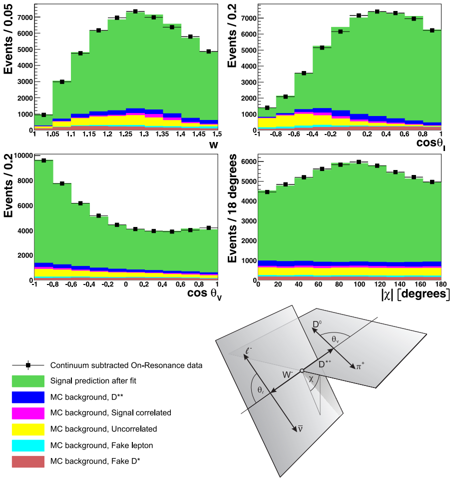
Hadronic current H_μ depends on a specific final state.

The differential decay rate for $B \to D^{(*)} \ell \bar{\nu}_\ell$ processes ($\ell = e, \mu$) are parametrized in terms of $w = v_B \cdot v_D = (m_B^2 + m_D^2 - q^2)/2 m_B m_D$, $(1 \leq w \leq w_{\max})$:

$$\frac{d\Gamma(\bar{B} \to D^* \ell \bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} m_{D^*}^3 \sqrt{w^2 - 1}\, P(w)\, |\mathcal{F}(w)|^2$$

$$\frac{d\Gamma(\bar{B} \to D \ell \bar{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} m_D^3 (w^2 - 1)^{3/2} (m_B + m_D)^2 |\mathcal{G}(w)|^2$$

In the heavy-quark limit at zero recoil, corresponding to $w = 1$, the form factors are normalized to $\mathcal{F}(1) = 1$ and $\mathcal{G}(1) = 1$. In the work [Nucl.Phys. B**530**, 153-181] the form factor $\mathcal{G}(w)$ is constrained and controlled by the overall scale $\mathcal{G}(1)$ and the slope ρ_D^2 at zero recoil to an accuracy better than 2% as well as $\mathcal{F}(w)$ with the two additional parameters R_1 and R_2.

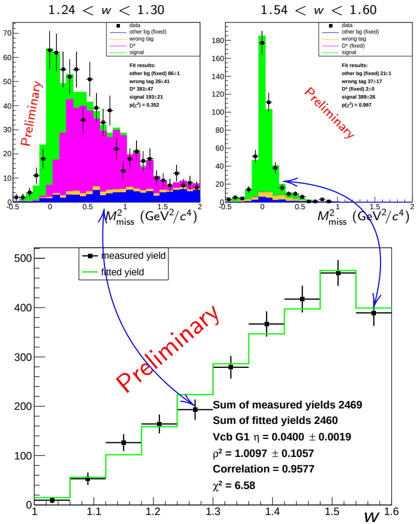


- Phys. Rev. D **82** (2010) 112007 [arXiv:1010.5620].
- Data sample is 710 fb^{-1}, 123427 ± 636 signal events.
- Fit in 40 bins of w, $\cos\theta_\ell$, $\cos\theta_V$ and χ to obtain phenomenological form factor parameters based on Nucl. Phys. B **530**, 153 (1998) [hep-ph/9712417].
- Fit result:

$$\begin{aligned} \mathcal{F}(1)|V_{cb}| &= (34.6 \pm 0.2 \pm 1.0) \times 10^{-3} \\ \rho^2_{D^*} &= 1.214 \pm 0.034 \pm 0.009 \\ R_1 &= 1.401 \pm 0.034 \pm 0.018 \\ R_2 &= 0.864 \pm 0.024 \pm 0.008 \\ \chi^2/\mathrm{ndf} &= 138.8/155 \end{aligned}$$

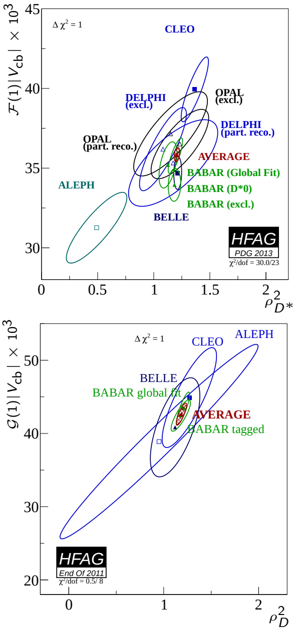
$$\mathcal{B}(B^0 \to D^{*-}\ell\bar{\nu}_\ell) = (4.58 \pm 0.03 \pm 0.26)\%$$

$\bar{B}^0 \to D^+ e\bar{\nu}$



- Results are preliminary.
- Data sample is 711 fb^{-1}.
- Inclusive semileptonic decays are used to correct tag efficiencies.
- D is reconstructed in multiple hadronic states.
- Branching fractions extracted:
  $\mathcal{B}(\bar{B}^0 \to D^+\ell\bar{\nu}) = (2.49 \pm 0.17) \times 10^{-2}$
  $\mathcal{B}(B^- \to \bar{D}^0\ell\bar{\nu}) = (2.70 \pm 0.19) \times 10^{-2}$
- Binned maximum likelihood fit to the M^2_{miss} distribution in 10 w regions.
- w distribution fit results:

Sample	$\eta_{\text{EW}}\mathcal{G}(1)\lvert V_{cb}\rvert$ 10^{-3}	ρ^2_D	corr.
$\bar{B}^0 \to D^+ e\bar{\nu}$	40.0±1.9±1.7	1.01±0.11±0.03	0.69
$\bar{B}^0 \to D^+ \mu\bar{\nu}$	40.7±2.1±1.7	1.08±0.12±0.03	0.71
$B^- \to \bar{D}^0 e\bar{\nu}$	43.7±1.9±1.7	0.91±0.10±0.01	0.71
$B^- \to \bar{D}^0 \mu\bar{\nu}$	46.7±1.9±1.8	1.08±0.09±0.01	0.68
$B \to D\ell\bar{\nu}$	42.6±1.0±1.4	1.00±0.05±0.02	0.49



$\mathcal{F}(1)|V_{cb}| = (35.90 \pm 0.11_{stat} \pm 0.44_{syst}) \times 10^{-3}$

	$\mathcal{F}(1)$	$\lvert V_{cb}\rvert \times 10^3$
Lattice QCD [PoS LATTICE2010, 311 (2010)]	0.908 ± 0.017	$39.54\pm0.50_{exp}\pm0.74_{th}$
Lattice QCD [Phys. Rev. D **89**, 114504 (2014)]	0.920 ± 0.013	$39.04\pm0.49_{exp}\pm0.56_{th}$
Sum rule [Phys. Rev. D **81**, 113002 (2010)]	0.866 ± 0.020	$41.47\pm0.52_{exp}\pm0.96_{th}$

$\mathcal{G}(1)|V_{cb}| = (42.64 \pm 0.72_{stat} \pm 1.35_{syst}) \times 10^{-3}$

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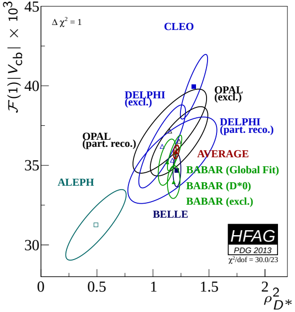
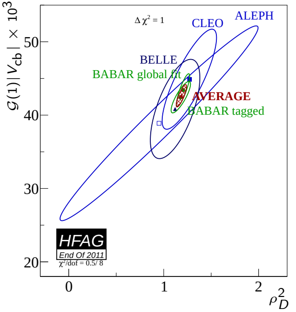
Lattice QCD [NPPS 140, 461-463 (2005)]	1.081 ± 0.024	$39.44\pm1.42_{exp}\pm0.88_{th}$
Sum rule [Phys.Lett.B**585**, 253-262 (2004)]	1.047 ± 0.020	$40.73\pm1.46_{exp}\pm0.78_{th}$

$|V_{cb}|$ average by HFAG





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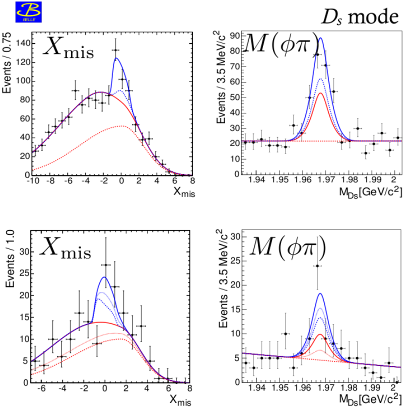
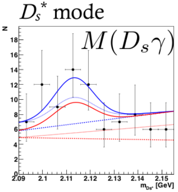
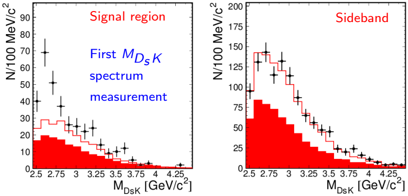
The tension between exclusive $|V_{cb}|$ with the new LQCD calculation of $\mathcal{F}(1)$ and inclusive $|V_{cb}| = (42.42 \pm 0.86) \times 10^{-3}$ from the recent global fit in the kinetic scheme [Phys. Rev. D **89**, 014022 (2014)] becomes now more significant ($\sim 3\sigma$).







$$X_{\rm mis} = \frac{E_{\rm beam} - E_{\rm vis} - |P_{\rm vis}|}{\sqrt{E^2_{\rm beam} - m^2_{B^+}}}$$

Data set 605 fb^{-1}

PRD **86**, 072007(R) (2012)
[arXiv:1207.6244]

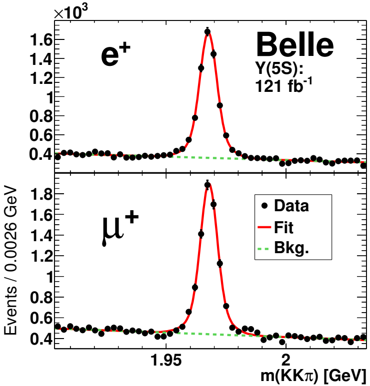
High mass region (above D_sK) of charm semileptonic decays is not well known. Theory can use new measurement to improve description of this region.

Select signal in $D_s \to \phi(K^+K^-)\pi$ mode, remaining particles must be consistent with B meson decay hypothesis in semileptonic mode.

X_c	$\mathcal{B} \times 10^4$	significance
$D_s^- K^+$	$3.0 \pm 0.9^{+1.1}_{-0.8}$	3.4σ
$D_s^{*-} K^+$	$2.9 \pm 1.6^{+1.1}_{-1.0}$	< 5.6 at 90% CL
Combined	$5.9 \pm 1.2 \pm 1.5$	6σ

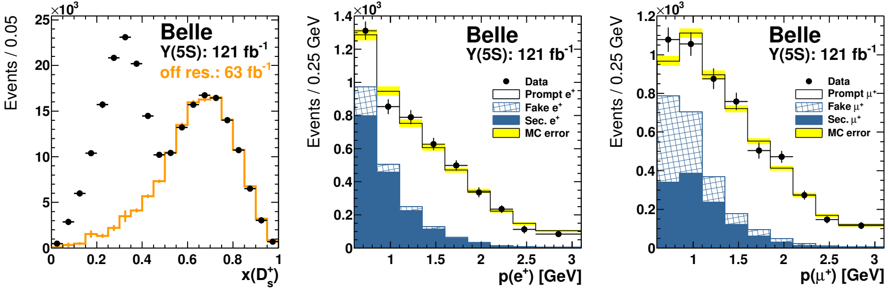
consistent with the BaBar result [PRL**107**(2011)041804]

$$\mathcal{B}(B^+ \to D_s^{(*)-} K^+ \ell \bar{\nu}_\ell) = [6.13^{+1.04}_{-1.03}(\text{stat.}) \pm 0.43(\text{syst.}) \pm 0.51(\mathcal{B}(D_s))] \times 10^{-4}$$



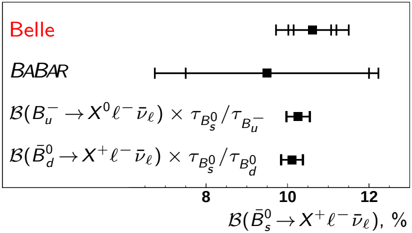
ℓ	$N(D_s^+\ell^+) \times 10^{-3}$	$\mathcal{R} \times 10^4$
e	4.26 ± 0.19	$428 \pm 20 \pm 13$
μ	4.76 ± 0.23	$470 \pm 23 \pm 16$

- 121 fb^{-1} @ $\Upsilon(5S)$ or $(7.1 \pm 1.3) \times 10^6$ $B_s^{0(*)}\bar{B}_s^{0(*)}$ pairs.
- B_s^0 meson is tagged by a D_s^+ meson reconstructed in the mode $D_s^+ \to \phi(K^+K^-)\pi$.
- To suppress continuum events low momentum D_s^+ meson is required $x(D_s^+) = p_{D_s^+}/p_{D_s^+}^{\max} < 0.5$
- Same charge for the lepton and D_s^+ meson to ensure they come from different B_s^0 mesons.
- Branching fraction from the $\mathcal{R} = N(D_s^+\ell^+)/N(D_s^+)$ ratio.
- Total number of selected D_s^+ mesons is $N(D_s^+) = (12.42 \pm 0.08) \times 10^4$ among which $N_{\text{cont}}(D_s^+) = (2.7 \pm 0.1) \times 10^4$ mesons are expected from continuum.

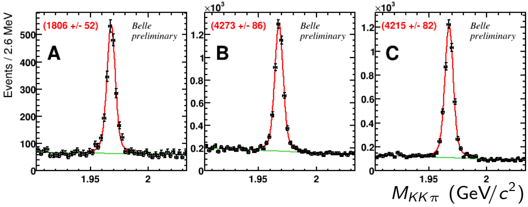


ℓ	$\mathcal{B}(\bar{B}^0_s \to X^+\ell^-\bar{\nu}_\ell)$, %
e	$10.1 \pm 0.6_{\rm stat} \pm 0.4_{\rm syst} \pm 0.6_{\rm ext}$
μ	$11.3 \pm 0.7_{\rm stat} \pm 0.5_{\rm syst} \pm 0.7_{\rm ext}$
Combined	$10.6 \pm 0.5_{\rm stat} \pm 0.4_{\rm syst} \pm 0.6_{\rm ext}$

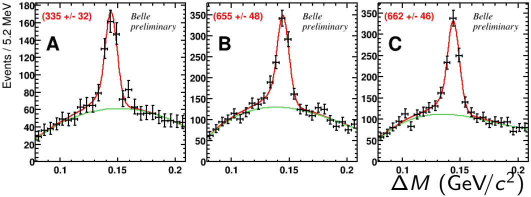
BABAR	3.2 fb^{-1}	PRD **85**, 011101 (2012)
Belle	121 fb^{-1}	PRD **87**, 072008 (2013)



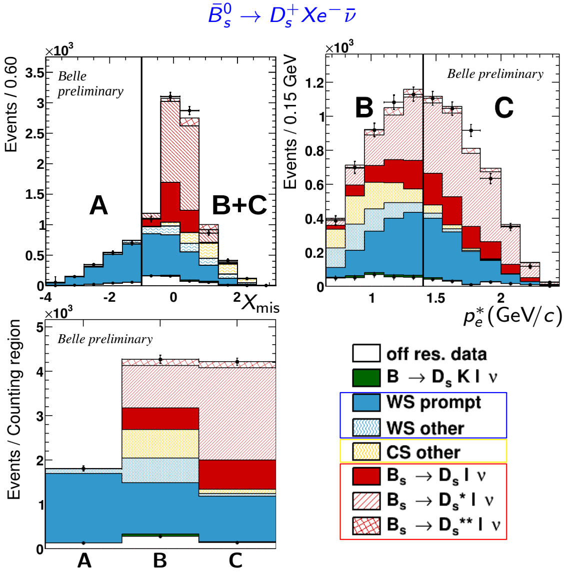
$\bar{B}_s^0 \to D_s^+ X e^- \bar{\nu}$



$\bar{B}_s^0 \to D_s^{*+} X e^- \bar{\nu}$



- 121 fb^{-1} @ $\Upsilon(5S)$.
- Number of B_s^0 is estimated from the luminosity $N_{B_s^0} = \mathcal{L}\,\sigma_{\Upsilon(5S)}(b\bar{b})\,f_s$. Main uncertainty comes from $f_s = 0.172 \pm 0.03$.
- D_s^+ meson reconstructed in the mode $D_s^+ \to \phi(K^+K^-)\pi$ and D_s^{*+} in $D_s^{*+} \to D_s^+\gamma$.
- To enhance semileptonic decays a lepton with the opposite charge to the reconstructed D_s^+ is required.
- Yields of D_s^+ and D_s^{*+} are from the fit to $M_{KK\pi}$ and $\Delta M = M_{KK\pi\gamma} - M_{KK\pi}$ distributions respectively.
- Fits are performed separately for the electron and muon modes.



The decay phase space is divided in three regions (A, B, C) using the variables

$$X_{\rm mis} = \frac{(E_B^* - E_{\rm vis}^*) - p_{\rm vis}^*}{p_B^*} \text{ and } p_\ell^*.$$

Extracted components

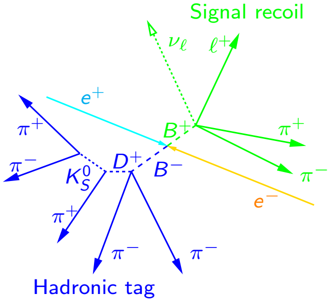
- Wrong B_s^0 assignment – lepton and D_s^+ are from different B_s^0.
- Secondary and fake leptons
- Signal

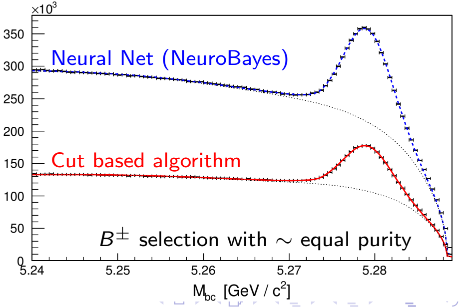
Combined result

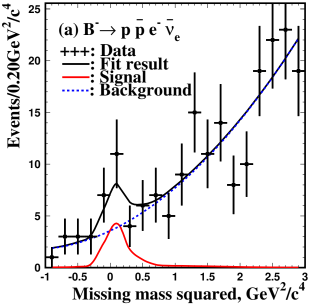
$\mathcal{B}(\bar{B}_s^0 \to D_s^+ X \ell^- \bar{\nu}) =$
$[8.2 \pm 0.2_{\rm stat} \pm 0.8_{\rm syst} \pm 1.5_{N_{B_s^0}}]\%$

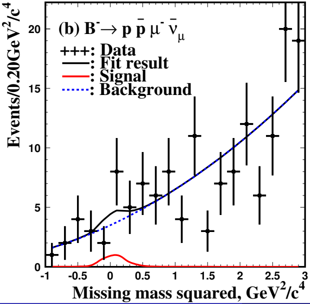
$\mathcal{B}(\bar{B}_s^0 \to D_s^{*+} X \ell^- \bar{\nu}) =$
$[5.4 \pm 0.4_{\rm stat} \pm 0.5_{\rm syst} \pm 1.0_{N_{B_s^0}}]\%$

- Recently new reconstruction procedure of B hadronic decays based on NeuroBayes package has been introduced in Belle. [NIM A**654**, 432 (2011)]
- New procedure tries to reconstruct B meson in more than 1100 exclusive hadronic decay channels.
- Compared to the previous cut based algorithm it offers roughly twice efficiency gain and about 2.1×10^6 of $B^\pm$ and 1.4×10^6 of B^0 with 711 fb^{-1} collected at $\Upsilon(4S)$ resonance.
- Hadronic tag has been calibrated with charm semileptonic decays with precision 4.2 % for B^+ and 4.5 % for B^0.

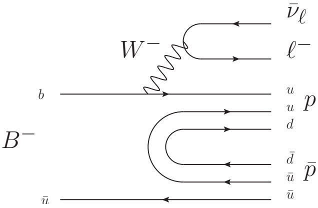








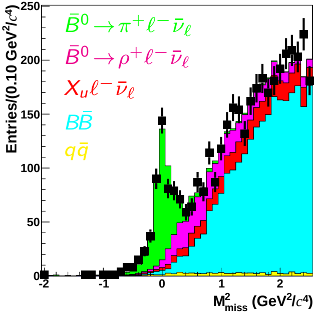
Phys. Rev. D**89**, 011101(R) (2013) [arXiv:1306.3353]

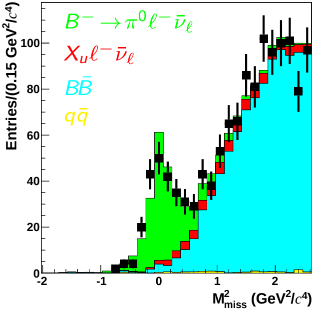


- No previous measurements
- U.L. from CLEO [PRD **68**, 012004 (2003)]: $\mathcal{B}(B^- \to p\bar{p}\ell^-\bar{\nu}_\ell) < 5.2 \times 10^{-3}$
- $\mathcal{B}$ prediction is $\sim 10^{-4}$ in [PLB **704**, 495 (2011)]
- Yield from unbinned maximum likelihood fit

Mode	$\mathcal{B}$ (10^{-6})	U.L. (10^{-6})
$B^- \to p\bar{p}e^-\bar{\nu}_e$	$8.2\,^{+3.7}_{-3.2} \pm 0.6$	13.8
$B^- \to p\bar{p}\mu^-\bar{\nu}_\mu$	$3.1\,^{+3.1}_{-2.4} \pm 0.7$	8.5
Combined sample	$5.8\,^{+2.4}_{-2.1} \pm 0.9$	9.6

$\bar{B} \to \pi \ell^- \bar{\nu}_\ell$ with hadronic tag at Belle





- PRD **88**, 032005 (2013) [arXiv:1306.2781]
- 711 fb^{-1} @ $\Upsilon(4S)$ + 79 fb^{-1} off-peak data
- Yield from binned maximum likelihood fit

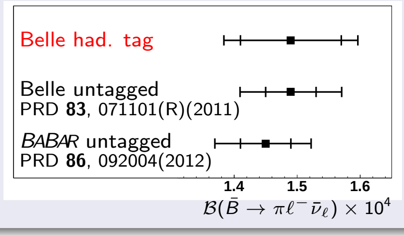
X_u	Yield	$\mathcal{B} \times 10^4$
π^+	462.6±27.7	$1.49 \pm 0.09 \pm 0.07$
π^0	232.2±22.6	$0.80 \pm 0.08 \pm 0.04$

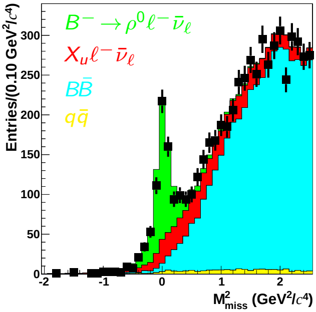
Test of isospin symmetry:

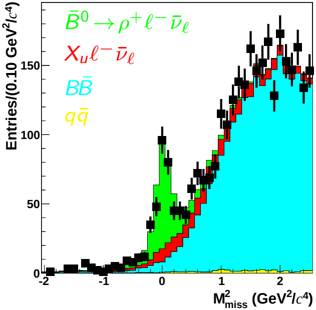
$$2 \times \frac{\mathcal{B}(B^- \to \pi^0 \ell^- \bar{\nu}_\ell)}{\mathcal{B}(\bar{B}^0 \to \pi^+ \ell^- \bar{\nu}_\ell)} \frac{\tau_{B^0}}{\tau_{B^+}} = 1.00 \pm 0.13$$

Combined branching fraction:

$$\mathcal{B}(\bar{B} \to \pi \ell^- \bar{\nu}_\ell) = (1.49 \pm 0.08_{\text{stat}} \pm 0.07_{\text{syst}}) \times 10^{-4}$$







- PRD **88**, 032005 (2013) [arXiv:1306.2781]
- 711 fb^{-1} @ $\Upsilon(4S)$ + 79 fb^{-1} off-peak data
- Yield from binned maximum likelihood fit

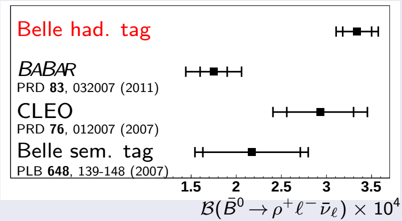
X_u	Yield	$\mathcal{B} \times 10^4$
ρ^+	343.3±28.3	$3.22 \pm 0.27 \pm 0.24$
ρ^0	621.7±35.0	$1.83 \pm 0.10 \pm 0.10$

Test of isospin symmetry:

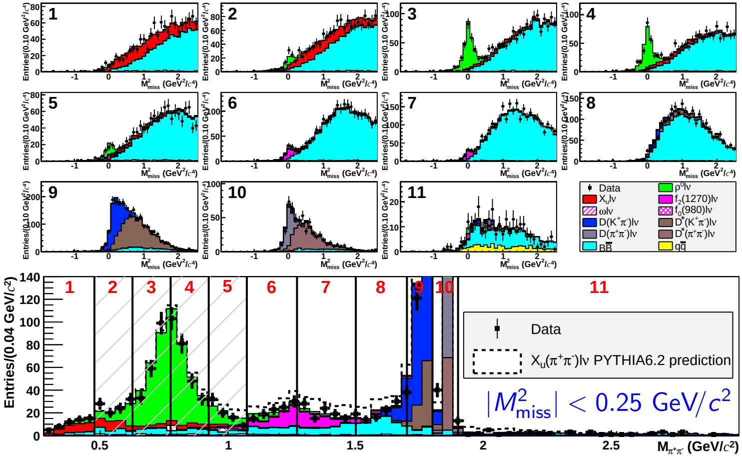
$$2 \times \frac{\mathcal{B}(B^- \to \rho^0\ell^-\bar{\nu}_\ell)}{\mathcal{B}(\bar{B}^0 \to \rho^+\ell^-\bar{\nu}_\ell)} \frac{\tau_{B^0}}{\tau_{B^+}} = 1.06 \pm 0.13$$

Combined branching fraction:

$$\mathcal{B}(\bar{B} \to \rho\ell^-\bar{\nu}_\ell) = (3.34 \pm 0.16_{\text{stat}} \pm 0.17_{\text{syst}}) \times 10^{-4}$$

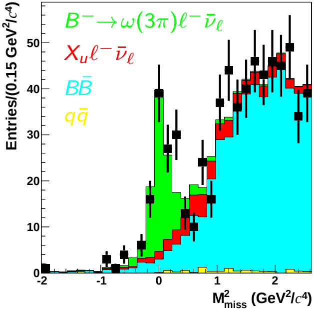


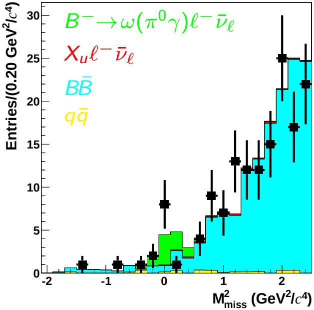
$\bar{B} \to \rho\ell^-\bar{\nu}_\ell$ with hadronic tag at Belle in bins of $M_{\pi\pi}$



- The extracted yield of the $B^- \to \rho^0\ell^-\bar{\nu}_\ell$ decay nicely agrees with the $M^2_{\rm miss}$ fit alone.
- Belle does not see the $\bar{B} \to X_u(\pi\pi)\ell^-\bar{\nu}_\ell$ non-resonance decay predicted by PYTHIA6.2 in data: $N^{\rm PYTHIA}_{\pi\pi\ell^-\bar{\nu}_\ell} = 334.9$, $N^{\rm fit}_{\pi\pi\ell^-\bar{\nu}_\ell} = 45.8 \pm 45.4$
- The extracted yield of the $B^- \to f_2\ell^-\bar{\nu}_\ell$ decay is 2-3 times higher than the ISGW2 model prediction: $N^{\rm ISGW2}_{f_2\ell^-\bar{\nu}_\ell} = 58.4$, $N^{\rm fit}_{f_2\ell^-\bar{\nu}_\ell} = 154.4 \pm 22.2$

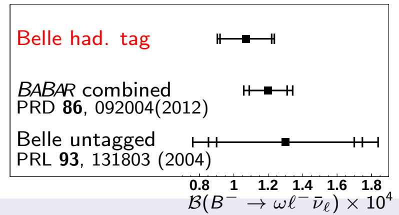
$B^- \to \omega \ell^- \bar{\nu}_\ell$ with hadronic tag at Belle

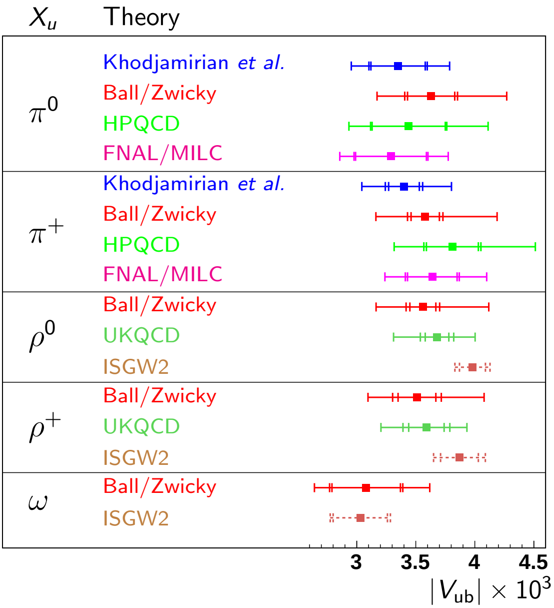




- PRD **88**, 032005 (2013) [arXiv:1306.2781]
- 711 fb^{-1} @ $\Upsilon(4S)$ + 79 fb^{-1} off-peak data
- Yield from binned maximum likelihood fit

X_u	Yield	$\mathcal{B} \times 10^4$
$\omega(3\pi)$	96.7±14.5	$1.07 \pm 0.16 \pm 0.07$
$\omega(\pi\gamma)$	9.0±4.0	$1.06 \pm 0.47 \pm 0.07$





Khodjamirian *et al.*
PRD **83**, 094031 (2011)

Ball/Zwicky
PRD **71**, 014015 (2005)
PRD **71**, 014029 (2005)

HPQCD
PRD **73**, 074502 (2006)

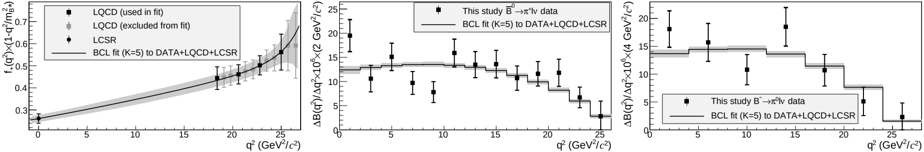
FNAL/MILC
PRD **79**, 054507 (2009)

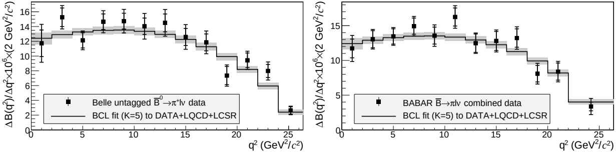
UKQCD
PLB **416**, 392 (1998)

ISGW2
PRD **52**, 2783 (1995)
Theory error is not available.

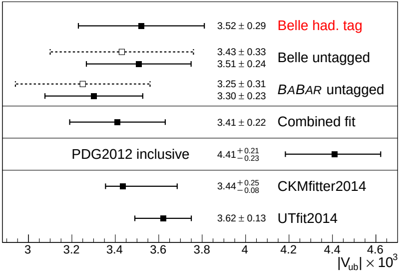
Model independent extraction of $|V_{ub}|$ from exclusive $\bar{B} \to \pi \ell^- \bar{\nu}_\ell$ decays

Simultaneous fit with the BCL parametrization [PRD79, 013008 (2009)] to the recent form factor predictions and the available $\bar{B} \to \pi \ell^- \bar{\nu}_\ell$ data provides the most precise value of $|V_{ub}|$.





$\chi^2/\text{ndf} = 44.74/44$



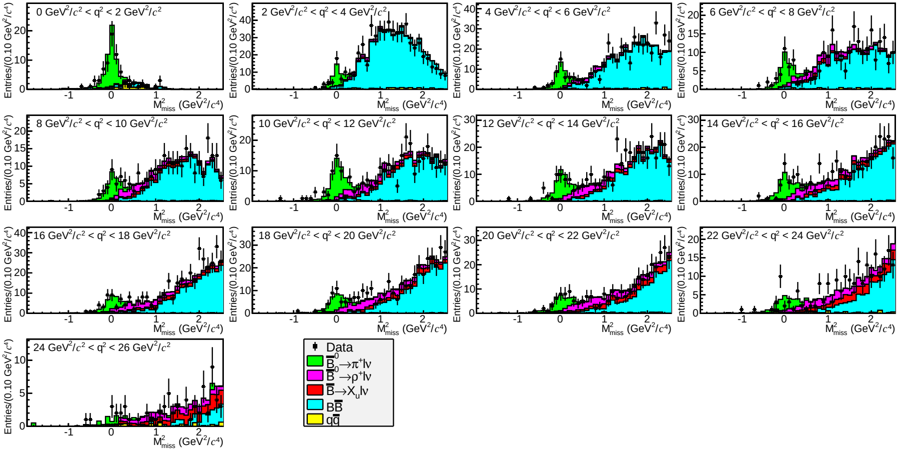
The dashed lines represent $|V_{ub}|$ values quoted in the original papers.

LCSR	JHEP **1205**, 092 (2012) [arXiv:1203.1359]
LQCD	PRD **79**, 054507 (2009) [arXiv:0811.3640]
Belle had. tag	PRD **88**, 032005 (2013) [arXiv:1306.2781]
Belle	PRD **83**, 071101(R) (2011) [arXiv:1012.0090]
BABAR	PRD **86**, 092004 (2012) [arXiv:1208.1253]

Conclusion

- Belle was decommissioned at 2010 but analysis of the collected data still improves our knowledge in B physics.
- Belle has significantly contributed to the $|V_{cb}|$ measurement.
- Belle has started to explore the poorly known region above 2.46 GeV/c^2 in hadronic recoil masses of semileptonic B decays.
- Belle has measured inclusive semileptonic decay rates of the B_s^0 meson.
- Using the new neural net based hadronic tag technique, Belle has measured exclusive branching fractions of a number of charmless semileptonic decays with low background and high accuracy.
- Using the pion modes, a recent LCSR calculation, lattice QCD results and a model-independent description of the hadronic form factor, a value of the CKM matrix element $|V_{ub}|$ is extracted.
- The tension between exclusive and inclusive measurements of $|V_{cb}|$ and $|V_{ub}|$ is now at 3σ level.

Backup



$$\chi^2/\text{ndf} = 813.165/1026$$

Differential branching fractions in bins of q^2 are also extracted!

Normalized decay rate $\Delta\zeta = \int d\Gamma/|V_{ub}|^2$ is based on form factor prediction from theory.

X_u	Theory	q^2 GeV/c^2	N^{fit}	$\Delta\mathcal{B}$ 10^{-4}	$\Delta\zeta$ ps^{-1}	$\|V_{ub}\|$ 10^{-3}
π^0	LCSR1	< 12	119.6 ± 16.2	0.423 ± 0.057	$4.59^{+1.00}_{-0.85}$	$3.35 \pm 0.23 \pm 0.09^{+0.36}_{-0.31}$
	LCSR2	< 16	168.2 ± 18.9	0.588 ± 0.066	$5.44^{+1.43}_{-1.43}$	$3.63 \pm 0.20 \pm 0.10^{+0.60}_{-0.40}$
	HPQCD	> 16	58.6 ± 10.5	0.196 ± 0.035	$2.02^{+0.55}_{-0.55}$	$3.44 \pm 0.31 \pm 0.09^{+0.59}_{-0.39}$
	FNAL				$2.21^{+0.47}_{-0.42}$	$3.29 \pm 0.30 \pm 0.09^{+0.37}_{-0.30}$
π^+	LCSR1	< 12	247.2 ± 18.9	0.808 ± 0.062	$4.59^{+1.00}_{-0.85}$	$3.40 \pm 0.13 \pm 0.09^{+0.37}_{-0.32}$
	LCSR2	< 16	324.2 ± 22.6	1.057 ± 0.074	$5.44^{+1.43}_{-1.43}$	$3.58 \pm 0.12 \pm 0.09^{+0.59}_{-0.39}$
	HPQCD	> 16	141.3 ± 16.0	0.445 ± 0.050	$2.02^{+0.55}_{-0.55}$	$3.81 \pm 0.22 \pm 0.10^{+0.66}_{-0.43}$
	FNAL				$2.21^{+0.47}_{-0.42}$	$3.64 \pm 0.21 \pm 0.09^{+0.40}_{-0.33}$
ρ^0	LCSR3	< 16	476.5 ± 30.5	1.431 ± 0.091	$13.7^{+3.4}_{-3.4}$	$3.56 \pm 0.11 \pm 0.09^{+0.54}_{-0.37}$
	UKQCD	full range	621.7 ± 35.0	1.834 ± 0.103	$16.5^{+3.5}_{-2.3}$	$3.68 \pm 0.10 \pm 0.10^{+0.29}_{-0.34}$
	ISGW2				$14.1\pm??$	$3.98 \pm 0.11 \pm 0.10^{+?.??}_{-?.??}$
ρ^+	LCSR3	< 16	268.8 ± 25.0	2.574 ± 0.239	$13.7^{+3.4}_{-3.4}$	$3.51 \pm 0.16 \pm 0.13^{+0.53}_{-0.36}$
	UKQCD	full range	343.3 ± 28.3	3.222 ± 0.266	$16.5^{+3.5}_{-2.3}$	$3.59 \pm 0.15 \pm 0.13^{+0.28}_{-0.33}$
	ISGW2				$14.1\pm??$	$3.87 \pm 0.16 \pm 0.15^{+?.??}_{-?.??}$
ω	LCSR3	< 12	61.3 ± 11.4	0.611 ± 0.113	$7.88^{+1.86}_{-1.86}$	$3.08 \pm 0.29 \pm 0.11^{+0.44}_{-0.31}$
	ISGW2	full range	96.7 ± 14.5	1.069 ± 0.160	$14.1\pm??$	$3.03 \pm 0.23 \pm 0.11^{+?.??}_{-?.??}$

LCSR1 PRD 83 (2011) 094031 HPQCD PRD 73 (2006) 074502 LCSR2 PRD 71 (2005) 014015 FNAL PRD 79 (2009) 054507
LCSR3 PRD 71 (2005) 014029 UKQCD PLB 416 (1998) 392 ISGW2 PRD 52 (1995) 2783

Note nice agreement between $\bar{B} \to \pi\ell^-\bar{\nu}_\ell$ and $\bar{B} \to \rho\ell^-\bar{\nu}_\ell$ modes with LCSR(2,3) prediction.

Matrix element of $B \to X_u \ell \bar{\nu}_\ell$ decay at first order

$$\mathcal{M}(B \to X_u \ell \bar{\nu}_\ell) = \frac{G_F}{\sqrt{2}} V_{\rm ub} L^\mu H_\mu, \;\; L^\mu = \bar{u}_\ell \gamma^\mu (1-\gamma^5) v_\nu$$

Hadronic current H_μ depends on specific final state.

The differential decay rate for $B \to \pi \ell \bar{\nu}_\ell$ process where $\ell = e, \mu$ can be expressed in terms of W boson momentum q:

$$\frac{\mathrm{d}\Gamma(B \to \pi \ell \bar{\nu}_\ell)}{\mathrm{d}q^2} = \frac{G_F^2 p_\pi^3}{24\pi^3} |V_{\rm ub}|^2 |f_+(q^2)|^2, \qquad (1)$$

where all QCD uncertainties are hidden in the $f_+(q^2)$ vector form factor. Model independent parametrization of $f_+(q^2)$, which satisfies unitarity, analyticity and perturbative QCD scaling has been suggested by Bourrely, Caprini and Lellouch (BCL) [PRD **79**, 013008 (2009)]:

$$f_+(q^2, \vec{b}) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{k=0}^{K} b_k \, z(q^2)^k, \quad z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad (2)$$

where $m_{B^*} = 5.325$ GeV/c^2 and $t_+ = (m_B + m_\pi)^2$. The optimal parameter $t_0 = 20.06$ GeV$^2/c^2$ provides fast series convergence since the physical q^2 region maps onto the disk $|z| < 0.28$. Unitarity and crossing symmetry properties of the form factor constrain the $\vec{b}$ parameters:

$$\sum_{j,k=0}^{K} B_{jk} b_j b_k \leq 1. \qquad (3)$$