Weak interactions of kaons from lattice QCD

BEACH 2014

July 21, 2014

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RBC and UKQCD Collaborations

Outline

- Lattice QCD in 2014
- First order electroweak:
  - f_K/f_π, $Kl3$
  - $K \rightarrow \pi\pi$ decay
- Second order electroweak
  - $K_L - K_S$ mass difference
  - Long distance parts of ε_K.
  - Rare kaon decays

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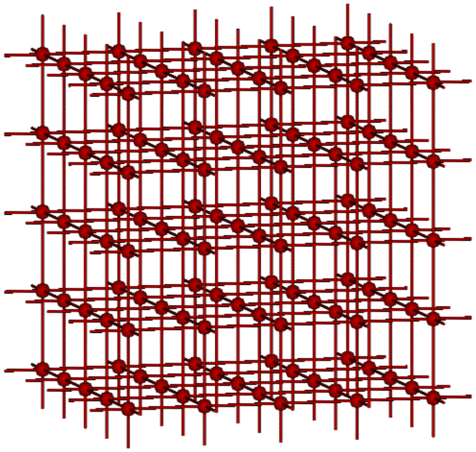
Lattice QCD

2014

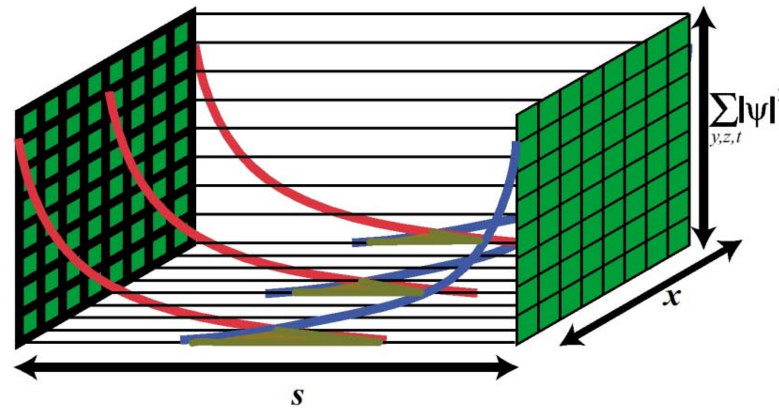
Lattice QCD



- First-principles treatment of low-energy, non-perturbative QCD.
- All approximations understood and controlled:
  - Non-zero lattice spacing: $a \to 0$.
  - Finite volume: $L \to \infty$
  - Typically neglect E&M and $m_u \neq m_d$, $\alpha_{\mathrm{EM}} << 1$
- Supports not only rough phenomenology but also accurate theoretical physics (where it can be applied).

Domain wall fermions

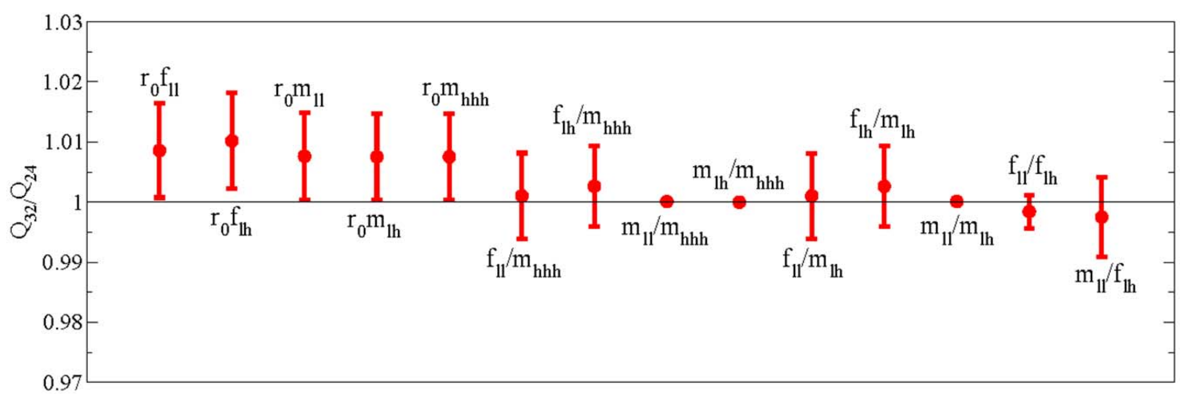
- Use 5-dimensional Wilson fermion action with open BC on $s = 0$ and $s = L_s - 1$ slice.
- 4-D chiral bound states form on the s = 0 and $L_s - 1$ walls.



- 5-D propagating states are large-action, lattice artifacts.
- 4-D states disappear as $p \rightarrow \pi / a$, solving the doubling problem.
- Accurate chiral symmetry at all energies, broken by left-right mixing: residual chiral symmetry breaking.

DWF at low energy

- At low energy $E << 1/a$, 5-D DWF theory looks like a chiral 4-D theory (QCD) with small chiral asymmetry:
  - Leading, dim-3 operator: $m_{\mathrm{res}}\,\overline{q}\,q$ (mass term)
  - Next leading dim-5 operator: $m_{\mathrm{res}}\,\overline{q}\,\sigma^{\upsilon\nu}F^{\upsilon\nu}q$ (clover term)
- Very small discretization errors:

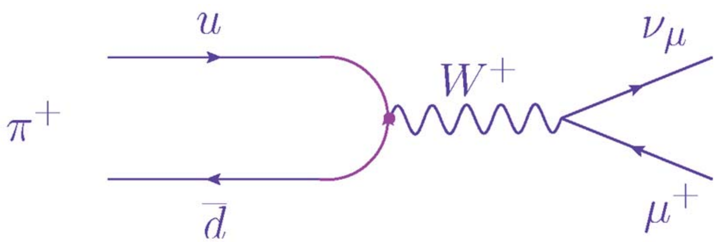


Ratios of dimensionless combinations of physical quantities computed using $1/a$ = 1.73 and 2.28 GeV.

Current state-of-the-art

- Physical m_π=135 MeV and L = 4 – 6 fm.
- Generate 48^3 x 96 and 64^3 x 128 ensembles.
- Complete set of measurements takes 5.3 hours on a 32-rack BG/Q machine (sustains 1 Pflops)
- Large collaboration essential:
  - Highly optimized code (64 threads, SPI comms., wide-vector FP)
  - Sophisticated algorithms (deflation, FG $(\Delta t)^3$ integrator)
  - Complex measurement strategies (NPR, G-parity BC, 4-pt functions, all-mode-averaging, all-to-all propagators)

Simple example: f_π



$$\langle 0|\bar{d}\gamma^5\gamma^\mu u|\pi^+(\vec{p})\rangle = f_\pi \frac{p^\mu}{\sqrt{4E_\pi(\vec{p})}}$$

$$f_\pi = N\sum_{\vec{r}} \frac{\langle A^0(\vec{r},t)O_\pi(t=0)\rangle}{\langle O_\pi^\dagger(t)O_\pi(t=0)\rangle^{\frac{1}{2}}} e^{m_\pi t/2}$$

- 2012 (elaborate chiral fit): $f_\pi = 127(3)_{stat}(3)_{sys}$ MeV
- 2013 (m_π=135 MeV): f_π = $130.0(0.3)_{stat}$ MeV (40 confgs.)
- Experiment: f_π = 130.4(0.04)(0.2) MeV

Semi-leptonic decays

- f_K / f_π
  - RBC/UKQCD (prelim.)

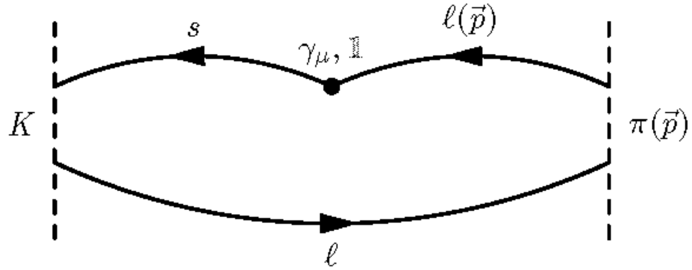
    $f_K / f_\pi = 1.199(4)_{\mathrm{stat}}(4)_{\mathrm{chi}}(2)_{\mathrm{FV}}$
  - FNAL/MILC (arXiv:1407.3772)

    $f_K / f_\pi = 1.1956(10)_{\mathrm{stat}}(+23/\text{-}14)_{\mathrm{cont}}$

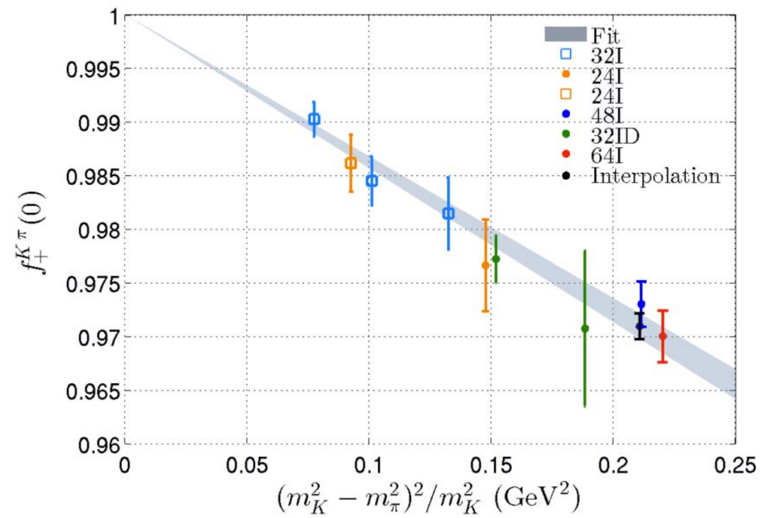
    $(10)_{\mathrm{FV}}\,(5)_{\mathrm{EM}}$
- $Kl3$: f_+
  - FNAL/MILC (PRL 112 (2014) 112001)

    $f_+(0) = 0.9704(24)_{\mathrm{chi\text{-}cont}}(22)_{\mathrm{sys}}$
  - ETMc, (preliminary)

    $f_+(0) = 0.9683(50)_{\mathrm{stat+fit}}(42)_{\mathrm{chiral}}$



RBC/UKQCD

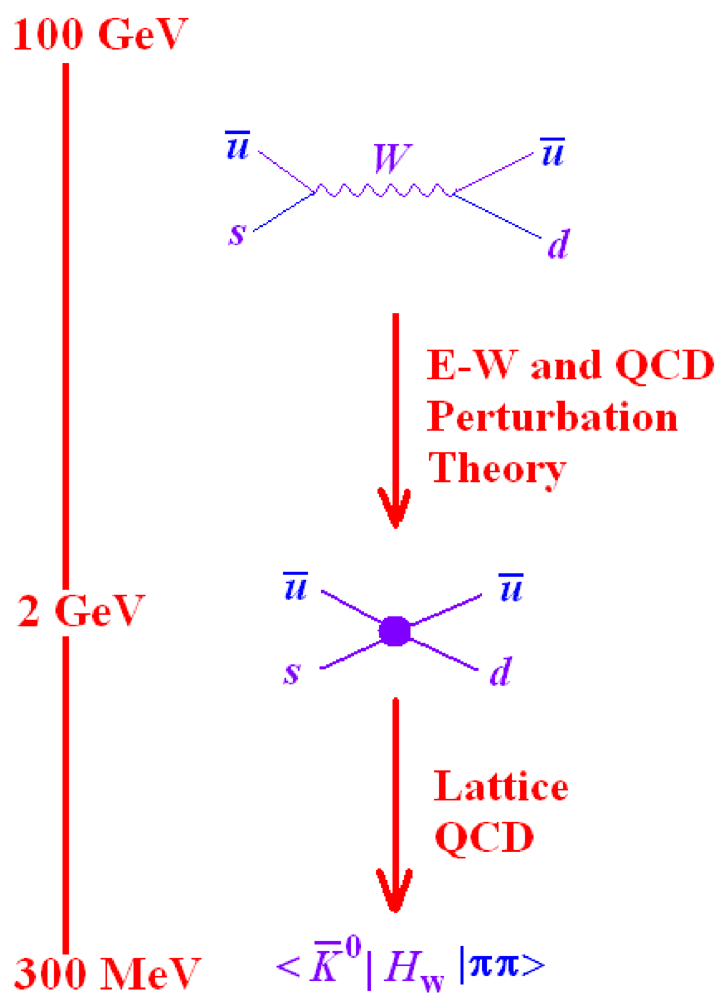


ΔS=1 Weak Interactions

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{ud}} \frac{V_{ts}^*}{V_{us}^*} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Four quark operators

- **Current-current operators**

$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**

$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**

$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

$K \rightarrow \pi\,\pi$ decay

$K \rightarrow \pi\,\pi$ phenomenology

- Final $\pi\pi$ states can have $I = 0$ or 2.

$$\begin{aligned} \langle \pi\,\pi(I = 2)|H_w|K^0\rangle &= A_2 e^{i\delta_2} & \Delta I = 3/2 \\ \langle \pi\,\pi(I = 0)|H_w|K^0\rangle &= A_0 e^{i\delta_0} & \Delta I = 1/2 \end{aligned}$$

- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| \left(\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right)$$

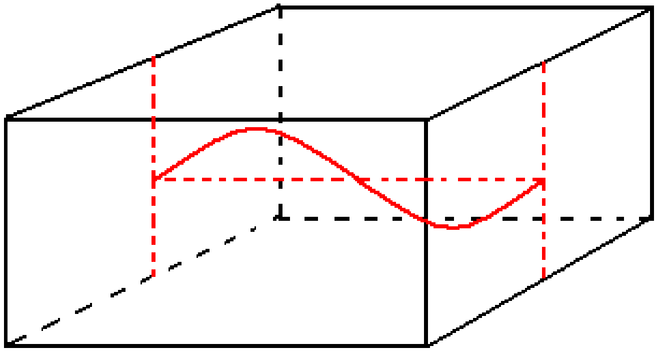
- $K^0 - K^0$ mixing gives indirect CP violation:

$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$

Lattice Aspects

Physical $\pi\pi$ states – Lellouch-Luscher

- Euclidean e^{-Ht} projects onto $|\pi\pi(\vec{p}=0)\rangle$
- Exploit finite-volume quantization.
- Adjust volume so 1st or 2nd excited state has correct p.

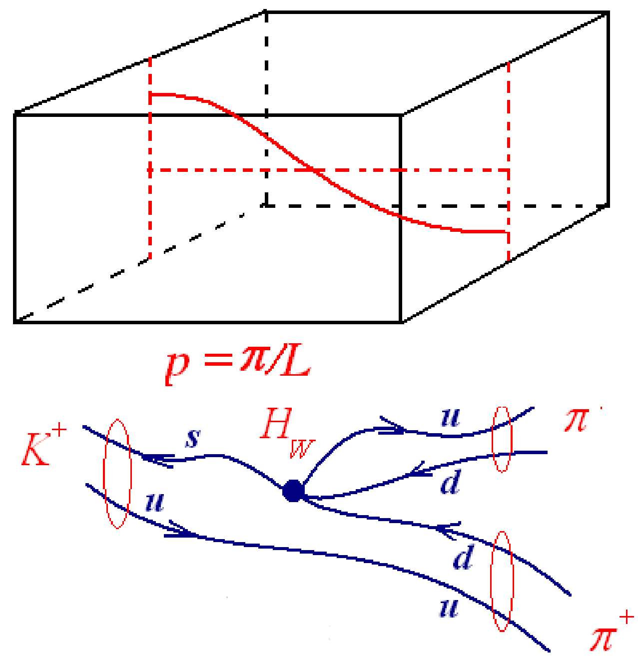


- Impose boundary conditions so ground state has physical p
  - $\Delta I = 3/2$: impose anti-periodic BC on d quark
  - $\Delta I = 1/2$: impose G-parity BC
- Correctly include π - π interactions, including normalization.

$\Delta I = 3/2$

$\Delta I = 3/2 \;\; K \rightarrow \pi\pi$

- Three operators contribute $O^{(27,1)}$, $O^{(8,8)}$ and $O^{(8,8)m}$.
- **Achieve essentially physical kinematics!**
  (146 configurations)
  - m_π = 142.9(1.1) MeV
  - m_K = 511.3(3.9) MeV
  - $E_{\pi\pi}$ = 492(5.5) MeV



- $\mathrm{Re}(A_2) = (1.436 \pm 0.063_{stat} \pm 0.258_{sys})\; 10^{-8}$ GeV

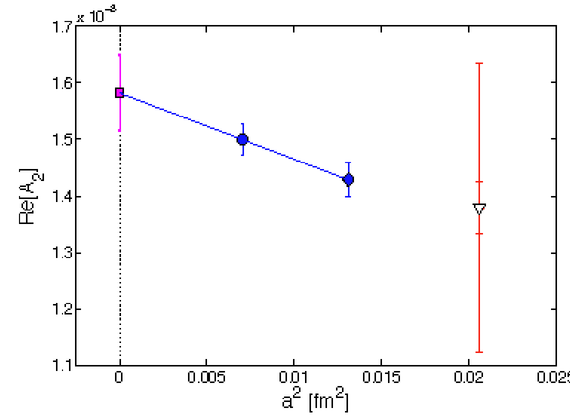
Experiment: 1.479(4) 10^{-8} GeV

- $\mathrm{Im}(A_2) = -(6.29 \pm 0.46_{stat} \pm 1.20_{sys})\; 10^{-13}$ GeV

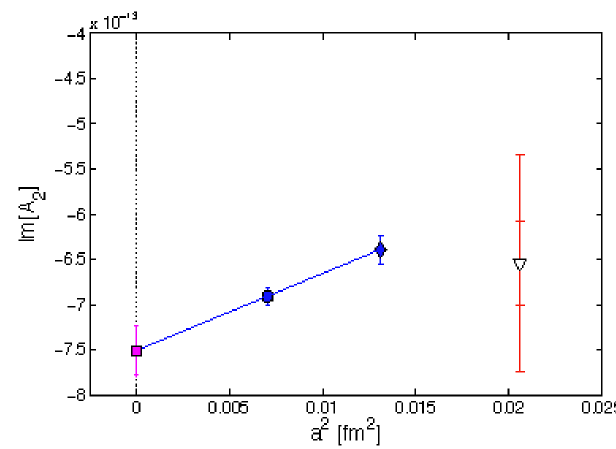
Δ I = 3/2: Next results

(Tadeusz Janowski)

- Use two new large ensembles to remove a^2 error (m_π=135 MeV, L=5.4 fm)
    - 48^3 x 96, 1/a=1.73 GeV
    - 64^3 x 128, 1/a=2.28 GeV



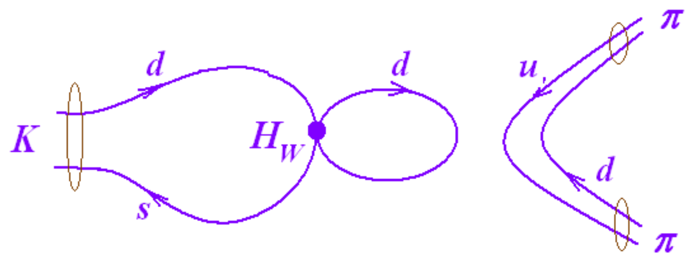
- First continuum results, (preliminary):
    - $\mathrm{Re}(A_2) = (1.583 \pm 0.067_{\mathrm{stat}})\times 10^{-8}$ GeV
    - $\mathrm{Im}(A_2) = -\,(7.51 \pm 27_{\mathrm{stat}})\times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2) = 1.479(4)\ 10^{-8}$ GeV



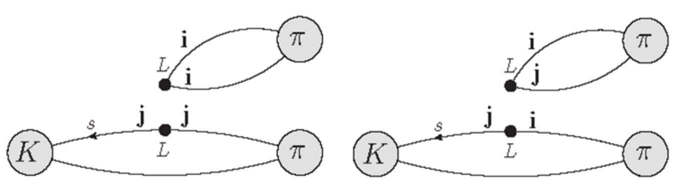
$$\Delta \mathbf{I} = \mathbf{1/2}$$

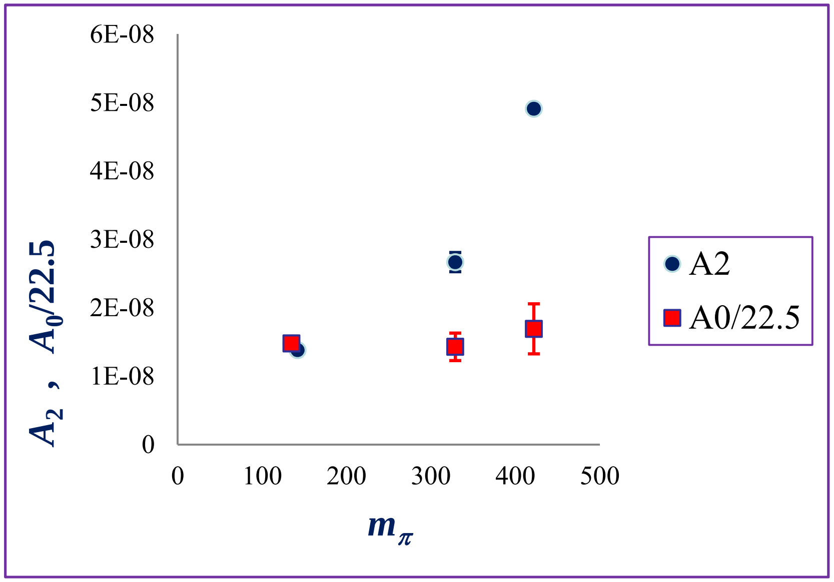
$\Delta I = 1/2$ $K \rightarrow \pi\pi$

- Made much more difficult by disconnected diagrams:



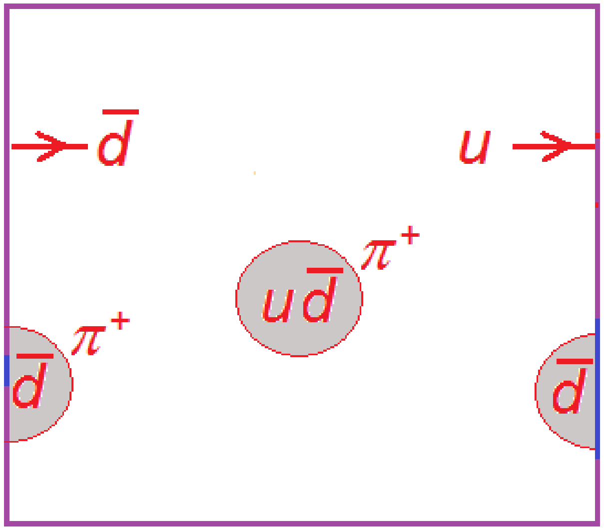
- Many more diagrams (48) than $\Delta I = 3/2$.
- Initial threshold decay calculation successful (Qi Liu)
  - Re (A_0): 25% stat errors
  - Im (A_0): 50% stat errors
- $\Delta I = 1/2$ rule "explained"





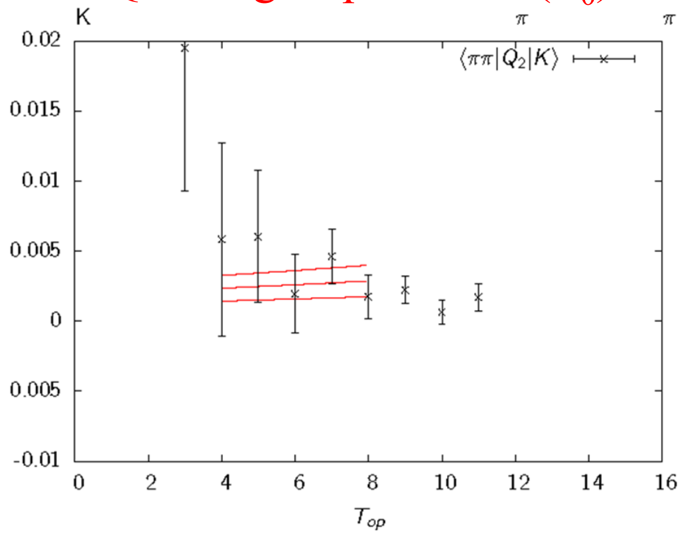
$\Delta I = 1/2$ $K \rightarrow \pi\pi$: Physical kinematics

- Goal is a 20% calculation of ε'/ε with all errors controlled
- Use 32^3 x 64 volume with $1/a = 1.37$ GeV
- Achieve $p = 205$ MeV from **G-parity** boundary conditions in 3 directions
- Requires new **G-parity** ensembles



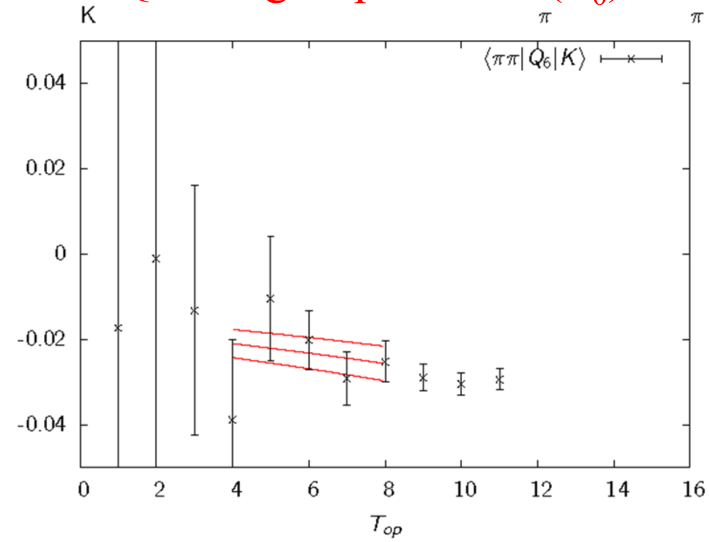
$\Delta I = 1/2$ $K \rightarrow \pi\pi$: Current status

Q2 - largest part of Re(A_0)



$$\langle \pi\pi_{I=0}|Q_2|K\rangle = (1.92 \pm 0.75)\times 10^{-3}$$

Q6 - largest part of Im(A_0)



$$\langle \pi\pi_{I=0}|Q_6|K\rangle = (-1.71 \pm 0.27)\times 10^{-3}$$

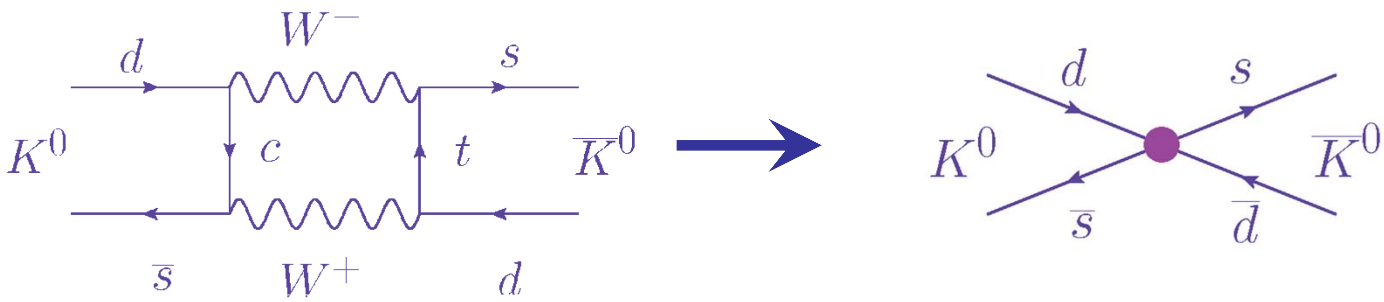
- 70 configurations
- Result expected in 1 year

$K_L - K_S$ mass difference

$K^0 - \overline{K}^0$ Mixing

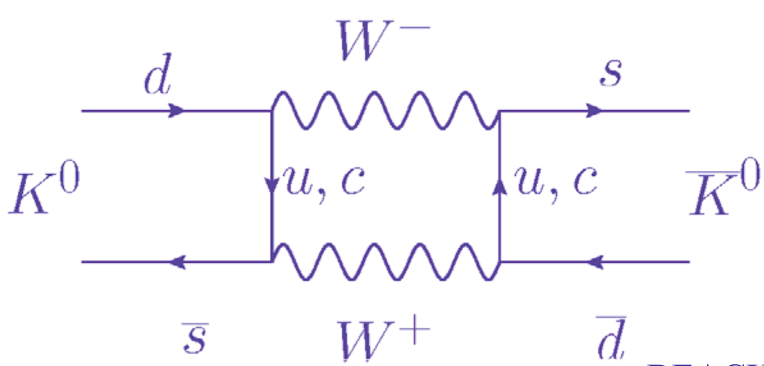
- CP violating: $p \sim m_t$

$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$



- CP conserving: $p \leq m_c$

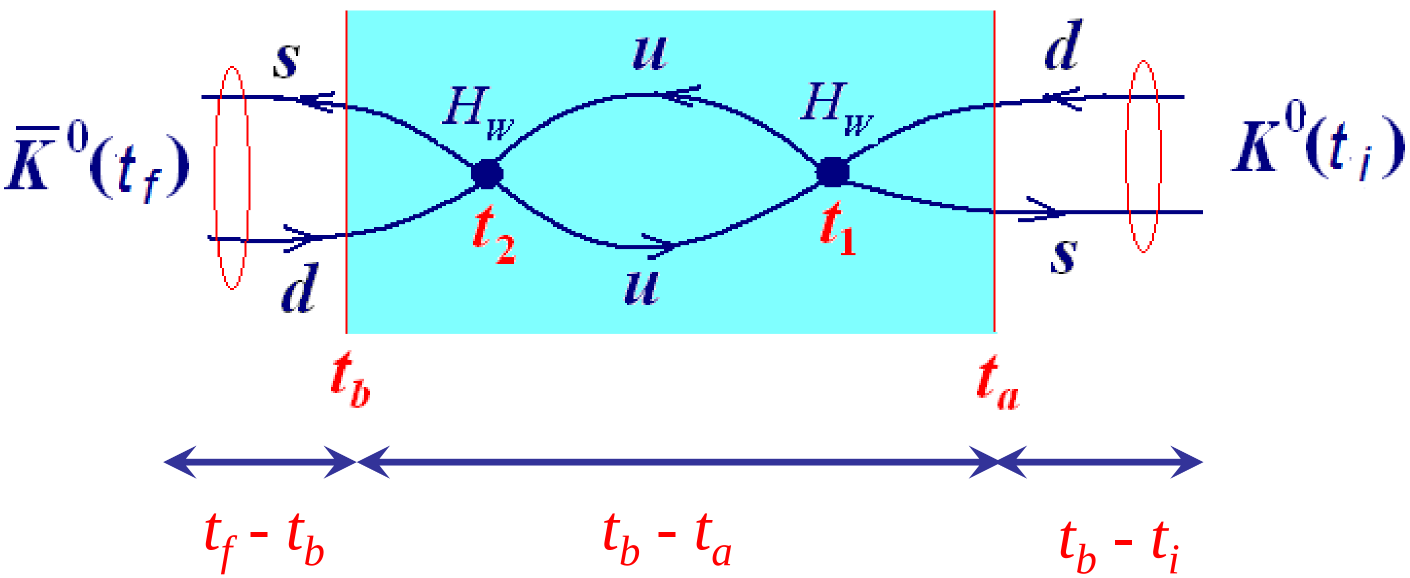
$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$



Lattice Version
(Jianglei Yu)

- Evaluate standard, Euclidean, 2nd order $K^0 - \overline{K^0}$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0'}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(\underbrace{-(t_b - t_a)}_{1.} \underbrace{- \frac{1}{M_K - E_n}}_{2.} \underbrace{+ \frac{e^{(M_K - E_n)(t_b - t_a)}}{M_K - E_n}}_{3.} \right)$$

1. Δm_K^{FV}
2. Uninteresting constant
3. Growing or decreasing exponential:
   states with $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_{n_0}} - \frac{E_{n_0}^2}{2 k_n M_K} \frac{d(\phi + \delta_0)}{dk} \bigg|_{m_K} V \, |\langle n_0 | H_W | K^0 \rangle|^2 \cot(\phi + \delta_0) \big|_{M_K}$$

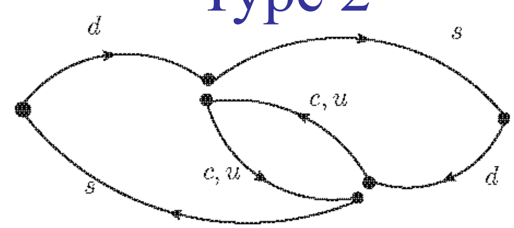
(N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda)

Lattice setup
(Jianglei Yu)

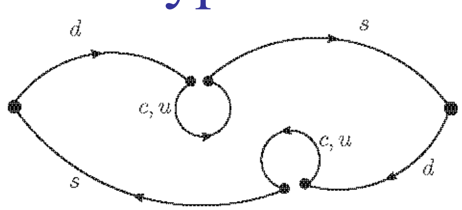
- Must include charm quark (GIM *u*–*d* cancellation)
- Two calculations performed
  - 16^3 x 32, m_p = 420 MeV, types 1 & 2 (arXiv:1212.5931)
  - 24^3 x 64, m_p = 330 MeV, all graphs (arXiv:1406.0916)

Type 1

Type 2



Type 3



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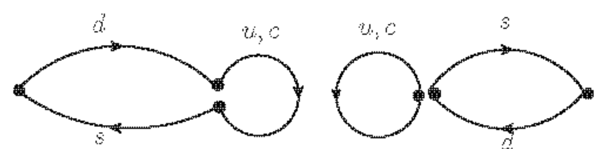


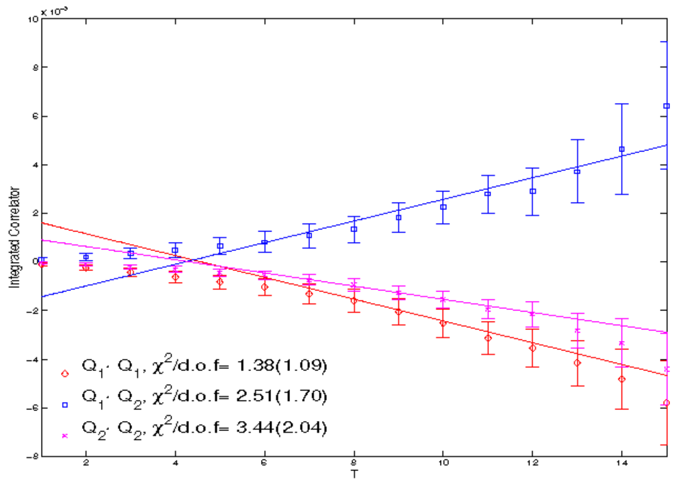
ΔM_K – results

Diagrams	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	ΔM_K
Type 1,2	1.485(80)	1.567(38)	3.678(56)	6.730(96)
All	0.754(42)	-0.16(15)	2.70(18)	3.30(34)

x 10^{-12} MeV

- Result:
  $\Delta M_K = 3.30(34)\ 10^{-12}$ MeV
- $\Delta M_K^{\mathrm{expt}} = 3.483(6)\ 10^{-12}$ MeV
- Agreement fortuitous!
- Unphysical, $m_\pi = 330$ MeV
- Active charm but $m_c\, a = 0.55$

ΔM_K – Next Steps
(Ziyuan Bai)



- M_π = 170 MeV
- $1/a$ = 1.35 MeV
- 250 configurations

Charm mass	ΔM_K
750 MeV	4.6(13)
592 MeV	3.8(17)

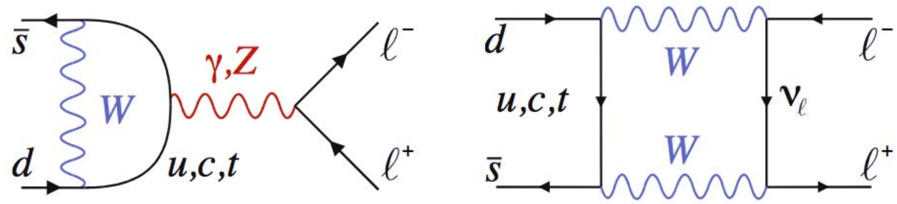
- Begin 32^3 x 64 calculation
- Case study for $2M_\pi < M_K$
- Prepare for 80^2 x 96 x 192, $1/a$=3.0 GeV.
- Long dist. contribution to ε_K being coded.

I	$E_{2\pi}$(MeV)	ΔM_K (10^{-12} MeV)
0	337.7(30)	-0.133(99)
2	343.5(25)	-6.54(25) x 10^{-4}

Rare Kaon Decays

Rare Kaon Decays

(Xu Feng, Antonin Portelli)



- Can lattice methods be of use for rare K decays?
  - $K \to \pi + l + \bar{l}$ or $K \to \pi + \nu + \bar{\nu}$
  - Semi-leptonic decays with ΔS=1 are second order in the standard model and hence sensitive to new physics.
- $K_L \to \pi^0 + l + \bar{l}$: determine the sign of the indirect CP violating amplitude.
- $K^+ \to \pi^+ + \nu + \bar{\nu}$: calculate the long distance ($l \geq 1/m_c$) part of charm contribution. Small (≈4%) but leading theoretical uncertainty.
- First calculations underway.

Outlook

- DWF with physical quark masses reproduce QCD at the ≤ 3% level on a 48^3 x 64 lattice.
- NPR gives continuum-like control of operator normalization and mixing.
- Theoretical advances allow rescattering effects to be correctly computed in Euclidean space (so far only for low energy $\pi - \pi$ states).
- Many critical quantities can now be computed:
  - $K \rightarrow \pi \pi$, ΔI=3/2 and 1/2, ε'/ε
  - $m_{K_L} - m_{K_S}$, long dist. contribution to ε
  - $K \rightarrow \pi l \bar{l}$, $K \rightarrow \pi \nu \bar{\nu}$
  - Quark effects in g_μ- 2 from HVP and HLbL at $O(\alpha^3)$