QCD tests with Kaons

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Abstract. Final results from an analysis of about 400 $K^{\pm} \to \pi^{\pm}\gamma\gamma$ rare decay candidates collected by the NA48/2 and NA62 experiments at CERN are presented. The results include a model-independent decay rate measurement and fits to Chiral Perturbation Theory (ChPT) description. The data support the ChPT prediction for a cusp in the di-photon invariant mass spectrum at the two pion threshold. The NA48/2 Collaboration at CERN has accumulated unprecedented statistics of rare kaon decays in the Ke4 modes $K^{\pm}_{e4} \to \pi^{+}\pi^{-}e\nu$ and $K^{00}_{e4} \to \pi^{0}\pi^{0}e\nu$ with one percent background contamination. The detailed study of the form factors brings new inputs to low energy QCD description and crucial tests of predictions from ChPT and lattice QCD calculations.

1. The NA48/2 and NA62-RK experiment

The NA48/2 beam line has been designed to deliver simultaneous narrow momentum band K^{+} and K^{-} beams derived from the primary 400 GeV/c protons extracted from the CERN SPS. Secondary beams with central momenta of 60 GeV/c (for NA48/2) or 74 GeV/c (for NA62-RK) were used. The beam kaons decayed in a fiducial volume contained in a 114 m long cylindrical vacuum tank. The momenta of charged decay products were measured in a magnetic spectrometer consisting of four drift chambers (DCHs), two upstream and two downstream of a dipole magnet. The dipole provided a horizontal transverse momentum kick of 120 MeV/c (for NA48/2) or 265 MeV/c (for NA62-RK) to charged particles. Each DCH was composed of eight planes of sense wires. A plastic scintillator hodoscope (HOD) producing fast trigger signals and providing precise time measurements of charged particles was placed after the spectrometer. Further downstream was a liquid krypton electromagnetic calorimeter (LKr), an almost homogeneous ionization chamber with an active volume of 7 m^3 of liquid krypton, $27X_0$ deep, segmented transversally into 13248 projective 2x2 cm^2 cells and with no longitudinal segmentation. The LKr information is used for photon measurements and charged particle identification. An iron/scintillator hadronic calorimeter and muon detectors were located further downstream. A detailed description of the detector can be found in[1].

2. The $K^{\pm} \to \pi^{\pm}\gamma\gamma$ decay

Measurements of radiative non-leptonic kaon decays provide crucial tests for the ability of the Chiral Perturbation Theory (ChPT) to explain weak low energy processes. In the ChPT

[1] The NA48/2 Collaboration: Cambridge, CERN, Dubna, Chicago, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien. The NA62-RK collaboration: Birmingham, CERN. Dubna, Fairfax, Ferrara, Firenze, Frascati, Mainz, Merced, Moscow, Napoli, Perugia, Pisa, Protvino, Roma I, Roma II, Saclay, San Luis Potosi, Stanford, Sofia, Torino,TRIUMF

framework, the $K^{\pm}(p) \to \pi^{\pm}(p_3)\gamma(q_1)\gamma(q_2)$ differential decay rate can be parametrized as[2]:

$$\frac{\partial^2\Gamma}{\partial x \partial y} = \frac{m_K}{(8\pi)^3}\left[z^2(|A+B|^2+|C|^2) + \left(y^2 - \frac{1}{4}\lambda(1,z,r_\pi^2)\right)^2 \cdot \left(|B|^2+|D|^2\right)\right] \tag{1}$$

where $z = m_{\gamma\gamma}^2/m_K^2$ and $y = p \cdot (q_1 - q_2)/m_K^2$. The dominant contribution at $O(p^4)$ comes from the loop term $A(z,\hat{c})$, including the pion and kaon loop amplitudes, the B and D terms are zero and the Wess-Zumino-Witten term C accounts for ~10% of the total decay rate. Higher order unitarity corrections from $K^{\pm} \to 3\pi^{\pm}$ decays, including the main $O(p^6)$ contribution as well as those beyond $O(p^6)$, have been found to contribute significantly to the BR (up to 30-40%) and to the shape of $M_{\gamma\gamma}$ spectrum[2]. The total decay rate is predicted to be $BR(K^{\pm} \to \pi^{\pm}\gamma\gamma) \sim 10^{-6}$. The only $K^{\pm} \to \pi^{\pm}\gamma\gamma$ experimental observation published so far is by the BNL E787 experiment[3] measuring the BR and $\hat{c}$:

$$BR(K^{\pm} \to \pi^{\pm}\gamma\gamma)_{E787} = (1.1 \pm 0.3_{stat} \pm 0.1_{syst}) \cdot 10^{-6} \qquad with \qquad \hat{c} = 1.8 \pm 0.6 \tag{2}$$

based on 31 K^+ decay candidates in the kinematic region 100 MeV/c< p_π < 180 MeV/c (p_π is the π^+ momentum in the K^+ rest frame). The value of the BR quoted in Eq.2 refers to the entire kinematical range and is obtained with a ChPT extrapolation using the value 1.8 for $\hat{c}$.

2.1. The data analysis

The measurements described here have been performed using minimum bias data sets collected during a 3-day special NA48/2 run in 2004 with 60 GeV/c $K^{\pm}$ beams, and a 3-month NA62 run in 2007 with 74 GeV/c $K^{\pm}$ beams. The effective kaon fluxes collected are similar, but the background conditions and resolution on kinematic variables differ significantly. Signal events are selected in the region z = $(m_{\gamma\gamma}/m_K)^2 > 0.2$ to reject the $K^{\pm} \to \pi^{\pm}\pi^0$ background peaking at z = 0.075. In total 149 (232) decays candidates are observed by NA48/2 (NA62), with backgrounds contaminations of 15.5 ± 0.7 (17.4 ± 1.1) dominated by $K^{\pm} \to \pi^{\pm}\pi^0(\pi^0)(\gamma)$ decays with merged photon clusters in the LKr calorimeter. The data spectra of the z kinematic variable, together with the signal and background expectations, are displayed in Fig. 1: they support the ChPT prediction of a cusp at the two-pion threshold. The values of the $\hat{c}$ parameter in the framework of the ChPT $O(p^4)$ and $O(p^6)$ parameterizations according to[2] have been measured by performing likelihood fits to the data. The uncertainties are dominated by the statistical ones while the systematic ones are mainly due to the background estimates uncertainties. The $O(p^6)$ parametrization involves a number of external inputs fixed as follows: the polynomial contribution terms are η_1 = 2.06, η_2 = 0.24 and η_3 = -0.26 as suggested in [2], while the

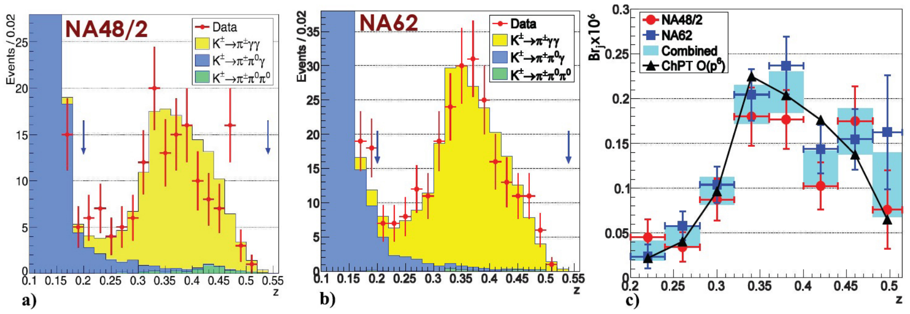

Figure 1. The $z = (m_{\gamma\gamma}/M_K)^2$ spectrum with MC expectations for signal, best fit, and backgrounds. On the right model-independent branching ratio B_j in z bins.

$K^{\pm} \rightarrow 3\pi^{\pm}$ amplitude parameters come from a fit to the experimental data [4]. Along with the separate 2004 and 2007 results, the combined NA48/2 and NA62 results are also presented in Tab.2. The measured BR, in agreement with the E787 one, improves the precision of the measurement by a factor ∼5. The obtained value of $\hat{c}$, for both the $O(p^4)$ and $O(p^6)$ fits, is in very good agreement with the previous measurement by E787[3]. The same parameter has been measured by NA48/2 using the decay $K^{\pm} \rightarrow \pi^{\pm}e^{+}e^{-}\gamma$, with a compatible value: $\hat{c} = 0.90 \pm 0.45$[5].

Table 1. Final results for the $\hat{c}$ and BR($\pi^{+}\gamma\gamma$) measurements.

	$\hat{c}$ $O(p^4)$ fit	$\hat{c}$ $O(p^6)$ fit	$BR_{MI}^{z>0.2}$(x10^6)	BR $O(p^6)$ (x10^6)
NA48/2[6]	1.37 ± 0.36	1.41 ± 0.40	$0.877 \pm 0.087_{stat} \pm 0.017_{syst}$	$0.910 \pm 0.072_{stat} \pm 0.022_{syst}$
NA62[7]	1.93 ± 0.27	2.10 ± 0.33	$1.088 \pm 0.093_{stat} \pm 0.027_{syst}$	$1.058 \pm 0.066_{stat} \pm 0.044_{syst}$
Combined[7]	1.72 ± 0.21	1.86 ± 0.25	$0.965 \pm 0.061_{stat} \pm 0.014_{syst}$	$1.003 \pm 0.051_{stat} \pm 0.024_{syst}$

3. The K$_{\mathrm{e4}}$ decay formalism

Four-body final state decays are described by five kinematic variables historically called, for K$_{\mathrm{e4}}$ decays, Cabibbo-Maksymowicz variables[8]: two invariant masses $S_{\pi} = M^2_{\pi\pi}$ and $S_e = M^2_{e\nu}$ and three angles ϑ_{π}, ϑ_e and φ. The hadronic current is described by form factors which can be developed in a partial wave expansion as suggested in [9] Limiting the expansion to S- and P-waves and considering a unique phase δ_p for all P-wave form factors, two axial (F, G) and one vector (H) complex form factors contribute to the transition amplitude: $F = F_s\ e^{i\delta_s} +\ F_p\ e^{i\delta_p} cos\theta_{\pi}$, $G = G_p\ e^{i\delta_p}$, $H = H_p\ e^{i\delta_p}$. From the differential rate study in the 5-dimensional space, four real form factors (F_s, F_p, G_p and H_p) and a single phase difference ($\delta = \delta_s - \delta_p$) have been measured, together with their energy variation with S_{π} and S_e, by the NA48/2 experiment[10]. In the neutral pion mode (K^{00}_{e4}), the differential rate depends on a single hadronic form factor F_s whose variation with (S_{π},S_e) is unknown and will be studied. No such study is available so far in the literature.

3.1. Form factor measurement

The form factor (FF) study requires a sample free of large radiative effects which can pollute the original kaon decay amplitude. The event density in the plane (S_{π},S_{e}) is proportional to F_s^2. The fit procedure minimizes a χ^2 expression in the two-dimensional space describing F_s by a polynomial expansion in $q^2 = (S_{\pi}/4m^2_{\pi+} - 1)$ and y^2=($S_e/4m^2_{\pi+}$):

$$F(q^2, y^2)_{q^2 \geq 0} = N(1 + aq^2 + bq^4 + cy^2)^2 \qquad F(q^2, y^2)_{q^2<0} = N(1 + dC(q^2) + cy^2)^2 \qquad (3)$$

The term $C(q^2) = \sqrt{|q^2/(1 + q^2)|}$ parameterize the cusp-like function. The results in Tab.2 are in good agreement with those obtained in a high statistics measurement of the corresponding form factor of the $K^{\pm}_{e4}$[10] mode see Fig.2b). Below $S_{\pi} = 4m^2_{\pi+}$, the observed deficit of events it is well described by a cusp-like function Fig.2a).

3.2. Measurement of the K^{00}_{e4} branching ratio

The K^{00}_{e4} rate is measured relative to the abundant $K^{00}_{3\pi}$ normalization channel. As the topologies of the two modes are similar the two samples are collected concurrently using the same trigger logic and a common selection is employed as far as possible. This leads to partial cancellation

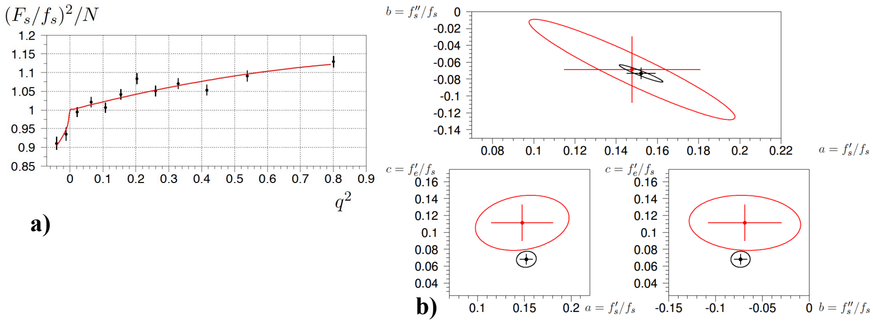

Figure 2. a) Ratio of the two q^2 distributions in equal population bins. b) FF description in 2-parameter planes obtained in K_{e4}^{00}[11] (red) and $K_{e4}^{\pm}$[10] (black).

Table 2. Best description of the $F_s(K_{00}^{e4})$ form factor using Eq.3

Fit parameter	values
a	$0.149 \pm 0.033_{stat} \pm 0.014_{syst}$,
b	$-0.070 \pm 0.039_{stat} \pm 0.013_{syst}$,
c	$0.113 \pm 0.022_{stat} \pm 0.007_{syst}$,
d	$-0.256 \pm 0.049_{stat} \pm 0.016_{syst}$.
χ^2	101.4/107=0.95

of the systematic effects and avoids relying on the absolute kaon flux measurement. After the selection described in [11] a sample of 65210 candidate is identified with 1% background dominated by $K_{3\pi}^{00}$ with a misidentified pion as an electron. The BR measurement [11], using the value $BR(K_{3\pi}^{00}) = (1.761 \pm 0.022)\%$ as normalization, is obtained:

$$BR(K_{e4}^{00}) = (2.552 \pm 0.010_{stat} \pm 0.010_{syst} \pm 0.032_{ext}) \cdot 10^{-5} \tag{4}$$

This measurement improves the current world average precision by one order of magnitude. Both total rate and form factor description are used to obtain the absolute FF value at $S_\pi = 4m_{\pi+}^2$, $S_e = 0$ ($q^2 = 0$, $y^2 = 0$)[11]:

$$f_s = 6.079 \pm 0.012_{stat} \pm 0.027_{syst} \pm 0.046_{ext},$$

where the dominating external error comes from uncertainties on the normalization mode $K_{3\pi}^{00}$ branching ratio, on the mean kaon life time and on $|V_{us}|$. This value shows some tension with the corresponding form factor of the $K_{e4}^{\pm}$ mode $f_s^{\pm} = 5.705 \pm 0.003_{stat} \pm 0.017_{syst} \pm 0.031_{ext}$[10].

References

[1] V. Fanti *et al.* [NA48 Collaboration], Nucl. Instrum. Meth. A **574**, 433 (2007).
[2] G. D'Ambrosio and J. Portoles, Phys. Lett. B **386**, 403 (1996)
[3] P. Kitching *et al.* [E787 Collaboration], Phys. Rev. Lett. **79**, 4079 (1997)
[4] J. Bijnens, P. Dhonte and F. Persson , Nucl. Phys. B **648**, 317 (2003)
[5] J. R. Batley *et al.* [NA48/2 Collaboration], Phys. Lett. B **659**, 493 (2008)
[6] J. R. Batley *et al.* [NA48/2 Collaboration], Phys. Lett. B **730**, 141 (2014)
[7] C. Lazzeroni *et al.* [NA62 Collaboration], Phys. Lett. B **732**, 65 (2014)
[8] N. Cabibbo and A. Maksymowicz, Phys. Rev. **137**, B438 (1965)
[9] A. Pais and S. B. Treiman, Phys. Rev. **168**, 1858 (1968).
[10] J. R. Batley *et al.* [NA48/2 Collaboration], Phys. Lett. B **715**, 105 (2012)
[11] J. R. Batley *et al.* [NA48/2 Collaboration], J. High Energy Phys. **08**, 159 (2014)