ELECTROMAGNETIC FORM FACTORS IN DUAL-LARGE N_c QCD

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AD-HOC MODELS FOR FORM FACTORS

NO SYSTEMATIC IMPROVEMENT

QCD SUM RULES

$F_{\pi}(q2)$ **OK**

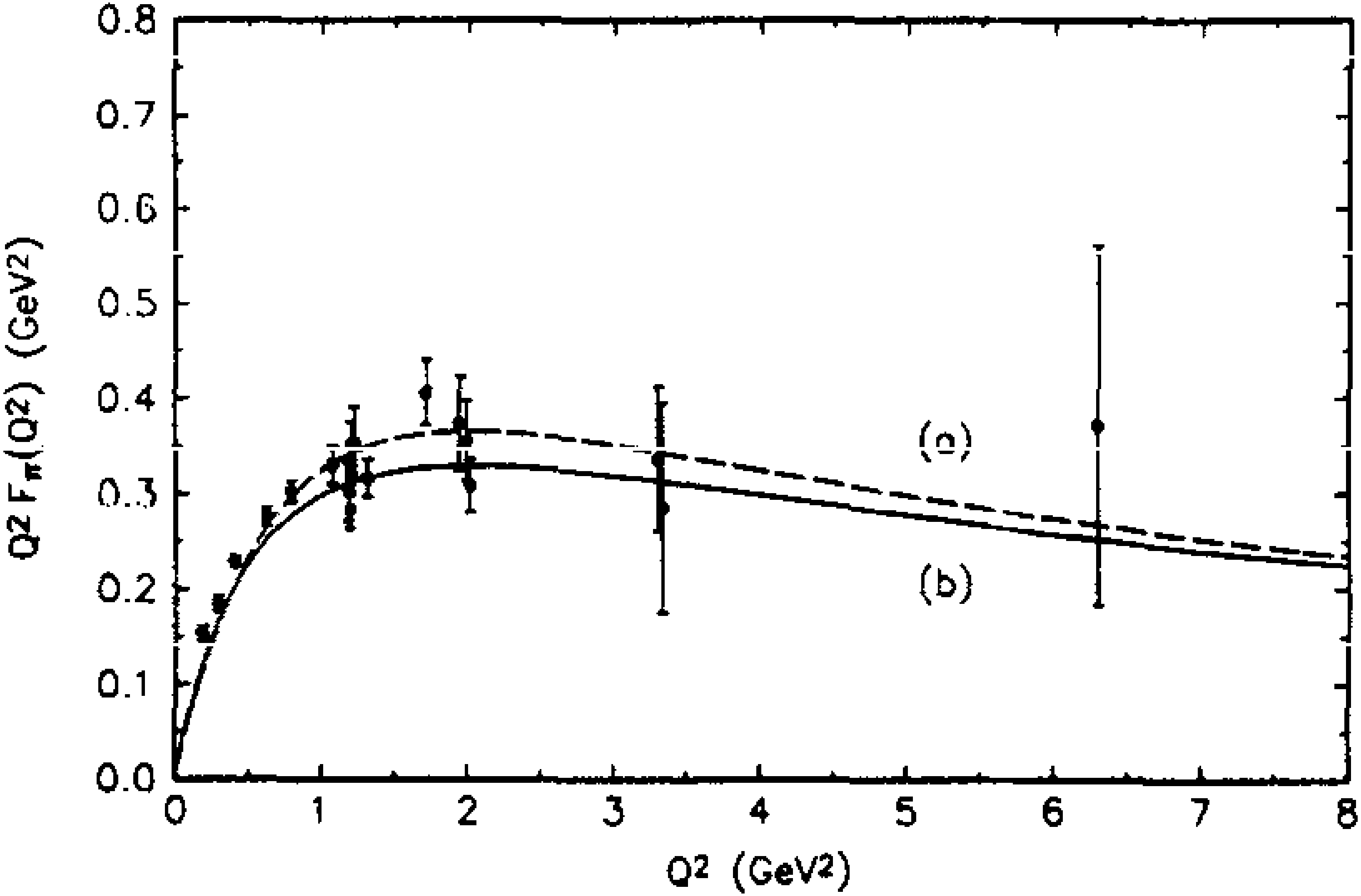


Fig. 1. The electromagnetic pion form factor at $T = 0$, determined from the QCD-FESR, Eq. (11), i.e. without invoking VMD (dashed curve (a)), compared with the result of the independent

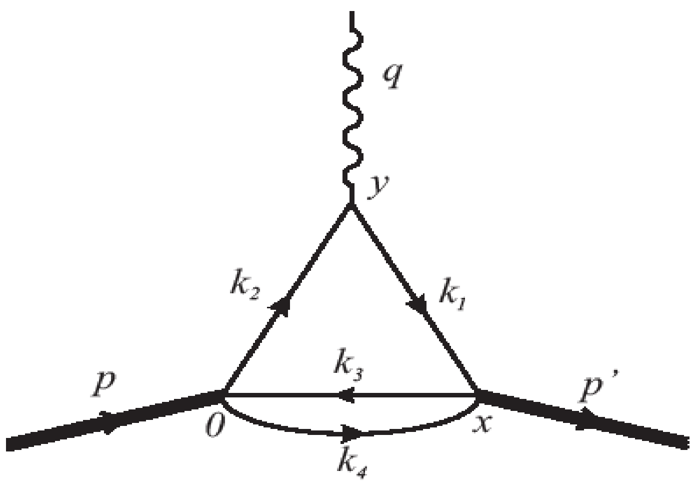
QCD SUM RULES

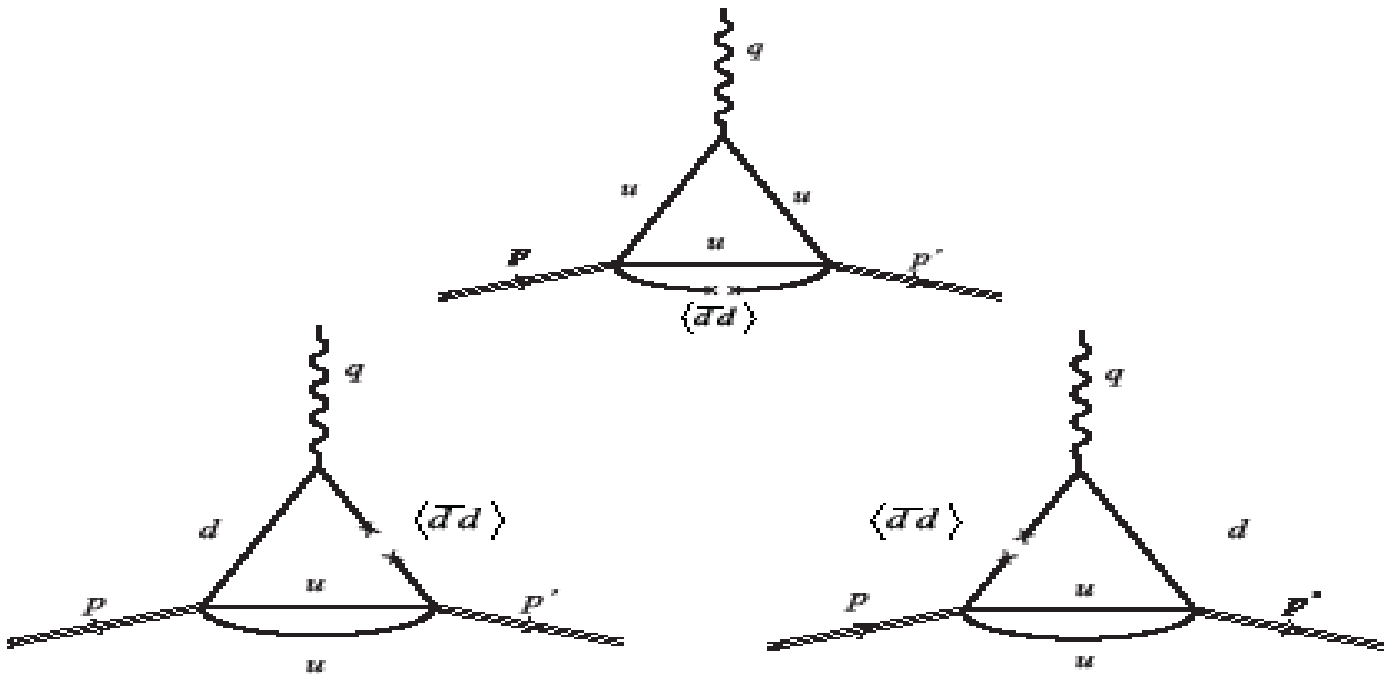
NUCLEON

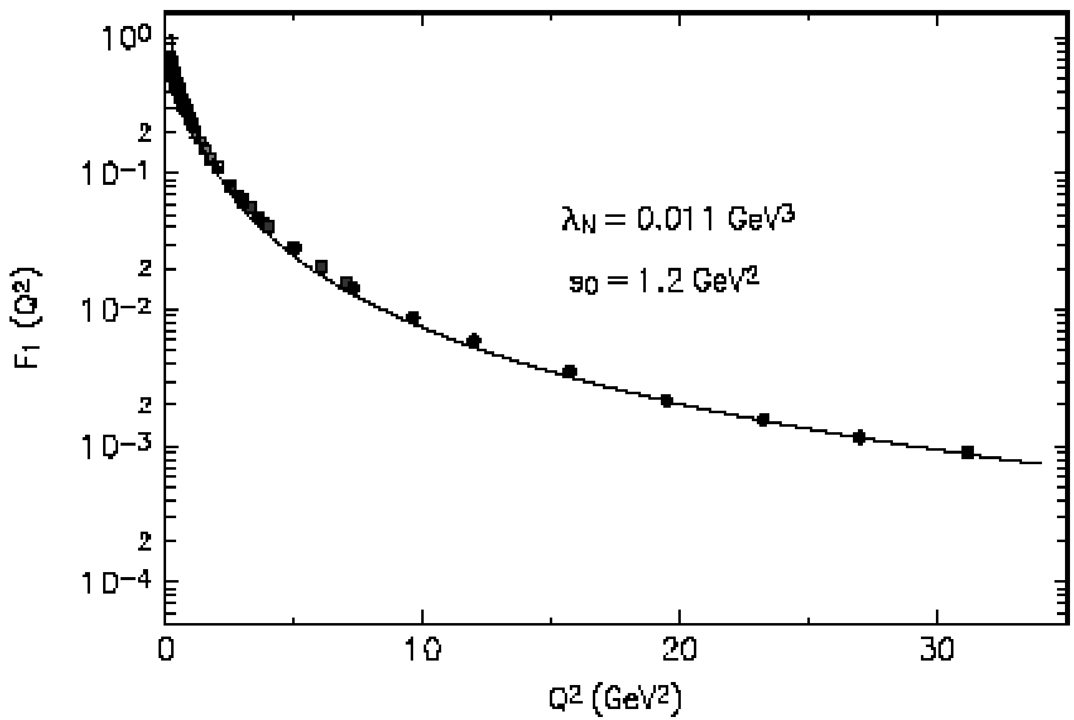
F_1 (q2) OK

$F_2(q^2)$ DISASTER

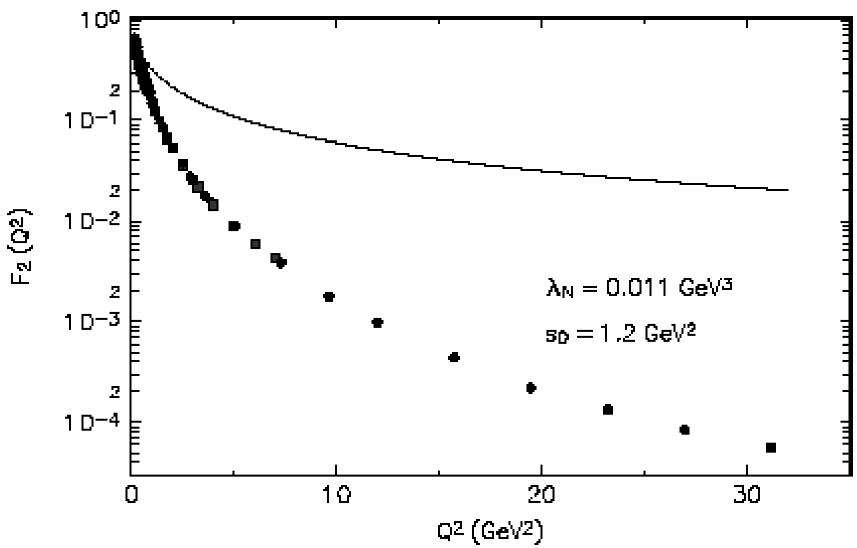
QCD SUM RULES $F_{1,2}(-q^2)$




q
u
u
u
p
p'
⟨$\bar{d}d$⟩
q
d
⟨$\bar{d}d$⟩
u
p
p'
u
q
⟨$\bar{d}d$⟩
d
u
p
p'
u



$\lambda_N = 0.011\ GeV^3$
$s_0 = 1.2\ GeV^2$
$F_1\ (Q^2)$
$Q^2\ (GeV^2)$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
2
0
10
20
30



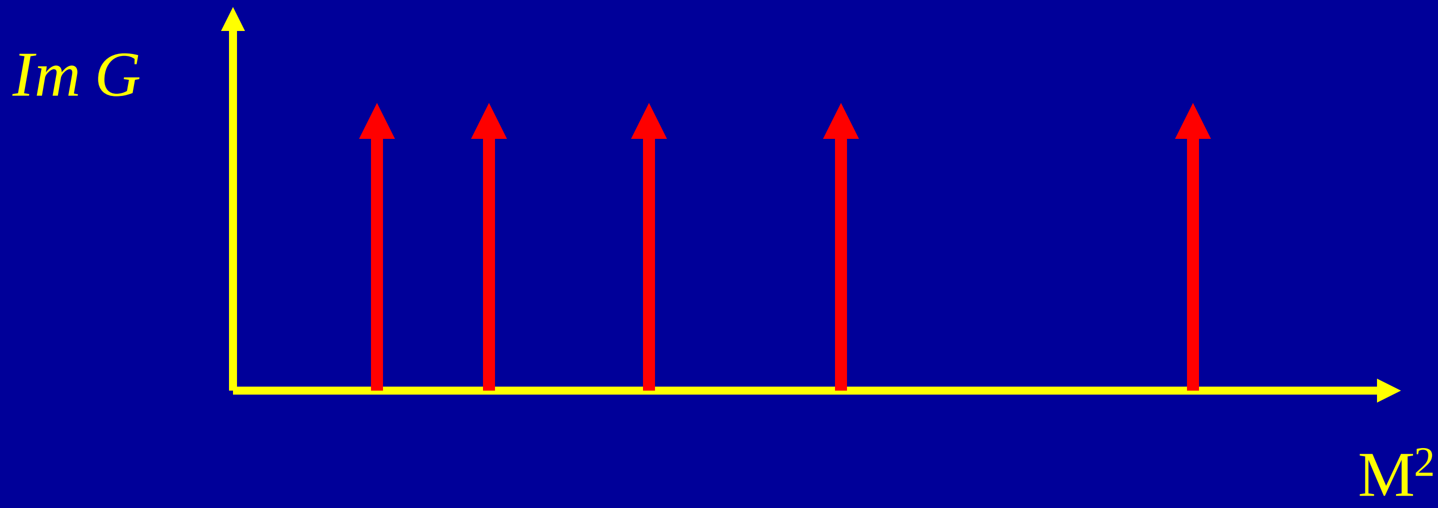
10^0
2
10^-1
2
10^-2
2
10^-3
2
10^-4
F2 (Q2)
λN = 0.011 GeV3
s0 = 1.2 GeV2
0
10
20
30
Q2 (GeV2)

SYSTEMATIC IMPROVEMENT

- **QCD_∞ : QCD IN THE LIMIT $N_c \rightarrow \infty$**

QCD $_\infty$

- *Lim* $N_c \rightarrow \infty$ ($N_c = 3$) (t'Hooft '74 & Witten '79)
- Spectrum: ∞ number of zero width resonances



QCD IN THE LIMIT $N_c \rightarrow \infty$

INFINITE NUMBER OF 0-WIDTH RESONANCES

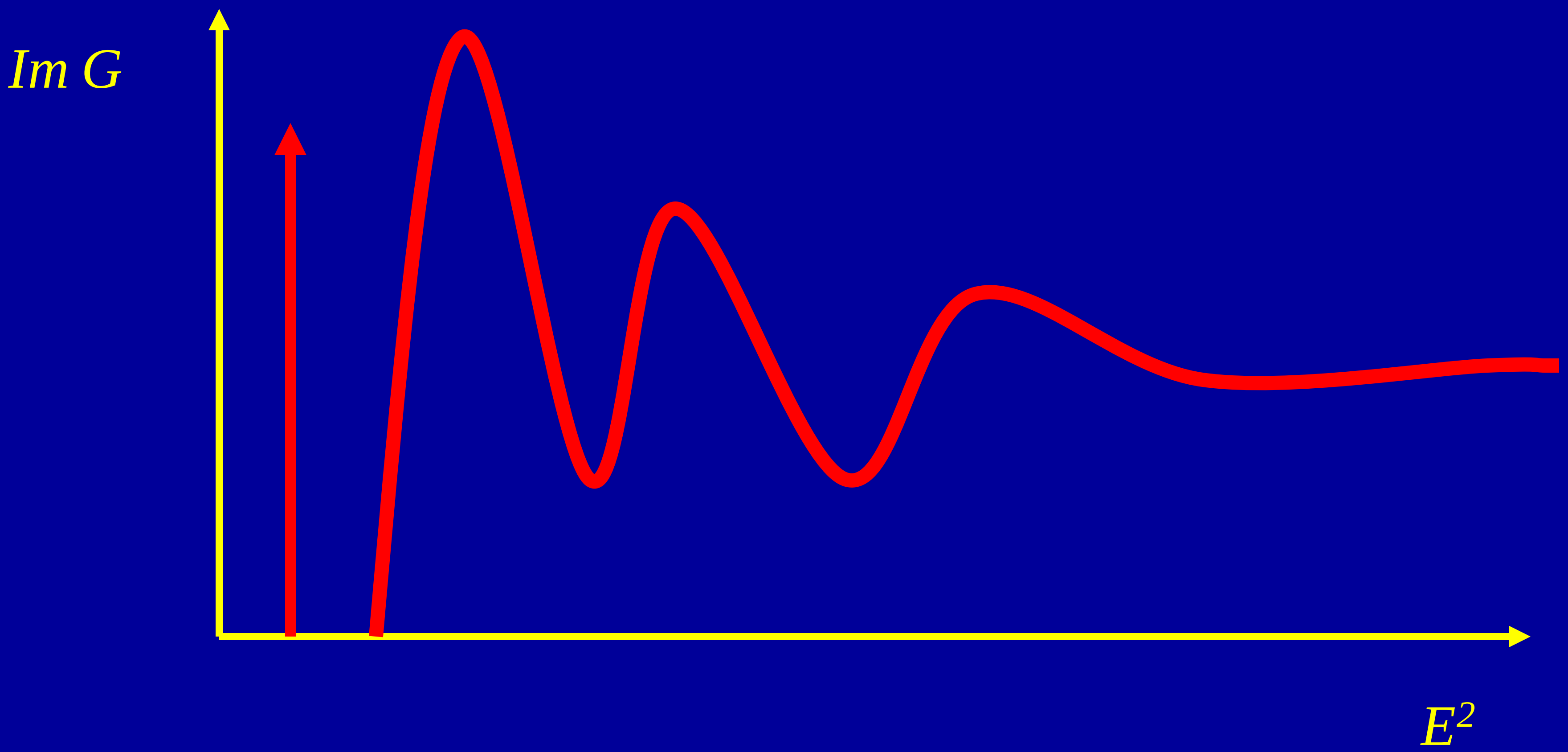
NEEDS A REALIZATION (MODEL) FOR HADRONIC MASSES & COUPLINGS

DUAL-RESONANCE MODEL (VENEZIANO)
DUAL QCD_{∞}

PION, NUCLEON, DELTA FORM FACTORS

CORRECTIONS DUE TO FINITE WIDTH (Γ/M)

RealisticSpectral Function



CORRECTIONS to $1/N_c$

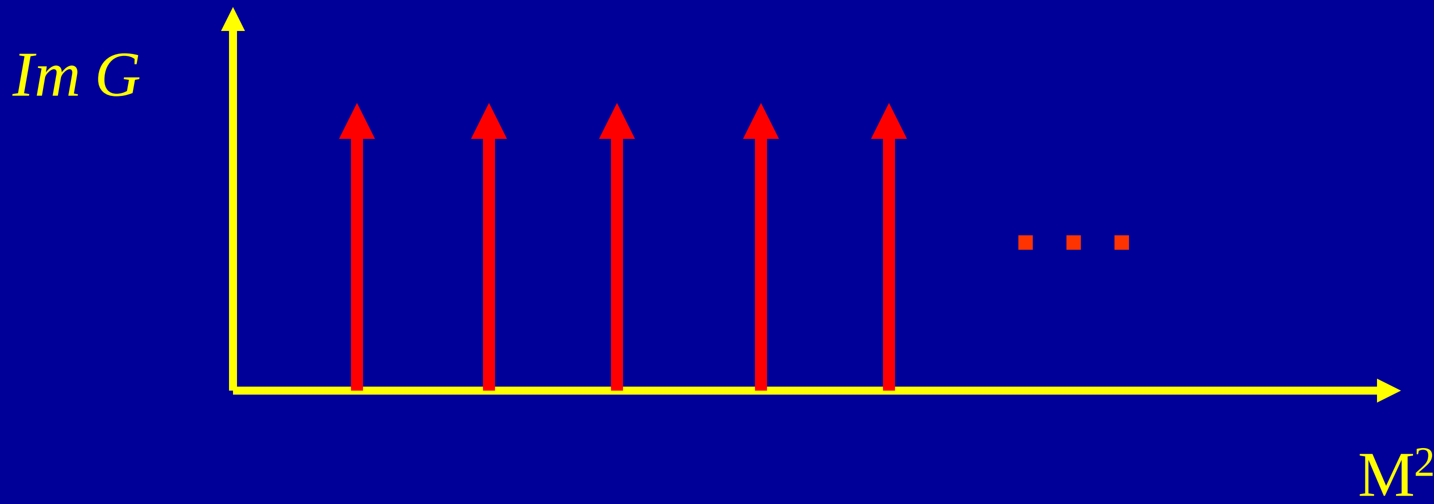
$$\Gamma / M \approx 10\ \%$$

Dual – QCD∞

- ∞ number of zero width resonances, equally spaced

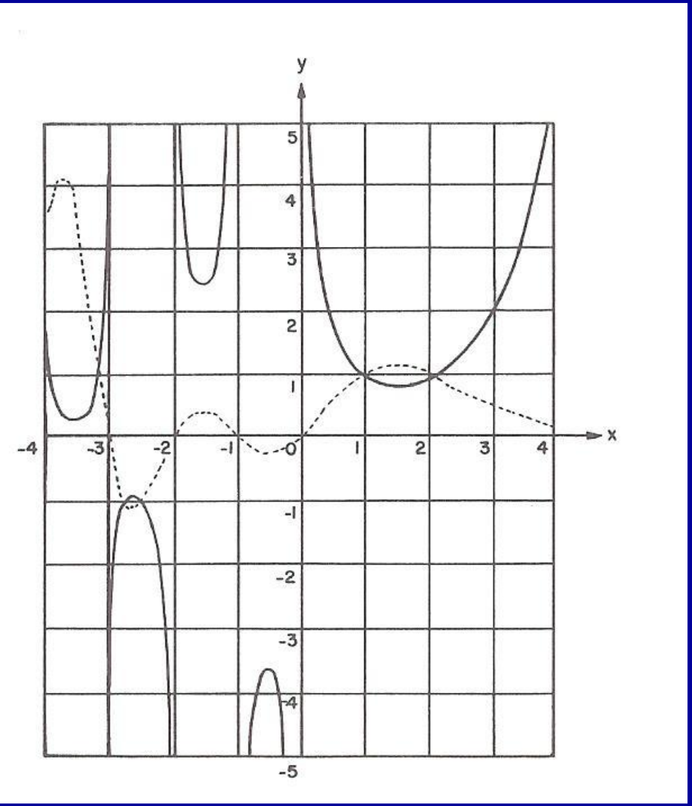
- Masses & couplings fixed to give an Euler Beta Function: $B(x,y) = \Gamma(x)\,\Gamma(y) / \Gamma(x+y)$

EQUALLY SPACED



1) Mass Spectrum: Linear Regge Trajectories
(equal spacing)

2) Functional Dependence: Euler β-Function



$$M_n^2 = M_0^2 + \mu^2 n \quad [\mu^2 = 1.35(4)\ \mathrm{GeV}^2]$$

Ruiz-Arriola et al.

$$M_n^2 = M_0^2 (1 + 2n)$$

$$\mu^2 = 1.20\ \mathrm{GeV}^2$$

CAD

$$< \pi(p_2)|J_\mu^{EM}|\pi(p_1) >= (p_1 + p_2)_\mu \ \ F_\pi(s)$$

$$F_\pi(s) = \sum_{n=0}^{\infty} \frac{C_n}{(M_n^2 - s)}$$

DUAL QCD$_\infty$

$$C_n = \frac{\Gamma(\beta - 1/2)}{\alpha' \sqrt{\pi}} \ \frac{(-1)^n}{\Gamma(n+1)} \ \frac{1}{\Gamma(\beta - 1 - n)}$$

$$\alpha' = 1/2{M_\rho}^2$$

$$\alpha_\rho(s) = 1 + \alpha'(s - M_\rho^2)$$

$$M_n^2 = M_\rho^2(1 + 2n)$$

$$F_\pi(s) = \frac{\Gamma(\beta - 1/2)}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{\Gamma(n+1)} \frac{1}{\Gamma(\beta - 1 - n)}$$

$$\times \frac{1}{[n + 1 - \alpha_\rho(s)]}$$

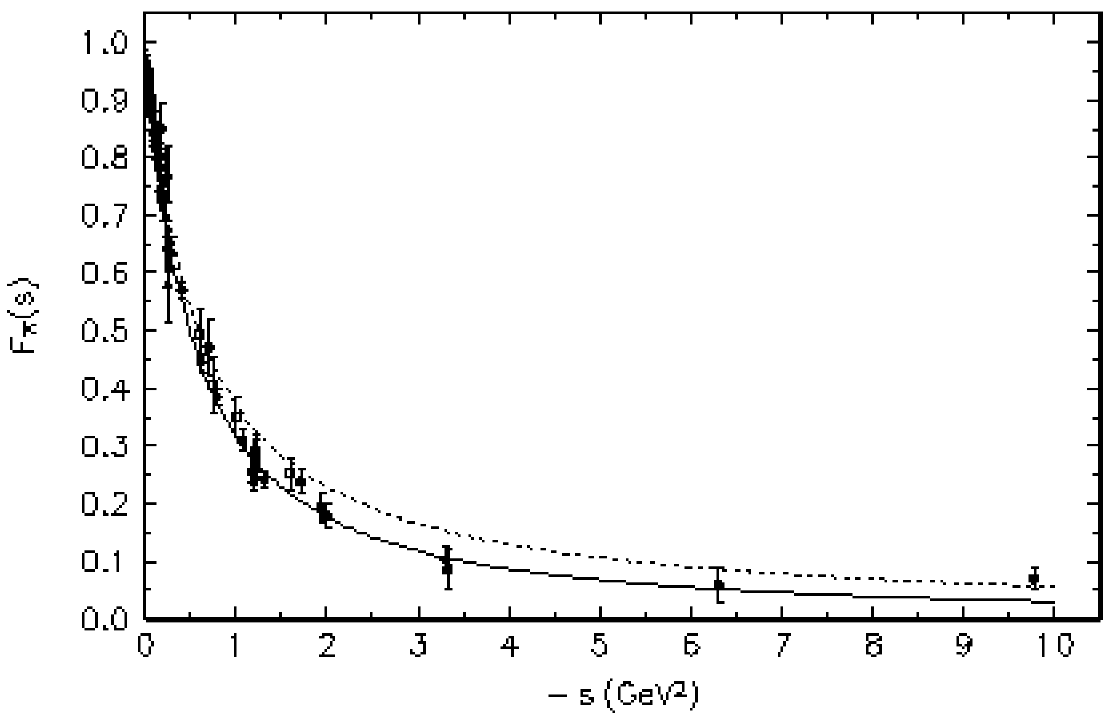
$$= \frac{1}{\sqrt{\pi}} \frac{\Gamma(\beta - 1/2)}{\Gamma(\beta - 1)} \; B(\beta - 1,\ 1/2 - \alpha' s)$$

$$\lim_{s \to -\infty} F_\pi(s) = (-\alpha'\, s)^{(1-\beta)}$$

$$Im\, F_\pi (s) \propto \sum_{n=0} (-1)^n\, \Gamma^{-1} (n+1)\, \Gamma^{-1}(\beta-1-n)\, \delta(M_n{}^2 - s)$$

$$\pi\, \delta(M_n{}^2 - s) \rightarrow \Gamma_n M_n / [(M_n{}^2 - s)^2 + \Gamma_n{}^2 M_n{}^2]$$

Figure 1



ONE FREE PARAMETER: β

$\beta=2$: VMD (ρ-) DOMINANCE: $\rightarrow 1/q^2$

$$\chi_F = 11\,,\ \langle r^2_\pi\rangle = 0.394\ \text{fm}^2,\ g_{\rho\pi\pi}\ /\ f\ \rho = 1$$

$$\langle r^2_\pi\rangle_{EXP} = 0.439(8)\ \text{fm}^2,\ g_{\rho\pi\pi}\ /\ f\ \rho|_{EXP} = 1.21(2)$$

$$\beta = 2.3 \qquad \rightarrow (1/q^2)^{1.3}$$

$$\chi_F = 1.4\,,\ \langle r^2_\pi\rangle = 0.436\ \text{fm}^2,\ g_{\rho\pi\pi}\ /\ f\ \rho = 1.2$$

How about the PQCD Logs ?

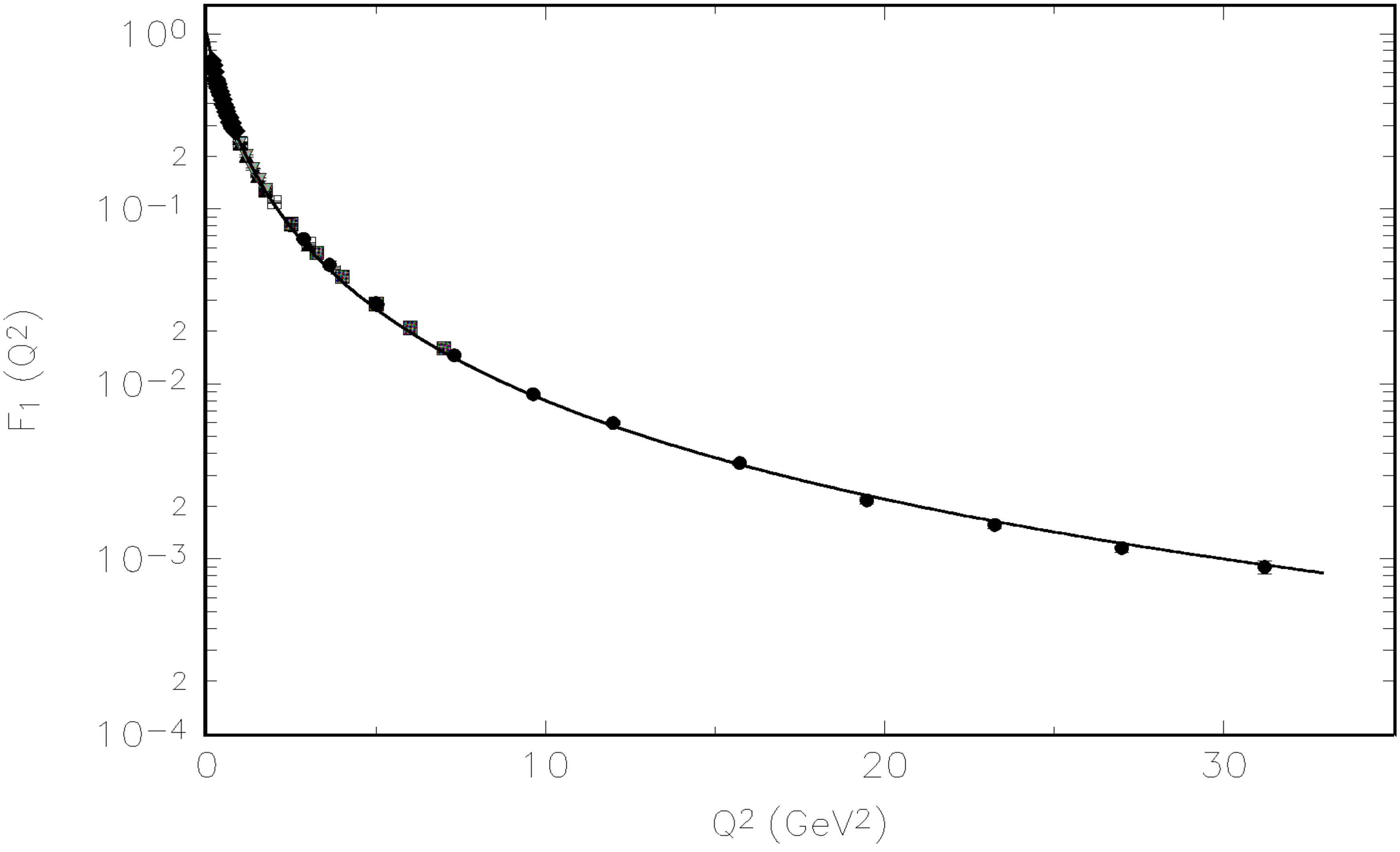
Masjuan, Ruiz Arriola, Broniowski: PRD 87 (2013)

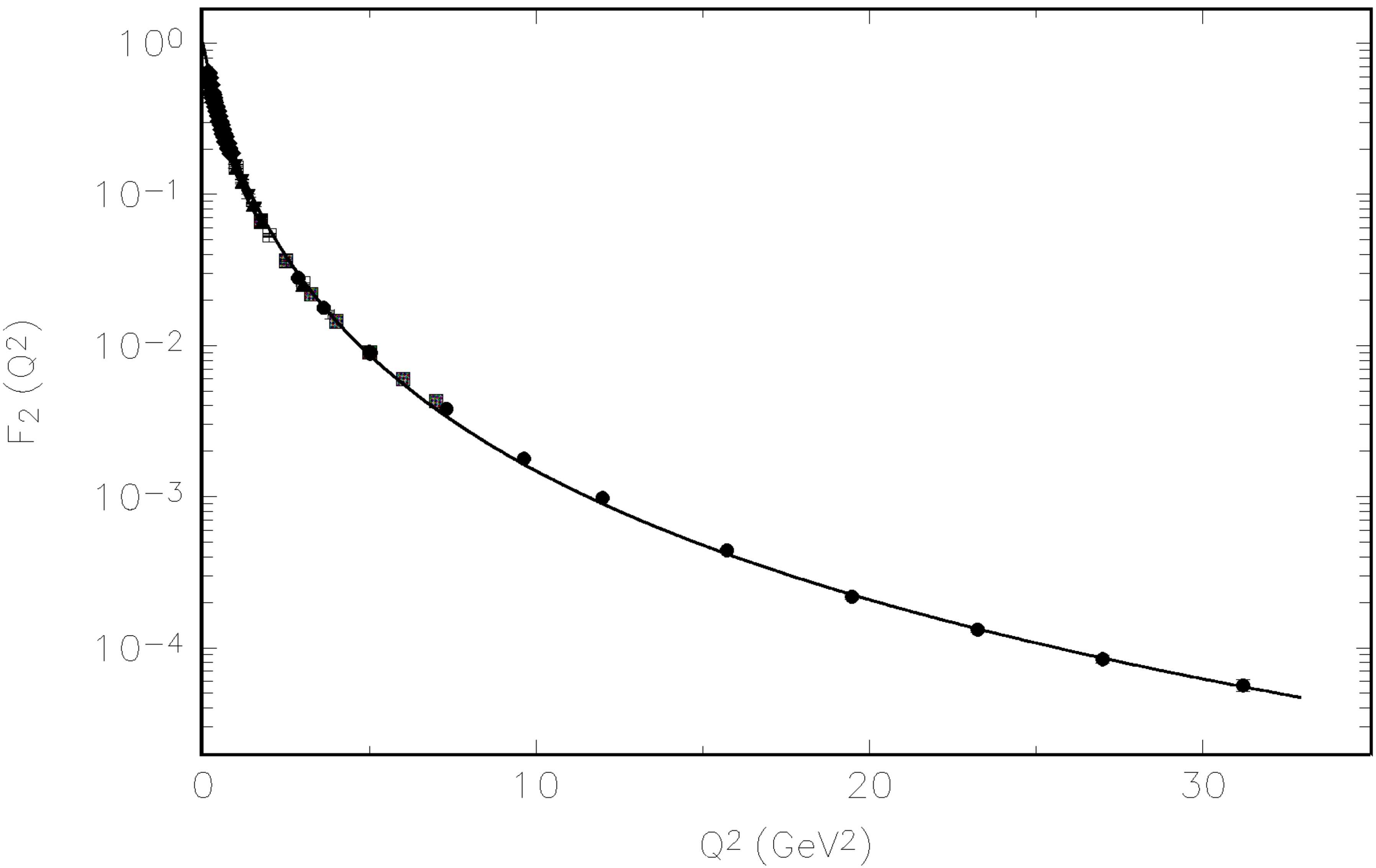
Nucleon Form Factors

- $F_1 (q^2) \rightarrow \beta_1$
- $F_2 (q^2) \rightarrow \beta_2$
- $G_M (q^2)$
- $G_E (q^2)$

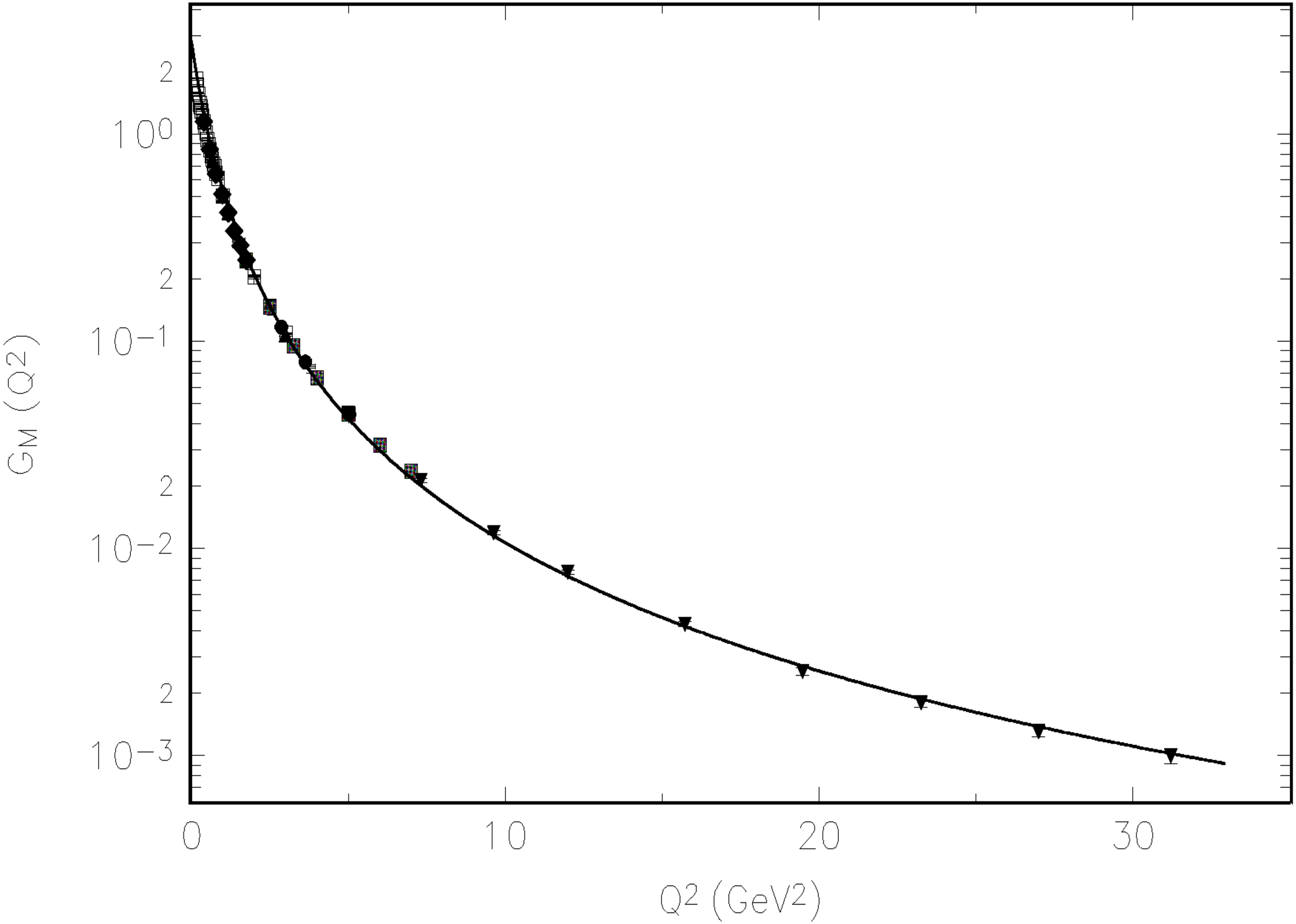
- $G_E (q^2) / G_M (q^2)$

Data base: E. J. Brash et al. Phys. Rev. D 65 (2002)

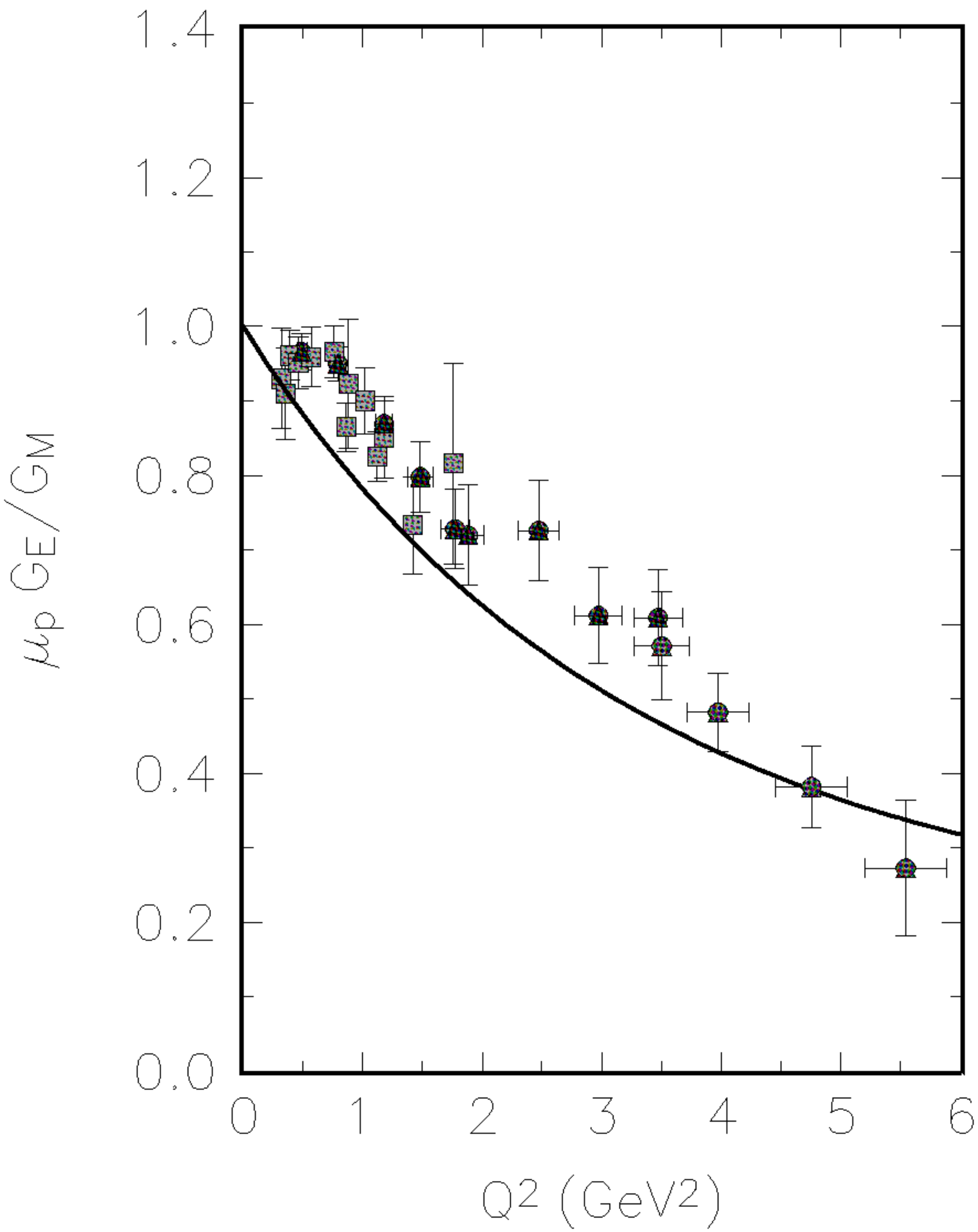




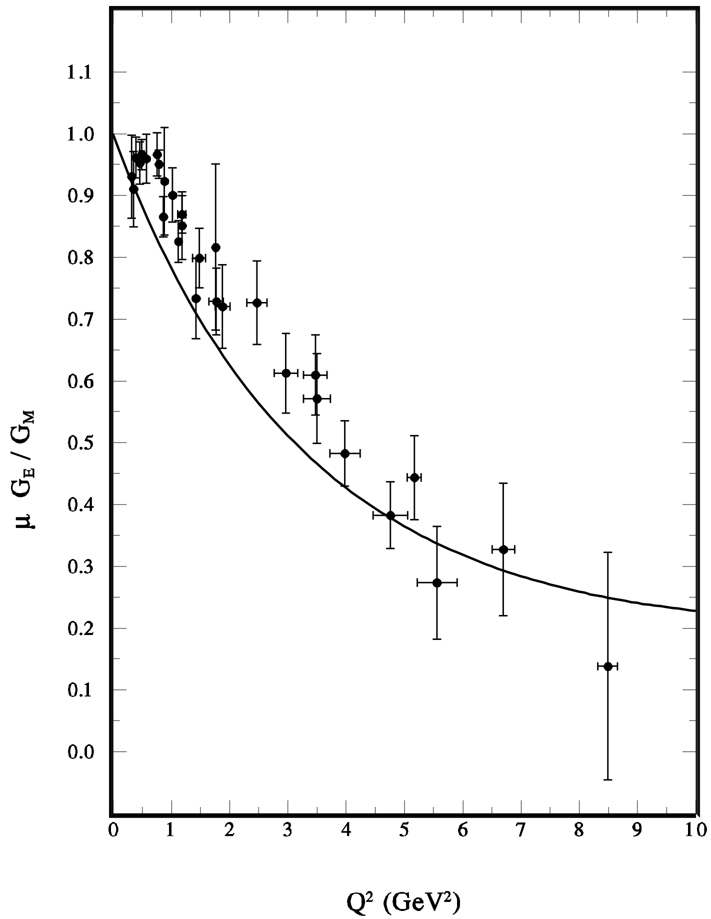
10^0
10^-1
10^-2
10^-3
10^-4
F2 (Q2)
0
10
20
30
Q2 (GeV2)



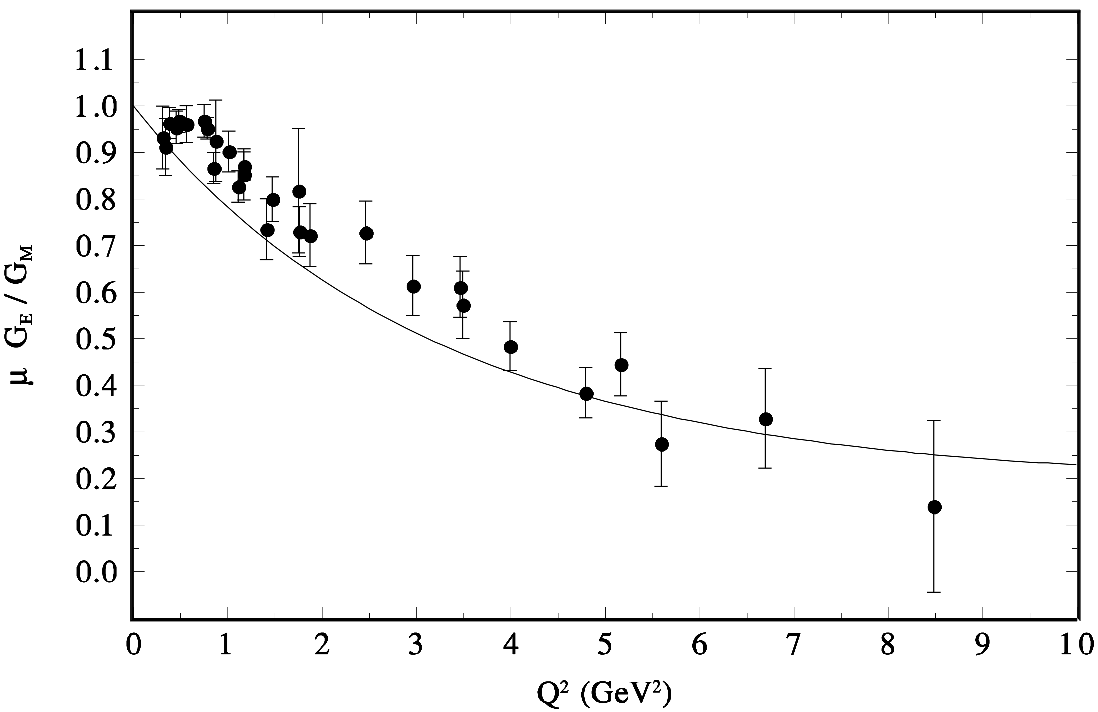
2
10^0
2
10^{-1}
2
10^{-2}
2
10^{-3}
G_M (Q^2)
0
10
20
30
Q^2 (GeV^2)



1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
$\mu_p G_E/G_M$
Q^2 (GeV^2)



μ G_E / G_M
Q² (GeV²)
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10



μ G_E / G_M
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0
1
2
3
4
5
6
7
8
9
10
Q² (GeV²)

(SACHS) PROTON RADII

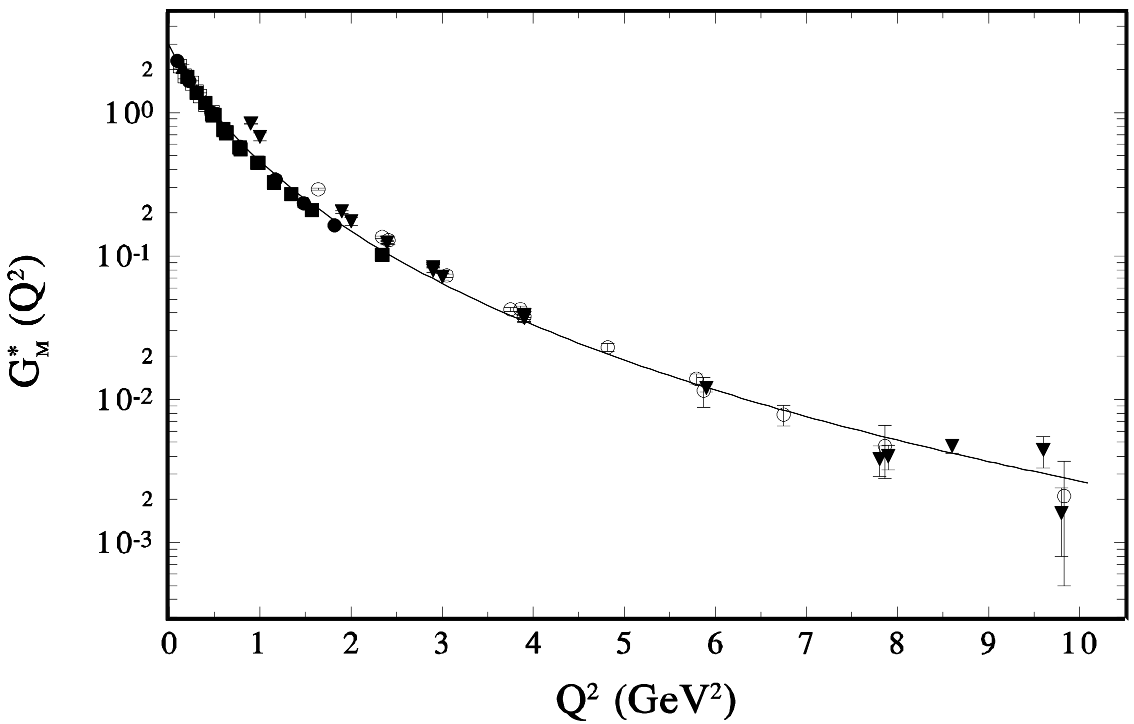
$$\langle r_E \rangle = 0.81 \text{ fm}$$

$$\langle r_M \rangle = 0.76 \text{ fm}$$

$$\langle r_E \rangle \approx \langle r_M \rangle \approx 0.8 \text{ fm}$$

FORM FACTORS OF Δ(1236)

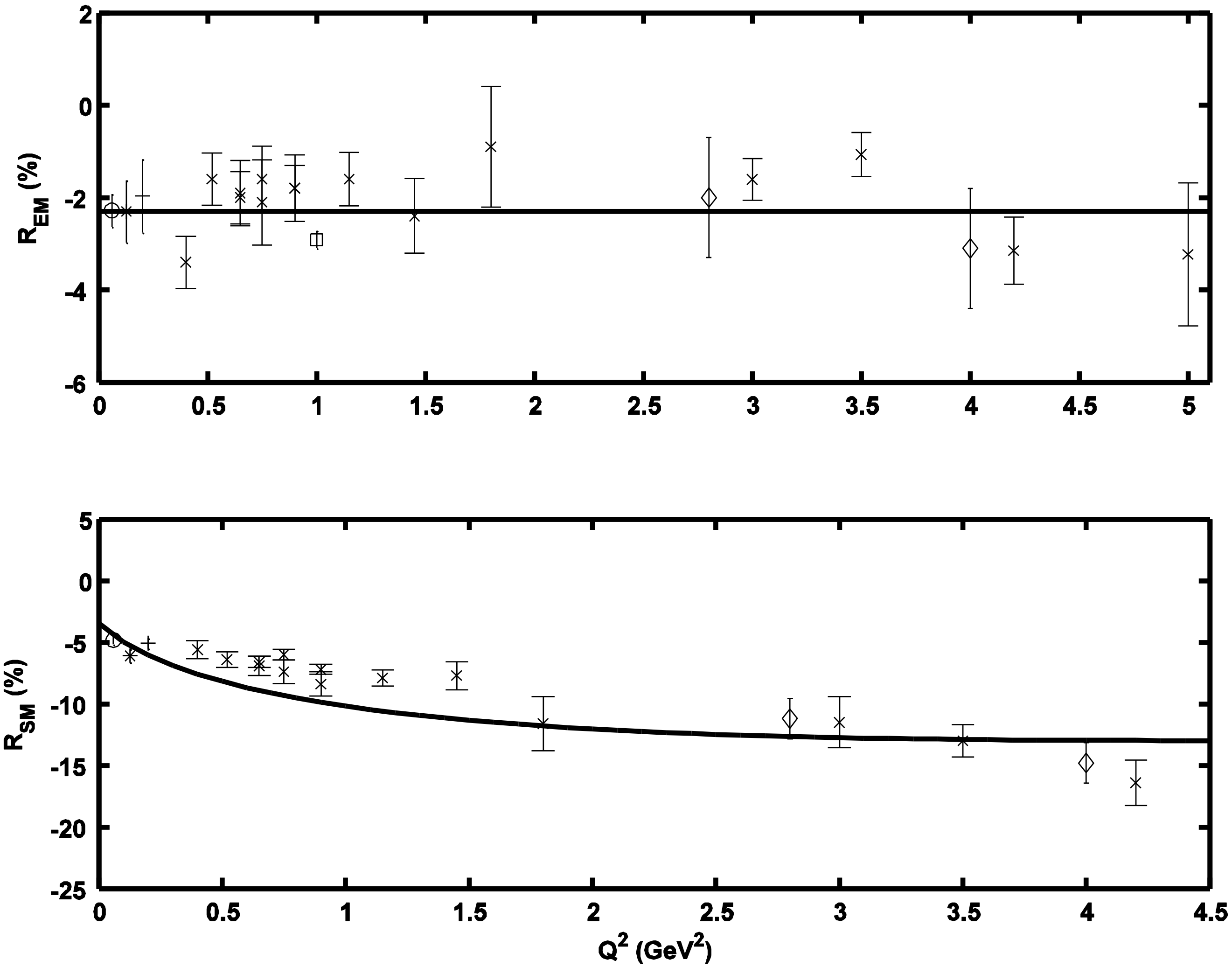
$G^{*}_{M}(q^2)$, $G^{*}_{E}(q^2)$, $G^{*}_{C}(q^2)$



G^*_M (Q^2)
Q^2 (GeV^2)

$$R_{EM}(Q^2) = - G^*_E(Q^2) / G^*_M(Q2)$$

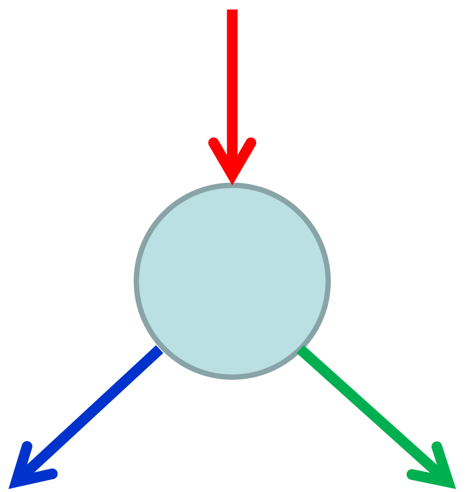
$$R_{SM}(Q^2) = - f(Q^2)\, G^*_C\,(Q^2) / G^*_M(Q2)$$



R_EM (%)
2
0
-2
-4
-6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
R_SM (%)
5
0
-5
-10
-15
-20
-25
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
Q^2 (GeV^2)

OFF-SELL THREE-POINT FUNCTION

$$F(p_1^2, p_2^2, p_3^2) = F(p_1^2) \cdot F(p_2^2) \cdot F(p_3^2)$$



CAD (1980-1983)

Broniowski, Masjuan, Ruiz-Arriola (2002 -)

THEORETICAL CALCULATION OF a_μ^{HAD}

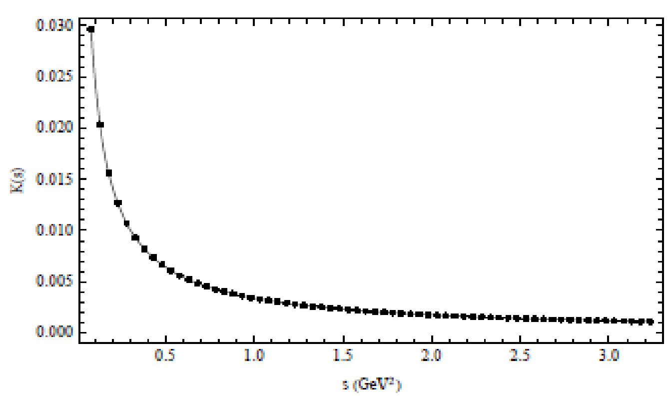
S. Bodenstein, CAD, K. Schilcher

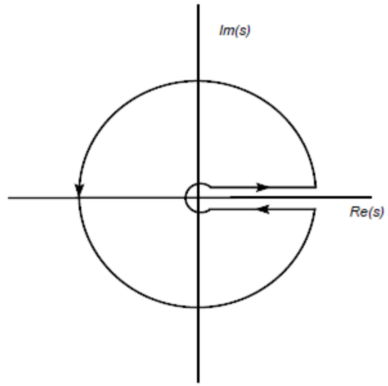
PRD 85, 014029 (2012)

$$a_\mu^{HAD} = \propto \int_{s_{th}}^{\infty} \frac{ds}{s} \, K(s) \; R(s) = \int_{s_{th}}^{s_0} \ldots + \int_{s_0}^{\infty} \ldots$$

$$s_0 \simeq (1.8 \, GeV)^2$$

$$K(s) \rightarrow K_1(s) = a_0 \; s + \sum_{n=1}^{3} \frac{a_n}{s^n}$$





$$\oint_C \frac{ds}{s}\, K_1(s)\, \Pi_{uds}(s) = 2\,\pi\, i\, Res \left[\frac{K_1(s)}{s}\, \Pi_{uds}(s) \right]_{s=0}$$

$$\begin{aligned} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(s)\, \Pi_{uds}(s)|_{PQCD} \;+\; 2\, i \int_{s_{th}}^{s_0} \frac{ds}{s} K_1(s)\, \mathrm{Im}\, \Pi_{uds}(s)|_{HAD} \\ = 2\,\pi\, i\, \mathrm{Res} \left[\frac{K_1(s)}{s}\, \Pi_{uds}(s) \right]_{s=0} \\ = 2\,\pi\, i\, \lim_{s\to 0} \sum_{n=1}^{3} \frac{a_n}{n!}\, \frac{d^n}{ds^n}\, \Pi_{uds}(s) \end{aligned}$$

$$a_\mu^{HAD}|_{uds} = 8\,\alpha_{EM}^2 \sum_{u,d,s} Q_i^2 \left\{ Res \left[\frac{K_1(s)}{s}\, \Pi_{uds}(s) \right]_{s=0} \right.$$

$$\left. -\ \frac{1}{2\pi i} \oint_{|s|=s_0} \frac{ds}{s}\, K_1(s)\, \Pi_{uds}(s)|_{PQCD} + \int_{s_0}^{\infty} \frac{ds}{s} K(s)\, \frac{1}{\pi} \mathrm{Im}\, \Pi_{uds}(s)|_{PQCD} \right\}$$

RESIDUE DOMINATED BY FIRST DERIVATIVE OF $\Pi_{uds}(s)$

CURRENTLY: Models, e.g.

(naive) VMD: $a_\mu^{HAD} = 644(6)\ \times\ 10^{-10}$

$DualQCD_\infty : a_\mu^{HAD} = 722(9)\ \times\ 10^{-10}$

FUTURE: LQCD, CHPT

CHPT: $L_{10}^r, L_9^r, C_{93}^r$.

$$a_\mu^{HAD}|_{c,b} \rightarrow PQCD$$

$$\Pi(s)|_{c,b} = \sum C_n \left[\frac{s}{4\overline{m}^2_{c,b}}\right]^n$$

$$a_\mu^{HAD}|_c = 14.4(1) \times 10^{-10}$$

$$a_\mu^{HAD}|_b = 0.29(1) \times 10^{-10}$$

LQCD: B. Chakraborty et al. arXiv: 1403.1778:

$$a_\mu^{HAD}|_c = 14.42(39) \times 10^{-10}$$

ETM Coll. F.Burger et al. JHEP 1402 (2014) 099