Dispersion formalism for $\gamma^*\gamma^* \to \pi\pi$

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THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

(preliminary) Work in collaboration with
Nils Asmussen and Marc Vanderhaeghen

Mainz, 9th April 2014

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Outline

- Warm up $\gamma\gamma\to\pi\pi$: main pieces
- $\gamma^*\gamma^*\to\pi\pi$
- Outlook

First look at $\gamma^*\gamma^*\rightarrow\pi\pi$ using dispersion relations

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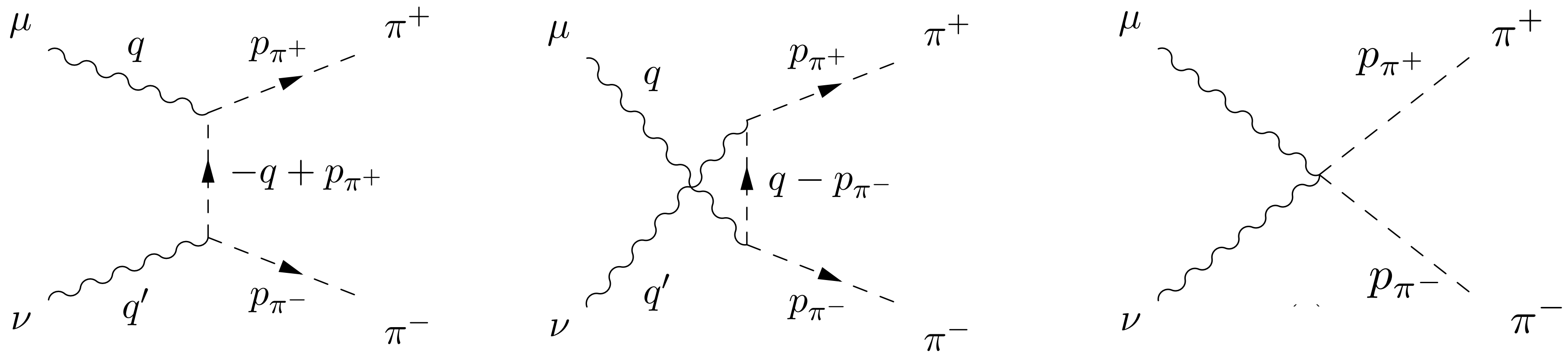
- Starting point: Low-energy theorem to build up the Born amplitude

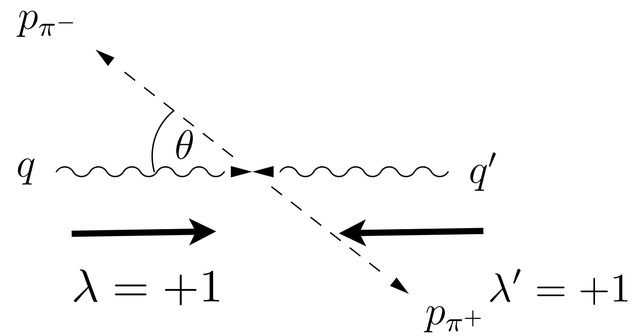


Warm up γγ→ππ: main pieces

a la Drechsel *et al* '99

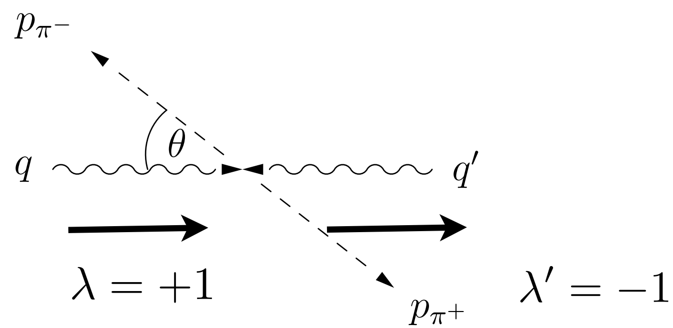
- Look first at γγ→ππ from dispersion relations and identify the most relevant pieces (goal is doubly virtual case)
- Starting point: Low-energy theorem to build up the Born amplitude

$$\mathcal{M}_{++}^{\mathrm{BORN}} = 2ie^2 \frac{1-\beta(s)^2}{1-\beta(s)^2\cos(\theta)^2}$$



$$\Lambda = \lambda - \lambda' = 0$$

$$\mathcal{M}_{+-}^{\mathrm{BORN}} = 2ie^2 \frac{\beta(s)^2\sin(\theta)^2}{1-\beta(s)^2\cos(\theta)^2}$$



$$\Lambda = \lambda - \lambda' = 2$$

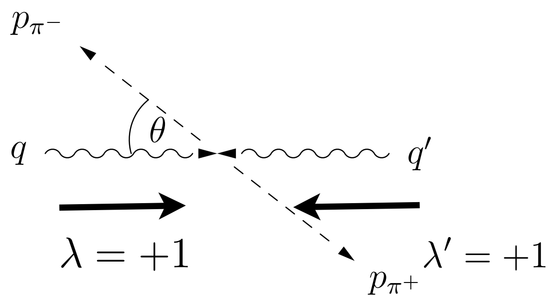
Warm up γγ→ππ: main pieces

a la Drechsel *et al* '99

- Decompose the amplitudes in Partial Waves

$$B_{J\Lambda} = \frac{1}{4ie^2}\int_{-1}^{1} d\cos\theta\,\sqrt{2J+1}\sqrt{\frac{(J-\Lambda)!}{(J+\Lambda)!}}P_J^{\Lambda}(\cos\theta)\mathcal{M}^{\Lambda}$$

$$\mathcal{M}_{++}^{\mathrm{BORN}} = 2ie^2\frac{1-\beta(s)^2}{1-\beta(s)^2\cos(\theta)^2}$$



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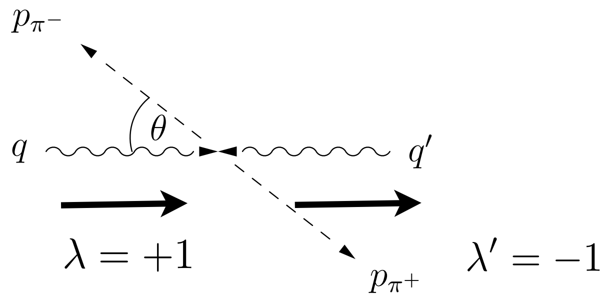
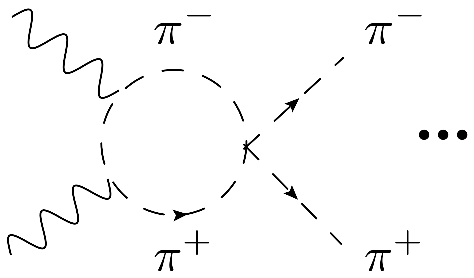


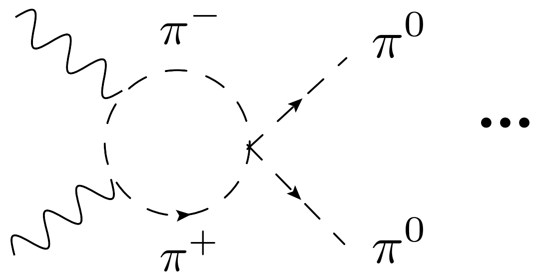
$$\Lambda = \lambda - \lambda' = 2$$

Warm up γγ→ππ: main pieces

a la Drechsel *et al* '99

- Include rescattering effects (FSI)





Warm up γγ→ππ: main pieces

a la Drechsel *et al* '99

- Include rescattering effects using Omnès representation
  - Only ππ (no KK..., no inelasticities)
  - Phase shifts from Peláez *et al.*
  - Left-hand cut with pion only

$$\mathrm{Im}\, F^{I}_{J\Lambda_\gamma}(\gamma\gamma\to\pi\pi) = \rho_{\pi\pi} F^{I*}_{J\Lambda_\gamma}(\gamma\gamma\to\pi\pi)\mathcal{I}^{I}_{J}(\pi\pi\to\pi\pi)$$

$$\phi_J^{I(\gamma\gamma\to\pi\pi)}(s) = \delta^{IJ}_{\pi\pi}(s) \qquad \text{for } 4m_\pi^2 < s < 4m_K^2$$

$$\Omega^I_J(s) = \exp\left[\frac{s}{\pi}\int_{s_0}^{\infty} \mathrm{d}s' \, \frac{\delta^I_J(s')}{s'(s'-s-\imath\epsilon)}\right]$$

(the right-hand cut is described by Omnès)

Warm up γγ→ππ: main pieces

a la Drechsel *et al* '99

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$$\operatorname{Im} F^I_{J\Lambda_\gamma}(\gamma\gamma\to\pi\pi) = \rho_{\pi\pi} F^{I*}_{J\Lambda_\gamma}(\gamma\gamma\to\pi\pi)\mathcal{I}^I_J(\pi\pi\to\pi\pi)$$

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Unitarized partial wave

Omnès (right-hand cut)

$$F^I_{J\Lambda}(s) = \Omega^I_J(s)\left\{B^I_{J\Lambda}(s)\operatorname{Re}\left[(\Omega^I_J)^{-1}(s)\right] - \frac{s(s-s_0)^{\frac{J}{2}}}{\pi} P\int_{s_0}^{\infty}\mathrm{d}s' \, \frac{B^I_{J\Lambda}(s')\operatorname{Im}(\Omega^I_J)^{-1}(s)}{(s'-s_0)^{\frac{J}{2}}(s'-s)}\right\}$$

Born (left-hand cut)

Warm up $\gamma\gamma\to\pi\pi$: main pieces

a la Drechsel *et al* '99

- Include rescattering effects using Omnès representation
  - Only $\pi\pi$ (no KK..., no inelasticities)
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Unitarized partial wave

Unitarized amplitude

$$F_{\Lambda=0}(s,\cos(\theta)) = \sum_{J\geq 0}\sqrt{2J+1}P_J(\cos(\theta))F_{J0}(s)$$

$$F_{\Lambda=2}(s,\cos(\theta)) = \sum_{J\geq 2}\sqrt{2J+1}\sqrt{\frac{(J-2)!}{(J+2)!}}P_J^2(\cos(\theta))F_{J2}(s)$$

$$\left(\frac{d\sigma}{d\cos(\theta)}\right)_{CM} = \frac{\beta(s)}{32\pi s}\left(|F_{\Lambda=0}|^2 + |F_{\Lambda=2}|^2\right)$$

Warm up γγ→ππ: main pieces

a la Drechsel *et al* '99

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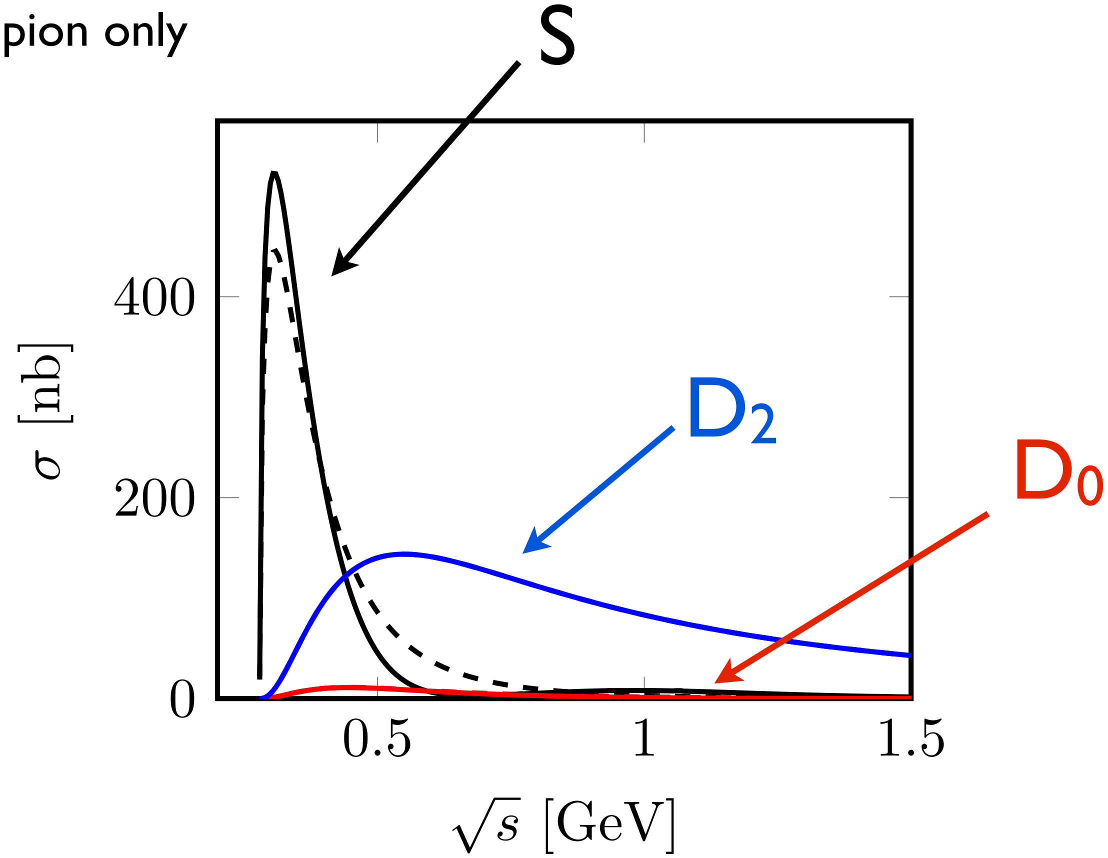


Warm up $\gamma\gamma\to\pi\pi$: main pieces

a la Drechsel *et al* '99

- Include $f_2(1270)$ resonance with a Breit-Wigner representation

$$\Gamma(f_2\to\pi\pi)=\frac{1}{40\pi}g^2_{f_2\pi\pi}\frac{\left(\sqrt{m^2_{f_2}/4-m^2_\pi}\right)^5}{m^4_{f_2}} \longleftarrow BR(f_2\to\pi\pi)=0.85$$

$$\Gamma_{f_2}=185\ \text{MeV}$$

$$\Gamma(f_2\to\gamma\gamma)=\frac{\pi\alpha^2}{5}g^2_{f_2\gamma\gamma}m_{f_2} \longleftarrow BR(f_2\to\gamma\gamma)=1.6\cdot10^{-7}$$

} [PDG]

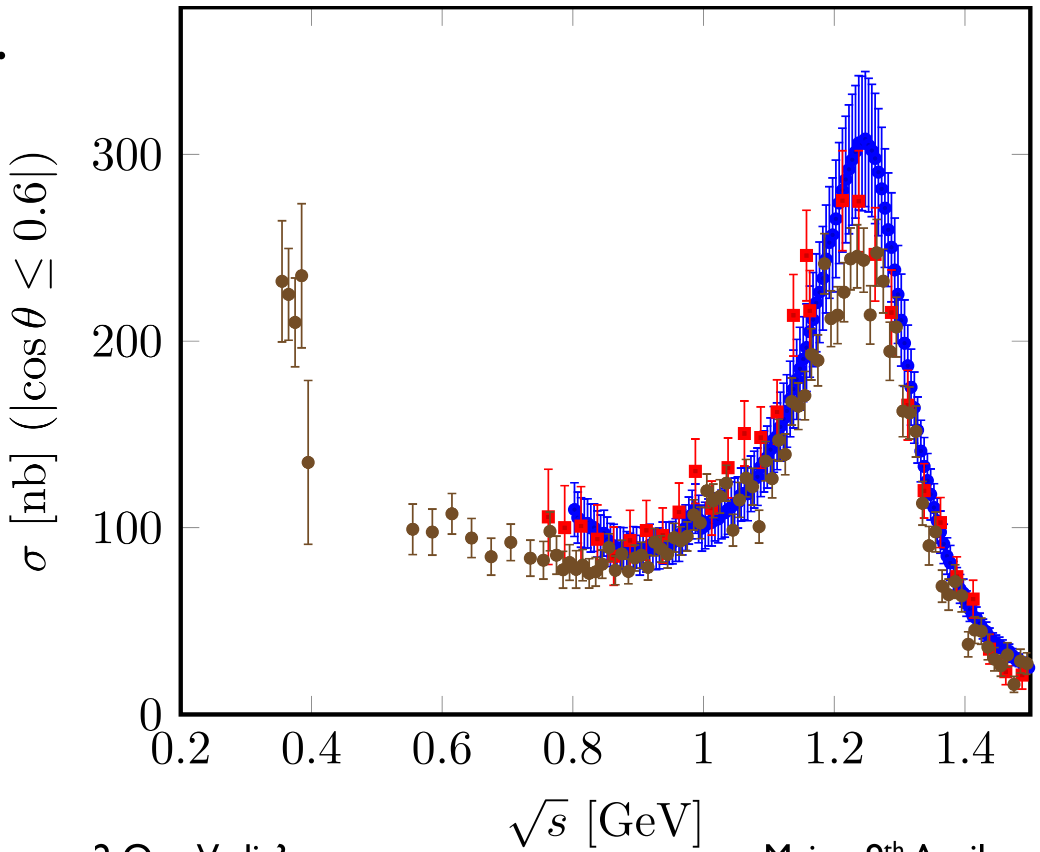
$$F^{(f_2)}_{J=2,\Lambda=2}(s)=-\pi\alpha\sqrt{\frac{2}{15}}\frac{g_{f_2\gamma\gamma}g_{f_2\pi\pi}}{m^2_{f_2}}\frac{s^2\beta^2}{s-m^2_{f_2}+im_{f_2}\Gamma(s)}$$

(notice no helicity 0 component)

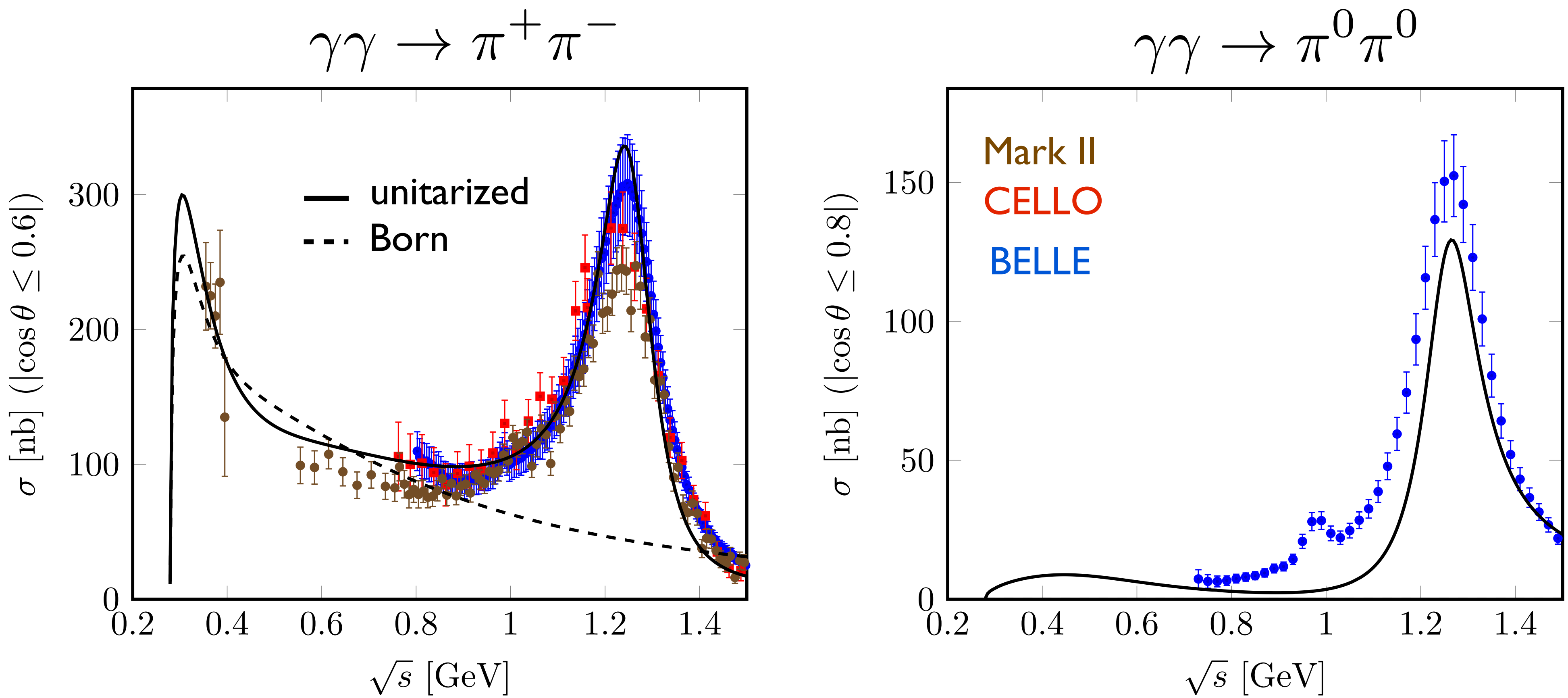
Warm up $\gamma\gamma \to \pi\pi$: main pieces

a la Drechsel *et al* '99

- Born diagrams + gauge invariance
- Rescattering of FS: Omnès representation
- Only $\pi\pi$ (no KK..., no inelasticities)
- Phase shifts from Peláez *et al.*
- Left-hand cut with pion only
- $f_2(1270)$ as Breit-Wigner resonance (I=0 from helicity 2 photons -no helicity=0 component- D_2 wave)



Warm up $\gamma\gamma \to \pi\pi$: main pieces



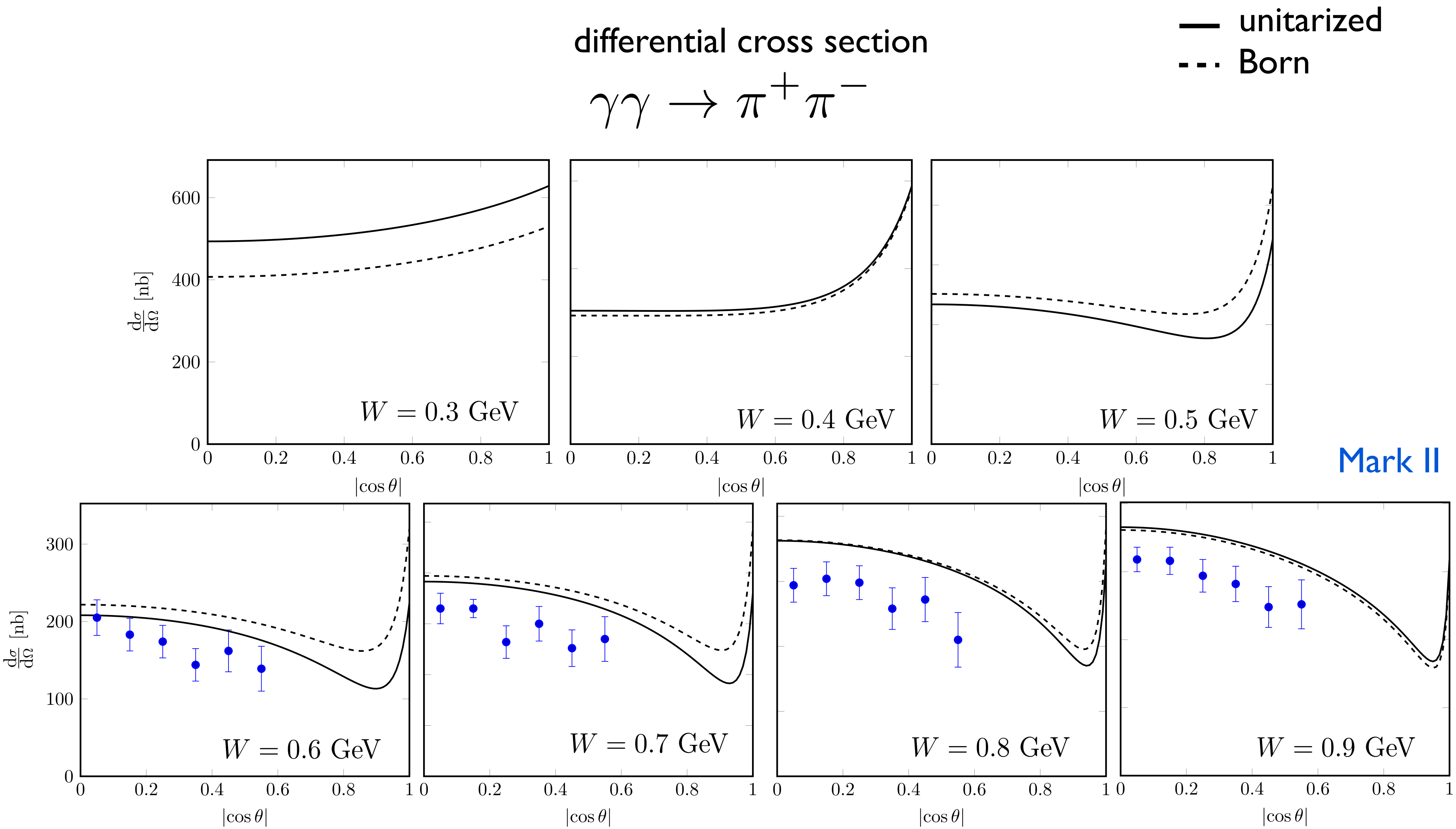
Improvements:
- KK threshold: better description around 1 GeV (less dependence on the phase shift)
- include helicity 0 f_2

[Pennington et al'08,'14]
[Mao et al, '09]
[García-Martin, '10]
[Hoferichter et al '11]

Warm up γγ→ππ: main pieces

differential cross section

$$\gamma\gamma \to \pi^+\pi^-$$



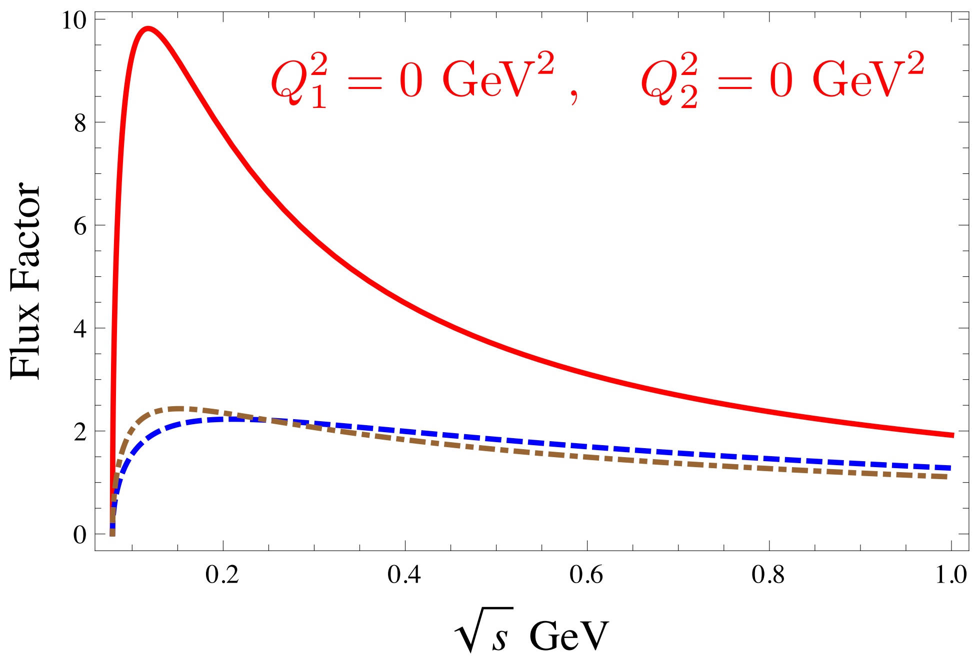
γ*γ*→ππ: flux factor

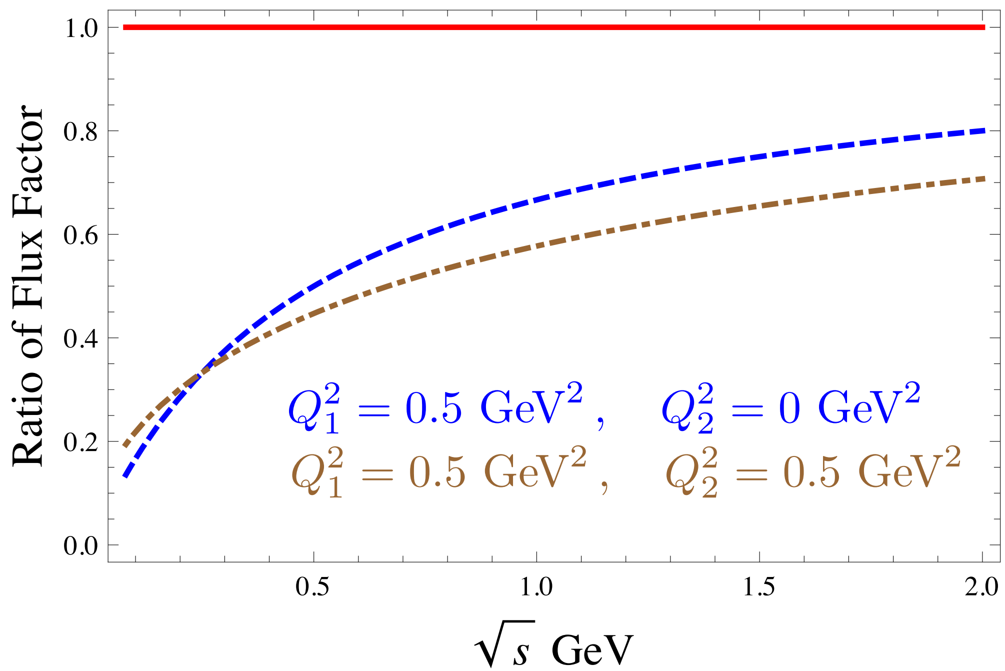
$$\left(\frac{d\sigma}{d\cos(\theta)}\right)_{CM} = \frac{\beta(s)}{64\pi\sqrt{X(s,Q_1^2,Q_2^2)}}\left(|F_{\Lambda=0}|^2 + |F_{\Lambda=2}|^2\right)$$

Flux factor

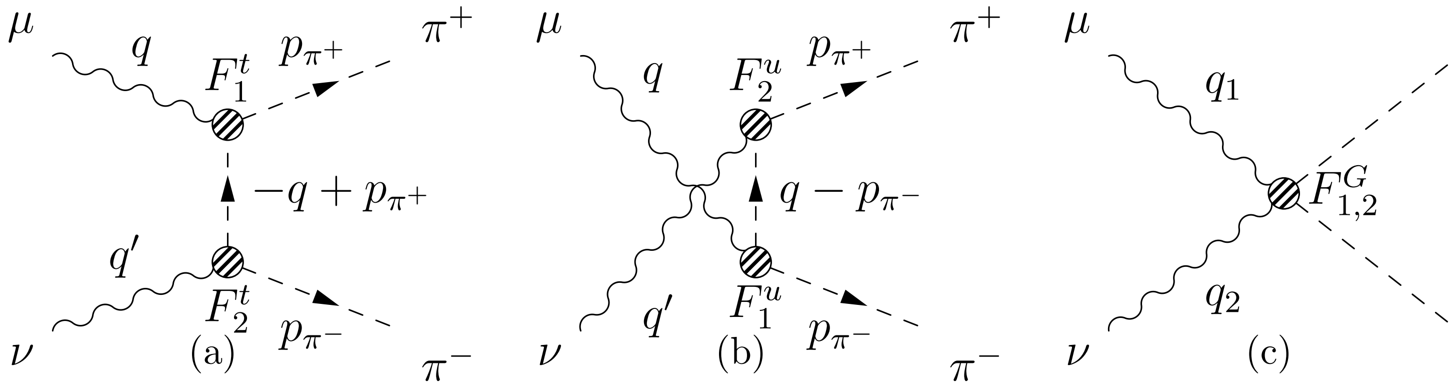
$$\beta = \sqrt{1 - \frac{4m_\pi^2}{s}}$$

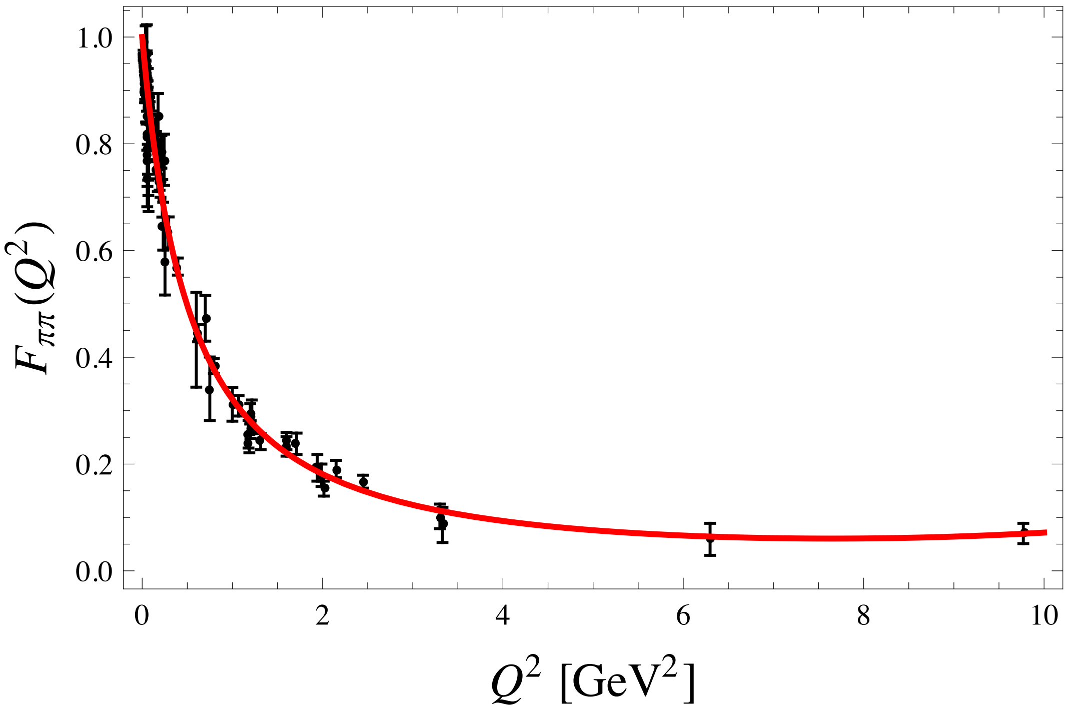
$$X(s,Q_1^2,Q_2^2) = \frac{1}{4}(s + Q_1^2 + Q_2^2)^2 - Q_1^2 Q_2^2$$





$\gamma^*\gamma^* \to \pi\pi$: form factor

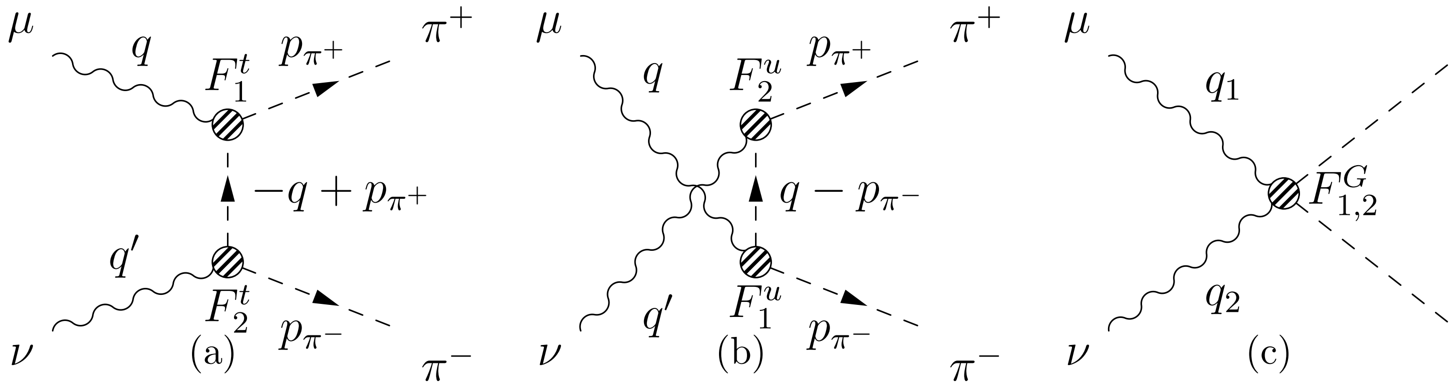




[P.M, S. Peris, JJ Sanz-Cillero, '08]

Brown '73; Bebek '74'76; Dally '77
Brauel '79; Amendolia (NA7)'86;
Horn '06'07; Tadevosyan '07

γ*γ*→ππ: amplitudes



$$\beta = \sqrt{1-\frac{4m_\pi^2}{s}}$$

$$\mathcal{M}_{++} = F_1 F_2 e^2 \left(2 - s\beta^2\nu \frac{\sin^2\theta}{\nu^2 - X\beta^2\cos^2\theta}\right)$$

$$X(s,Q_1^2,Q_2^2) = \frac{1}{4}(s+Q_1^2+Q_2^2)^2 - Q_1^2Q_2^2$$

$$\mathcal{M}_{+-} = F_1 F_2 e^2 s\beta^2\nu \frac{\sin^2\theta}{\nu^2 - X\beta^2\cos^2\theta}$$

$$\nu(s,Q_1^2,Q_2^2) = \frac{1}{2}(s+Q_1^2+Q_2^2)$$

F_1, F_2 form factors

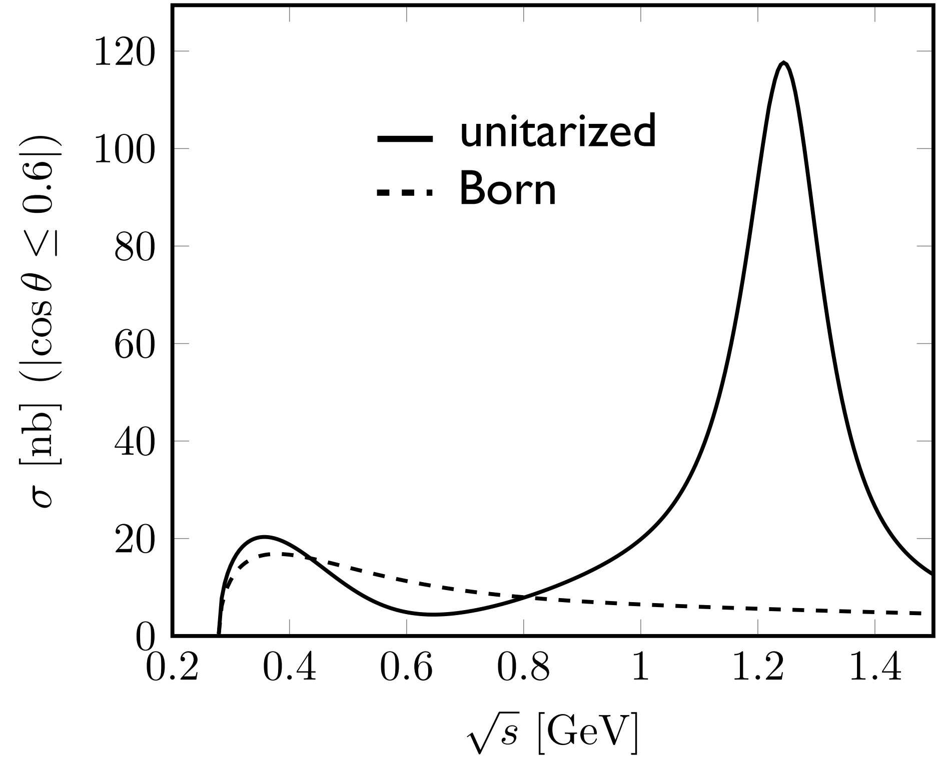
$$\mathcal{M}_{00} = 2F_1F_2e^2\sqrt{Q_1^2Q_2^2}\,\frac{-\nu + s\beta^2\cos^2\theta}{\nu^2 - X\beta^2\cos^2\theta}$$

both photons are longitudinal

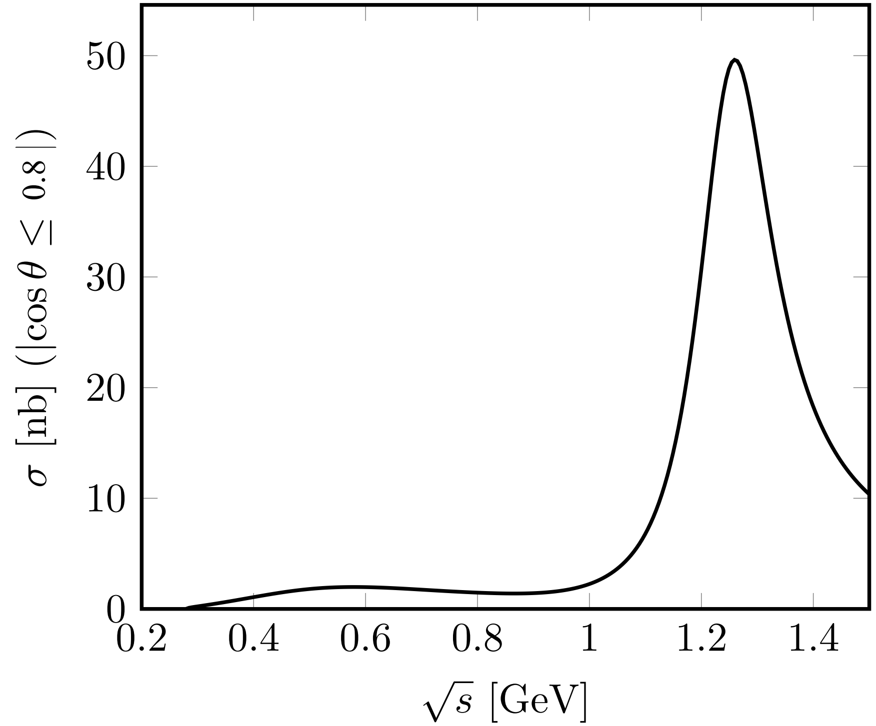
$\gamma^*\gamma \to \pi\pi$: main pieces

$$Q_1^2 = 0.5\ \text{GeV}^2\,, \quad Q_2^2 = 0$$

$\gamma^*\gamma \to \pi^+\pi^-$



$\gamma^*\gamma \to \pi^0\pi^0$



Improvements:
- KK threshold: better description around 1 GeV (less dependence on the phase shift)
- include helicity 0 f_2

[Moussallam '13]

$\gamma^*\gamma^*\to\pi\pi$: main pieces

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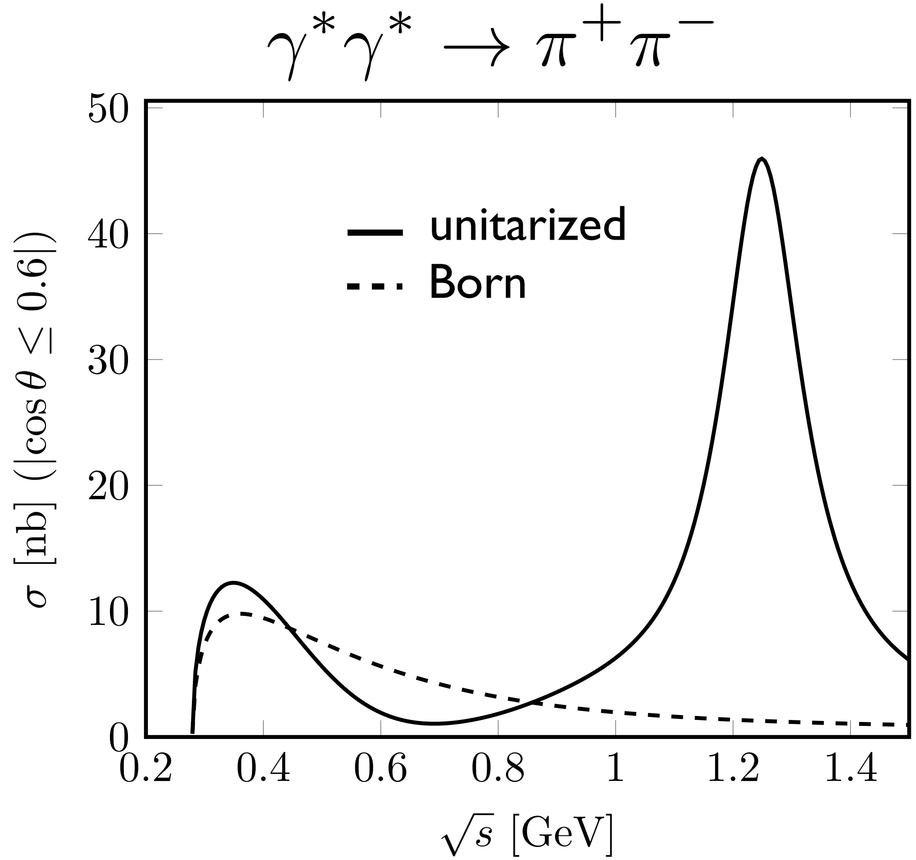

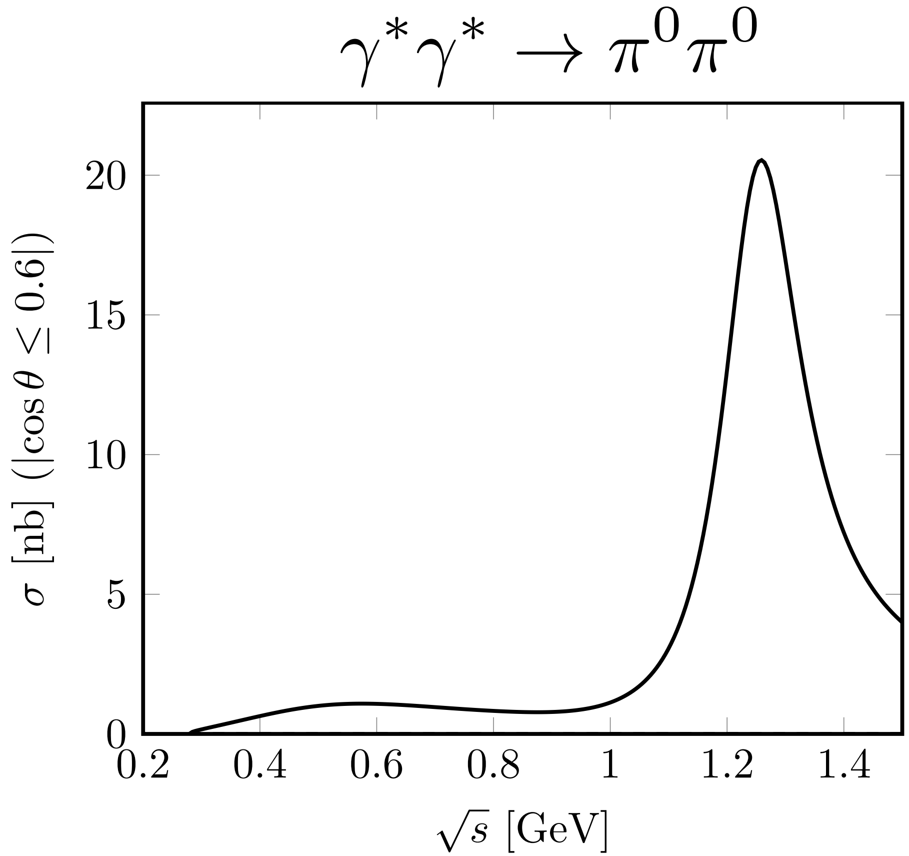

New feature: appearance of $\mathcal{M}_{00}$

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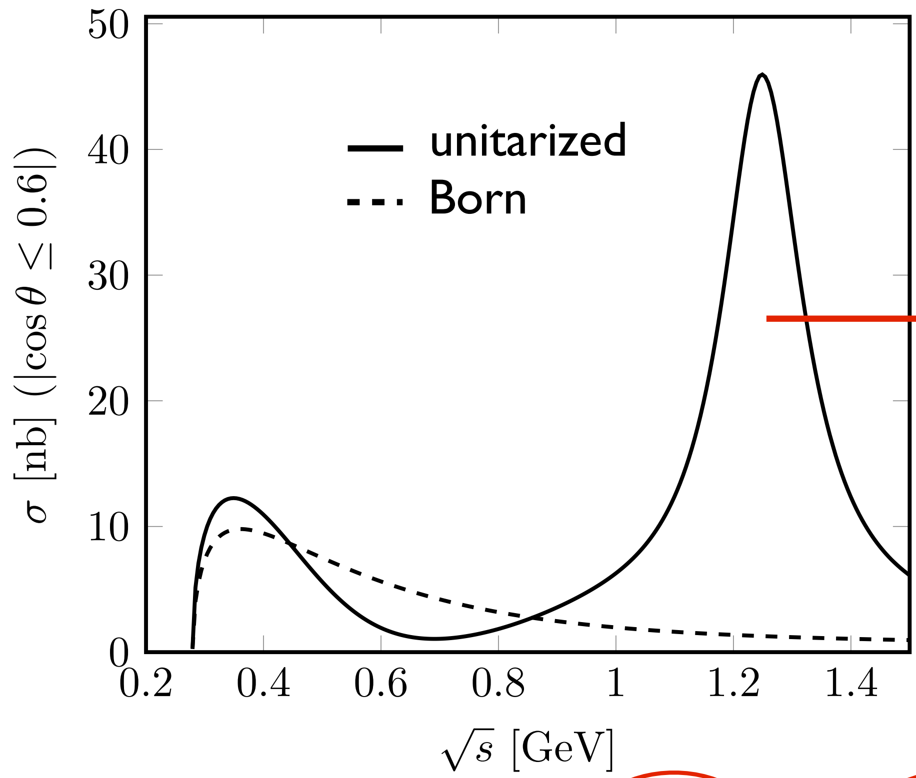
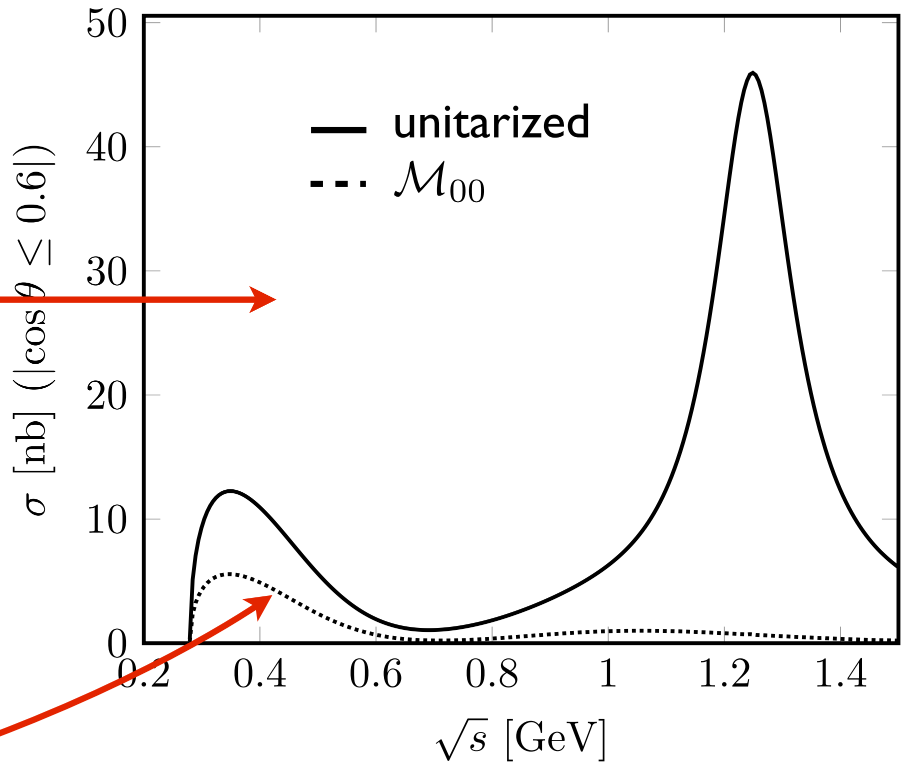
$\gamma^*\gamma^*\to\pi\pi$: main pieces

$$Q_1^2 = 0.5\ \mathrm{GeV}^2\,,\quad Q_2^2 = 0.5\ \mathrm{GeV}^2$$

$$\gamma^*\gamma^* \to \pi^+\pi^-$$





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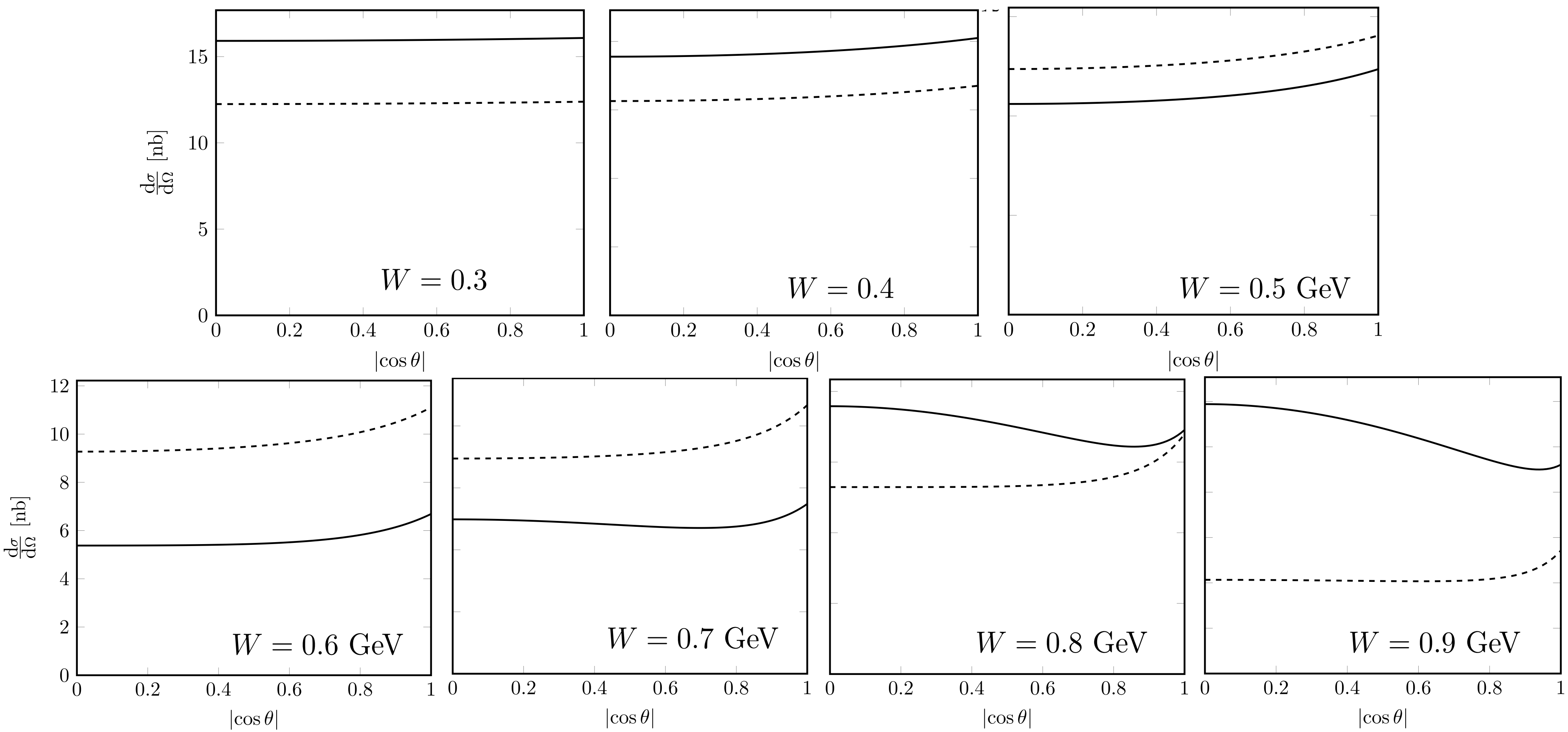
γ*γ*→ππ: main pieces

$$\gamma\gamma \to \pi^+\pi^-$$

$$Q_1^2 = 0.5\ \text{GeV}^2\,, \quad Q_2^2 = 0.5\ \text{GeV}^2$$

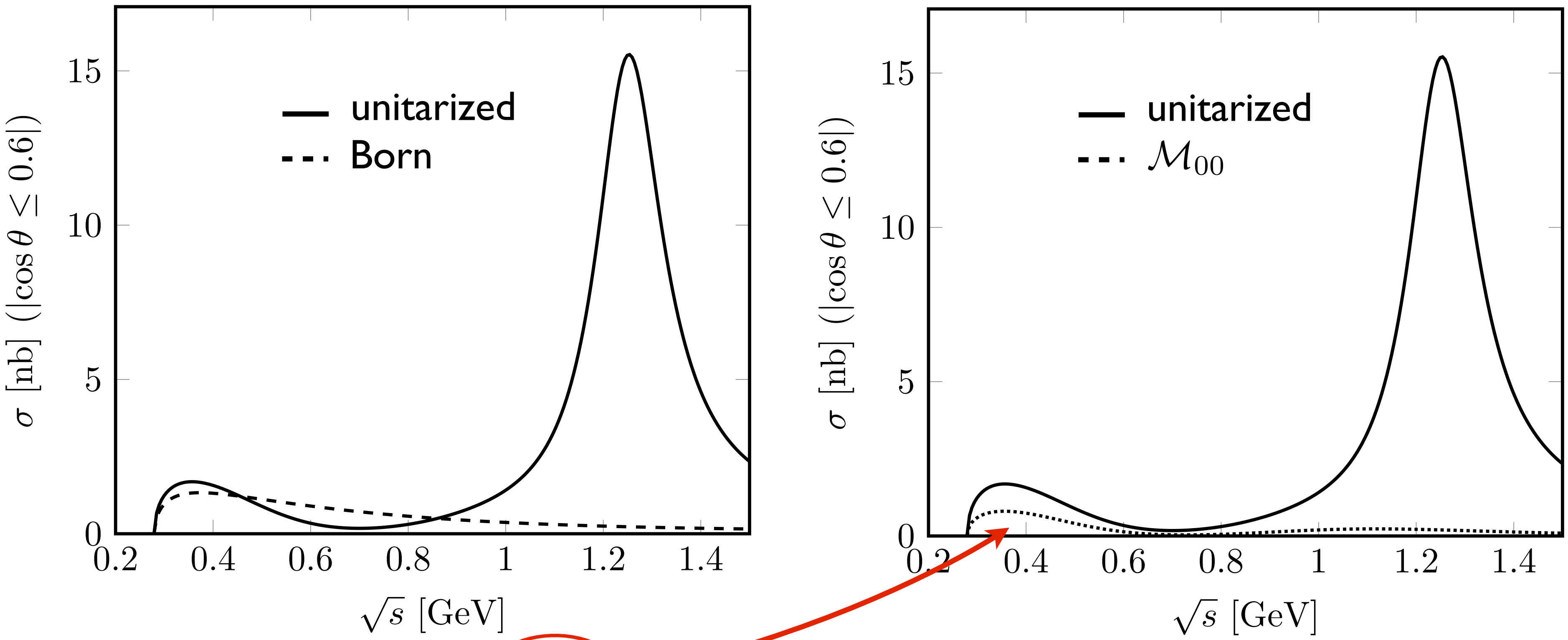
— unitarized
- - - Born



γ*γ*→ππ: main pieces

$$Q_1^2 = 1\ \text{GeV}^2\,,\quad Q_2^2 = 1\ \text{GeV}^2$$

$$\gamma^*\gamma^* \to \pi^+\pi^-$$



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Improvements:

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$\gamma^*\gamma^* \to \pi\pi$: conclusions

- First glance into $\gamma^*\gamma^* \to \pi\pi$
- Longitudinal photon contribution
- Interesting region around and up to $Q^2 = 1\ \mathrm{GeV}^2$

Improvements:
- KK threshold: better description around 1 GeV (less dependence on the phase shift)
- include helicity 0 f_2
- Use data at low-energies for subtracting

Thank you!

Warm up γγ→ππ: subtraction