Influence of temperature dependent shear-viscosity on the elliptic flow at back- and forward rapidities in ultrarelativistic heavy-ion collisions

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with:
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Introduction and Motivation

Perfect Liquid I.

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Strongly Interacting Low-Viscosity Matter Created in Relativistic Nuclear Collisions

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Substantial collective flow is observed in collisions between large nuclei at BNL RHIC (Relativistic Heavy Ion Collider) as evidenced by single-particle transverse momentum distributions and by azimuthal correlations among the produced particles. The data are well reproduced by perfect fluid dynamics. A calculation of the dimensionless ratio of shear viscosity η to entropy density s by Kovtun, Son, and Starinets within anti-de Sitter space/conformal field theory yields $\eta/s = \hbar/4\pi k_B$, which has been conjectured to be a lower bound for any physical system. Motivated by these results, we show that the transition from hadrons to quarks and gluons has behavior similar to helium, nitrogen, and water at and near their phase transitions in the ratio η/s. We suggest that experimental measurements can pinpoint the location of this transition or rapid crossover in QCD.

DOI: 10.1103/PhysRevLett.97.152303 PACS numbers: 12.38.Mh, 24.10.Nz, 25.75.Nq, 51.20.+d

PRL **97,** 152303 (2006) PHYSICAL REVIEW LETTERS week ending 13 OCTOBER 2006

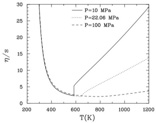


FIG. 3. The ratio η/s as a function of T for water with s normalized such that $s(T = 0) = 0$. The curves correspond to fixed pressures, one of them being the critical pressure, and the others being greater (100 MPa) and the other smaller (10 MPa). Below the critical pressure there is a jump in the ratio, and above the critical pressure there is only a broad minimum. They were constructed using data from NIST and CODATA.

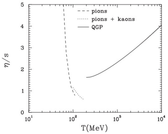
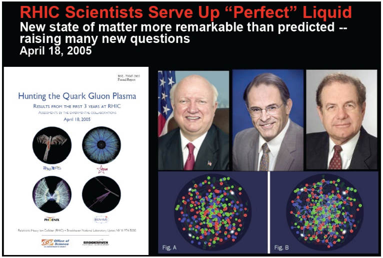
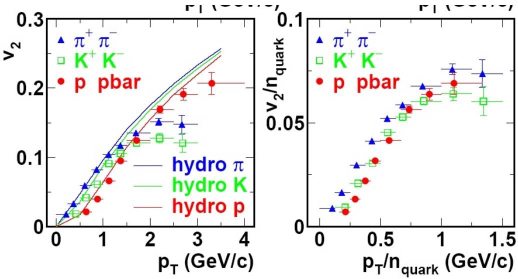


FIG. 4. The ratio η/s for the low temperature hadronic phase and for the high temperature quark-gluon phase. Neither calculation is very reliable in the vicinity of the critical or rapid crossover temperature.

Perfect Liquid II.





PRL 94, 111601 (2005) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2005

Viscosity in Strongly Interacting Quantum Field Theories from Black Hole Physics

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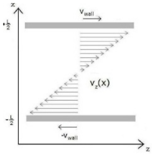
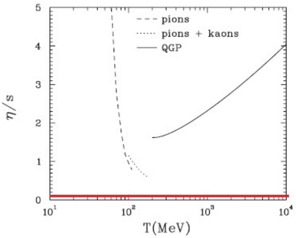
The ratio of shear viscosity to volume density of entropy can be used to characterize how close a given fluid is to being perfect. Using string theory methods, we show that this ratio is equal to a universal value of $\hbar/4\pi k_B$ for a large class of strongly interacting quantum field theories whose dual description involves black holes in anti–de Sitter space. We provide evidence that this value may serve as a lower bound for a wide class of systems, thus suggesting that black hole horizons are dual to the most ideal fluids.

DOI: 10.1103/PhysRevLett.94.111601 PACS numbers: 11.10.Wx, 04.70.Dy, 11.25.Tq, 47.75.+f

Shear Viscosity





Kinetic theory

$$\pi^{xz}_{NS} = -\eta \frac{\partial v_z(x)}{\partial x}$$

$$\eta \simeq 1.2T/\sigma = 1.2T\lambda_{mfp} n$$

$$\eta/s \approx T\lambda_{mfp}$$

(Dilute) Gases may have $\lambda \to \infty$ or $\sigma \to 0$ hence $\eta \to \infty$ ideal gas!

$\eta_{Air} \approx 1.8 \times 10^{-5}\mathrm{Pa.s}\,(T \sim 20C^o)$

Fluids (liquids), $\lambda \to 0$ or $\sigma \to \infty$ hence $\eta \to 0$ ideal fluid (liquid)!

$\eta_{Water} \approx 50 \times \eta_{Air} = 9 \times 10^{-4}\mathrm{Pa.s}$

$\eta_{gas}(T)$ *increases* with increasing T

$\eta_{liquid}(T)$ *decreases* with increasing T

Csernai et.al. PRL (2006)

$$\left(\frac{\eta}{s}\right)_\pi = \frac{15}{16\pi}\frac{f_\pi^4}{T^4}$$

$$\left(\frac{\eta}{s}\right)_{QCD} = \frac{5.12}{g^4 \ln(2.42g)} > 1$$

$$\left(\frac{\eta}{s}\right)_{ADS/CFD} = \frac{1}{4\pi}$$

f_π pion decay constant

g QCD coupling constant

Ideal Fluids I.

Conservation laws for a simple (single component) perfect fluid (no dissipation)

$$\partial_\mu N_0^\mu = 0 \quad \textbf{charge conservation} \quad \Rightarrow \textbf{1 eq.}$$
$$\partial_\mu T_0^{\mu\nu} = 0 \quad \textbf{energy-momentum conservation} \quad \Rightarrow \textbf{4 eqs.}$$

Perfect fluid decomposition with respect to u^μ

$$N_0^\mu = n_0 u^\mu$$
$$T_0^{\mu\nu} = e_0 u^\mu u^\nu - p_0 \Delta^{\mu\nu}$$
$$n_0 = N_0^\mu u_\mu \quad \textbf{(net)charge density}$$
$$e_0 = T_0^{\mu\nu} u_\mu u_\nu \quad \textbf{energy density}$$
$$p_0 = -\frac{1}{3}\Delta_{\mu\nu} T_0^{\mu\nu} \quad \textbf{equilibrium pressure}$$

- The time-like normalized flow velocity is $u^\mu(t,\vec{x})$, where $u^\mu u_\mu = 1$
- Projection tensor $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$, where $g^{\mu\nu} = diag(1,-1,-1,-1)$
- We have 5 equations for 6 unknowns *not closed*: $n_0(1), e_0(1), p_0(1)$ and $u^\mu(3)$.

Ideal Fluids II.

- The assumption of local thermal equilibrium provides *closure*:

Equation of State (EoS)

$$p_0 = p_0(e_0, n_0) \qquad \text{EoS} \Rightarrow \mathbf{1\ eq.}$$

and/or $p(T, \mu)$ or $s = s(e, n)$.

- $S_0^\mu = s_0 u^\mu$, where $s_0 = S_0^\mu u_\mu$, and for continuous solutions

$$\partial_\mu S_0^\mu = 0$$

entropy is maximum in local thermal equilibrium

Thermodynamics

$$\begin{aligned} Ts &= e + p - \mu n \\ T\dot{s} &= \dot{e} - \mu \dot{n} \\ \dot{p} &= s\dot{T} + n\dot{\mu} \end{aligned}$$

- The fundamental thermodynamic relations are derived from, $T\partial_\mu(su^\mu) = \partial_\mu(eu^\mu) + p(\partial_\mu u^\mu) - \mu\partial_\mu(nu^\mu)$

Dissipative Fluids I.

Conservation laws for a simple (single component) dissipative fluid

$$\partial_\mu N^\mu = 0$$ **charge conservation** $\Rightarrow$ **1 eq.**

$$\partial_\mu T^{\mu\nu} = 0$$ **energy-momentum conservation** $\Rightarrow$ **4 eqs.**

General decomposition $N^\mu = N_0^\mu + \delta N^\mu$ and $T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu}$

$$N^\mu = n u^\mu + V^\mu$$

$$T^{\mu\nu} = e u^\mu u^\nu - (p + \Pi)\Delta^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu}$$

$$n = N^\mu u_\mu$$ **charge density**

$$e = T^{\mu\nu} u_\mu u_\nu$$ **energy density**

$$p = -\frac{1}{3}\Delta_{\mu\nu} T^{\mu\nu}$$ **isotropic pressure**

$$V^\mu = \Delta^{\mu\alpha} N_\alpha$$ **charge flow**

$$W^\mu = \Delta^{\mu\alpha} u^\beta T_{\alpha\beta}$$ **energy-momentum flow**

$$\pi^{\mu\nu} = \left[\frac{1}{2}\left(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\alpha\nu}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}\right] T_{\alpha\beta}$$ **stress tensor**

- We only have 5 equations for 17 unknowns, $n(1), e(1), p(1), u^\mu(3)$ and $V^\mu(3), W^\mu(3), \pi^{\mu\nu}(5)$.

Dissipative Fluids II.

Simplifications (I): Matching to equilibrium and the EOS

$$n = n_0, \quad e = e_0, \quad p(e,n) = p_0(e_0, n_0) + \Pi$$

- $\Pi = p - p_0 = -\frac{1}{3}\Delta_{\mu\nu}\delta T^{\mu\nu}$ is the bulk viscosity
- $T = T_0$ and $\mu = \mu_0$, while $s = s_0 + \delta s$!

Simplifications (II): Fixing the Local Rest Frame

$$u^\mu_E = N^\mu / n \quad \Leftrightarrow V^\mu = 0 \quad \Rightarrow q^\mu = W^\mu \qquad \textbf{Eckart}$$

$$u^\mu_L = T^{\mu\nu} u_{L\nu}/e \quad \Leftrightarrow W^\mu = 0 \quad \Rightarrow q^\mu = -\frac{e+p}{n} V^\mu \qquad \textbf{Landau \& Lifsitz}$$

- Now, we are left with 14 unknowns! $n(1), e(1), u^\mu(3)$ and $\Pi(1), q^\mu(3), \pi^{\mu\nu}(5)$.
- The definition of entropy is also modified $S^\mu \equiv S^\mu_0 + \delta S^\mu = (s_0 + \delta s)u^\mu + \Phi^\mu$

2nd law of thermodynamics

$$\partial_\mu S^\mu = -\frac{q^\mu}{T}\left(\frac{1}{T}\partial_\mu T - \dot{u}_\mu\right) - \frac{\Pi}{T}\partial_\mu u^\mu + \frac{\pi^{\mu\nu}}{T}\partial_\mu u_\nu \geq 0$$

Dissipative Fluids III.

Solution (I): The relativistic Navier-Stokes equations

$$\Pi_{NS} = -\zeta \, \nabla_\mu u^\mu$$
$$q^\mu_{NS} = -\kappa \, T \frac{T\,n}{e+p} \, \nabla^\mu \left(\frac{\mu}{T}\right)$$
$$\pi^{\mu\nu}_{NS} = 2\,\eta \, \nabla^{\langle\mu} u^{\nu\rangle}$$

- $\zeta \geq 0, \kappa \geq 0, \eta \geq 0$ coefficient of bulk viscosity, thermal conductivity and shear viscosity.
- Now, the equations of fluid dynamics are closed, but the relativistic Navier-Stokes theory leads to accausal signal propagation and stability issues.

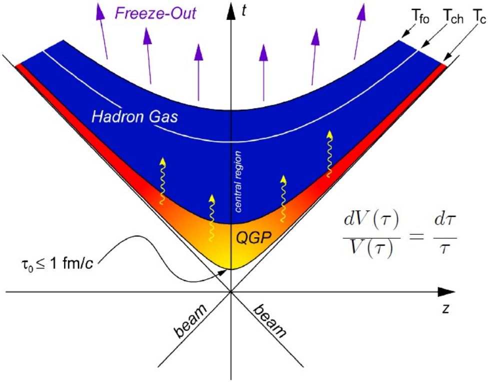
Solution (II): Relaxation equations (Israel 1976, Israel and Stewart 1979)

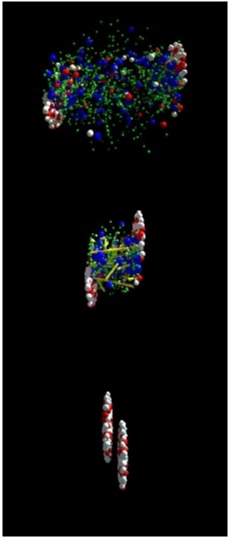
$$\tau_\Pi \dot{\Pi} + \Pi = \Pi_{NS} + l_{\Pi q} \nabla_\mu q^\mu$$
$$\tau_q \Delta^\mu_\alpha \dot{q}^\alpha + q^\mu = q^\mu_{NS} + l_{q\Pi} \nabla^\mu \Pi - l_{q\pi} \Delta^\mu_\alpha \partial_\nu \pi^{\alpha\nu}$$
$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = \pi^{\mu\nu}_{NS} + l_{\pi q} \nabla^{\langle\mu} q^{\nu\rangle}$$

- The relaxation equations determine the time evolution of Π, q^μ, and $\pi^{\mu\nu}$
- The relativistic Navier-Stokes theory appears if the relaxation times and length scales $\tau_i \to 0$, $l_i \to 0$ with ζ, η and κ_q fixed

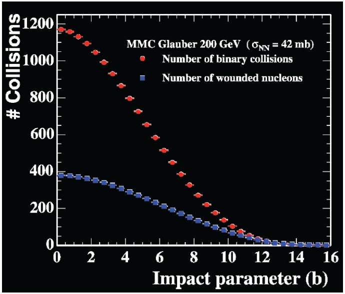
Parameters and Setup

Space-time evolution of matter in heavy-ion collisions.





Initial condition - Transverse profile



- Nr. wounded nucleons $N_{WN} \sim 2A$
- Nr. binary collisions $N_{BC} \sim A^{4/3}$
- $n_{BC}(x, y, b) = \sigma_{NN} T_A(x + b/2, y)\, T_B(x - b/2, y)$
- $T_A(x, y) = \int_{-\infty}^{\infty} dz\, \rho_A(x, y, z)$
- $\rho_A(\mathbf{r}) = \frac{\rho_0}{1+\exp[(r - R_A)/d]}$

Initial conditions

- Centrality selection according to multiplicity
- $f_{BC}(n_{BC}, n_{WN}) = n_{BC} + c_1 n_{BC}^2 + c_2 n_{BC}^3$
- $e_T(\tau_0, x, y, b) = C_e(\tau_0)\, f(n_{BC}, n_{WN})$

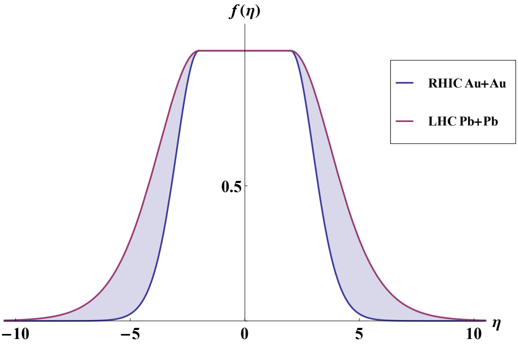
RHIC

- ideal fluid: $e_0 = 17.0$ GeV/fm^3
- LH-LQ and HH-LQ: $e_0 = 15.8$ GeV/fm^3
- LH-HQ and HH-HQ: $e_0 = 14.9$ GeV/fm^3

LHC

- ideal fluid: $e_0 = 57.5$ GeV/fm^3
- LH-LQ and HH-LQ: $e_0 = 54.5$ GeV/fm^3
- LH-HQ and HH-HQ: $e_0 = 49.5$ GeV/fm^3

Initial condition - Longitudinal profile

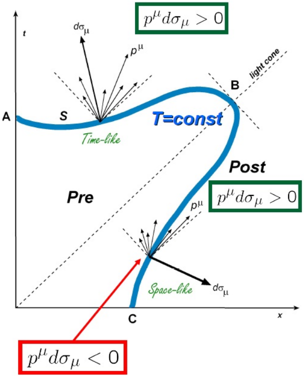


$$e_L(\eta) = \exp\left(-2c_\eta\sqrt{1+\frac{(|\eta|-\eta_0)^2}{2c_\eta\sigma_\eta^2}\Theta\left(|\eta|-\eta_0\right)}+2c_\eta\right) \quad \text{where} \quad \eta_0 = 2.0$$

RHIC Au+Au 200 GeV $c_\eta = 4.0$ $\sigma_\eta = 1.0$

LHC Pb+Pb 2760 GeV $c_\eta = 2.0$ $\sigma_\eta = 1.8$

Freeze-out



Correction by Bugaev 1996
$\Theta(p^\mu d\sigma_\mu)$
Gradual freeze-out (Grassi et al. 1995, Csernai et.al. 1999)

Cooper-Frye freeze-out for particle i

$$E\frac{dN}{d^3\mathbf{k}} = \frac{g_i}{(2\pi)^3}\int_\Sigma d\sigma^\mu k_\mu f_i(x,\mathbf{k})$$

where

$$f_i(x,k) = f_{0i}\left[1 + \left(1 \mp \frac{(2\pi)^3}{g_i} f_{0i}\right)\frac{k_i^\mu k_i^\nu \pi_{\mu\nu}}{2T^2(e+p)}\right]$$

and

$$f_{0i}(x,k) = \frac{g_i}{(2\pi)^3}\left[\exp\left(\frac{k_i^\mu u_\mu - \mu_i}{T}\right) \pm 1\right]^{-1}$$

- Constant Temperature hypersurface, T_{dec}
- Constant Knudsen number hypersurface

$$\text{Kn} = \tau_\pi \theta \sim \frac{\lambda_{mfp}}{L} \sim 1$$

where e.g., $Kn_{dec} = 0.8 \simeq T_{dec} \in [115, 165]$

Equations of motion

Conserved Quantities

$$T^{\mu\nu} = e_0 u^\mu u^\nu - p_0 \Delta^{\mu\nu} + \pi^{\mu\nu},$$
$$N^\mu = 0, \quad \Pi = 0, \quad q^\mu = 0$$

Equations of motion - Denicol et al. PRD(2012)

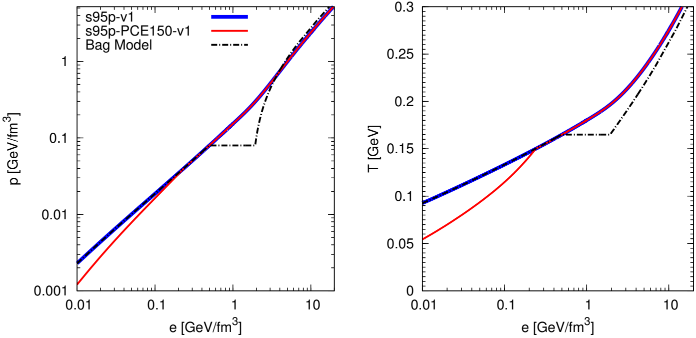
$$\partial_\mu N^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0,$$
$$\tau_\pi D\pi^{\mu\nu} = 2\eta_s \sigma^{\mu\nu} - \pi^{\mu\nu} - \tau_\pi \left(\pi^{\lambda\mu} u^\nu + \pi^{\lambda\nu} u^\mu\right) Du_\lambda$$
$$- \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi_\lambda^{\langle\mu} \sigma^{\nu\rangle\lambda}$$
$$+ 2\tau_\pi \pi_\lambda^{\langle\mu} \omega^{\nu\rangle\lambda} + \varphi_7 \pi_\lambda^{\langle\mu} \pi^{\nu\rangle\lambda}$$

$\tau_\pi = 5\eta_s/(e+p)$, $\tau_\pi = \frac{5}{3}\lambda_{mfp}$, $\delta_{\pi\pi} = (4/3)\,\tau_\pi$, $\tau_{\pi\pi} = (10/7)\,\tau_\pi$, $\varphi_7 = (9/70)\,/p_0$

Initial values

- Energy density at $\tau_0 = 1$ fm/c is $e(\tau_0, x, y, \eta, b) = e_T(\tau_0, x, y, b)\, e_L(\eta)$
- Initial velocities: $v_x = v_y = 0$, Bjorken scaling flow $v_z = \frac{z}{t}$ that is $v_\eta = 0$
- Initial shear-stress: $\pi^{\mu\nu} = 0$

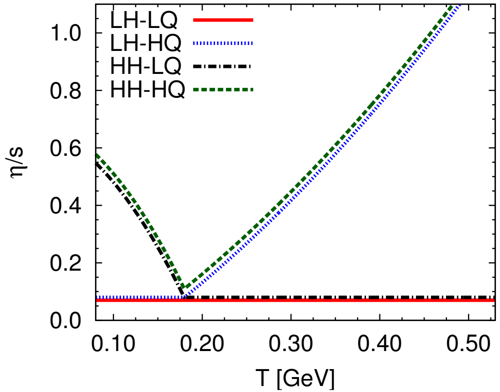
Equation of State

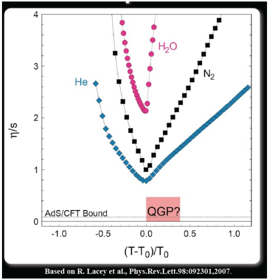


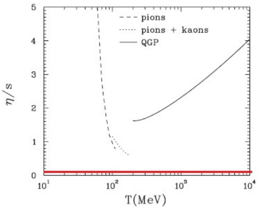
EoS - Lattice parametrization by Petreczky and Huovinen Nucl. Phys. A (2010)

- Chemical Equilibrium (s95p-v1)
- (Partial) Chemical Equilibrium with chemical freeze-out at $T_{ch} = 150$ MeV (s95p-PCE150-v1)
- Hadron Resonance Gas (HRG) includes all hadronic states up to $m \sim 2$ GeV

Temperature dependent η/s I.



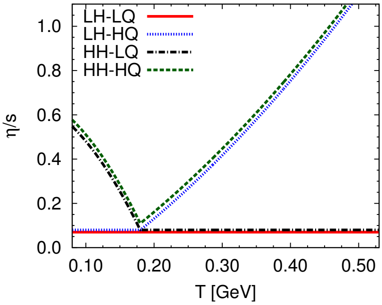


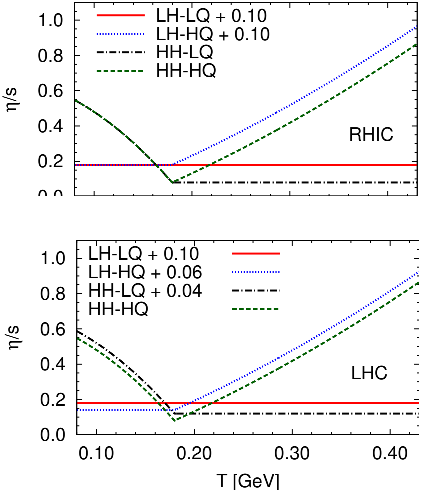


Niemi et al. PRL (2011)

- Can we separate hadronic viscosity from the QGP viscosity?

Temperature dependent η/s II. - Scaled Parametrizations

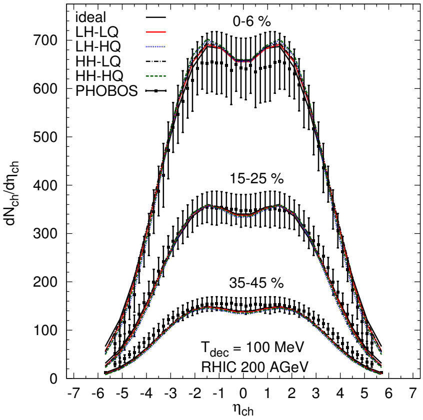


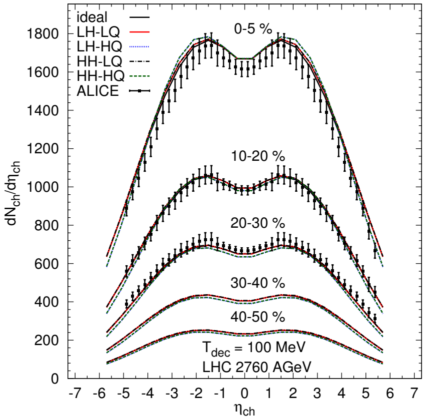


- Can we study the influence of viscosity at and near the QCD-transition point ?

Results I. - Spectra and flow at RHIC and at the LHC

Multiplicity

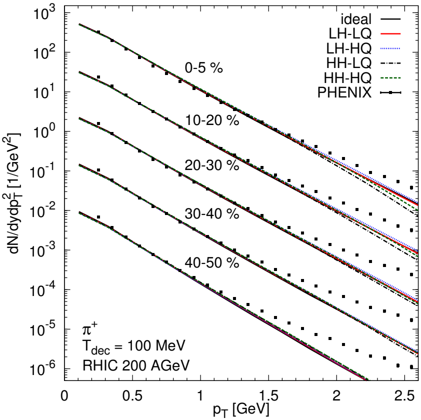


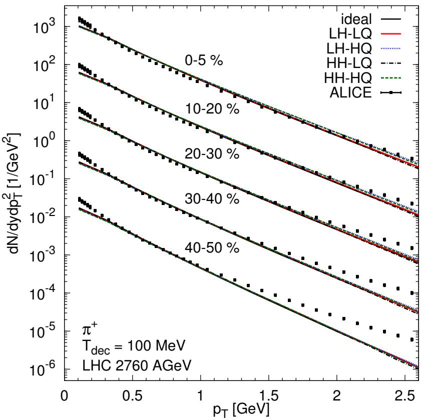


- Multiplicity at RHIC independent of (η/s)-parametrization and decoupling temperature $T_{dec} \in [80, 140]$ MeV for EoS s95p-PCE150-v1

- Multiplicity at the LHC independent of (η/s)-parametrization and decoupling temperature $T_{dec} \in [80, 140]$ MeV for EoS s95p-PCE150-v1

Pion Spectra

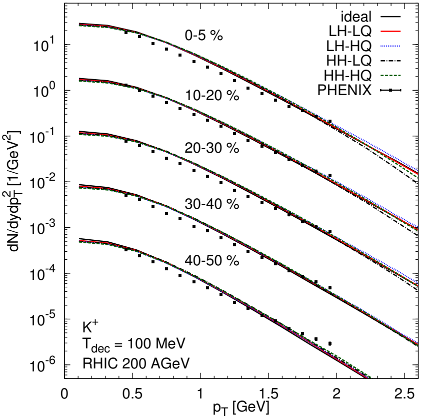


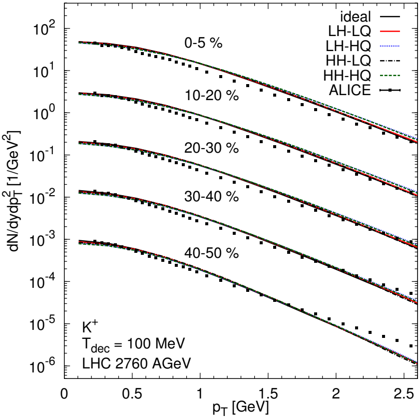


- Spectra at RHIC independent of (η/s)-parametrization

- Spectra at the LHC independent of (η/s)-parametrization

Kaon Spectra

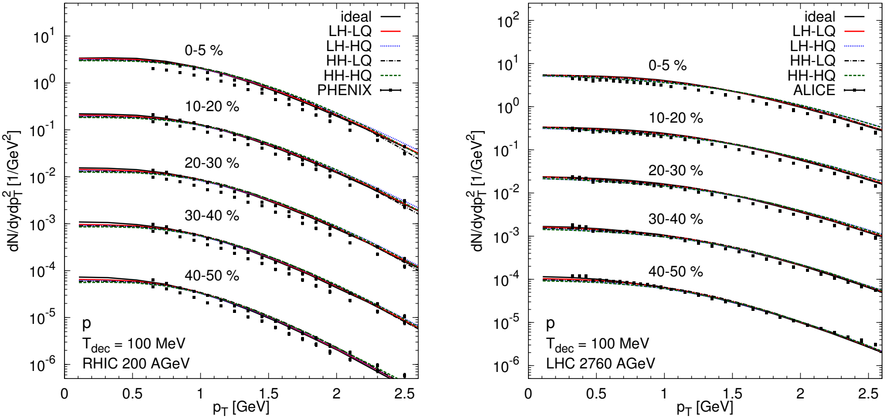




- Spectra at RHIC independent of (η/s)-parametrization

- Spectra at the LHC independent of (η/s)-parametrization

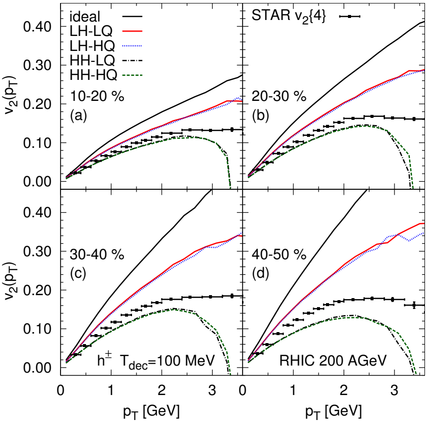
Proton Spectra

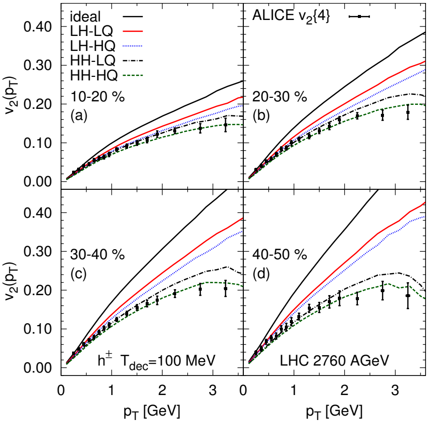


- Spectra at RHIC independent of (η/s)-parametrization

- Spectra at the LHC independent of (η/s)-parametrization

Elliptic Flow I.

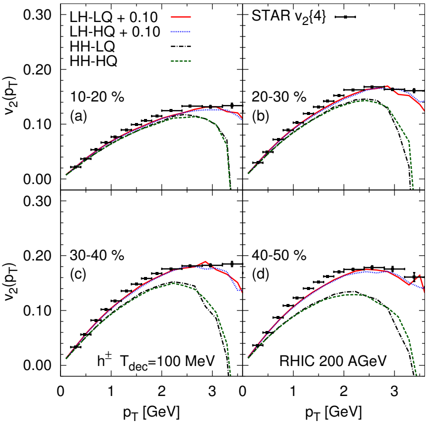


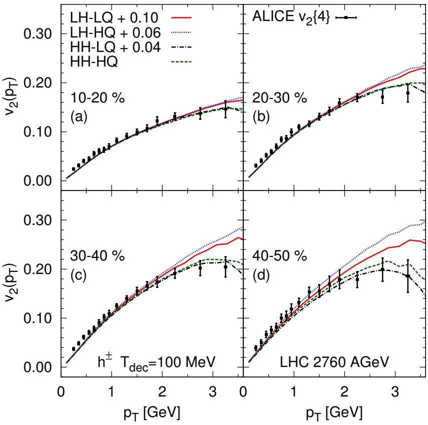


- $v_2(p_T)$ at RHIC decreases and separates according to the low-temperature (η/s) but independent of the high-temperature (η/s)-parametrization

- $v_2(p_T)$ at the LHC equally depends on both the low- and the high-temperature (η/s)-parametrizations

Elliptic Flow I. - Scaled

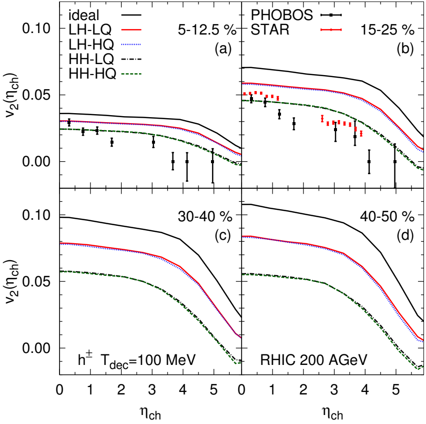


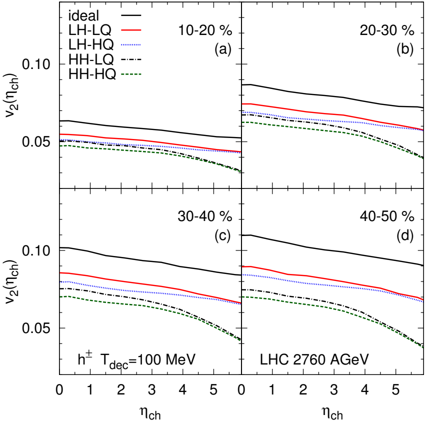


- Scaling the low-temperature (η/s)-parametrizations, the QCD-transition region also shows its effect on $v_2(p_T)$

- Differences are modest but increasing with centrality and $v_2(p_T)$ is ordered by increasing hadronic viscosity

Elliptic Flow II.

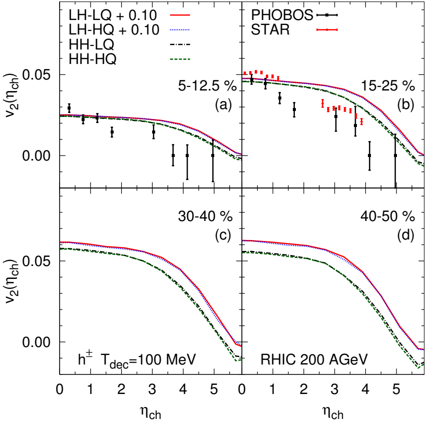


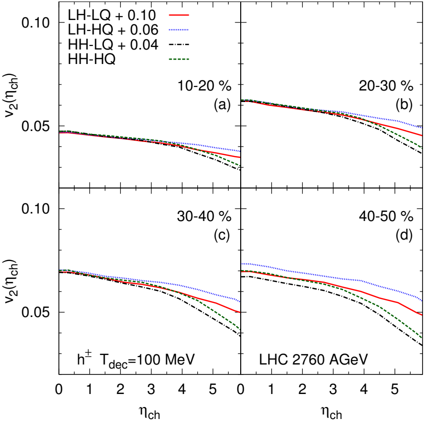


- $v_2(\eta_{ch})$ at RHIC decreases and separates according to the low-temperature (η/s) similarly to $v_2(p_T)$ at mid-rapidity!

- With increasing centrality and rapidity, $v_2(\eta_{ch})$ at the LHC, starts to behave like the matter at RHIC

Elliptic Flow II. - Scaled

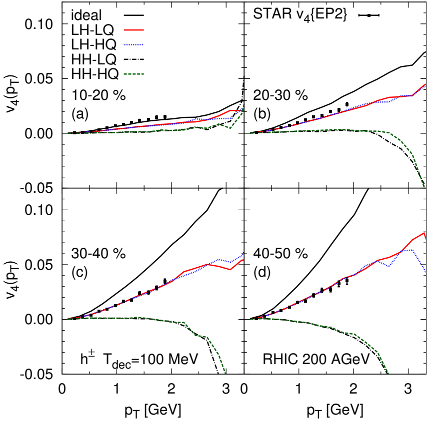


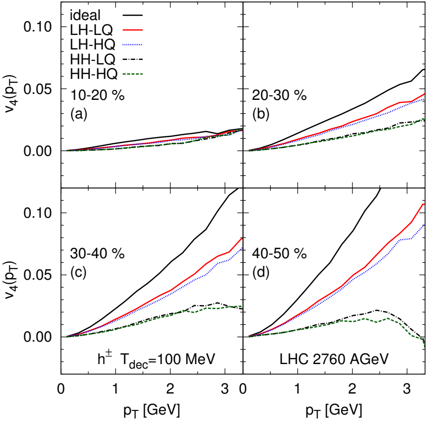


- Scaled (η/s)-parametrizations only show differences with increasing centrality and rapidity

- Scaled (η/s)-parametrizations show only modest differences with increasing centrality and rapidity

Quadrangular Flow I.

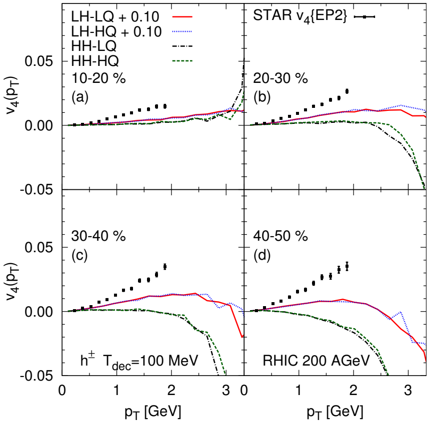


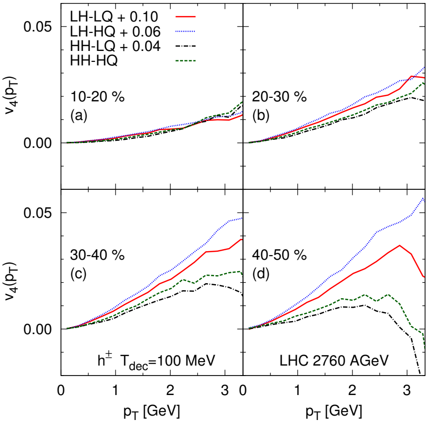


- $v_4(p_T)$ behaves similarly to $v_2(p_T)$ (at RHIC)
- Difference in experimental and theoretical plots, due to different methods!

- $v_4(p_T)$ at the LHC behaves similarly to the $v_2(p_T)$ at RHIC (note the grouping)
- $v_4(p_T)$ is more sensitive to the low-temperature part than $v_4(p_T)$

Quadrangular Flow I. - Scaled

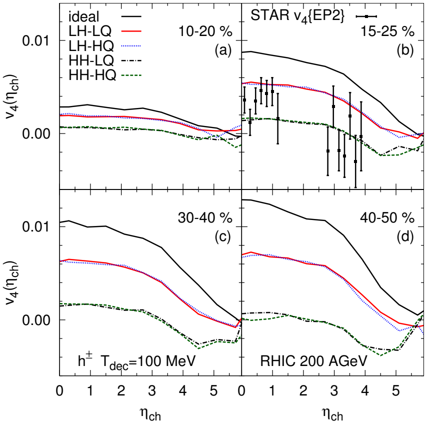


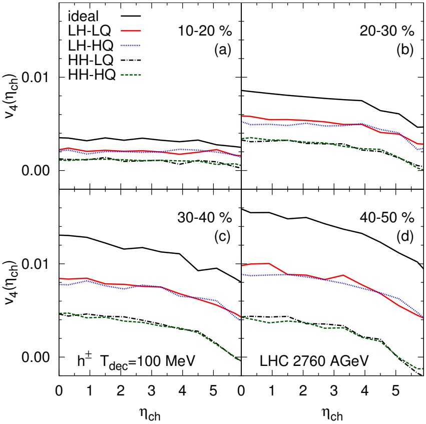


- $v_4(p_T)$ behaves similarly to $v_2(p_T)$

- $v_4(p_T)$ is more sensitive than $v_2(p_T)$ to the different (η/s) parametrizations

Quadrangular Flow II.



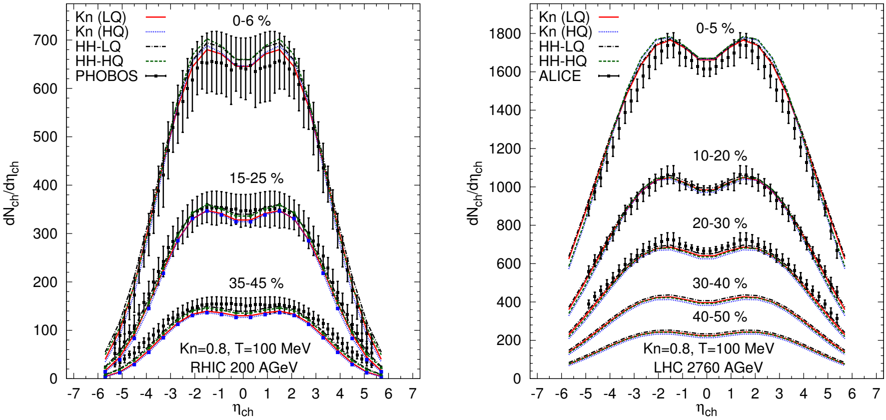


- $v_4(\eta_{ch})$ at RHIC decreases and separates according to the low-temperature (η/s) similarly to $v_2(\eta_{ch})$

- The same observation holds at the LHC!

Different freeze-out criterion

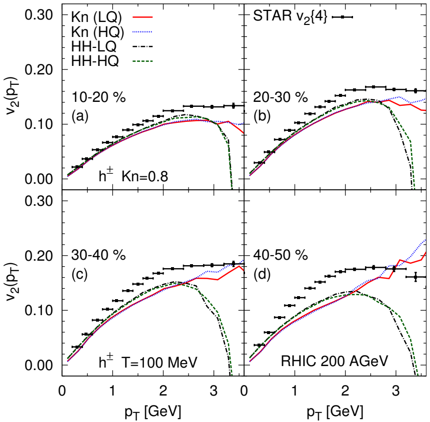
Multiplicity II.

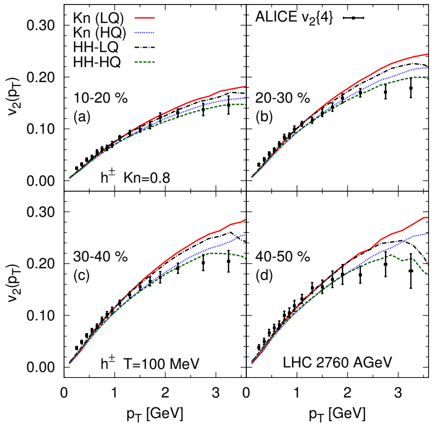


- Multiplicity at RHIC independent of freeze-out criteria $T_{dec} = 100$ MeV $\sim Kn_{dec} = 0.8$

- Multiplicity at the LHC independent of freeze-out criteria $T_{dec} = 100$ MeV $\sim Kn_{dec} = 0.8$

Elliptic Flow III.

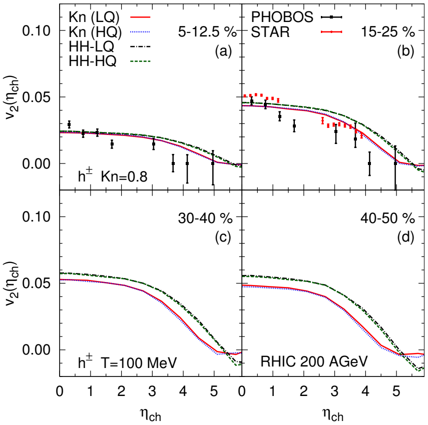


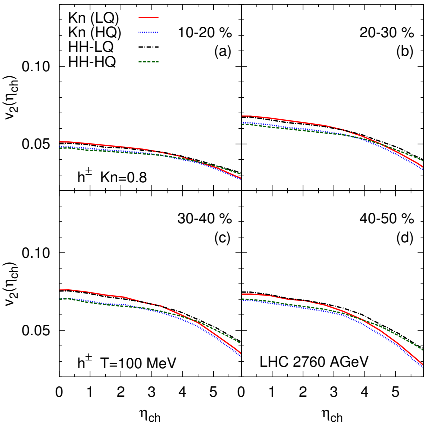


- $v_2(p_T)$ shows little sensitivity to the decoupling criteria

- Similarly at the LHC!

Elliptic Flow III.





- $v_2(\eta_{ch})$ at RHIC shows some sensitivity at large rapidity

- Similarly at the LHC!

Conclusions ...

- At RHIC the suppression of the elliptic flow, v_2, strongly depends on the viscosity at and after the QCD transition (hadronic viscosity)
- At the LHC the viscous suppression of elliptic flow, v_2, at midrapidity is affected by both hadronic and QGP viscosities
- Towards back- and forward rapidities, hadronic viscosity becomes more and more dominant and so the matter at the LHC behaves like at RHIC
- v_n as function of transverse momentum, rapidity, centrality and collision energy provide a way to distinguish between different parametrizations of $(\eta_s/s)(T)$
- Decoupling at constant temperature and at constant Knudsen number lead to very similar anisotropies at midrapidity, but at lower collision energies the results may be more sensitive to the freeze-out criterion.