Energy–momentum tensor on the lattice from the gradient flow

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- H.S., Prog. Theor. Exp. Phys. (2013) 083B03 [arXiv:1304.0533 [hep-lat]]
- M. Asakawa, T. Hatsuda, E. Itou, M. Kitazawa, H.S. (FlowQCD Collaboration), Phys. Rev. D90 (2014) 011501 [arXiv:1312.7492 [hep-lat]]
- H. Makino and H.S., Prog. Theor. Exp. Phys. (2014) 063B02 [arXiv:1403.4772 [hep-lat]]
- M. Asakawa, T. Hatsuda, T. Iritani, E. Itou, M. Kitazawa, H.S. (FlowQCD Collaboration), in progress

- An attempt to define/understand the energy–momentum tensor (EMT) in QFT in the non-perturbative level.
- (Hopefully) applicable to, thermodynamics, transport coefficients, momentum/spin structure, conformal field theory, dilaton physics, . . .
- The title is just a combination of:
  - Applications of the gradient flow in lattice QCD.
  - Complex Langevin simulation of theories with complex action.
  - Signal-to-noise problem in baryon correlation functions.
  - Energy-momentum tensor on the lattice.
  - Scaling behaviour of QCD-like theories from short to long distances.
  - Conformal/dilation symmetry in continuum theories.
- Naturally, you wonder how “from” comes in.
- So, let me start by recalling why EMT on the lattice is difficult. . .

Energy–momentum tensor (EMT) on the lattice

- Lattice regularization breaks the Poincaré symmetry.
- Assuming the hypercubic symmetry, in QCD, one has to determine seven(!) Z_i's for the correct normalization and the consecration law of EMT in $a \to 0$ (Caracciolo et al. (1989)):

$$\{T_{\mu\nu}\}_R(x) = \sum_{i=1}^{7} Z_i \mathcal{O}_{i\mu\nu}(x)|_{\text{lattice}} - \text{VEV},$$

where

$$\mathcal{O}_{1\mu\nu}(x) \equiv \sum_{\rho} F^a_{\mu\rho}(x) F^a_{\nu\rho}(x), \qquad \mathcal{O}_{2\mu\nu}(x) \equiv \delta_{\mu\nu} \sum_{\rho,\sigma} F^a_{\rho\sigma}(x) F^a_{\rho\sigma}(x),$$

$$\mathcal{O}_{3\mu\nu}(x) \equiv \bar{\psi}(x) \left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu \right) \psi(x), \quad \mathcal{O}_{4\mu\nu}(x) \equiv \delta_{\mu\nu} \bar{\psi}(x) \overleftrightarrow{\not{D}} \psi(x),$$

$$\mathcal{O}_{5\mu\nu}(x) \equiv \delta_{\mu\nu} m_0 \bar{\psi}(x) \psi(x),$$

and, rotational non-covariant ones:

$$\mathcal{O}_{6\mu\nu}(x) \equiv \delta_{\mu\nu} \sum_{\rho} F^a_{\mu\rho}(x) F^a_{\mu\rho}(x), \qquad \mathcal{O}_{7\mu\nu}(x) \equiv \delta_{\mu\nu} \bar{\psi}(x) \gamma_\mu \overleftrightarrow{D}_\mu \psi(x).$$

- Michele Pepe and Leonardo Giusti: New approach to determine some Z_i's

Yang–Mills gradient flow (Lüscher, (2009∼))

- The Yang–Mills gradient flow is an evolution of the gauge field $A_\mu(x)$ wrt to a fictitious time $t \in \mathbb{R}$, according to

$$\partial_t B_\mu(t,x) = -g_0^2 \frac{\delta S_{\mathrm{YM}}}{\delta B_\mu(t,x)} = D_\nu G_{\nu\mu}(t,x) = \Delta B_\mu(t,x) + \cdots ,$$

  where the initial value is

$$B_\mu(t=0,x) = A_\mu(x).$$

  $G_{\mu\nu}(t,x)$ is the field strength at flow time t,

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)], \qquad D_\mu = \partial_\mu + [B_\mu, \cdot].$$

- This is a sort of diffusion equation with which the diffusion length in x-space is $\sim \sqrt{8t}$
- But, why this can be relevant to lattice EMT???
- The key is the UV finiteness of the gradient flow

- Yang–Mills gradient flow

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu(t,x), \qquad B_\mu(t=0,x) = A_\mu(x),$$

where the term with α_0 is introduced to suppress the gauge mode. The formal solution is

$$B_\mu(t,x) = \int d^D y \left[K_t(x-y)_{\mu\nu} A_\nu(y) + \int_0^t ds\, K_{t-s}(x-y)_{\mu\nu} R_\nu(s,y) \right],$$

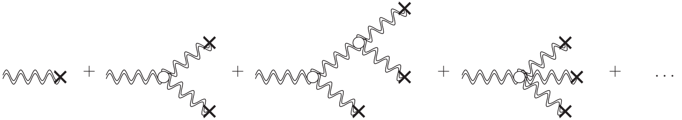
where K is the heat kernel (for $\alpha_0 = 1$)

$$K_t(x)_{\mu\nu} = \delta_{\mu\nu} \int_p e^{ipx} e^{-tp^2} = \delta_{\mu\nu} \frac{1}{(4\pi t)^{D/2}} e^{-\frac{x^2}{4t}},$$

and R denotes non-linear terms

$$R_\mu = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] + (\alpha_0 - 1)[B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]].$$

Pictorially, (double lines: K, crosses: A_μ, white circles: R),

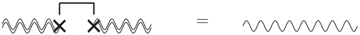


Perturbative expansion of the gradient flow

- Quantum correlation function of the flowed gauge field

$$\langle B_{\mu_1}(t_1, x_1) \cdots B_{\mu_n}(t_n, x_n) \rangle ,$$

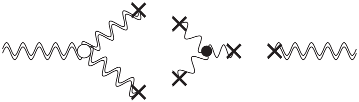
is obtained by taking the quantum expectation value of the initial value $A_\mu(x)$. For example, the contraction of two A_μ's



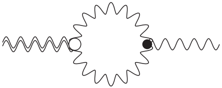
produces the free propagator of the flowed field (in the Feynman gauge)

$$\left\langle B_\mu^a(t, x) B_\nu^b(s, y) \right\rangle_0 = \delta^{ab} g_0^2 \delta_{\mu\nu} \int_p e^{ip(x-y)} \frac{e^{-(t+s)p^2}}{p^2} .$$

Similarly, for (black circle: Yang–Mills vertex)



we have the loop flow-line Feynman diagram



backup: Gauge invariance of the gradient flow

- Under the infinitesimal gauge transformation (no $B_t(t,x)$; in 4D sense),

$$B_\mu(t,x) \to B_\mu(t,x) + D_\mu\omega(t,x),$$

the flow equation

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu(t,x),$$

changes to

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu(t,x) - D_\mu(\partial_t - \alpha_0 D_\nu \partial_\nu)\omega(t,x).$$

- Choosing $\omega(t,x)$ as

$$(\partial_t - \alpha_0 D_\nu \partial_\nu)\omega(t,x) = -\delta\alpha_0 \partial_\nu B_\nu(t,x), \qquad \omega(t=0,x)=0,$$

α_0 can be changed accordingly

$$\alpha_0 \to \alpha_0 + \delta\alpha_0.$$

That is, $B_\mu(t,x)$'s corresponding to different α_0's are related by a gauge transformation

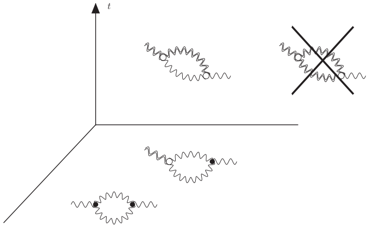
- Gauge invariant quantity (in 4D sense) is independent of α_0

UV finiteness of the gradient flow I (Lüscher–Weisz (2011))

- Correlation function of the flowed gauge field

$$\langle B_{\mu_1}(t_1, x_1) \cdots B_{\mu_n}(t_n, x_n) \rangle , \qquad t_1 > 0, \ldots, t_n > 0,$$

  when expressed in terms of renormalized parameters, is UV finite without the wave function multiplicative renormalization
- All order proof using a local $D + 1$-dimensional field theory



- No bulk ($t > 0$) counterterm: because of the Gaussian damping factor $\sim e^{-tp^2}$ in the propagator.
- No boundary ($t = 0$) counterterm besides Yang–Mills ones: because of a BRS symmetry

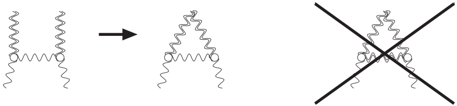
UV finiteness of the gradient flow II

- Correlation function of the flow gauge field

$$\langle B_{\mu_1}(t_1, x_1) B_{\mu_2}(t_2, x_2) \cdots B_{\mu_n}(t_n, x_n) \rangle , \qquad t_1 > 0, \ldots, t_n > 0,$$

remains finite even for the equal-point product

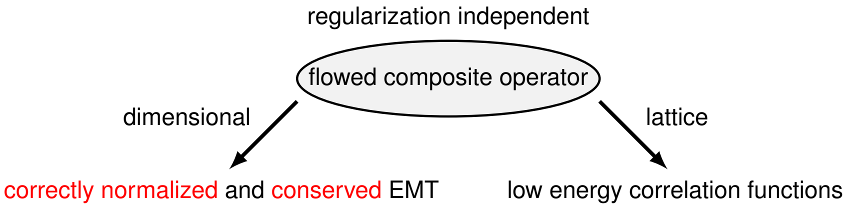
$$t_1 \longrightarrow t_2, \qquad x_1 \longrightarrow x_2,$$



- The new loop always contains the Gaussian damping factor $\sim e^{-tp^2}$ which makes integral finite; no new UV divergences arise
- Composite operators of the flowed gauge field $B_\mu(t, x)$ are renormalized UV finite quantities, although the flowed field is a certain combination of the bare gauge field
- Such UV finite quantities must be independent of the regularization

Our basic idea

- We try to bridge dimensional regularization which preserves the translational invariance and non-perturbative lattice regularization, by using UV finite flowed composite operators.
- Schematically,



EMT in dimensional regularization

- EMT in dimensional regularization is simple and explicit, because it preserves the translational invariance:

$$\{T_{\mu\nu}\}_R(x) = \frac{1}{g_0^2}\left\{\mathcal{O}_{1\mu\nu}(x) - \frac{1}{4}\mathcal{O}_{2\mu\nu}(x)\right\} + \frac{1}{4}\mathcal{O}_{3\mu\nu}(x) - \frac{1}{2}\mathcal{O}_{4\mu\nu}(x) - \mathcal{O}_{5\mu\nu}(x) - \text{VEV},$$

where

$$\mathcal{O}_{1\mu\nu}(x) \equiv \sum_\rho F^a_{\mu\rho}(x)F^a_{\nu\rho}(x), \qquad \mathcal{O}_{2\mu\nu}(x) \equiv \delta_{\mu\nu}\sum_{\rho,\sigma} F^a_{\rho\sigma}(x)F^a_{\rho\sigma}(x),$$

$$\mathcal{O}_{3\mu\nu}(x) \equiv \bar{\psi}(x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\psi(x), \qquad \mathcal{O}_{4\mu\nu}(x) \equiv \delta_{\mu\nu}\bar{\psi}(x)\overleftrightarrow{\not{D}}\psi(x),$$

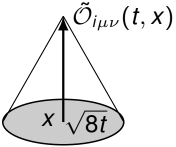
$$\mathcal{O}_{5\mu\nu}(x) \equiv \delta_{\mu\nu} m_0 \bar{\psi}(x)\psi(x),$$

- This is correctly normalized and conserved (for $\mathcal{O}_{\text{ext}} \notin R$, $\mathcal{O}_{\text{int}} \in R$):

$$\left\langle \mathcal{O}_{\text{ext}} \int_R \mathrm{d}^D x\, \partial_\mu \{T_{\mu\nu}\}_R(x)\, \mathcal{O}_{\text{int}} \right\rangle = -\left\langle \mathcal{O}_{\text{ext}}\, \partial_\nu \mathcal{O}_{\text{int}} \right\rangle.$$

Small flow-time expansion

- But how can we relate a flowed composite operator in $t > 0$ (heaven) and local composite operators in 4D (down to the earth)?
- Small flow-time expansion (Lüscher–Weisz (2011))



$$\tilde{\mathcal{O}}_{i\mu\nu}(t,x) = \left\langle \tilde{\mathcal{O}}_{i\mu\nu}(t,x) \right\rangle + \zeta_{ij}(t) \left[\mathcal{O}_{j\mu\nu}(x) - \langle \mathcal{O}_{j\mu\nu}(x) \rangle \right] + O(t).$$

Inverting this relation,

$$\mathcal{O}_{i\mu\nu}(x) - \langle \mathcal{O}_{j\mu\nu}(x) \rangle = \lim_{t\to 0} \left\{ \left(\zeta^{-1}\right)_{ij}(t) \left[\tilde{\mathcal{O}}_{j\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}_{j\mu\nu}(t,x) \right\rangle \right] \right\}.$$

- If we know the $t \to 0$ behavior of the coefficients $\zeta_{ij}(t)$, the LHS can be extracted.
- Antonio Rago with Agostino Patella and Luigi Del Debbio: Non-perturbative method to determine $\zeta_{ij}(t)$ in the context of EMT.

Renormalization group argument and the perturbation theory

- We are interested in the $t \to 0$ behavior of the coefficients $\zeta_{ij}(t)$ in

$$\tilde{\mathcal{O}}_{i\mu\nu}(t,x) = \left\langle \tilde{\mathcal{O}}_{i\mu\nu}(t,x) \right\rangle + \zeta_{ij}(t) \left[\mathcal{O}_{j\mu\nu}(x) - \langle \mathcal{O}_{j\mu\nu}(x) \rangle\right] + O(t).$$

  Lüscher–Wise (2011) moreover says: The associated coefficients … satisfy a renormalization group equation that allows their exact asymptotic behavior at small t to be worked out analytically.

- When $\tilde{\mathcal{O}}_{j\mu\nu}(t,x)$ are indep. of renormalized parameters,

$$\left(\mu \frac{\partial}{\partial \mu}\right)_0 \zeta_{ij}(t) = 0,$$

  and $\zeta_{ij}(t)$ are indep. of the renormalization scale q, when expressed in terms of running parameters. We may take, for example, $q = 1/\sqrt{8t}$, and

$$\zeta_{ij}(t)\,[g, m; \mu] = \zeta_{ij}(t) \left[\bar{g}(1/\sqrt{8t}), \bar{m}(1/\sqrt{8t}); 1/\sqrt{8t}\right].$$

- For $t \to 0$, $\bar{g}(1/\sqrt{8t}) \to 0$ for the asymptotic freedom and perturbation theory is justified

Flow of fermion fields

- A possible choice (Lüscher (2013))

$$\partial_t \chi(t,x) = [\Delta - \alpha_0 \partial_\mu B_\mu(t,x)]\,\chi(t,x), \qquad \chi(t=0,x) = \psi(x),$$
$$\partial_t \bar{\chi}(t,x) = \bar{\chi}(t,x)\left[\overleftarrow{\Delta} + \alpha_0 \partial_\mu B_\mu(t,x)\right], \qquad \bar{\chi}(t=0,x) = \bar{\psi}(x),$$

where

$$\Delta = D_\mu D_\mu, \qquad D_\mu = \partial_\mu + B_\mu,$$
$$\overleftarrow{\Delta} = \overleftarrow{D}_\mu \overleftarrow{D}_\mu, \qquad \overleftarrow{D}_\mu \equiv \overleftarrow{\partial}_\mu - B_\mu.$$

- Unfortunately, the flowed fermion field requires the wave function renormalization:

$$\chi_R(t,x) = Z_\chi^{1/2}\chi(t,x), \qquad \bar{\chi}_R(t,x) = Z_\chi^{1/2}\bar{\chi}(t,x),$$
$$Z_\chi = 1 + \frac{g^2}{(4\pi)^2} C_2(R) 3 \frac{1}{\epsilon} + O(g^4),$$

although composite operators of $\chi_R(t,x)$ are UV finite

Ringed fermion fields

- To avoid the complication associated with Z_χ, we introduce

$$\mathring{\chi}(t,x) = \mathcal{C}\frac{\chi(t,x)}{\sqrt{t^2\left\langle\bar{\chi}(t,x)\overleftrightarrow{\not{D}}\chi(t,x)\right\rangle}} = \mathcal{C}\frac{\chi_R(t,x)}{\sqrt{t^2\left\langle\bar{\chi}_R(t,x)\overleftrightarrow{\not{D}}\chi_R(t,x)\right\rangle}} = \chi_R(t,x)+O(g^2),$$

where

$$\mathcal{C} \equiv \sqrt{\frac{-2\dim(R)N_{\mathrm{f}}}{(4\pi)^2}},$$

and similarly for $\bar{\chi}(t,x)$.

- Since Z_χ is cancelled out in $\mathring{\chi}(t,x)$, composite operators of $\mathring{\chi}(t,x)$ and $\mathring{\bar{\chi}}(t,x)$ are UV finite

EMT from the gradient flow

- Small flow-time expansion:

$$\tilde{\mathcal{O}}_{i\mu\nu}(t,x) = \left\langle \tilde{\mathcal{O}}_{i\mu\nu}(t,x) \right\rangle + \zeta_{ij}(t)\left[\mathcal{O}_{j\mu\nu}(x) - \langle \mathcal{O}_{j\mu\nu}(x)\rangle\right] + O(t).$$

  We consider following composite operators of flowed fields:

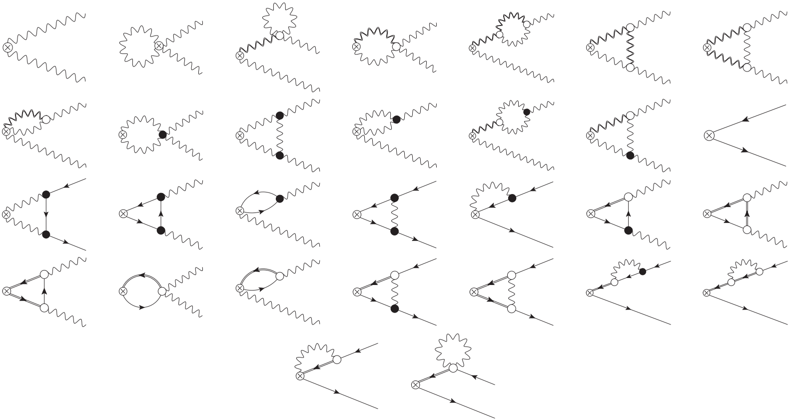
$$\begin{aligned}
\tilde{\mathcal{O}}_{1\mu\nu}(t,x) &\equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x),\\
\tilde{\mathcal{O}}_{2\mu\nu}(t,x) &\equiv \delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x),\\
\tilde{\mathcal{O}}_{3\mu\nu}(t,x) &\equiv \mathring{\bar{\chi}}(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\mathring{\chi}(t,x),\\
\tilde{\mathcal{O}}_{4\mu\nu}(t,x) &\equiv \delta_{\mu\nu}\mathring{\bar{\chi}}(t,x)\overleftrightarrow{\not{D}}\mathring{\chi}(t,x),\\
\tilde{\mathcal{O}}_{5\mu\nu}(t,x) &\equiv \delta_{\mu\nu}m\mathring{\bar{\chi}}(t,x)\mathring{\chi}(t,x).
\end{aligned}$$

- We compute $\zeta_{ij}(t)$ to the one-loop order and substitute

$$\mathcal{O}_{i\mu\nu}(x) - \langle \mathcal{O}_{i\mu\nu}(x)\rangle = \lim_{t\to 0}\left\{\left(\zeta^{-1}\right)_{ij}(t)\left[\tilde{\mathcal{O}}_{j\mu\nu}(t,x) - \left\langle \tilde{\mathcal{O}}_{j\mu\nu}(t,x)\right\rangle\right]\right\},$$

  in the expression of EMT in dimensional regularization

- To the one-loop order, we have to evaluate following flow-line Feynman diagrams:



Master formula

- Gathering all the above elements, we have

$$\{T_{\mu\nu}\}_R(x) = \lim_{t\to 0}\Big\{c_1(t)G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x) + \left[c_2(t) - \frac{1}{4}c_1(t)\right]\delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x)$$
$$+ c_3(t)\mathring{\bar{\chi}}(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\mathring{\chi}(t,x)$$
$$+ \left[c_4(t) - 2c_3(t)\right]\delta_{\mu\nu}\mathring{\bar{\chi}}(t,x)\overleftrightarrow{\not{D}}\mathring{\chi}(t,x) + c_5'(t)\mathring{\bar{\chi}}(t,x)\mathring{\chi}(t,x) - \mathrm{VEV}\Big\},$$

where (for the MS scheme; $\ln\pi \to \gamma_E - 2\ln 2$ for $\overline{\mathrm{MS}}$)

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - b_0\ln\pi - \frac{7}{8}\frac{1}{(4\pi)^2}\left[\frac{11}{3}C_2(G) - \frac{12}{7}T(R)N_{\mathrm{f}}\right],$$
$$c_2(t) = \frac{1}{8}\frac{1}{(4\pi)^2}\left[\frac{11}{3}C_2(G) + \frac{11}{3}T(R)N_{\mathrm{f}}\right],$$
$$c_3(t) = \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(R)\left[\frac{3}{2} + \ln(432)\right]\right\},$$
$$c_4(t) = \frac{1}{8}d_0\bar{g}(1/\sqrt{8t})^2,$$
$$c_5'(t) = -\bar{m}(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}C_2(R)\left[3\ln\pi + \frac{7}{2} + \ln(432)\right]\right\},$$

Master formula

- and

$$b_0 = \frac{1}{(4\pi)^2}\left[\frac{11}{3}C_2(G) - \frac{4}{3}T(R)N_\mathrm{f}\right], \qquad d_0 = \frac{1}{(4\pi)^2}6C_2(R).$$

- Correlation functions of the RHS of the master formula can be computed non-perturbatively by using lattice regularization
- The coefficients $c_i(t)$ are universal, i.e., indep. of the lattice transcription
- Ideally, one should first take the continuum limit $a \to 0$ to restore the "universality" and then take $t \to 0$
- Practically, we cannot simply take $a \to 0$ and may take t as small as possible in the window,

$$a \ll \sqrt{8t} \ll \frac{1}{\Lambda}.$$

Thus the usefulness with presently-accessible lattice parameters is not obvious a priori. . .

Application to the thermodynamics of $SU(3)$ pure Yang–Mills theory

- Asakawa–Hatsuda–Itou–Kitazawa–H.S. (FlowQCD Collaboration)
- Thermal average of diagonal elements of EMT: the trace part,

$$\langle \varepsilon - 3p \rangle_T = -\left\langle \{T_{\mu\mu}\}_R(x) \right\rangle_T ,$$

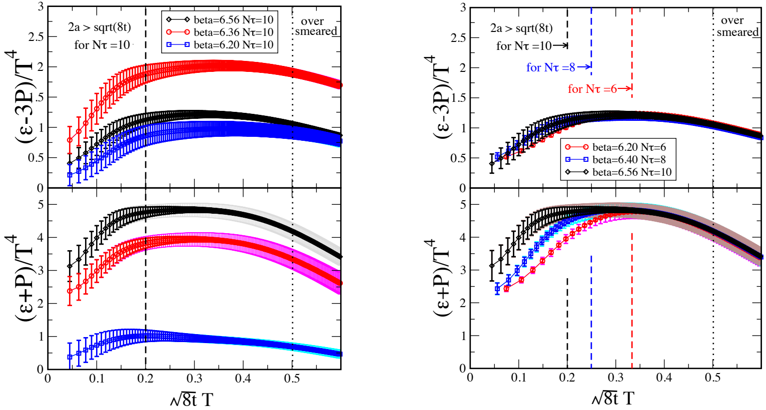
  and the traceless part (the entropy density),

$$\langle \varepsilon + p \rangle_T = -\left\langle \{T_{00}\}_R(x) \right\rangle_T + \frac{1}{3} \sum_{i=1,2,3} \left\langle \{T_{ii}\}_R(x) \right\rangle_T .$$

- Thermodynamical quantities are obtained by the expectation value of EMT just at that temperature T (no integration wrt the temperature)
- We do not need to compute renormalization factors Z_i
- Experiment setting
  - Wilson plaquette action
  - $N_s^3 \times N_\tau = 32^3 \times (6, 8, 10, 32)$, $\beta = 5.89$–6.56, ~ 300 configurations
  - Wilson flow: 4th order Runge–Kutta with $\epsilon/a^2 = 0.025$
  - Scale setting: $\beta \leftrightarrow a\Lambda_{\overline{\text{MS}}}$ from ALPHA Collaboration, aT_c at $\beta = 6.20$ from Boyd et al.
  - 4-loop running coupling in the $\overline{\text{MS}}$ scheme
  - Clover field strength $G_{\mu\nu}^a(x)$

Application to the thermodynamics of $SU(3)$ pure Yang–Mills theory

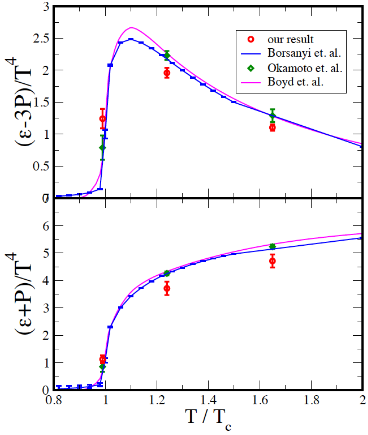
- Thermal expectation values versus the flow time $\sqrt{8t}$



- We observe stable behavior for $2a < \sqrt{8t} < 1/(2T))$ which indicates (!!!) the $t \to 0$ limit

Application to the thermodynamics of $SU(3)$ pure Yang–Mills theory

- Continuum limit (from values at $\sqrt{8t}T = 0.40$)



Boyd et. al.
NPB469,419 (1996)

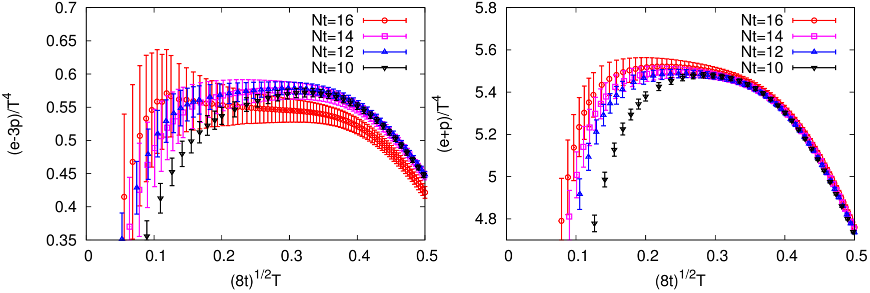
Okamoto et. al. (CP-PACS)
PRD60, 094510 (1999)

Borsanyi et. al.
JHEP 1207, 056 (2012)

- That our simple method produces results being consistent with past comprehensive studies (within 2σ) indicates that our reasoning is correct. This finding encouraged us very much!

New results (preliminary)

- Asakawa–Hatsuda–Iritani–Itou–Kitazawa–H.S. (FlowQCD Collaboration)
- Twice finer lattice!: $N_s^3 \times N_\tau = 64^3 \times (10, 12, 14, 16, 64)$, $\beta = 6.4$–7.4, ~ 2000 (~ 500 for vacuum with $\beta = 7.4$) configurations



- Wider stable regions and much less errors.
- More convincing results are expected!

Summary and prospects

- We developed a formula that relates a correctly-normalized conserved EMT and composite operators defined through the gradient flow:

$$\{T_{\mu\nu}\}_R(x) = \lim_{t\to 0}\Big\{c_1(t)G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x) + \left[c_2(t) - \frac{1}{4}c_1(t)\right]\delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x)$$
$$+ c_3(t)\mathring{\bar{\chi}}(t,x)\left(\gamma_\mu\overleftrightarrow{D}_\nu + \gamma_\nu\overleftrightarrow{D}_\mu\right)\mathring{\chi}(t,x)$$
$$+ [c_4(t) - 2c_3(t)]\,\delta_{\mu\nu}\mathring{\bar{\chi}}(t,x)\overleftrightarrow{\not{D}}\mathring{\chi}(t,x) + c_5'(t)\mathring{\bar{\chi}}(t,x)\mathring{\chi}(t,x) - \text{VEV}\Big\}.$$

- Correlation functions of RHS can be computed by lattice Monte Carlo simulations
- Possible obstacle would be

$$a \ll \sqrt{8t}$$

- The measurement of one-point functions in the finite temperature shows encouraging results; the method appears to be promising even practically
- The conservation law of EMT is still needed to be demonstrated by Monte Carlo simulations

Summary and prospects

- Further physical applications: EoS of QCD, viscosities, momentum/spin structure of nucleons, critical exponents in low-energy conformal field theory, dilaton physics, . . .
- By using a similar idea as above, can we construct other Noether currents such as the chiral current or the SUSY current on the lattice??? Usefulness???
- Further (even theoretical) application of the gradient flow, on the basis of its UV finiteness?