Renormalization of the energy-momentum tensor with the Wilson flow

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Introduction

The lattice is a powerful tool to investigate non perturbative properties of gauge theories.

- However it explicitly breaks the Poincaré group at finite lattice spacing.
- A restoration of the related invariances can be recovered only in the continuum limit
- Space-time symmetries are broken and the associated ward identities are violated: special care is needed to construct a renormalized energy-moment tensor.

Small flow time expansion $\sqrt{8t} \ll L$

Consider an operator $\phi(t,x)$ at positive flowtime with small flowtime expansion of the type:

$$\phi(t,x) = \langle \phi(t,x) \rangle + c(t)\phi^{RGI}(x) + \mathcal{O}(t)$$

- $\langle \phi(t,x) \rangle$ can be set to be equal to zero in infinite volume limit.
- $\phi^{RGI}(x)$ is a Renormalization Group Invariant operator of dimension 4.
- $c(t)$ is order $\mathcal{O}(t^0)$
- $\mathcal{O}(t)$ contains operator of at least dimension 6

If $c(t)$ is known and $\mathcal{O}(t)$ is small we can extract $\phi^{RGI}(x)$

How to evaluate $c(t)$?

- Perturbative approach (Suzuki's talk)
- Non-Perturbative approach

Evaluate $c(t)$ from two point functions

The key observation is that the RGI part of ϕ^{RGI} will not depend on t

$$\phi(t,x) = \langle \phi(t,x) \rangle + c(t)\phi^{RGI}(x) + \mathcal{O}(t)$$

- If we evaluate the connected correlator with a probe ψ at time $s > t$

$$c(t)\langle \phi^{RGI}(x)\psi(s,y)\rangle_{c,L} + \mathcal{O}(t) = \langle \phi(t,x)\psi(s,y)\rangle_{c,L}$$

- It is them possible to define a proper logarithmic derivative that doesn't depend on $\phi^{RGI}(x)$

$$\gamma_{eff}(t,s,L) \equiv -2t\frac{d}{dt}\log\langle \phi(t,x)\psi(s,y)\rangle_{c,L} = -2t\frac{d}{dt}\log c(t) + \mathcal{O}_{s,L}(t)$$

- Look for a region of parameters (s and L) in which

$$\gamma_{eff}(t,s,L) = \gamma(t)$$

Energy-momentum tensor

- Consider the spin-2 component of the EMT

$$Y_{\mu\nu}(t,x) = G^a_{\mu\rho} G^a_{\nu\rho}(t,x) - \frac{\delta_{\mu\nu}}{4} G^a_{\rho\sigma} G^a_{\rho\sigma}(t,x) \sim c_Y(t) \left(T_{\mu\nu}(x) - \frac{\delta_{\mu\nu}}{4} T_{\rho\rho} \right) + \mathcal{O}(t)$$

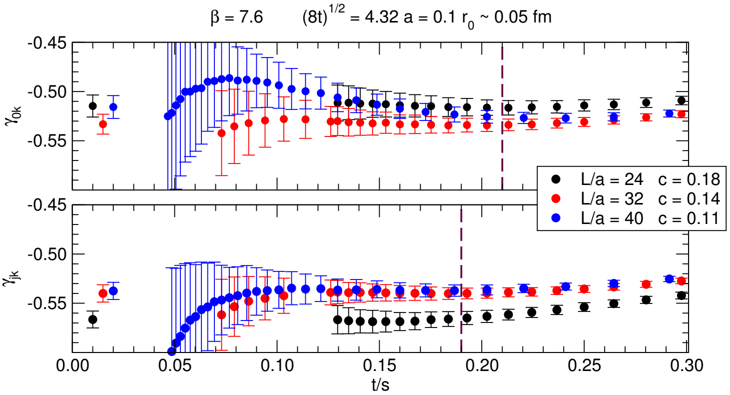
- We evaluate the two points functions

$$\begin{aligned} \gamma_{0k}(t,s,L) &= -2t \frac{d}{dt} \log \sum_k \langle Y_{0k}(t,0) Y_{0k}(s,0) \rangle_L \\ \gamma_{jk}(t,s,L) &= -2t \frac{d}{dt} \log \sum_{jk} \langle \tilde{Y}_{jk}(t,0) \tilde{Y}_{jk}(s,0) \rangle_L \end{aligned}$$

where $\tilde{Y}_{jk}$ is the traceless part of Y_{jk}

$\gamma(t)$ fot the EMT from the spin-2

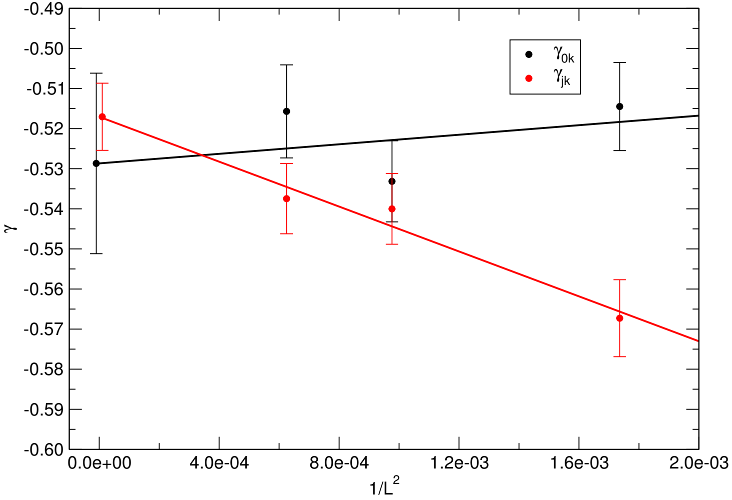
The dependence on the probe:



Open-SF boundary conditions
3000 measures @ L=24; 4228 measures @ L=32 ; 1000 measures @ L=40 Wilson action + tree-level improvement boundary terms
Tree-level improved observable

$\gamma(t)$ fot the EMT from the spin-2

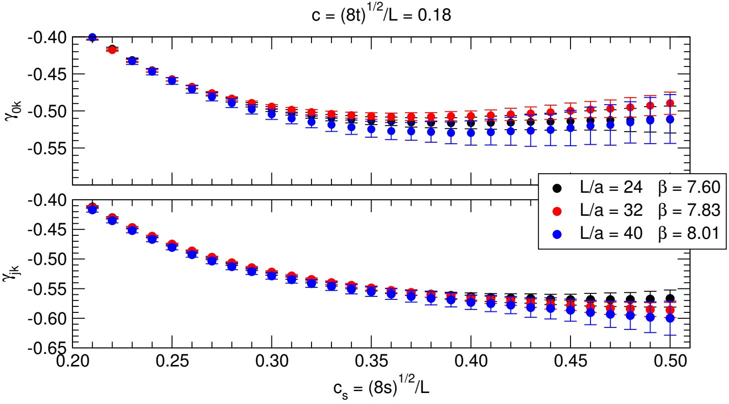
Infinite volume extrapolation



with a final estimate of $\gamma = 0.53(2)$

$\gamma(t)$ fot the EMT from the spin-2

Approach to the continuum limit



Evaluate $c(t)$ from one point functions

Derivative with respect to the linear size of the box

- In Hamiltonian formalism:

$$\frac{d}{dL}\langle\phi(t,0)\rangle_L = -\frac{1}{2}\sum_\mu \int (\prod_{\nu\neq\mu} dx_\nu)\,\langle\{T_{\mu\mu}(x)|_{x_\mu=L/2} + T_{\mu\mu}(x)|_{x_\mu=-L/2}\}\phi(t,0)\rangle_{L,c}\ .$$

- In the $\sqrt{8t} \ll L$ regime, one can use the small flow time expansion in the 2-point functions above:

$$\frac{d}{dL}\langle\phi(t,0)\rangle_L = \\ -\frac{1}{2}c(t)\sum_\mu \int (\prod_{\nu\neq\mu} dx_\nu)\,\langle\{T_{\mu\mu}(x)|_{x_\mu=L/2} + T_{\mu\mu}(x)|_{x_\mu=-L/2}\}\phi(0)\rangle_{L,c} + \mathcal{O}(t)$$

- By integrating this:

$$\langle\phi(t,0)\rangle_{L_2} - \langle\phi(t,0)\rangle_{L_1} = -\frac{1}{2}c(t)\int_{L_1}^{L_2} d\ell \sum_\mu \int (\prod_{\nu\neq\mu} dx_\nu)\,\langle\cdots\rangle_{\ell,c} + \mathcal{O}(t)$$

Evaluate $c(t)$ from one point functions

- Ratios of the smallflow time coefficient can be easily extracted from finite volume effects

$$\frac{\langle\phi(t,0)\rangle_{L_2} - \langle\phi(t,0)\rangle_{L_1}}{\langle\phi(t_0,0)\rangle_{L_2} - \langle\phi(t_0,0)\rangle_{L_1}} = \frac{c(t)}{c(t_0)} + \mathcal{O}(t)$$

- And for anomalous dimension:

$$\gamma_{eff}(t, L_1, L_2) = -2t\frac{d}{dt}\log|\langle\phi(t,0)\rangle_{L_2} - \langle\phi(t,0)\rangle_{L_1}| = -2t\frac{d}{dt}\log|c(t)| + O(t) \ .$$

- Look for a region of parameters (L_1 and L_2) in which

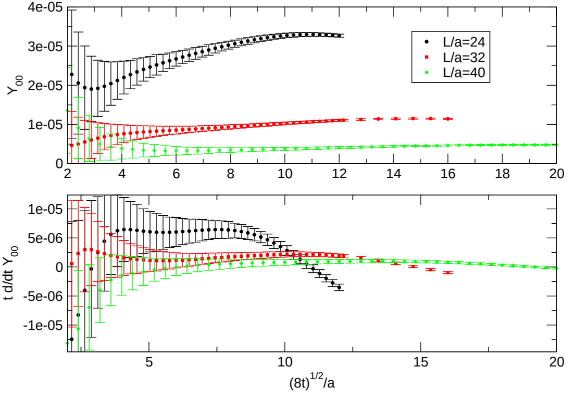
$$\gamma_{eff}(t, L_1, L_2) = \gamma(t)$$

- We measure on the lattice point function (for spin-2 component the infinite-volume limit vanishes)

$$\gamma_{00}(t, L) = -2t\frac{d}{dt}\log\langle Y_{00}(t,0)\rangle_L$$

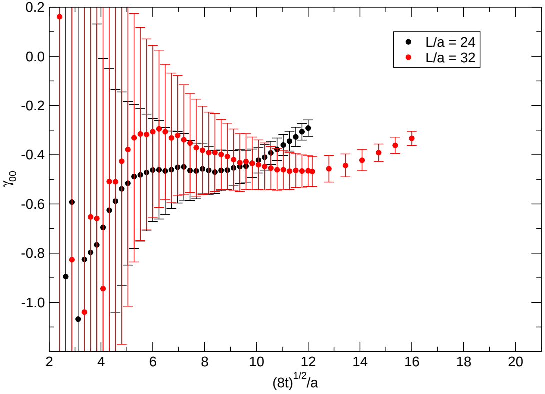
$\gamma(t)$ fot the EMT from the spin-2

for the case of the one point function $\gamma_{00}(t, L) = -2t\frac{d}{dt}\log\langle Y_{00}(t, 0)\rangle_L$



$\gamma(t)$ fot the EMT from the spin-2

for the case of the one point function $\gamma_{00}(t, L) = -2t \frac{d}{dt} \log\langle Y_{00}(t, 0)\rangle_L$



A comparison with the perturbative approach

We can compare our findings for γ and g_W^2 with the prediction of

- From the perturbative expansion for

$$c = g^2_{\overline{MS}}(1 + 2b_0 s_1 g^2_{\overline{MS}}) \text{ and } g_W^2 = g^2_{\overline{MS}}(1 + c_1 g^2_{\overline{MS}})$$

  we can evaluate the value of γ in perturbation theory as function of g_W^2

$\gamma = \frac{\mu}{c}\frac{\partial}{\partial\mu}c$	Non-perturbative	tree-level	one-loop
	-0.53(2)	-0.32482(14)	-0.2621(1)

- Performing the scale setting $t = \frac{(0.0602(48))^2}{8\Lambda^2_{\overline{MS}}}$ we can compare g_W^2

Non-perturbative	tree-level	one-loop
2.3315(10)	2.55(7)	2.21(6)

Caveat: both our estimates are at fixed lattice spacing

Thanks to Hiroshi Suzuki and Etsuko Itou

Conclusions and perspectives

- Renormalization-group invariant operators can be represented in terms of positive flowtime operators via the small flowtime expansion.
- We have developed a possible nonperturbative strategy to extract the Wilson coefficient of the small flowtime expansion.
- This procedure can be embedded in a modified step scaling (one or two scale problem), which we will study in detail.
- Investigations with the spin-2 part of the EMT suggest that the procedure is expensive but viable.
- We will use this strategy for the trace of the EMT as well.

Appendix: A different approach

- The energy-moment tensor can be obtained as a linear combination of all the operators with dimension not greater than four allowed by the symmetries.
  This program was articulated in great detail by Caracciolo Curci Pelissetto Menotti in 1988 1990

$$T^R_{\mu\rho} = \sum_i c_i \left\{ \hat{T}^{(i)}_{\mu\rho} - \langle \hat{T}^{(i)}_{\mu\rho} \rangle \right\} \ ,$$

- How to evaluate the c_i?

$$\delta_\alpha A_\mu(x) = \alpha_\rho(x) F_{\rho\mu}(x)$$

- Put the probe at positive flow time

$$\langle \delta_{x,\rho} P_T \rangle = -\langle P_T \partial_\mu T_{\mu\rho}(x) \rangle$$

- RHS doesn't renormalize, LHS renomalizes only multiplicatively

$$Z_\delta \langle \delta_{x,\rho} P_T \rangle = -\langle P_T \partial_\mu T^R_{\mu\rho}(x) \rangle$$

- How to fix the Z_δ?

$$Z_\delta \int_V d^D y \ \ \delta_{y,\rho} P_T(x) = \partial_\rho P_T(x) + O\left(e^{-\frac{r^2}{4(T)}} \right)$$