Gluon TMDs and Quarkonium Production in Unpolarised and Polarised Proton-Proton Collisions

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Collaboration with W. den Dunnen, C. Lorcé, C. Pisano, M. Schlegel, H.S. Shao
Part I

Generalities on gluon TMDs
Gluon distributions

Experimental and theoretical investigations of gluons inside hadrons focused so far on their longitudinal momentum and helicity distributions:

\[ g(x, \mu_F) \]: unpolarised gluons with a collinear momentum fraction \( x \) in unpolarised nucleons

\[ \Delta g(x, \mu_F) \]: circularly polarised gluons with a collinear momentum fraction \( x \) in polarised nucleons

Gluon Transverse Momentum Dependent pdfs (TMDs) can be nonzero. Example: for nonzero \( k_T \), the gluons can be polarised even if the nucleons are unpolarised (\( h_\perp g_1 \) vs. \( \Delta g \)).

Nontrivial property that received much more attention in the quark sector:

\[ \rightarrow \] Boer-Mulders effect

Once \( h_\perp g_1 \) is known, polarised processes in high-energy hadron-hadron collisions (dominated by \( gg \) fusion) become accessible even with unpolarised hadron beams!

Prime example: the LHC!
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Beyond collinear factorisation

- Observed final-state $q_T$ from "intrinsic" $k_T$ from initial partons
- Novel kind of factorisation w.r.t. the collinear one
- Additional degree of freedom of the partonic motion

TMD factorisation from gluon-gluon process:

$$q_T \ll Q_H$$ is free of $q_T$

$$d\sigma = \left(\frac{2\pi}{\sqrt{s}}\right)^4 \int d^2k_1^T d^2k_2^T \delta^2(k_1^T + k_2^T - q_T) H_{\mu\rho}(H_{\nu\sigma})^* \times \Phi_{\mu\nu} g(x_1, k_1^T, \zeta_1, \mu) \Phi_{\rho\sigma} g(x_2, k_2^T, \zeta_2, \mu) dR + O(q_T^2 Q_H^2)$$

Proven for SIDIS + pp reactions with colour singlet final states

Collins; Ji, Ma, Qiu; Rogers, Mulders, ...

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Beyond collinear factorisation

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\[ \Phi_g^{\mu\nu}(x_1, k_1T, \zeta_1, \mu) \Phi_g^{\rho\sigma}(x_2, k_2T, \zeta_2, \mu) d\mathcal{R} + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right) \]
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- Proven for SIDIS + $pp$ reactions with colour singlet final states
Gluon TMDs in unpolarised protons

Gauge-invariant definition:

\[ \Phi_{\mu\nu}^g(x, k_T, \zeta, \mu) \equiv \int d(\xi \cdot P) \frac{d^2 \xi}{(2\pi)^3} T(xP \cdot n) \frac{1}{2(xP + k_T) \cdot \xi} \langle \mathbf{P} | F_{n\nu}^{\mu} (0) | \mathbf{U}_{n\{0, \xi\}} \rangle | \xi \cdot P = 0 \]

The gauge link \( \mathbf{U}_{n\{0, \xi\}} \) renders the matrix element gauge invariant and runs from 0 to \( \xi \) via \( -\infty \) along the \( n \) direction.

Parametrization:

\[ \Phi_{\mu\nu}^g(x, k_T, \zeta, \mu) = -\frac{1}{2} x \{ g_{\mu\nu} T f_{g1}(x, k_T, \mu) - (k_\mu T k_\nu T - g_{\mu\nu} T k_T^2) h_{\perp g1}(x, k_T, \mu) \} + \text{suppr.} \]

\( f_{g1} \): TMD distribution of unpolarised gluons

\( h_{\perp g1} \): TMD distribution of linearly polarised gluons

[Helicity-flip distribution]
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- **Parametrisation:**

\[
\Phi_{g}^{\mu\nu}(x, k_T, \zeta, \mu) = -\frac{1}{2x} \left\{ g_T^{\mu\nu} h_{1}^{\perp g}(x, k_T, \mu) - \left( \frac{k_T^{\mu} k_T^{\nu}}{M_P^2} + g_T^{\mu\nu} \frac{k_T^{2}}{2M_P^2} \right) h_{1}^{\perp g}(x, k_T, \mu) \right\} + \text{suppr.}
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\( f_1^{g} \): TMD distribution of unpolarised gluons

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[Helicity-flip distribution]

Glue-Interactions in an arbitrary process (colorless final state)

dσgg(qT ≪ Q) ∝

( ∑
λa,λb
Hλaλb H∗λaλb ) C[f1g f1g]

→ F1  →  helicity non-flip, azimuthally independent, survives qT-integration

+ ( ∑
λ
Hλ,λ H∗−λ,−λ ) C[w2 h1⊥ h1⊥]

→ F2  →  double helicity flip, azimuthally independent

+ ( ∑
λa,λb
Hλa,λb H∗−λa,λb ) C[w3 f1g h1⊥g] + {a ↔ b}

→ F3  →  single helicity flip, cos(2φ) [sin(2φ)]-modulation

+ ( ∑
λ
Hλ,−λ H∗−λ,λ ) C[w4 h1⊥g h1⊥g]

→ F4  →  double helicity flip, cos(4φ) [sin(4φ)]-modulation

illustrative: helicity space (helicity amplitudes) → fully diff. cross section: 4 structures
Visualisation of $h_1 \perp g$


The ellipsoid axis lengths are proportional to the probability of finding a gluon with a linear polarization in that direction.

A single constraint: a positivity bound

$|h_1 \perp g| \leq \frac{2 M^2}{\vec{p} T f g_1}$

This bound is saturated by a number of models.
Visualisation of $h_1 \perp g$

- Gaussian form for $h_1 \perp g$ [left: $h_1 \perp g > 0$; right: $h_1 \perp g < 0$]
Visualisation of $h_1^\perp g$

- Gaussian form for $h_1^\perp g$ [left: $h_1^\perp g > 0$; right: $h_1^\perp g < 0$]

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- A single constraint: a positivity bound $|h_1 \perp g| \leq 2M_p^2 / \vec{p}_T^2 f_1^g$
Visualisation of $h_1^\perp g$

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Part II

Ideas to extract gluon TMDs at colliders
Beside being the QCD background for $H_0$ studies in the $\gamma\gamma$ channel, $pp \rightarrow \gamma\gamma X$ is an interesting process to study gluon TMDs. Only colour-singlet particles in the final state (also true for $ZZ$ and $\gamma Z$). But contaminations from the $q\bar{q}$ channel (particularly at RHIC).

Quark TMDs gluon TMDs at $O(\alpha_s^2)$. Only $F_4$ (i.e., the cos($4\phi$) modulation) is purely gluonic. Huge background from $\pi^0 \rightarrow \gamma\gamma$ isolation cuts are needed.

J.W Qiu, M. Schlegel, W. Vogelsang, PRL 107, 062001 (2011)
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Low $P_T$ quarkonia and TMDs
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Polarized gluon studies with charmonium and bottomonium at LHCb and AFTER

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PHYSICAL REVIEW D 86, 094007 (2012)

$\eta_c$ production at one-loop


$\chi_c$ production at one-loop

J.P. Ma, J.X. Wang, S. Zhao, PLB737 (2014) 103-108

Cannot tune $Q$: $Q \simeq m_Q$

Experimentally very difficult

First $\eta_c$ production study at collider ever, only released last month for $P_{\eta_c} > 6$ GeV

LHCb, 1409.3612
Low $P_T$ quarkonia and TMDs

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- **Low $P_T$ C-even quarkonium production is a good probe of $h^\perp_1g$**
- **Affect the low $P_T$ spectra:**
  \[
  \frac{1}{\sigma} \frac{d\sigma(\eta_Q)}{dq_T^2} \propto 1 - R(q_T^2) \quad \text{and} \quad \frac{1}{\sigma} \frac{d\sigma(\chi_{Q,0})}{dq_T^2} \propto 1 + R(q_T^2)
  \]
  
  ($R$ involves $f^g_1(x, k_T, \mu)$ and $h^\perp_1g(x, k_T, \mu)$)

\[
\sigma^{-1} \frac{d\sigma}{dq_T^2} \text{ (GeV}^{-2})
\]

\[
\langle p_T^2 \rangle = 1 \text{ GeV}^2
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- **$\eta_c$ production at one-loop** J.P. Ma, J.X. Wang, S. Zhao, PRD88 (2013) 1, 014027.
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- $\chi_{c0,2}$ factorisation issue? ↔ CO-CS mixing J.P. Ma, J.X. Wang, S. Zhao, PLB737 (2014) 103-108
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- $\eta_c$ production at one-loop \cite{Ma2013}

- $\chi_{c0,2}$ factorisation issue? $\iff$ CO-CS mixing \cite{Ma2014}

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**Equation:**
\[
\sigma^{-1} \frac{d\sigma}{dq_T^2} \quad \begin{array}{c}
\chi_0 \\
(h_1^\perp g = 0) \\
\chi_2 \\
\eta_Q
\end{array}
\]

**Graph:**
- $\langle p_T^2 \rangle = 1$ GeV$^2$
- $q_T$ (GeV)
- $\sigma^{-1} d\sigma / dq_T^2$ (GeV$^{-2}$)
- $h_1^\perp g = 0$
- $\chi_0$
- $\chi_2$
- $\eta_Q$

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**References:**
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- J.P. Lansberg (IPNO) Gluon TMDs and Quarkonium Production
Part III

Quarkonium + photon
$Q + \text{isolated } \gamma$: interesting but ...

- At high energy, 2 gluons in the initial states: no quark
Q + isolated γ: interesting but ...

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R.Li and J.X. Wang, PLB 672,51,2009
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\[ JPL, \text{ PLB 679,340,2009.} \]

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![Graph showing the distribution of event rates.](image)

All this is certainly interesting but TMD factorisation is most likely not applicable because of colour in the final state (either COM or gluons)
**Q + γ:** back-to-back and both isolated

Representative diagrams contributing to the hadroproduction of a $Q$ in association with a photon at orders $\alpha_s^2 \alpha$, $\alpha_s^3 \alpha$ (a, b, c), $\alpha_s^4 \alpha$ (d, e, f).
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  - smearing effect small for $P_T \gg \langle k_T \rangle$
- (c)-(f) populate $\Delta \phi_{Q-\gamma} < \pi$ [even $\Delta \phi \rightarrow 0$ for (c) and (d) at large $P_T$]
$Q + \gamma$: back-to-back and both isolated

- The studies is of an isolated quarkonium back-to-back with an (isolated) photon selects the Born contributions to $Q + \gamma$.
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- The “back-to-back” requirement also limits the DPS contributions [a priori evenly distributed in $\Delta \phi$]
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- The studies is of an isolated quarkonium back-to-back with an (isolated) photon selects the Born contributions to \( Q + \gamma \)

- The “back-to-back” requirement also limits the DPS contributions [a priori evenly distributed in \( \Delta \phi \)]

- Unique candidate to pin down the gluon TMDs
  - gluon sensitive process
  - colourless final state (virtue of isolation): TMD factorisation applicable
  - small sensitivity to QCD corrections (most of them in the TMD evolution)
Expected rates for back-to-back $Q + \gamma$

Direct back-to-back Onium + $\gamma$ at $\sqrt{s} = 14$ TeV

$\mu_R = \mu_F = m_{onium}^T$ | $m_Q = m_{onium}/2$  
$\ |Y| < 0.5; |\cos\theta_{CS}| < 0.45$

$q-q$ (x 50)  
$g-g$

Color Singlet  
Color Octet

$\langle O^{1S_0^{[8]}}(\Upsilon) \rangle = 0.1$ GeV$^3$  
$\langle O^{3S_1^{[8]}}(\Upsilon) \rangle = 0.01$ GeV$^3$  
$\langle O^{1S_0^{[8]}}(J/\psi) \rangle = 0.2$ GeV$^3$  
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- $g-g$ contribution:
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  - $\langle O^{1S_0}[8] (J/\psi) \rangle = 0.02 \text{ GeV}^3$
  - $\langle O^{3S_1}[8] (J/\psi) \rangle = 0.002 \text{ GeV}^3$

- $q\bar{q}$ contribution:
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- At 14 TeV, $\sigma(J/\psi|\Upsilon + \gamma, Q > 20\text{GeV}) \simeq 100\text{fb}$; about half at 7 TeV

$\mu_R=\mu_F=m_{\text{onium}} \quad m_Q=m_{\text{onium}}/2$

$|Y| < 0.5; |\cos\theta_{CS}| < 0.45$

$q\bar{q}$ (x 100)

g-g

$<O_1^{\text{S}_0}[8](\Upsilon) >= 0.1 \text{ GeV}^3$

$<O_3^{\text{S}_1}[8](\Upsilon) >= 0.01 \text{ GeV}^3$

$<O_1^{\text{S}_0}[8](J/\psi) >= 0.02 \text{ GeV}^3$

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Expected rates for back-to-back $Q + \gamma$


Direct back-to-back Onium + $\gamma$ at sqrt(s)=14 TeV

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<tr>
<th>Color Singlet</th>
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<tr>
<td>$\mu_R=\mu_F=m_{onium}$</td>
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- At 14 TeV, $\sigma(J/\psi|\Upsilon + \gamma, Q > 20\text{GeV}) \approx 100\text{fb}$; about half at 7 TeV
- With the $\mathcal{L} \approx 20 \text{ fb}^{-1}$ of pp data on tape, one expects up to 2000 events
The $q_T$-differential cross section involves $f_1^g(x, k_T, \mu_F)$ and $h_1^{\perp g}(x, k_T, \mu_F)$

$$
\frac{d\sigma}{dQ dY d^2q_T d\Omega} = \frac{C_0(Q^2 - M_Q^2)}{s Q^3 D} \left\{ F_1 C [f_1^g f_1^g] + F_3 \cos(2\phi_{CS}) C [w_3 f_1^g h_1^{\perp g} + x_1 \leftrightarrow x_2] + F_4 \cos(4\phi_{CS}) C [w_4 h_1^{\perp g} h_1^{\perp g}] \right\} + O\left(\frac{q_T^2}{Q^2}\right)
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back-to-back $Q + \gamma$ and the gluon TMDs

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$S_{q_T}^{(0)} = \frac{C[f_1^g f_1^g]}{\int dq_T^2 C[f_1^g f_1^g]}$: does not involve $h_{1\perp}^g$ [not always the case]
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- $S_{q_T}^{(4)} = \frac{F_4 C[w_4 h_1^g h_1^g]}{2 F_1 \int dq_T^2 C[f_1^g f_1^g]}$:

$S_{q_T}^{(4)} \neq 0 \Rightarrow$ nonzero gluon polarisation in unpolarised protons!
back-to-back $Q + \gamma$ and the gluon TMDs

The $\vec{q}_T$-differential cross section involves $f^g_1(x, \vec{k}_T, \mu_F)$ and $h_1^{\perp g}(x, \vec{k}_T, \mu_F)$

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Results with UGDs as Ansätze for TMDs

$S_{q_T}^{(0)}(x, k_T)$ from the $q_T$-dependence of the yield.
Results with UGDs as Ansätze for TMDs

$S_{q_T}^{(0)} : f_1^g(x, k_T)$ from the $q_T$-dependence of the yield.

$S_{q_T}^{(4)} : \int dq_T S_{q_T}^{(4)}$ should be measurable [$\mathcal{O}(1 - 2\%)$: ok with 2000 events]
Results with UGDs as Ansätze for TMDs

\[ S_{q_T}^{(0)}(0) : f_1^g(x, k_T) \] from the \( q_T \)-dependence of the yield.

\[ S_{q_T}^{(4)} : \int dq_T S_{q_T}^{(4)} \] should be measurable [\( \mathcal{O}(1 - 2\%) \): ok with 2000 events]

\[ S_{q_T}^{(2)} : \] slightly larger than \( S_{q_T}^{(4)} \)
Part IV

Quarkonium + Z boson
Rates similar for $\Upsilon + Z$ and $J/\psi + Z$  [Same for $Q + \gamma$ for $Q \gtrsim 20$ GeV]
**Y + Z cross sections**


- Rates similar for $\Upsilon + Z$ and $J/\psi + Z$ [Same for $Q + \gamma$ for $Q \gtrsim 20$ GeV]

\[
\frac{d\sigma}{dP_T} \times Br (fb/GeV)
\]

\[
sqrt(s)=8 \text{ TeV} \quad \mu_R=\mu_F=m_Z \quad |y_{J/\psi}| < 2.4
\]

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Potential probe of gluon TMDs as well
Rates similar for $\Upsilon + Z$ and $J/\psi + Z$ [Same for $Q + \gamma$ for $Q \gtrsim 20$ GeV]

- Potential probe of gluon TMDs as well
- Rate clearly smaller than $Q + \gamma$ even at low $P_T$
$Y + Z$ and TMDs

- $Y + Z @ \sqrt{s} = 14 \text{ TeV}$;
- $Q = 120 \text{ GeV}, Y = 0, \theta = \pi/2$
Y + Z and TMDs

- Y + Z @ $\sqrt{s} = 14$ TeV;
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$S^{(n)}_{qT}$ smaller than for $Q + \gamma$ [one can integrate up to larger $q_T$, though]
Y + Z and TMDs

- $Y + Z @ \sqrt{s} = 14$ TeV;
- $Q = 120$ GeV, $Y = 0, \theta = \pi/2$

$S^{(n)}(\sqrt{s_{cm}}=14$ TeV, $Q=120$ GeV, $Y=0, q_T, \theta=\pi/2)$ vs. $q_T$ for the process $p+p \rightarrow Y+Z+X$

$S^{(n)}(\sqrt{s_{cm}}=14$ TeV, $Q=120$ GeV, $Y=0, q_T, \theta=\pi/2)$ vs. $q_T$ for the process $p+p \rightarrow Y+Z+X$

- $S_{q_T}^{(n)}$ smaller than for $Q + \gamma$ [one can integrate up to larger $q_T$, though]
- Naturally large $Q$: interest to study the scale evolution?
Conclusions and Outlooks

- **TMD studies in the gluon sector** are very promising.
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J.P. Lansberg (IPNO)
Gluon TMDs and Quarkonium Production
October 20, 2014 21 / 21
Conclusions and Outlooks

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- Di-photon production is perhaps more tractable but very challenging where the rates are high
- **Back-to-back $J/\psi + \gamma$ and $\Upsilon + \gamma$** is certainly at reach
  - Already a couple of thousand **events on tapes**
  - $f_1^g(x, k_T, \mu)$ and $h_{1g}^\perp(x, k_T, \mu)$ can be determined **separately**
  - $Q$ can even be **tuned → gluon TMD evolution**
Conclusions and Outlooks

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  - $Q$ can even be tuned $\rightarrow$ gluon TMD evolution.
- Low $P_T$ onium and SSA of onium+photon studies could be done with **A Fixed-Target Experiment at the LHC: AFTER@LHC**
  [see talk by L. Massacrier on Friday, S11, 9h35]
Part V

Backup
$S_{qT}^{(0)}$: Model predictions for $\Upsilon + \gamma$ production at $\sqrt{s} = 14$ TeV

$Q = 20$ GeV, $Y = 0$, $\theta_{CS} = \pi/2$

Models for $f_1^g$: assumed to be the same as for Unintegrated Gluon Distributions

- **Set B**: B0 solution to CCFM equation with input based on HERA data
  Jung et al., EPJC 70 (2010) 1237

- **KMR**: Formalism embodies both DGLAP and BFKL evolution equations
  Kimber, Martin, Ryskin, PRD 63 (2010) 114027

- **CGC**: Color Glass Condensate Model
  Dominguez, Qiu, Xiao, Yuan, PRD 85 (2012) 045003
  Metz, Zhou, PRD 84 (2011) 051503
Model predictions for $\Upsilon + \gamma$ production at $\sqrt{s} = 14$ TeV

$Q = 20$ GeV, $Y = 0$, $\theta_{CS} = \pi/2$

$h_1^\perp g$: predictions only in the CGC: in the other models saturated to its upper bound

$S_{q_T}^{(2,4)}$ smaller than $S_{q_T}^{(0)}$: can be integrated up to $q_T = 10$ GeV

$$2.0\% \ (\text{KMR}) < \left| \int d^2 q_T S_{q_T}^{(2)} \right| < 2.9\% \ (\text{Gauss})$$

$$0.3\% \ (\text{CGC}) < \int d^2 q_T S_{q_T}^{(4)} < 1.2\% \ (\text{Gauss})$$

Possible determination of the shape of $f_1^g$ and verification of a non-zero $h_1^\perp g$
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

$q\bar{q}' \rightarrow \gamma^* W \rightarrow J/\psi W$ and $q\bar{q}' \rightarrow g^* W \rightarrow J/\psi W$ are very similar

why?
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

$q\bar{q}' \rightarrow \gamma^* W \rightarrow J/\psi W$ and $q\bar{q}' \rightarrow g^* W \rightarrow J/\psi W$ are very similar. Why?

Let us simplify and look at $q\bar{q}' \rightarrow \gamma^* \rightarrow J/\psi$ vs. $q\bar{q}' \rightarrow g^* \rightarrow J/\psi$. The cross sections are well-known:

-CSM: \[ \hat{\sigma}^{[1]} \] via $\gamma^* = \left(4\pi\alpha\right)^2 e^2 q e^2 Q M_3 Q_s \delta \left(x_1 x_2 - M_2 Q/s\right) |R(0)|^2 \]

-COM: \[ \hat{\sigma}^{[8]} \] via $g^* = \left(4\pi\alpha S\right)^2 \pi^2 27 M_3 Q_s \delta \left(x_1 x_2 - M_2 Q/s\right) \langle O Q(3 S[1]) \rangle \]

The ratio gives:

\[ \hat{\sigma}^{[1]} / \hat{\sigma}^{[8]} = \frac{6}{\alpha^2} e^2 q e^2 Q \langle O Q(3 S[1]) \rangle / \langle O Q(3 S[8]) \rangle = 2 N_c (2J + 1) |R(0)|^2 / 4 \pi \]

Colour factor: 2
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

$q\bar{q}' \rightarrow \gamma^* W \stackrel{3 S_1^{[1]}}{\rightarrow} J/\psi W$ and $q\bar{q}' \rightarrow g^* W \stackrel{3 S_1^{[8]}}{\rightarrow} J/\psi W$ are very similar

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Let us simplify and look at

$q\bar{q}' \rightarrow \gamma^* \rightarrow J/\psi$ vs. $q\bar{q}' \rightarrow g^* \rightarrow J/\psi$

The cross sections are well-known:

$$\hat{\sigma} \left[ 3 S_1^{[1]} \right]_{\gamma^*} \rightarrow J/\psi \longrightarrow \frac{6 \alpha^2 e_q^2}{\left( 3 S_1^{[1]} \right)_{\hat{O} Q} \left( 3 S_1^{[8]} \right)_{\hat{O} Q}} \alpha^2 S \left[ \langle O Q \left( 3 S_1^{[1]} \right)_{\hat{O} Q} \rangle \langle O Q \left( 3 S_1^{[8]} \right)_{\hat{O} Q} \rangle \right] \frac{1}{2 N_c (2J+1)} \left| R \left( 0 \right) \right|^2 \frac{1}{4 \pi}$$

Colour factor: $2 N_c (2J+1)$
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

$q\bar{q}' \rightarrow \gamma^* W \rightarrow J/\psi W$ and $q\bar{q}' \rightarrow g^* W \rightarrow J/\psi W$ are very similar why?

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The cross sections are well-known:

- **CSM**: $\hat{\sigma}_{\gamma^*}^{[1]} = \frac{(4\pi\alpha)^2 e_q^2 e_Q^2}{M_Q^3 s} \delta (x_1 x_2 - M_Q^2 / s) |R(0)|^2$

- **COM**: $\hat{\sigma}_{g^*}^{[8]}$
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

$q\bar{q}' \to \gamma^* W \to J/\psi W$ and $q\bar{q}' \to g^* W \to J/\psi W$ are very similar why?

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- **COM:** $\hat{\sigma}^{[8]}_{g^*} = \frac{(4\pi\alpha_s)^2 \pi}{27M_Q^3 s} \delta \left( x_1 x_2 - M_Q^2 / s \right) \langle \mathcal{O}_{Q\left(3S_1^{[8]}\right)} \rangle$
Discussion: CSM via $\gamma^* vs. COM via g^*$

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- **COM**: \( \hat{\sigma}_{g^*}^{[8]} = \frac{(4\pi\alpha S)^2\pi}{27M_Q^3 s} \delta \left( x_1 x_2 - M_Q^2 / s \right) \langle O_Q(3S_1^{[1]} ) \rangle \)

The ratio gives:

\[
\frac{\hat{\sigma}_{\gamma^*}^{[1]}}{\hat{\sigma}_{g^*}^{[8]}} = \frac{6\alpha^2 e_q^2 e_Q^2 \langle O_Q(3S_1^{[1]} ) \rangle}{\alpha_s^2 \langle O_Q(3S_1^{[8]} ) \rangle} = \frac{2N_c(2J + 1) |R(0)|^2}{4\pi} \]

\( \langle O_Q(3S_1^{[1]} ) \rangle = 2N_c(2J + 1) |R(0)|^2 \)
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\]

Colour factor: $2N_c$
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

$$\frac{\hat{\sigma}_{\text{via } \gamma^*}^{[1]}}{\hat{\sigma}_{\text{via } g^*}^{[8]}} = \frac{6\alpha^2 e_q^2 e_Q^2 \langle O_Q(3S_1^{[1]}) \rangle}{\alpha_s^2 \langle O_Q(3S_1^{[8]}) \rangle}$$

The ratio depends on the initial quark, $q$, on $\alpha_s$ at $\mu_R \approx m_Q$ and on the ratio of the non-perturbative coefficients. For $J/\psi$ production in $u\bar{u}$ fusion and for $\langle O_{J/\psi}(3S_1^{[8]}) \rangle = 2 \times 10^{-3} \text{GeV}^3$, the ratio CSM vs. COM is $2/3$.

For $\Upsilon$ production, it is about the same (e.g. smaller but $\alpha_s$ also smaller and $|R(0)|^2$ larger).

If we add the $W$ emission, the charge factor changes and $\mu_R$: $O(m_Q) \rightarrow O(m_W)$...

General conclusion: For production processes involving light quarks, the CSM via off-shell photon competes with the COM via off-shell gluon.
The ratio depends on the initial quark, \( q \), on \( \alpha_s \) at \( \mu_R \approx m_Q \) and on the ratio of the non-perturbative coefficients.
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\]

- The ratio depends on the initial quark, $q$, on $\alpha_s$ at $\mu_R \simeq m_Q$ and on the ratio of the non-perturbative coefficients.

- For $J/\psi$ production in $u\bar{u}$ fusion and for $\langle O_{J/\psi} \left( ^3S_1^{[8]} \right) \rangle = 2.2 \times 10^{-3}$ GeV$^3$, the ratio CSM vs. COM is $2/3$.
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

\[
\frac{\hat{\sigma}^{[1]}_{\text{via } \gamma^*}}{\hat{\sigma}^{[8]}_{\text{via } g^*}} = \frac{6\alpha_s^2 e_q^2 e_Q^2 \langle \mathcal{O}_Q(3S_1^{[1]}) \rangle}{\alpha_s^2 \langle \mathcal{O}_Q(3S_1^{[8]}) \rangle}
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  \[\rightarrow \text{This explains our results for } J/\psi + W\]
Discussion: CSM via $\gamma^*$ vs. COM via $g^*$

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- For $J/\psi$ production in $u\bar{u}$ fusion and for $\langle O_{J/\psi}(3S_1^{[8]}) \rangle = 2.2 \times 10^{-3}$ GeV$^3$, the ratio CSM vs. COM is $2/3$.
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$\rightarrow$ This explains our results for $J/\psi + W$.

General conclusion:

For production processes involving light quarks, the CSM via off-shell photon competes with the COM via off-shell gluon.